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Shear Control in Corner-Gusset Connections by Means of Virtual Gussets and the Uniform Force Method

Brandt Saxey[1*] and Rafael Sabelli[2]

[1]Technical Director, CoreBrace, West Jordan, UT
[2]Senior Principal, Director of Seismic Design, Walter P. Moore, San Francisco, CA
[*]brandt.saxey@corebrace.com (corresponding author)

ABSTRACT

The Uniform Force Method (UFM) is a powerful tool to facilitate rational, economical design of gusset plates. As presented in literature, the baseline case for the UFM has gusset-flange interface forces centered on the centroid of the gusset edges; deviations from this are allowed for existing connections and accounted for by considering eccentricity. Bracing connections with high force demand, such as those encountered in seismic design, cause local shear in both the beam and the column. The UFM is often used to analyze forces in these connections. By adapting the UFM to minimize shear in the connected members, additional economy can be achieved. This adaptation will be presented through 5 key principles.

Keywords: Uniform Force Method, Gusset Design, Braced Frame Design, Seismic Design, Sustainability

INTRODUCTION

Design of brace connections requires consideration of local forces induced in the surrounding framing members. Although the discussion of these forces initially focused on beam midspan connections (Fortney and Thornton, 2015), such forces also occur at beam-column-brace connections. These local forces are often missed by analysis methods that neglect connection dimensions. The Uniform Force Method (UFM) (Thornton, 1991; Muir and Thornton, 2014; AISC, 2017) is commonly used to analyze forces at gusset-plate connections. It is a powerful tool that permits designers to proportion and optimize connections. This study presents an adaptation of the UFM specifically developed to allow the designer to proportion and analyze connections to reduce required member shear strength and thus to reduce the instances of connections requiring web doublers. Similar to the UFM as originally developed, the methods presented in this paper rely on the lower bound theorem as presented by Thornton (1984) for similar connections, demonstrating adequate strength through investigation of an advantageous load path in a ductile connection and examining forces at gusset edges. The method presented here is a supplement to the UFM, showing necessary dimensions to eliminate the need for web reinforcement.

The presented adaptation of the UFM depends on the lower-bound theorem to justify the use of a relatively simple design model. While the design model is not intended to produce forces matching those of more sophisticated analytical models, the methods are expected to result in designs with adequate strength. Finite element analyses have been performed which confirm this strength. Additionally, the FEA produce force distributions similar to those determined using the design model.

This paper focuses exclusively on the design of braced-frame-connections to achieve the required strength of each component for a defined set of forces. The design forces considered are brace axial forces, beam axial, and beam shear forces, Inelastic deformation demands related to brace buckling are not considered; as such, the methods are appropriate for wind design and for the design of buckling-restrained braced frames but are not sufficient for providing the required ductility of special concentrically braced frames.

The paper begins with a brief introduction to the UFM. The adaptation of the UFM, as presented in Sabelli, et al (2021) is then presented, organized around a set of five key principles.

INTRODUCTION TO THE UNIFORM FORCE METHOD

The Uniform Force Method (UFM) (AISC, 2017) is a commonly employed method of analyzing traditional gussets. The UFM utilizes the gusset dimensions 2α and 2β, as well as the beam and column eccentricities e_b and e_c as shown in Figure 1 and discussed below. In an optimally-proportioned gusset designed using the UFM, the dimensions α and β represent the centroids of the gusset interfaces at the beam and column, as well as the centroids of the forces acting on those joints. (Moments are addressed for the analysis of existing bracing connections when the optimal bracing proportions are not met.)

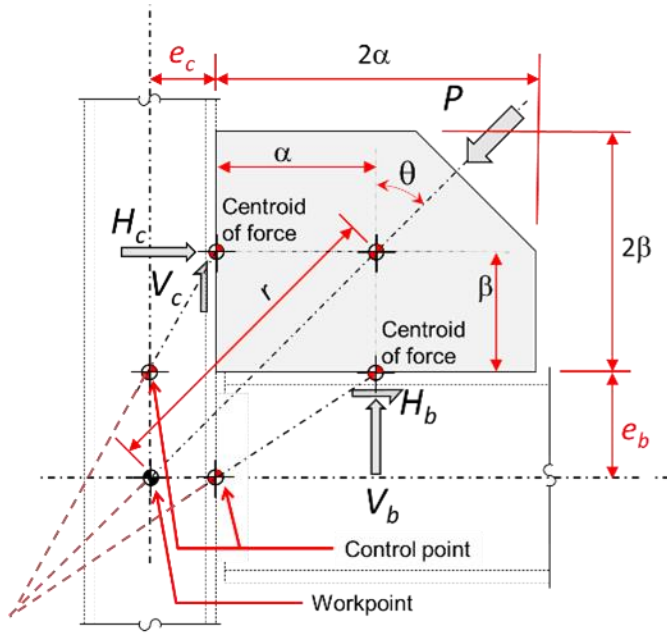

Figure 1. Uniform Force Method dimensions

The forces on the beam comprise a vector passing through the control point at the centroid of the beam-to-column connection. Likewise, the forces on the column comprise a vector passing through the control point at the column centerline at the elevation consistent with the gusset-to-beam interface (at the top of flange elevation for the condition shown in Figure 1). These forces satisfy statics in that the vertical forces (V_b and V_c) add up to the vertical component of the brace force and the horizontal forces (H_b and H_c) add up to the horizontal component of the brace force. Additionally, the vector sum of the beam vector and the column vector are equivalent to the brace vector and intersect on the brace centerline (see red extensions in Figure 1) such that no moment is required in any of the members outside of the connection region.

Optimal Proportioning

Optimal proportioning of the gusset eliminates the need for moments at the interfaces with the beam and the column by aligning the centroids of the gusset with the locations of the forces. For this to be the case, the dimensions α and β must conform to Equation (1):

$$\alpha - \beta \tan\theta = e_b \tan\theta - e_c \quad (1)$$

where

α	=	Distance from column face to centroid of Uniform Force Method force acting on beam flange
β	=	Distance from beam flange to centroid of Uniform Force Method force acting on column face
θ	=	brace angle from vertical

Force Equations

The UFM defines the distance from the workpoint to the centroid of the gusset r, shown in Figure 1, as follows:

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \quad (2)$$

where

e_b = Eccentricity from beam flange to beam centerline, equal to half the beam depth

e_c = Eccentricity from column flange to column centerline, equal to half the column depth

The forces on the gusset interfaces are given by the equations:

$$V_c = \frac{\beta}{r} P \quad (3)$$

$$H_c = \frac{e_c}{r} P \quad (4)$$

$$V_b = \frac{e_b}{r} P \quad (5)$$

$$H_b = \frac{\alpha}{r} P \quad (6)$$

where

P = Brace axial force

V_c = Column vertical force

H_c = Column horizontal force

V_b = Beam vertical force

H_b = Beam horizontal force

The normal forces (V_b and H_c) induce shear in the respective members. These normal forces are typically assumed to be uniformly distributed, such that the member shear due to brace forces accumulates from zero outside the gusset region to the maximum value (V_b or H_c) at the control point.

FIVE PRINCIPLES OF THE ADAPATATION OF THE UNIFORM FORCE METHOD

Sabelli et al. (2021) present an adaptation of this Uniform Force Method that facilitates optimized designs, specifically eliminating the need for shear reinforcement of the beam and column. The adaptation can be understood through five principles:

1. Use of a virtual gusset to establish magnitudes and centroids of shear and normal forces on the gusset interfaces.
2. Optimal proportioning of the virtual gusset.
3. Optimal sizing of the virtual gusset to control member shear by means of the dimension r.
4. Moments induced when actual gusset dimensions do not match the virtual gusset dimension.
5. Sizing of actual gusset to be significantly smaller than the virtual gusset without increasing member shear.

Principle 1 –The Virtual Gusset

The location, magnitude, and direction of the gusset resultant forces on the beam and column can be established by means of a virtual gusset. The centroid of this virtual gusset defines the location of the centroid of the forces acting on the beam and the column per the Uniform Force Method.

Principle 2 – The Optimally Proportioned Virtual Gusset

Optimal proportioning of the virtual gusset requires that the centroids of the virtual gusset align with the locations of the forces. Thus, the dimensions α and β must conform to Equation (1). As shown in Sabelli, et al, Equation 1 can be combined with Equation 2 to express the centroid dimensions of the virtual gusset in terms of r, which can be expressed as:

$$\alpha = r \sin\theta - e_c \quad (7)$$

$$\beta = r\cos\theta - e_b \tag{8}$$

Thus, although three virtual dimensions (r, α and β) are used in the equations below, they are constrained to each other, and a single variable (r) is selected in the design. The dependent virtual dimensions α and β locate the centroids of the forces acting on the beam flange and column flange.

Principle 3 – The Optimally Sized Virtual Gusset

Selection of the dimension r determines the size of the virtual gusset as shown in Equations (7) and (8). As such, it also determines the distribution of the shear and normal forces acting on the gusset interfaces. Equations (3) and (6) can be rewritten so that, like Equations (4) and (5), they are in terms only of dimensions of the frame geometry and brace angle (eliminating α and β):

$$V_c = \left(\cos\theta - {e_b}/{r}\right)P \tag{9}$$

$$H_b = \left(\sin\theta - {e_c}/{r}\right)P \tag{10}$$

Arranging Equations (4), (5), (9), and (10) into those causing shear force and normal force (the latter of which causes member shear) produces the following:

Table 1. Shear and Normal Forces on Gusset Interface

	Normal Force	**Shear Force**
Vertical Force	$V_b = \frac{e_b}{r}P$	$V_c = \left(\cos\theta - {e_b}/{r}\right)P$
Horizontal Force	$H_c = \frac{e_c}{r}P$	$H_b = \left(\sin\theta - {e_c}/{r}\right)P$

The dimension r can be selected so that the member shear in the beam (at the column face) and in the column (at the beam flange elevation) is limited. The greater the dimension r, the more force is transferred through shear forces (acting parallel to the member) and the less through normal forces, thus reducing the shear in the frame members.

It follows that a minimum dimension of r can be selected such that the shear capacity of the members is not exceeded. These equations are derived in Sabelli, et al and are as follows:

$$r \geq r_{minCol} = \frac{e_c P}{\phi V_{n,col}} \tag{11}$$

$$r \geq r_{minBm} = \frac{e_b P}{U_c \phi V_{n,bm}} \tag{12}$$

Equation (11) provides that $\phi V_{n,col} \geq H_c$ indicating that the shear capacity of the column web is sufficient for the force producing column shear. Similarly, Equation (12) provides that $U_c \phi V_{n,bm} \geq V_b$, where U_c is the ratio of the beam connection shear strength to the beam shear strength (taken ≤ 1.0), and indicates that this shear capacity is greater than the resulting beam shear demand. The larger value of r controls the design:

$$r \geq \max\left(r_{\min Col}, r_{\min Bm}\right) \tag{13}$$

It is noted that Equation (11) and (12) are provided for the case, as shown in Figure 1, with a single gusset at a beam-column joint. Sabelli, et al. provide equations with the apportionment required for cases with multiple gussets at the beam-column joint and for additional shear forces present in the member.

Principle 4 –Moments Induced When Actual Gusset ≠ Virtual Gusset

The actual gusset dimensions need not match the dimensions of the virtual gusset described by Equation (1). In such cases, a moment, and only a moment, is induced at the gusset interface due to the moment-arm between virtual and actual centroids of the interfaces. These moments are given in Equations (14) and (15) below:

$$M_c = H_c\left(\beta - \bar{\beta}\right) \tag{14}$$

$$M_b = V_b\left(\alpha - \bar{\alpha}\right) \tag{15}$$

As used here, α and β are dimensions of the virtual gusset complying with Equation (1) and $\bar{\alpha}$ and $\bar{\beta}$ represent dimensions of the actual gusset (i.e. the half-length of the of the beam and column interfaces respectively). See Figure 2.

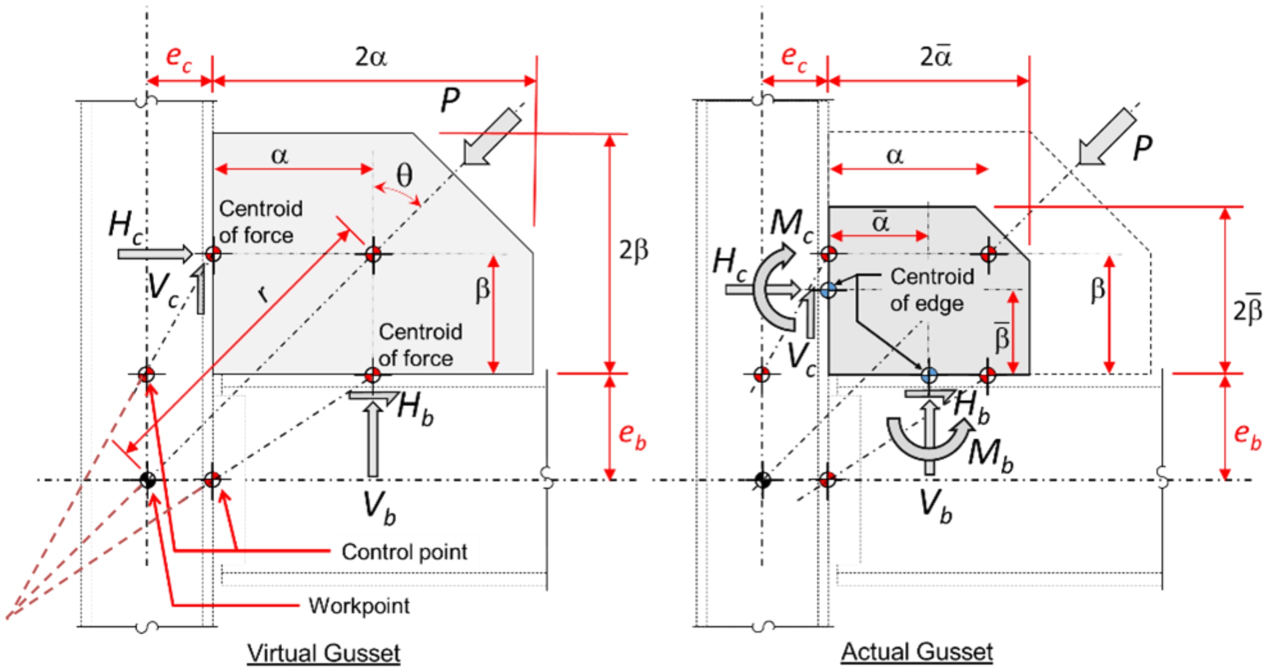

Figure 2. Virtual and actual gussets

The larger the deviation of the actual dimension from the virtual dimension, the larger the moment that must be resisted at the gusset interface.

While the member shear at the control point remains unchanged, the moment at the interface changes the member shear between the gusset tip and the control point. The normal stress at the beam interface is a combination of the shear force, V_b given in Equation (5), uniformly distributed over the gusset length, $2\bar{\alpha}$. with the effect of moment at this interface, as given in Equation (15). The resulting diagrams of member shear and distributed normal stress, are shown in Figure 3 where sections B1 and B2 are taken at the column interface and at the mid-length of the beam interface respectively. Figure 3 shows the case where the value of $\bar{\alpha}$ is such that the shear at section B2 is equal to the shear at section B1. The case shown in Figure 3 is for the idealized situation where the shear capacity at B1 is equal to B2 (that is, the connection strength is at least equal to beam shear strength) and the resulting $U_c = 1.0$. When $U_c < 1.0$ a different stress diagram will result as shown in Sabelli et al.

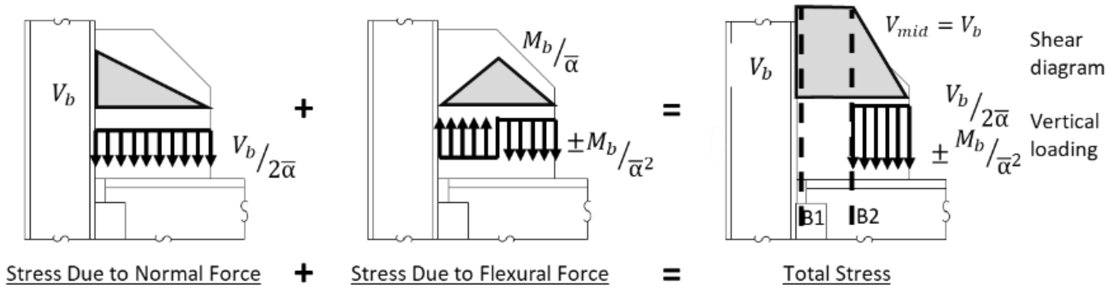

Figure 3. Stress distribution in beam due to normal force and moment.

Principle 5 – Actual Gusset Can Be Significantly Smaller Than Virtual Gusset Without Increase in Member Shear

The actual gusset can be significantly smaller than the virtual gusset without causing excessive member shear. For the condition at the beam interface, if $\overline{\alpha} \neq \alpha$ a moment exists at the beam as indicated in Equation (15). The shear at section B2, shown in Figure 3, is:

$$\begin{aligned} V_{mid} &= \frac{V_b}{2} + \frac{M_b}{\overline{\alpha}} \\ &= V_b\left(\frac{\alpha}{\overline{\alpha}} - \frac{1}{2}\right) \end{aligned} \tag{16}$$

This shear must be less than the beam shear capacity:

$$|V_{mid}| \leq \phi V_{n,bm} \tag{17}$$

Note that check is performed away from the connection and so the U_c factor is not applied.

The minimum dimension $\overline{\alpha}$ can be determined by combining Equations (16) and (17):

$$\overline{\alpha} \geq \frac{\alpha}{\left|\frac{\phi V_{n,bm}}{V_b}\right| + \frac{1}{2}} \text{ for } r \geq r_{\text{minBm}} \text{ or Uc} < 1.0 \tag{18}$$

This minimum dimension should be compared to that required for the brace-to-gusset connection.

If the virtual dimension $r = r_{min,Bm}$, Equation (18) can be combined with $U_c = V_b/\phi V_{n,bm}$ (where V_b is the lesser of the connection strength and the beam shear strength) and simplified to:

$$\overline{\alpha} \geq \frac{\alpha}{\frac{1}{U_C} + \frac{1}{2}} \text{ for } r = r_{\text{min}Bm} \text{ and } U_c < 1.0 \tag{19}$$

If the connection strength is equal to the beam shear strength ($U_c = 1.0$) then Equation (19) simplifies further to:

$$\overline{\alpha} \geq \frac{2}{3}\alpha \text{ for } r = r_{minBm} \text{ and } U_c = 1.0 \tag{20}$$

Equations 19 and 20 provide the minimum value of the gusset dimension $\overline{\alpha}$ for the typical case of the virtual gusset sized to limit the beam shear to the maximum value that can be resisted at the control point at the face of the column. A connection with U_c = 1. 0 and gusset sized per Equation 20 can be used to illustrate the relationship between the virtual and actual gusset in terms of both the beam shear and the forces and stresses at the beam-gusset interface. Figure 4 shows the virtual gusset on the left and the actual gusset on the right. The stress corresponding to the force V_b spread uniformly over the virtual gusset length is shown on the virtual gusset diagram below that. The corresponding beam shear diagram is also shown (assuming no other shears than those due to the brace force) in the connection region. The diagram of the actual gusset is similar, but instead of the force V_b being spread over the entire (actual) gusset length, the combined effects of V_b and M_b (as shown in Figure 3) result in the force V_b being spread over half the actual gusset length. The force V_b is thus spread over the central third of what was the virtual gusset. The maximum shear at the control point remains unchanged, but this value is maintained over half the actual gusset length. Because of the higher concentration of the force in an actual gusset 2/3 of the virtual gusset size, the required gusset thickness may be correspondingly greater than would be needed if the size of the virtual gusset were used for the actual gusset. This is discussed in the next section.

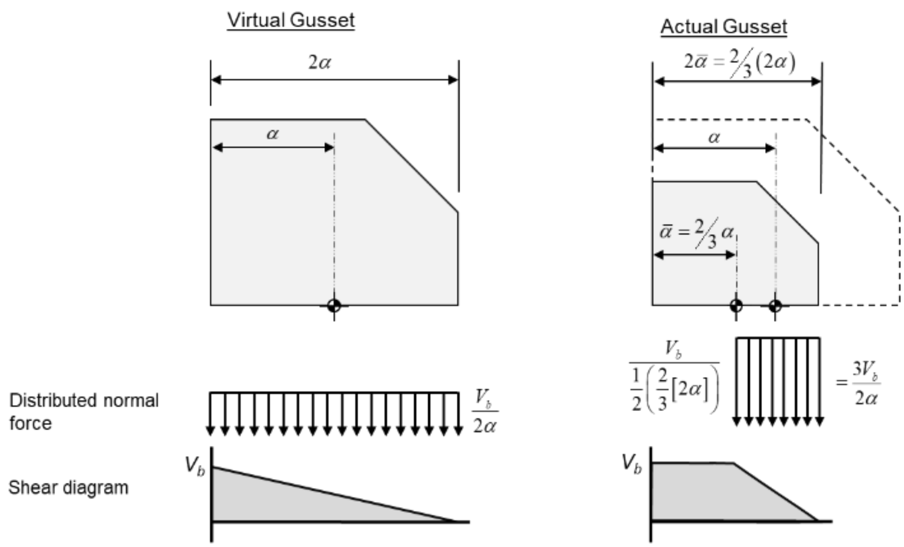

Figure 4. Distribution of normal force and corresponding shear diagrams for virtual and actual gussets.

The above example is provided for the case where the dimension r in Equation (13) is controlled by the shear strength of the beam (i.e. $r_{minBm} > r_{minCol}$). If a larger value of r is used, the value of $\bar{\alpha}$ from Equation (18) will produce the minimum length of the gusset. While not derived here, similar values for $\bar{\beta}$ can be derived for the gusset length at the column interface. These derivations are given in Sabelli et al.

ADDITIONAL CONSIDERATIONS

In addition to the minimum beam and column shear strengths, the column and beam should be evaluated for local web crippling and web yielding for a required strength (R_u) due to the combined effects of normal force and moment over a bearing length N of $\bar{\beta}$ or $\bar{\alpha}$, respectively:

$$R_u = V_{mid} = H_c \left(\frac{\beta}{\bar{\beta}} - \frac{1}{2}\right) \tag{21}$$

$$R_u = V_{mid} = V_b \left(\frac{\alpha}{\bar{\alpha}} - \frac{1}{2}\right) \tag{22}$$

The gusset and its connection must also be checked for the combined effects of moment, horizontal, and vertical forces at both the column interface (M_c, H_c, *and* V_c). and the beam interface (M_b, H_b, and V_b). Von mises yield criterion may be used to determine the minimum gusset thickness (t_g) considering forces at the interface with the column. The reader is referred to Sabelli et al for the appropriate equations.

Sabelli, et al. provides an example of the presented method. A validation of the example was provided by means of a FEM which showed good results.

CONCLUSIONS

The uniform force method (UFM) provides an effective means to design gussets for braced frames. An extension of the UFM has been presented which adds further economy to the design of braced frame gussets. This method relies on the lower bound theorem and shows good comparison with FEM analysis.

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