Canadian Conference - Pacific Conference on Earthquake Engineering 2023 Vancouver, British Columbia

June 25th - June 30th, 2023



Seismic design of concrete deck arch bridges considering asynchronous ground motion effects

Qi Zhang^{1,2}, Saqib Khan³, Jhordy Rodriguez⁴ and M. Shahria Alam^{5*}

¹Post Doctoral Fellow, University of British Columbia, Kelowna, BC, Canada

ABSTRACT

This study investigates several key factors regarding the seismic design of concrete deck arch bridges with a focus on the effects of asynchronous ground motions. The research compares response spectrum analysis (RSA)-based results with those from nonlinear time-history analysis (NTHA) for deck arch bridges. Such structures exhibit large stiffness variability due to significantly different heights of spandrel columns connecting the deck to the arch rib. The study further investigates the effects of asynchronous ground motions by comparing results from asynchronous ground motion inputs with those from uniform ground motions. When conducting this comparison, 4 combinations of Site Classes are considered in the NTHA: (1) uniform Site Class A, (2) uniform Site Class C, (3) one-half of the bridge supports on Site Class A and another half on Site Class C, (4) main span supported on Site Class A ground and the rest of the spans supported on Class C sites. Ratios of displacement demands of asynchronous ground motions to uniform ground motions are calculated. It is found that typical RSA-based demands used for regular girder bridges are not suitable for predicting the force and deformation demands of concrete deck arch bridges in the transverse direction. The asynchronous ground motions excite bridges' global rotational mode shapes and generate larger displacement demands compared with uniform ground excitation. When comparing the displacement demands from asynchronous ground motions with those from uniform ground motion, asynchronous ground motions can generate up to 50% or higher demands in the transverse direction. Thus, for the seismic design of concrete deck arch bridges under asynchronous ground motions, the time-history analysis cannot be replaced by RSA or uniform ground excitation.

Keywords: arch bridges, deck arch, seismic design, asynchronous ground motion, non-linear time history analysis

INTRODUCTION

Concrete arch structures are frequently used in bridges. There are mainly three types of concrete arch bridges, which are deck arch, through arch bridge, and half-through arch. For deck arch bridges, decks are located above the crown of the arch, and they directly support the traffic loads. They are also known as true or perfect arches. In a through-arch bridge, the deck is located at the arch springing line. The third type of concrete arch bridge is called half-through arch, which is a form between the deck arch and through arch bridges, where the deck is at an elevation between the deck arch and the through-arch [1]. Among the three types of concrete arch bridges, the deck arch bridge is the most popular option [2] and they are often built to span deep valleys. In a deck arch bridge, under vertical load paths, the main arch, spandrel columns and piers are mainly under compressive forces. Under lateral loads, the main arches, spandrel columns and piers are subject to combined bending, shear and axial forces.

The seismic design of arch bridges is an important issue but has received little attention in bridge design codes and guidelines compared with girder-type bridges. Khan et al. [2] investigated the use of the direct displacement-based design (DDBD) procedure on arch bridges. The authors concluded that DDBD can be used in predicting peak chord rotation demands on the bridge piers. It was suggested that although the arch displacement may be underestimated in DDBD, since the arch displacements are small compared with deck displacement, the DDBD procedure is still valid in arch bridge design. On the

²Instructor, British Columbia Institute of Technology, Burnaby, BC, Canada

³Founding Partner and Principal Engineer, Spannovation Consulting Limited, Langley, BC, Canada

⁴PhD Student, University of British Columbia, Kelowna, BC, Canada

⁵Professor and Tier-1 Principal's Research Chair, University of British Columbia, Kelowna, BC, Canada

^{*}shahria.alam@ubc.ca (Corresponding Author)

topic of asynchronous ground motions, Bi et al. [3] studied concrete-filled steel tubular (CFST) arch bridges under spatially varying ground motions. More recently, shake table tests and analytical studies were conducted by Liu and Zhang [4]; the authors concluded that spatially varying effects can cause more severe damage compared with that from uniform ground motions.

Currently, there are limited studies on deck arch bridges under asynchronous ground motions at multiple supports. This study investigates two issues faced in the analysis and design of deck arch bridges. The first issue is if a standard RSA method could be reliably used in the design of the deck arch bridge and whether the results would be conservative. Second, this study compares the results from time-history analyses under uniform and asynchronous ground motions to examine the effects of the asynchronous ground motions. To achieve the two goals, three prototype deck arch bridges are studied using response spectrum analyses, uniform ground motion time history analyses and asynchronous ground motion time history analyses. The results from time history analyses are compared between uniform and asynchronous ground motions at different piers in terms of displacement demands. The results from time history analyses are also compared with that from response spectrum analyses.

CHARACTERISTICS OF THE ARCH BRIDGES AND SITE CONDITIONS

This study is based on three prototype arch bridges that were studied by Khan et al. [2]. The seismic hazards corresponding to two principal site conditions are considered. The deck arch bridges were previously designed as per the Canadian Highway Bridge Design Code [5] and analyzed for fragilities by Aldabagh et al. [6]. The three bridges each have total span lengths of 72m, 132m and 188m and a constant rise-to-span ratio. They were named B1, B2 and B3, with B1 being the shortest bridge and B3 being the longest bridge. Because of the constant rise-to-span ratio, the heights of the bridges increase from B1 to B2 and B3. The length of the longest pier of each of the three bridges is 21.6 m (B1), 38.7 m (B2), and 56.0 m (B3). For all three bridges, they are composed of 28 m long continuous spans supported by piers and spandrel columns. At the abutments, the structure is restrained from transverse movements perpendicular to the traffic direction. However, abutments are considered unrestrained in the bridges' longitudinal direction. Rotational degrees of freedom are released at abutments. For piers, full fixity is assumed at the foundation level. The elevation view of the three bridges is shown in Figure 1. In Figure 1, the abutments, piers and spandrel columns are named sequentially from left to right. In the naming convention, the first letter and first number represent the bridge number B1, B2 and B3. The second letter indicates the type of substructure, where A represents abutment and P represents a pier or a spandrel column. The last number represents the sequence from left to right of the supports, which starts from 0 at the west abutment. For example, the second pier of bridge B2 is named B2P2 and the first abutment of B3 is named B3A0.

The bridge superstructures are supported by an arch, spandrel columns, and piers. At the central span of each bridge, four spandrel columns are designed using solid square reinforced concrete sections. The length of the central spandrel columns of B1, B2 and B3 are relatively short, their lengths being 6 m, 8 m and 10 m, respectively. Other long spandrel columns and piers are designed with hollow square reinforced concrete sections. The cross-section dimensions of these spandrel columns and piers range from 1.5 m x 1.5 m to 2.2 m x 2.2 m. The wall thickness of the hollow sections is 0.3 m for all sections. The slenderness ratios (column height to cross-section dimension ratio) of these spandrel columns and piers range from 4.0 to 18.7. The longitudinal reinforcement ratio of these piers and columns varies from 1.1% to 1.8% of the cross-section area. They are well-confined sections with lateral reinforcement proving confinement factors between 1.5 to 1.7. The slenderness ratios and provided lateral reinforcement are intended to ensure these vertical elements are governed by ductile flexural failure mode rather than brittle shear failure. The main arches are designed as capacity-protected elements and would therefore remain essentially elastic during earthquake events. The detailed section design of the piers and columns is summarized in Table 1. The column and pier naming convention in Table 1 are consistent with Figure 1.

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Bridge B1	B1P1, B1P6	B1P2, B1P5	B1P3, B1P4	
Cross section dimension	1.5 m	1.8 m	1.5 m	<u> </u>
Longitudinal rebar ratio	1.5%	1.8%	1.7%	
Bridge B2	B2P1, B2P3, B2P6, B2P8	B2P2, B2P7	B2P4, B2P5	<u> </u>
Cross section dimension	1.8 m	2.2 m	1.5 m	<u> </u>
Longitudinal rebar ratio	1.8%	1.7%	1.6%	
Bridge B3	B3P1, B3P3, B3P8, B3P10	B3P2, B3P9	B3P4, B3P7	B3P5, B3P6
Cross section dimension	2.2 m	3 m	1.8 m	1.8 m
Longitudinal rebar ratio	1.7%	1.2%	1.8%	1.1%

Table 1. Bridge pier and column design parameters.

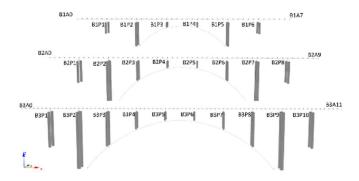


Figure 1. Elevation view of deck arch bridges.

It is assumed that the arch bridge foundations are supported by soils in different site classes. When performing seismic design and analysis, the seismic hazards are functions of the site conditions. As per the Canadian Highway Bridge Design Code [5, 7], site properties are characterized as 6 groups ranging from Site Class A to Site Class F, representing hard rock, rock, very dense soil and soft rock, stiff soil, soft soil, and other soils. Arch bridges are usually built in good ground conditions ranging from dense soil to hard rock. Therefore, in the parametric study, only Site Class C and Site Class A conditions are used. Site Class A represents a site with hard rock and shear wave average velocity greater than 1500 m/s. Site Class C represents a site with very dense soil and soft rock. Its shear wave average velocity ranges from 360 m/s to 760 m/s.

To investigate the effects of asynchronous ground motions, four different combinations of the Site Classes are considered in this study (shown in Figure 2). In Figure 2, the spandrel columns are removed for clarity and only piers and arches that would be subjected to ground motions are shown. Uniform and nonuniform site conditions are considered at the foundation levels; therefore, Figure 2 applies to all three bridges. Although B2 and B3 have more spans compared with B1, the additional spans are supported by spandrel columns. The number of foundations is the same for B1, B2 and B3. The letters in Figure 2 represent the ground classifications of Site Class A and Site Class C. The site condition combination 1 and 2 represent uniform Site Class conditions, whereas combination 1 has Site Class A for all piers and abutments. Similarly, in combination 2, all piers are founded on Site Class C ground. Combinations 3 and 4 are used to analyze the effect of asynchronous ground motions. In combination 3, half of the piers are constructed in Site Class C while the other half of the piers are constructed in Site Class A. In combination 4, abutments and adjacent piers to abutments are founded on Site Class C while the arches and the piers adjacent to the arch are founded on Site Class A. In this case, the assumption is that due to the significant elevation difference between these supports, the ground condition is stiffer at a lower elevation. These combinations of site classes cannot be properly considered in a typical RSA. These are investigated through nonlinear NTHA.

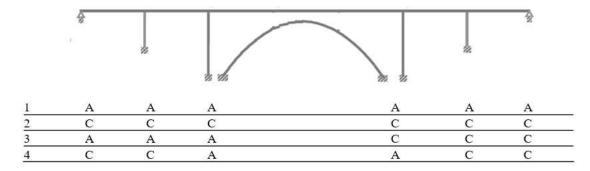


Figure 2. Combinations of Site Classes for time history analyses.

GLOBAL DISPLACEMENTS FROM RESPONSE SPECTRUM ANALYSIS (RSA)

As discussed above, this study includes two Site Classes, which are Site Class A representing hard rock and Site Class C representing very dense soil and soft rock. The two spectra were previously used in the study by Rodríguez et al. [8]; these were obtained from the National Building Code of Canada (NBCC) [9]. The two spectra are shown in Figure 3. Response spectrum analyses of each of the three bridges under each of the two spectra are performed, which results in 6 sets of results. The global displacement results used in this section are the relative displacement between the top of the piers / spandrel columns and the foundations. For spandrel columns, the definition of the foundation is the nearest support to that column.

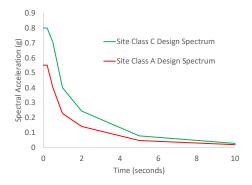


Figure 3 Design response spectra

The two response spectra in Figure 3 are used as the basis to generate asynchronous ground motions. In Figure 3, there are generally three zones that define the ratios of Site Class C to Site Class A spectral accelerations. In the short time period range between 0 and 0.2 seconds, the ratio is 1.45. In between 0.2 seconds and 0.5 seconds, the ratio increases from 1.45 to 1.75 and remains at about 1.75 until the period reaches 2 seconds. After 2 seconds, the ratio reduces gradually from 1.75 to 1.49 at the end of the chart at 10 seconds. Thus, it is expected that for most medium to long-span bridges with fundamental periods beyond 0.5 seconds, the seismic demands from Site Class C would be about 1.75 times that from Site Class A.

As part of the RSA, mode shapes of the bridges are first investigated. The vibration periods of the three bridges are presented in Table 1. From B1 to B2 and B3, because the rise-to-span ratio is kept constant, the total length and height of the bridge increase from B1 to B3. This leads to an increase in bridge vibration periods from B1 to B2 and B3.

Table 1 Bridge vibration periods									
Modes	B1	B2	В3						
Longitudinal mode	0.758 sec	1.283 sec	1.448 sec						
Transverse mode	0.766 sec	1.499 sec	2.549 sec						

In engineering design practice, response spectrum analyses are usually performed to determine structural member sizes, which means that initial design is usually based on elastic forces or reduced forces according to ductility/force reduction factors. For irregular bridges, many design codes require design verification using more sophisticated methods such as NTHA. The concern is that elastic RSA may not be able to capture the seismic responses of irregular bridges. In this study, elastic response spectrum analyses are first performed to determine the displacement demands. The displacement demands from RSA at the top of each pier are reported in Table 2 and Table 3 for site class A and site class C, respectively. Since all bridges are symmetric structures, only half of the pier displacements are presented. When comparing the displacements from Site Class A and Site Class C, in both the transverse and longitudinal directions, the Site Class C displacements are about 1.72 to 1.75 times that of the Site Class A displacements. Due to the significant difference between Site Class A and Site Class C would be more conservative than considering the asynchronous ground motions combining Site Class C and Site Class A. The following sections on time-history analysis results will further discuss this issue.

Table 2 RSA Displacements - Site Class A Spectrum

	Transv	erse displ	lacement.	, m			Longitudinal displacement, m					
	A0	P1	P2	Р3	P4	P5	A0	P1	P2	Р3	P4	P5
B1	0.00	0.012	0.038	0.054	-	-	0.038	0.031	0.035	0.032	-	-
B2	0.00	0.019	0.064	0.107	0.135	-	0.069	0.063	0.066	0.069	0.062	-
В3	0.00	0.026	0.091	0.172	0.244	0.286	0.086	0.083	0.085	0.086	0.083	0.070

Table 3 RSA Displacements - Site Class C Spectrum

	Transverse displacement, m						Longitudinal displacement, m					
	A0	P1	P2	Р3	P4	P5	A0	P1	P2	P3	P4	P5
B1	0.00	0.02	0.07	0.10	-	-	0.07	0.06	0.06	0.06	-	-
B2	0.00	0.03	0.11	0.19	0.23	-	0.12	0.11	0.12	0.12	0.11	-
В3	0.00	0.04	0.16	0.30	0.42	0.49	0.15	0.14	0.15	0.15	0.15	0.12

When comparing displacements of different bridges, for instance, comparing B1 with B3 in the transverse direction, B3 has higher displacement demands due to its longer span length and taller piers (smaller stiffness). When comparing the two bridges in the longitudinal direction, although B3 has higher longitudinal displacements, B3 also has a more significant difference in longitudinal displacement between midspan and abutments compared to B1.

When comparing the displacements of the same bridge in transverse and longitudinal directions, it can be seen that the longitudinal and transverse displacements of B1 are of the same order of magnitude, the difference being generally less than 5 mm. This is somewhat expected because the vibration periods of B1 in both longitudinal and transverse directions are similar. For B2, the transverse displacement at midspan is about double that of the longitudinal displacement. For B3, the transverse displacement at midspan is more than three times that of the longitudinal displacements. This trend is also logical, since for B2 and B3, the transverse vibration modes have significantly longer periods compared with the longitudinal period. It should be noted that the displacements reported in this section are global displacements including the deformation of the arches.

GLOBAL DISPLACEMENTS FROM NONLINEAR TIME HISTORY ANALYSIS (NTHA)

To investigate the effects of asynchronous ground motions, nonlinear time history analyses are performed by applying both uniform and asynchronous ground motions at different supports of the bridges using the combinations shown in Figure 2. The seed ground motions used in this study were previously used in the study by Rodríguez et al. [8]. These comprise crustal, subcrustal and subduction earthquake sources, representing high seismic zones from northern Vancouver Island in Canada to Northern California in the United States. Two horizontal orthogonal records are used in each NTHA. All the selected ground motions were scaled using the linear scaling method following the requirements in NBCC such that the inherent signal characteristics of the ground motions are not modified. In the linear scaling method, a scaling period range is required, which is defined as the range between 0.2 times the fundamental period to 2 times the fundamental period of the structure. Within this period range, the NBCC linear scaling technique requires that the mean response spectrum of each set of time histories does not deviate more than 10% from the target design spectrum. For each scaled ground motion, the response spectra must equal or exceed the target response spectra over the appropriate period ranges. This method stipulates the use of a single scaling factor, which should be the geometric mean of the two orthogonal directions of the same earthquake motions.

The purpose of scaling the ground motions to Site Class A and C spectra is to be able to generate appropriate asynchronous ground motions for NTHA. When performing asynchronous ground motion NTHA, the same set of motion is scaled based on the site Class A and C spectra and applied to the bridge models. The asynchronous ground motion NTHA does not mix ground motions from significantly different earthquakes from different events as that would be an unrealistic scenario. The response spectra from the unscaled records vs Site Class A and C spectra are plotted in Figure 4.

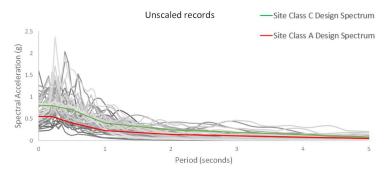


Figure 4 Response spectra from the unscaled records vs Site Class A and C spectra

Subsequent to the ground motion scaling, the nonlinear time history analyses are performed in the software SeismoStruct [10]. In the nonlinear analyses, the piers and spandrel columns are modelled as nonlinear elements incorporating lumped parameter plastic hinges. All other elements such as arches and superstructures are modelled as linear elastic elements because these are designed as capacity-protected components for seismic design purposes. Based on the nonlinear time history analyses, global displacement demands at key locations of the bridges are extracted for further synthesis. The global displacement demands are defined as the absolute value of the relative deformation between the top of the column and the corresponding foundation. Whereas for spandrel columns that are not directly connected to a foundation, the nearest foundation support displacement is used to calculate the relative displacement. The reason for using the relative displacement is to remove the ground displacements from the structural deformation calculation. Since the bridge deck and arch have large stiffness in the bridge's longitudinal direction, the displacements in the longitudinal direction are relatively uniform; the focus of this paper is, therefore, on the global transverse response of the bridges.

EFFECTS OF ASYNCHRONOUS GROUND MOTIONS ON DISPLACEMENTS DEMANDS

To study the effects of asynchronous ground motions, the time-history analysis results from asynchronous ground motions and uniform ground motions are compared. The results from response spectrum analyses are also presented. The average transverse displacements of the piers and spandrel columns of the three bridges are plotted from Figure 5 to Figure 7. In each of the figures, the column displacements from NTHA in the 4 combinations are compared along with those from the two response spectrum analyses. From all time history analysis results, it is seen that the mid-spans of the bridges do not have the highest transverse displacement. This is due to the design of the strong arch which has large stiffness and resistance and remains elastic in earthquake events.

When evaluating the NTHA results against the response spectrum analyses, it is seen that NTHA combination 1 and combination 2 demands are generally in agreement with what is predicted by using the response spectrum method. Combination 1 represents the scenario where all foundations are founded on Site Class A rock, while Combination 2 represents the scenario where all foundations are founded on Site Class C soil. For B1, the RSA results are very close to those from NTHA analyses for the uniform ground motion excitations. Similarly, for B2 and B3, the uniform ground motion NTHA results are in reasonable agreement with what is predicted using response spectrum analyses, with response spectrum-based values being somewhat conservative. Comparing asynchronous ground motion NTHA results with RSA results, the RSA demands for both B1 and B2 are underestimated. For B3, the RSA demands are mostly less than those of NTHA with a few exceptions. Based on this comparison, the RSA method fails to give reliable displacement demands for asynchronous ground motions, even if the higher design spectrum is used. For uniform ground motions, RSA may generally underestimate displacement demands; however, it was found to overestimate displacement demands for deck arch bridges studied in this paper.

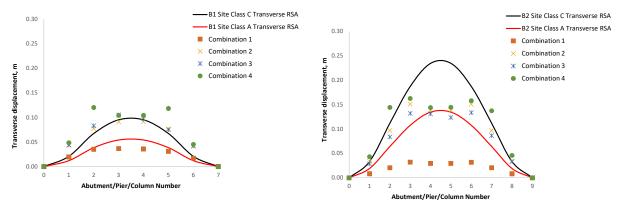


Figure 5 Bridge B1 transverse displacement

Figure 6 Bridge B2 transverse displacement

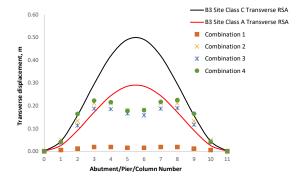


Figure 7 Bridge B3 transverse displacement

When comparing the 4 sets of NTHA results, it is seen that the demands from combination 4 are particularly high. In this Site Class combination, the arch foundation and the pier adjacent to the arch (supporting main spans) are founded on Site Class A rock while the rest of the piers are located on Site Class C (higher seismic accelerations), thus, this set of asynchronous ground motions excites the global rotational (torsional) mode of the bridge, resulting in higher displacements compared with all other combinations.

To quantify the effects of the asynchronous ground motion, Table 4 presents the ratios of asynchronous ground motion displacements (Combinations 3 and 4) to those of a Site Class C uniform ground motion analysis (Combination 2). These ratios are calculated by dividing the displacements from Combinations 3 and 4 by those from Combination 2. When the ratio is smaller than 1.0, it indicates that using the Site Class C design spectrum is conservative in predicting the displacement for that pier or spandrel column. The ratios greater than 1.0 mean that there are amplification effects from the asynchronous motions. The highest predicted ratio is 1.55 for B1P2, meaning that the asynchronous motions scenario introduced 55% additional displacement demand to this pier. If the designer ignores the asynchronous ground motion effects, the structures would not be designed for adequate seismic demands.

Table 4 Displacement ratios of asynchronous to uniform ground motion analyses (normalized by Site Class C)

	Site Combination	P1	P2	Р3	P4	P5	P6	P7	P8	Р9	P10
B1	Combination 3	1.02	1.07	1.16	1.06	0.97	1.02	NA	NA	NA	NA
	Combination 4	1.12	1.55	1.15	1.14	1.53	1.10	NA	NA	NA	NA
B2	Combination 3	0.84	0.87	0.87	0.95	0.89	0.89	0.89	0.98	NA	NA
	Combination 4	1.27	1.49	1.07	1.04	1.04	1.05	1.42	1.35	NA	NA
В3	Combination 3	0.74	0.85	0.89	0.88	0.91	0.86	0.89	0.90	0.88	0.86
	Combination 4	0.83	1.23	1.06	1.02	0.98	0.98	1.03	1.07	1.24	0.82

CONCLUSIONS

This research investigates the effects of asynchronous ground motions on the seismic performance of concrete deck arch bridges. The results from RSA are first compared with those obtained from NTHA. The study then compares the results from uniform and asynchronous ground motions through NTHA. Four combinations of Site Classes are considered in the NTHA for this comparison, including uniform Site Class A, uniform Site Class C, half of the bridge supports on Site Class A and half on Site Class C, and the main span supported on Site Class A with the rest of the spans supported on Site Class C. The deformation profiles of the bridges in the transverse and longitudinal directions are presented and compared. The following conclusions can be drawn from this study:

The RSA method, typically used for regular girder bridges, is not suitable for predicting the deformed shape of concrete deck arch bridges in the transverse direction. The RSA method predicts that the midspans of the arch bridges have the largest transverse deformation, but this was not supported by the NTHA results. Based on the NTHA, in many of the cases, the midspan of the bridges does not have the highest transverse displacement.

In the transverse direction, asynchronous ground motions excite the bridges' global rotational/torsional mode shapes, resulting in larger displacement demands compared to uniform ground excitation. In some cases, asynchronous ground motions can generate 55% higher demands when compared to the higher intensity uniform ground motion (Site Class C). Therefore, for the seismic design of concrete deck arch bridges under asynchronous ground motions, time-history analysis is necessary and cannot be substituted by RSA or uniform ground excitation.

Within the parameters examined in this study, it was found that asynchronous ground motions do not increase seismic demands in the longitudinal direction, likely due to the large stiffness of the arch and deck in the longitudinal direction.

ACKNOWLEDGMENTS

The financial contribution of Spannovation Consulting through the Mitacs Accelerate program was critical to conduct this research and is gratefully acknowledged.

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