



Utilizing Hybrid Simulations in the Calibration of Numerical Elements in Nonlinear Seismic Analysis

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ABSTRACT

Nonlinear hysteretic models are frequently used in seismic analysis of structures in order to simulate the behavior of key structural components such as beam-column joints and energy dissipation elements. The calibration of the model parameters plays an important role in ensuring the accuracy of seismic analysis. In current research and engineering practice, a widely accepted calibration method is to tune the parameters of a nonlinear hysteretic model such that the simulated hysteresis curve matches the test data of a single component tested under a standardized quasistatic loading history. However, due to the idealized loading profile of this type of test, previous research has found that even a well-calibrated set of parameters might result in significant errors in estimating the global dynamic response of a structural system. These observations indicated that there might be a weak relevance between the accuracy of the calibrated model and the accuracy of the system-level structural model under the ground excitations. This relationship between the accuracy of the calibration and the accuracy of the system-level model is termed as calibration relevance in this study. In this paper, a calibration method is proposed utilizing hybrid simulations which provides a more realistic loading history for calibration compared to conventional cyclic tests. A framework that uses virtual experiments to evaluate the calibration relevance of the calibration methods is also proposed incorporating uncertainties in the hysteretic model parameters. A case study is conducted numerically for two single-degree-of-freedom systems, and the results demonstrate that the proposed hybrid-simulation-based calibration method has a higher calibration relevance than that of the conventional method.

Keywords: nonlinear seismic analysis, calibration relevance, nonlinear hysteretic model, hybrid simulation, uncertainty quantification

INTRODUCTION

While nonlinear time-history analysis has gradually become a routine in assessing the seismic performance of civil structures, the accuracy of analysis results is still poorly understood due to the complexity of nonlinear structural behavior and the scarcity of available reference data [1]. This issue can be directly observed in previous seismic response blind prediction contests where enormous errors could be seen in numerical analyses conducted by researchers and professional engineers [2, 3]. The significant level of error and variation in the predicted results by different contestants can be considered as the impact of modeling uncertainties.

Efforts have been made to study modeling uncertainties in the past two decades. Ibarra [4] propagated modeling uncertainties in parameters of a deteriorating hysteretic model using the first-order second-moment method and identified two parameters that most influenced the collapse capacity of a system. Kwon and Elnashai [5] compared the impacts of uncertainties in material properties and ground motion records on vulnerability curves of a reinforced concrete structure. Afterwards, frameworks have been proposed to use Monte Carlo simulations with the incremental dynamic analysis (IDA) technique for incorporating modeling uncertainties [6-8]. Vamvatsikos [9] further proposed an incremental accelerogram-wise Latin hypercube sampling (LHS) method to efficiently estimate the effect of modeling uncertainties. Pourreza et al. [10] proposed another efficient approach which incorporated Monte Carlo simulations and quadratic response surfaces and also used the endurance time analysis method as an efficient alternative to the IDA.

Most of the mentioned literature focused on investigating the effects of uncertainties in hysteretic model parameters but paid no attention to the calibration procedure, which is the main source of uncertainties in hysteretic model parameters. Hysteretic models are frequently used in structural seismic analysis to simulate the nonlinear hysteretic behavior of key structural components such as beam-column hinges and energy dissipation devices. Calibration of the hysteretic model parameters is a crucial step in predicting the nonlinear seismic response of structures. As defined by Li and Mahadevan [11], “*The purpose of model calibration is to adjust a set of parameters associated with a computational model so that the agreement between model prediction and experimental observation is maximized.*” For complicated systems such as structural models for nonlinear seismic analysis, a direct calibration based on experimental data at the system level is generally impossible due to the high cost of system-level tests. Therefore, the model parameters of a complicated system are usually calibrated based on experimental data obtained from tests conducted at lower levels of complexity [11]. An illustration of four structural testing environments with different levels of complexity is shown in Figure 1: (I) Shake table tests have the most realistic test data but are least affordable; (II) Hybrid simulations are an efficient testing method where key structural components are experimentally tested and integrated with the numerical model of the rest of the structural system; (III) Quasistatic tests of a single component can be done by applying displacement histories of the corresponding component in the system-level numerical model; (IV) Cyclic test of a single component under a standardized loading protocol. The complexity becomes lower and significant information is lost from environment I to IV as illustrated in Figure 1.

A widely accepted practice in calibrating hysteretic model parameters in structural models is to utilize standardized cyclic tests of a single component (environment IV). The underlying assumption is that there is a high relevance between environments I and IV, i.e., a well-tuned hysteretic model that accurately predicts standardized cyclic test results can also produce the same level of accuracy in estimating the system-level structural dynamic responses. However, due to the low complexity of environment IV, the assumption of high relevance has been challenged by previous research, which has shown that minor errors between calibrated models and reference data in environment IV that researchers generally accept may lead to amplified errors at the system level structural dynamic response in environment I or II [1, 12-14]. Therefore, the interest in investigating the calibration relevance has emerged in recent years. Calibration relevance is defined as the capacity of the calibration method to focus on such relevant features of the structural model [1]. Zsarnóczay and Baker [1] proposed a framework of virtual experiments to investigate calibration relevance in order to evaluate different calibration methods. Zhang et al. [15] further incorporated a rigorous uncertainty quantification framework to study calibration relevance. The framework by Zhang et al. [15] utilized the method of LHS to propagate uncertainties of hysteretic model parameters and a metamodel technique for estimating calibration relevance and conducting global sensitivity analysis [15]. However, these two initial attempts that formulize the problem of calibration relevance still concentrated on the calibration in environment IV. As an efficient and affordable testing method, hybrid simulations can be used as an alternative component calibration method which can potentially achieve a higher calibration relevance due to the higher level of complexity which includes the interactions of substructures and more realistic loading histories. Now hybrid simulations are becoming easier to implement with the recent development of a generalized simulation framework called UT-SIM [16, 17], which integrates diverse numerical integration and substructure modules and experimental specimens. Moreover, a few multi-element hybrid simulations have been conducted recently [13, 18, 19] using the University of Toronto Ten Element Hybrid Simulation Platform (UT10) [12], which provides more advanced options for hybrid simulations.

This study proposed a calibration method that utilized hybrid simulations. A framework is also developed to evaluate the calibration relevance of a calibration method incorporating the uncertainty in hysteretic model parameters. As an initial exploration, purely numerical analyses are conducted for two SDOF systems with different periods as a case study. The framework of virtual experiments is implemented where the reference and simulation hysteretic models are chosen as the Steel4 [20] and Steel02 [21] materials, respectively, in OpenSees [22]. The parameters of the Steel4 model and the distributions of Steel02 parameters are adopted from [15]. The calibration relevance is evaluated based on the correlation between the calibration error quantifying the discrepancy between the reference and simulation hysteresis curves in calibrations and the global error measuring the error of global peak displacements under ground motion excitations. The responses of SDOF systems generated by nonlinear time history analyses (NTHA) with the reference Steel4 material represent the experimental results of hybrid simulations. The analysis results show that the proposed hybrid-simulation-based calibration method has a much higher calibration relevance than that of the cyclic-test-based method. This study provides a preliminary numerical demonstration of the superiority of the newly proposed calibration method. Hybrid simulations of real specimens will be conducted in the future to provide realistic data for evaluating the new calibration method.

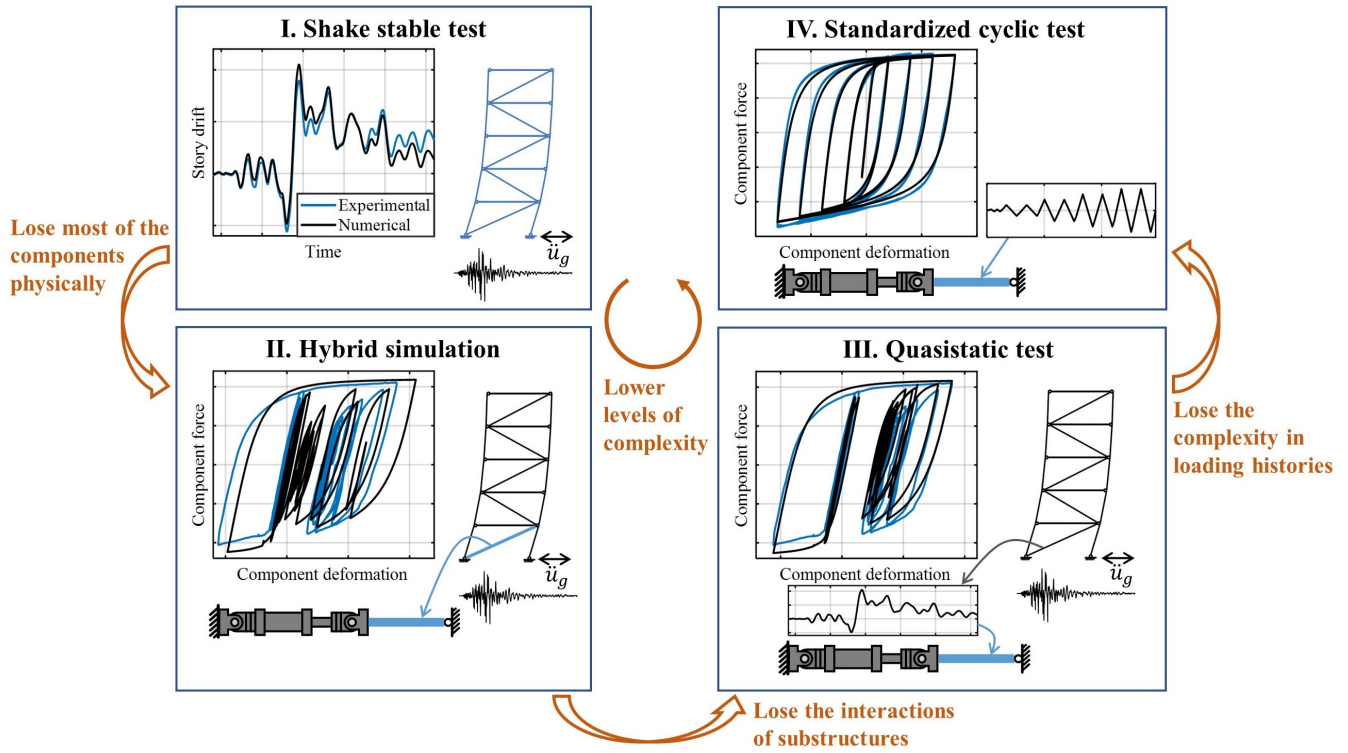


Figure 1. Four testing environments with different levels of complexity.

OVERALL FRAMEWORK

Figure 2 shows the framework for evaluating the calibration relevance. Uncertainties of hysteretic model parameters are incorporated into the framework by assuming the parameters of hysteretic models are random variables, and using the realization of the random variables in the numerical analysis. The outputs of the numerical models are then compared with the experimental test data, and their discrepancies are quantified as three types of errors: (i) calibration error of the cyclic test, $\varepsilon_{CAL,C}$; (ii) calibration error of hybrid simulation, $\varepsilon_{CAL,H}$; and (iii) global error, ε_{GLO} . The calibration error measures the discrepancies between the numerical and experimental hysteresis curves, while the global error quantifies the error in global peak responses of the structural system under ground motion excitations. By applying a large number of sampled hysteretic model parameters based on predefined distributions, the corresponding number of data points of the three errors can be obtained. In the last step of the framework, Spearman's rank correlation coefficient (r_s) between the calibration and global errors are computed as the calibration relevance. Spearman's rank correlation coefficient is a nonparametric (distribution-free) statistic which is a measure of monotone association [23]. It should be noted that, ideally, ε_{GLO} should be calculated based on the most realistic type of tests in environment I. But due to the high cost of shake table tests, hybrid simulations are used as an alternative to obtaining the reference global response at the system level. Thus, among the conducted hybrid simulations under different ground motions, one or more hybrid simulations will be selected and considered as calibration cases, and the rest hybrid simulations will be used as global cases.

In this study, the framework of virtual experiments has been implemented to conduct a series of purely numerical analyses. In this framework, a high-fidelity numerical model will be used as the reference case to represent real experiments, while another type of numerical model will be used as the simulation case such that the error of numerical models can be captured by the difference between the reference and simulation models [1]. The response of the reference models represents the hybrid simulation results in this study. As an initial investigation of the newly proposed calibration method, a case study of SDOF systems is presented in the next section.

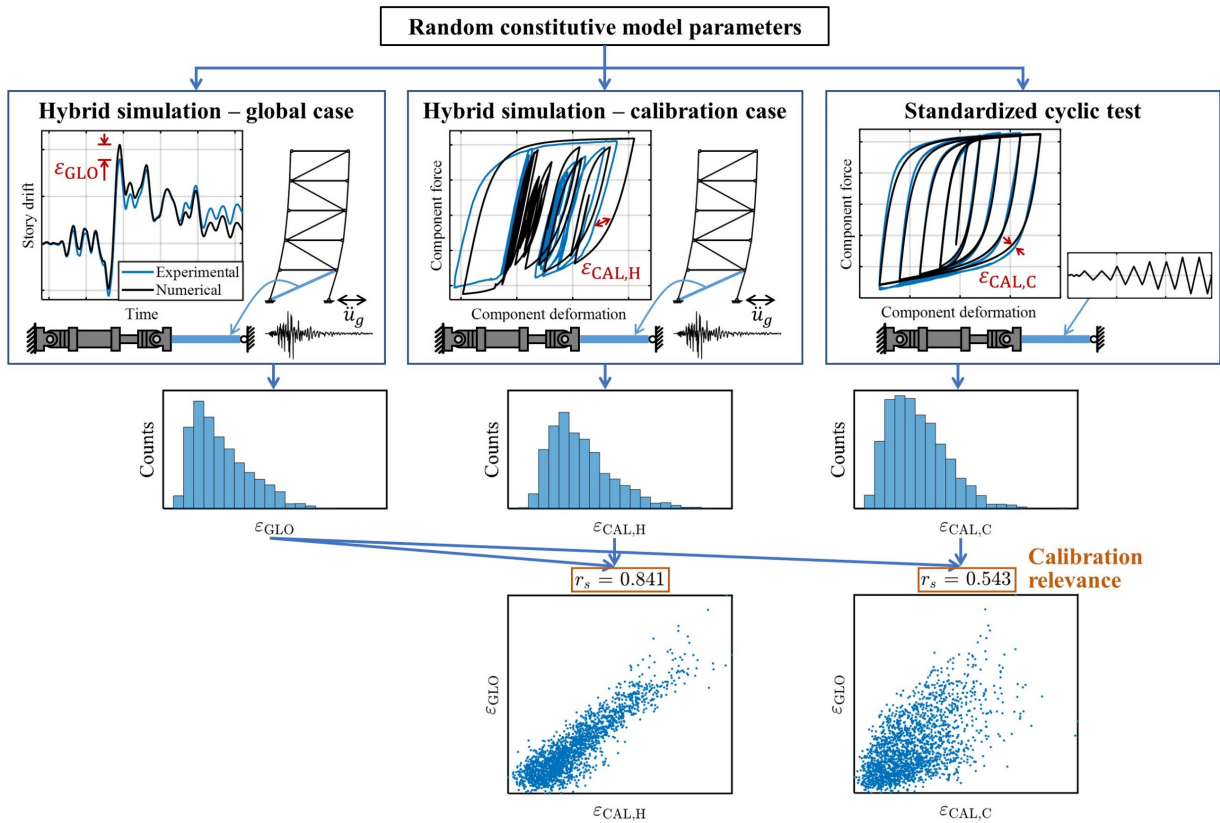


Figure 2. Framework of evaluating calibration relevance.

CASE STUDY OF SDOF SYSTEMS

In this study, numerical analyses of two SDOF systems with different fundamental periods are used as a case study to demonstrate the calibration relevance evaluation framework and investigate the hybrid-simulation-based calibration method.

Reference and simulation models in the framework of virtual experiments

In the framework of virtual experiments, a numerical model with high fidelity is chosen as the reference model, and a simplified one is chosen as the simulation model. The discrepancy between the real response of a structure and the numerical estimation is captured by comparing the results from the reference and the simulation models [1]. The SDOF models are used to simulate the hysteresis behavior of BRBs under ground motion excitations. The reference and simulation models are implemented utilizing the Steel4 and Steel02 materials in OpenSees, respectively. The parameters of the reference Steel4 material, as shown in Table 1, and the probabilistic model of the Steel02 parameters, as shown in Table 2, are determined based on [15]. The detailed meaning of the parameters of Steel02 and Steel4 can be found in [21] and [20], respectively. The initial elasticity (E_0) and the yield strength (f_y) of Steel02 are considered deterministic and their values are the same as those of the reference Steel4 model. The other eight parameters of Steel02, as listed in Table 2, are random variables and are assumed to be independent with each other. The randomness in the Steel02 parameters represents the uncertainty in the process of calibration [15]. In order to propagate the uncertainties in the random variables of Steel02 parameters, 1000 sets of model parameters are generated by using the LHS method. Figure 3 shows the hysteresis curves corresponding to these 1000 Steel02 models and the reference Steel4 model.

Table 1. Steel4 parameters

	E_0	f_y	b_k	R_0	r_1	r_2	b_i	b_l	ρ_i	R_i	l_{yp}
Tension	200 GPa	490 MPa	0.2%	20	0.92	0.15	1%	0	0.2	3	1
Compression			1.5%	13	0.88						

Table 2. Distributions of input variables for Steel02

Variable	Distribution	Mean	Standard deviation	Bounds
b	Truncated Gaussian	0.01	0.001	[0.005, 0.015]
R_0	Uniform	-	-	[13, 20]
cR_1	Truncated Gaussian	1	0.1	[0, 0.92]
cR_2	Truncated Gaussian	0.15	0.2	[0.05, 1]
a_1	Uniform	-	-	[0.01, 0.025]
a_2	Uniform	-	-	[0.5, 0.6]
a_3	Uniform	-	-	[0.01, 0.03]
a_4	Uniform	-	-	[1, 3]

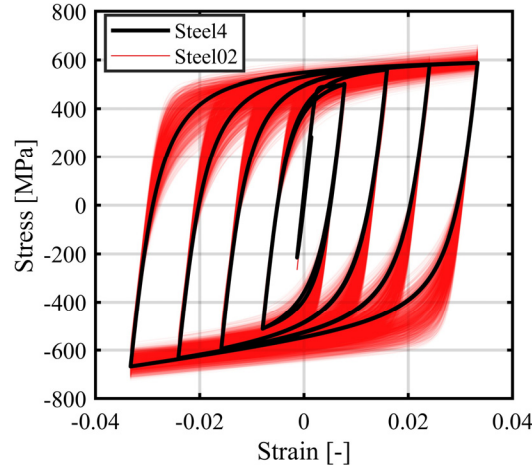


Figure 3. Hysteresis curves of the Steel4 and Steel02 models.

SDOF models

Two SDOF systems with initial periods T_0 of 0.5 and 1 sec are considered in this study. Another key parameter that dominates the dynamic response of a nonlinear SDOF system is the strength ratio η defined as the ratio between the yielding force and the weight of the system:

$$\eta = \frac{F_y}{mg} \quad (1)$$

where g is the gravity acceleration. The strength ratio is chosen as 0.1 and 0.05 for the SDOF systems with $T_0 = 0.5$ sec and $T_0 = 1$ sec, respectively. For each SDOF system, 200 unscaled ground motion records have been selected from the PEER NGA-West2 database [24] for NTHA to cover various characteristics of the hysteresis curves such as the magnitude of peak response, the number of loading cycles, the partial reloading behavior, etc. In order to consider a wide range of peak responses of the SDOF systems, five performance levels of the maximum displacement D_{max} are defined and are equally represented by the selected ground motions. The performance levels are (1) $2D_y < D_{max} < 4D_y$; (2) $4D_y < D_{max} < 6D_y$; (3) $6D_y < D_{max} < 8D_y$; (4) $8D_y < D_{max} < 10D_y$; (5) $10D_y < D_{max} < 12D_y$, where D_y is the yielding displacement. Peak displacements are estimated by the reference Steel4 model. Viscous damping with a damping ratio of 2% is assumed in the numerical models.

Calibration error

The goal of a calibration process is to identify a group of parameters of the simulation model that will minimize the discrepancy with the reference. Thus, a calibration process can be considered as an optimization problem, as shown in this mathematical expression:

$$\hat{\theta} = \operatorname{argmin}_{\theta} L(\mathbf{y}_r, \mathbf{y}_s(\theta)) \quad (2)$$

where $\hat{\theta}$ is the target set of optimized parameters for the simulation model, L is a loss function that quantifies the discrepancy between the reference outputs \mathbf{y}_r and the simulation outputs $\mathbf{y}_s(\theta)$. In this study, two calibration methods are considered, based on cyclic tests and hybrid simulations, respectively. The loss functions of calibration, i.e., the calibration errors of the two calibration methods are quantified as below:

$$\varepsilon_{\text{CAL,C}}(\boldsymbol{\theta}) = \sqrt{\frac{\sum_{i=1}^n [F_{s,i}(\boldsymbol{\theta}) - F_{r,i}]^2}{\sum_{i=1}^n F_{r,i}^2}} \quad (3)$$

$$\varepsilon_{\text{CAL,H}}(\boldsymbol{\theta}) = \sqrt{\frac{\sum_{i=1}^n [D_{s,i}(\boldsymbol{\theta}) - D_{r,i}]^2}{\sum_{i=1}^n D_{r,i}^2}} + \sqrt{\frac{\sum_{i=1}^n [F_{s,i}(\boldsymbol{\theta}) - F_{r,i}]^2}{\sum_{i=1}^n F_{r,i}^2}} \quad (4)$$

where n is the number of ground motion steps; $D_{s,i}$ and $F_{s,i}$ represent the simulation displacement and force, respectively, at step i ; $D_{r,i}$ and $F_{r,i}$ represent the reference displacement and force, respectively, at step i ; and $\boldsymbol{\theta}$ represents the eight random variables of the Steel02 material, i.e., $\boldsymbol{\theta} = [b, R_0, cR_1, cR_2, a_1, a_2, a_3, a_4]$. These two equations are composed of normalized root-mean-square errors of the forces and displacements. In Eq. (3), since the simulation of a cyclic test is based on a predefined loading profile, the discrepancy between the reference and simulation models will only occur in forces. While in hybrid simulations, the calibration error exists in both displacements and forces. It should be noted that in Eq. (4), the weights of the two parts of the calibration error are assumed to be equal. Generally, the displacement error is larger than the force error, given the fact that displacements are more difficult to estimate. Therefore, by using equal weights on the two portions of $\varepsilon_{\text{CAL,H}}$, the optimization process will put more emphasis on minimizing displacement error, which is desirable because displacements are important engineering demand parameters in the seismic assessment of structures.

Another method to utilize hybrid simulations for calibration is to use the displacement history generated from a hybrid simulation as the loading profile for a quasistatic test. This method is similar to a conventional cyclic test but with a realistic loading profile. The calibration error of this method is denoted as $\varepsilon_{\text{CAL,Q}}$ which is computed using Eq. (3).

Global error

Global errors are used to quantify the error of the system-level structural model under ground motion excitations. Peak displacement is an important and commonly used engineering demand parameter. Thus in this study, the global error is defined as the root-mean-squared log-relative difference between the simulated and reference peak displacements:

$$\varepsilon_{\text{GLO}}(\boldsymbol{\theta}) = \sqrt{\frac{1}{n} \sum_{i=1}^n \left[\log \frac{PD_{s,i}(\boldsymbol{\theta})}{PD_{r,i}} \right]^2} \quad (5)$$

where n is the number of selected ground motions; $PD_{s,i}$ and $PD_{r,i}$ represent the peak displacements of the simulation and reference models under the i^{th} ground motion.

Evaluation of calibration relevance

Utilizing the 1000 sampled $\boldsymbol{\theta}$ generated previously, there will be 1000 realizations of the simulation SDOF models with a specific T_0 . For every realization of the simulation models, 200 NTHA were conducted based on the selected ground motions. Based on the displacement histories generated from the NTHA, 200 quasistatic analyses were also conducted on the realizations of the models. Considering the reference models and two different T_0 values, 400,400 NTHA have been conducted in this study. Additionally, 401,401 quasistatic analyses including 1001 cyclic analyses have been done. Figure 4 shows the data preparation for the evaluation of calibration relevance where each block represents the hysteresis data from one analysis.

Among the 200 ground motions, N_{CAL} ground motions can be selected randomly as calibration cases. The reference results under these ground motions represent the real response from hybrid simulations, and the corresponding simulation results represent the numerical response which is used to match the reference data in the process of calibrations. Based on the results of the N_{CAL} selected ground motions, the HS-based calibration error is defined as the square root of the sum of the squares of the calibration errors of every ground motion:

$$\varepsilon_{\text{CAL,H},\alpha} = \sqrt{\sum_{i \in \alpha} \varepsilon_{\text{CAL,H},i}^2}, \alpha \subset S, |\alpha| = N_{\text{CAL}}, S = \{1, 2, \dots, 200\} \quad (5)$$

where α with the size of N_{CAL} is a set of the indices of selected ground motions for calibration; $\varepsilon_{\text{CAL,H},i}$ can be calculated using Eq. (4). $\varepsilon_{\text{CAL,Q},\alpha}$ can be calculated in the same manner. After selecting N_{CAL} ground motions as the calibration cases, the remaining ground motions are then used as the global cases to generate global errors denoted as $\varepsilon_{\text{GLO},S-\alpha}$. The calibration and global errors are functions of $\boldsymbol{\theta}$ essentially, thus are random variables as well. Then the three types of calibration relevance $r_{s,C}$, $r_{s,H}$, and $r_{s,Q}$ which are the correlation between the global error ($\varepsilon_{\text{GLO},S-\alpha}$) and the three types of calibration errors ($\varepsilon_{\text{CAL,C}}$,

$\varepsilon_{\text{CAL,H}}$, and $\varepsilon_{\text{CAL,Q}}$), respectively, can be computed. Due to the randomness in selecting a group of N_{CAL} ground motions, the three types of calibration relevance ($r_{s,C}$, $r_{s,H}$, and $r_{s,Q}$) are uncertain as well. For each value of N_{CAL} , distributions of calibration relevance can be generated. Examples of these distributions with N_{CAL} equals to four and ten for the SDOF model with $T_0 = 0.5$ sec are shown in Figure 5.

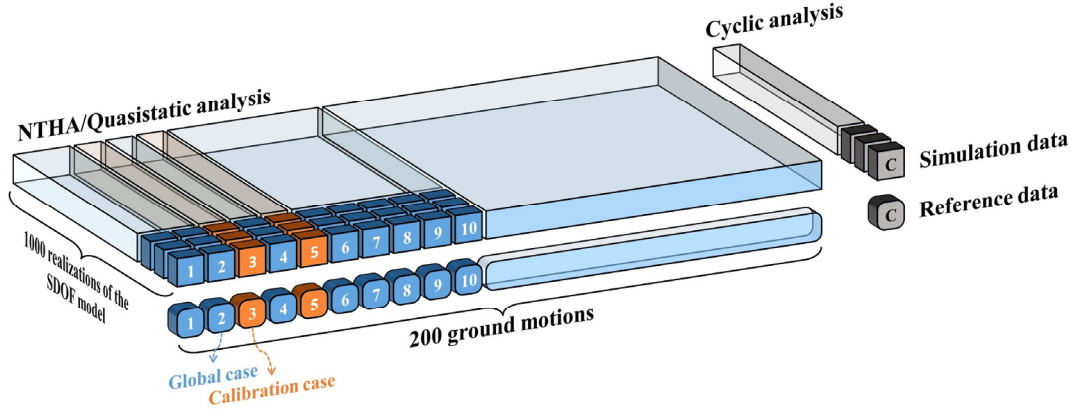


Figure 4. Data preparation for the evaluation of calibration relevance

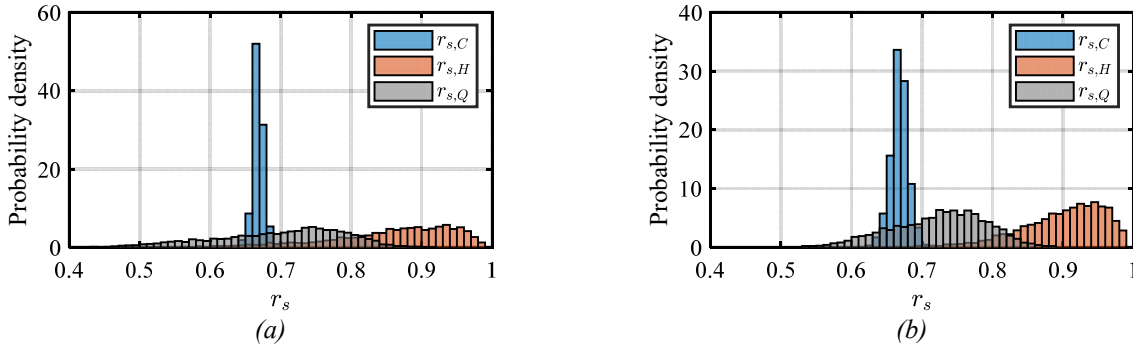


Figure 5. Distributions of $r_{s,C}$, $r_{s,H}$, and $r_{s,Q}$ (a) $N_{\text{CAL}} = 4$, (b) $N_{\text{CAL}} = 10$.

Based on the distribution of calibration relevance, the corresponding confidence interval can be calculated for each N_{CAL} . Figure 6 shows the results of the evaluation of the calibration relevance of the three calibration methods based on the two SDOF models. The medians of $r_{s,H}$ and $r_{s,Q}$ are increasing and their 95% confidence intervals are narrowing down with the increase of N_{CAL} ; a trend of their convergence can also be observed. The median of $r_{s,C}$ is relatively stable, while its confidence interval becomes slightly wider with a larger N_{CAL} . This is due to the fact that with more ground motions selected for calibration cases, fewer ground motions will be used for computing the global error. In this case, a larger variation in the global error will be obtained with different selections of ground motions. For example, if 190 out of 200 ground motions are selected for calibration, there will be only ten ground motions for the global cases. And there is apparently a large variation of the global error calculated by randomly selecting ten out of 200 ground motions. The most crucial observation from the evaluation results is that with two or more ground motions selected for calibration, $r_{s,H}$ is significantly higher than $r_{s,C}$ and $r_{s,Q}$. This demonstrates the superiority of using hybrid simulations for hysteretic model calibration, at least for these two SDOF systems. Another interesting finding is that $r_{s,Q}$ is much lower than $r_{s,H}$. This reveals that by only introducing complicated loading profiles may not increase the calibration relevance of quasistatic tests with predefined loading histories. The high complexity level of the environment of hybrid simulations is the key factor in establishing a superior calibration method. Additionally, the results of the two SDOF systems are very similar which indicates that the conclusions are independent of fundamental periods of structures.

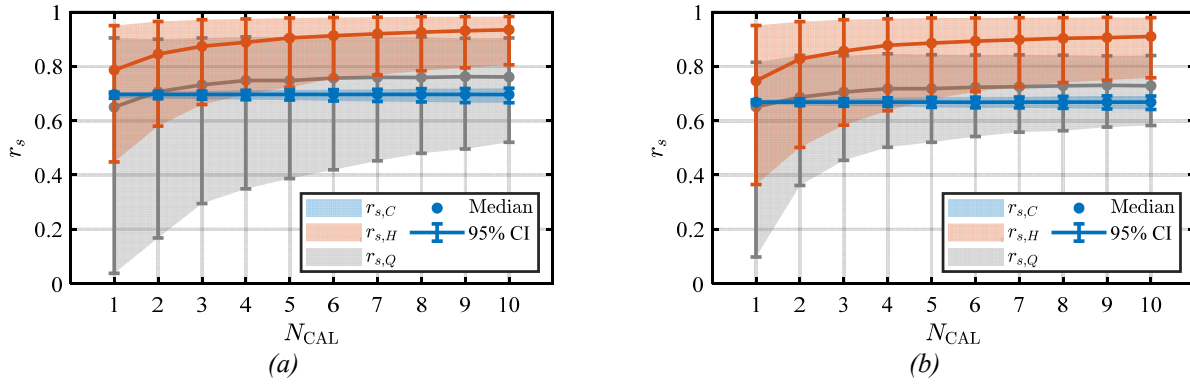


Figure 6. Evaluation of calibration relevance (a) $T_0 = 0.5$ sec; (b) $T_0 = 1$ sec

CONCLUSIONS

The widely accepted calibration method based on standardized cyclic tests for hysteretic models in nonlinear structural seismic analysis has been challenged previously due to the large unproportionate error in estimating the global response at the system level. This is fundamentally caused by the low complexity level of the calibration environment compared to the system-level modeling environment. A calibration method is proposed utilizing hybrid simulation data, which is obtained in a much more complicated environment. A framework for evaluating calibration methods is also developed incorporating the uncertainties in hysteretic model parameters. A purely numerical study on two SDOF systems with different periods is conducted utilizing the framework of virtual experiments. The results show that the proposed hybrid-simulation-based method has a much large calibration relevance than that of the conventional cyclic-test-based method. This numerical study serves as a preliminary analysis for future hybrid simulations on real specimens to further investigate the newly proposed calibration method.

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