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Accounting for corrosion in computing the seismic response of reinforced concrete beams

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ABSTRACT

To assess the seismic safety of existing reinforced concrete structures, the effect of corrosion must be addressed. The purpose of the present work is to build a numerical model including the influence of corrosion on the dynamic response of reinforced concrete structures.

The challenge of such a model is to account for material nonlinearities, while keeping a computational cost low enough to perform computations at the structural scale. Therefore, the proposed model is based on fiber beam elements. The fiber beam element method has proved to be efficient to compute the seismic response of structures. By adding a scale to classical beam elements, fiber beam elements enable to compute material strains and stresses at the cross-section level instead of using a generalized constitutive law. Thus, a higher accuracy is achieved at the local level.

The present work enhances fiber beam elements to account for the mechanical effects of corrosion. A specific constitutive law is used for corroded steel rebars, along with a reduced resisting steel cross-section. Damage of concrete induced by the swelling of the corrosion products is computed at the beam's cross-section level. A projection technique is used on the damage field to reduce the number of elements in the cross-section and increase the computational efficiency of the model. Finally, the random distribution of corrosion pits is characterized and included in the model.

The numerical results are first compared to experimental quasi-static cyclic tests performed on corroded RC beams. The contribution of each enhancement is analyzed. The model is then used to predict the dynamic behavior of corroded RC beams, tested on a shaking table.

Keywords: Earthquake engineering; Reinforced concrete; Corrosion; Finite element method; Fiber beam element.

INTRODUCTION

To characterize the seismic behavior of existing reinforced concrete (RC) structures, the effect of pathologies must be addressed. Among these pathologies, corrosion of the steel reinforcement is one of the most threatening. If the quasi-static behavior of reinforced concrete with corroded rebars is widely addressed in the literature, the dynamic behavior of corroded RC structural elements needs further investigation.

For this purpose, an experimental campaign on corroded RC beams has been conducted [1, 2]. The purpose of the present paper is to develop a numerical model able to address the dynamic behavior of structures affected by corrosion. The results of the experimental tests will be used as a validation reference for the numerical results, at the scale of the beams. Thus, the first section gives a short synthesis of the main features of the experimental campaign necessary to understand the numerical case-studies.

The numerical model aims at carry out dynamic computations at the structural scale. Thus, the fiber beam element method has been chosen for its efficiency [3, 4]. By replacing the generalized constitutive law by the computation of material strains and stresses at the scale of the cross-section, the fiber beam element method enables the access to the local behavior of the materials.

By suitably targeting the effects of corrosion to be represented in the model, it is possible to account for corrosion without drastically increase the computational time. The proposed enhancements to account for corrosion are detailed in the second section. A specific constitutive law is used for corroded steel, as well as a reduced cross-section for the steel rebars. At the relatively low corrosion rates of the experimental campaign (up to 10% in mass loss), the main other effect of corrosion is the damage of concrete induced by the rebar swelling. A modelling strategy is proposed to determine this initially cracked stage of concrete, before the occurrence of the quasi-static or dynamic loading. Finally, the effect of non-uniform corrosion is also addressed.

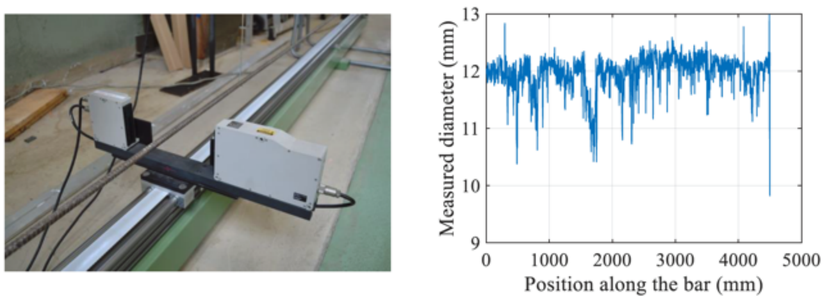
Once the method to account for corrosion is explained, the numerical set-up to represent the experimental tests is described. First, the numerical results obtained with the model for the non-corroded beams are presented. Eventually, the QS and DYN responses of the corroded RC beams are presented.

SUMMARY OF THE EXPERIMENTAL CAMPAIGN

To address the dynamic behavior of RC beams with corroded reinforcements, an experimental campaign has been conducted [1, 2]. The specimen were RC beams of 4.5 m length and with a 20x40 cm cross-section. Different corrosion configurations were tested. In the present paper, only the configuration with the corroded longitudinal rebars is developed. Three corrosion rates were achieved by an accelerated corrosion process: 3.1%, 6.3%, and 9.4%. To quantify the non-uniformity of corrosion, the distribution of the steel cross section diameters after corrosion were measured (section "Loss of diameter measurements"). The RC beams with corroded rebars were submitted to two types of tests. Quasi-static cyclic tests (QS) were performed by the means of an actuator, and dynamic tests (DYN) by the means of a shaking table (section "Quasi-static and dynamic tests"). Details on the experimental campaign necessary to understand the background of the numerical model are provided below. For further information on the experimental campaign and its results, the interested reader is invited to refer to [1, 2].

Loss of diameter measurements

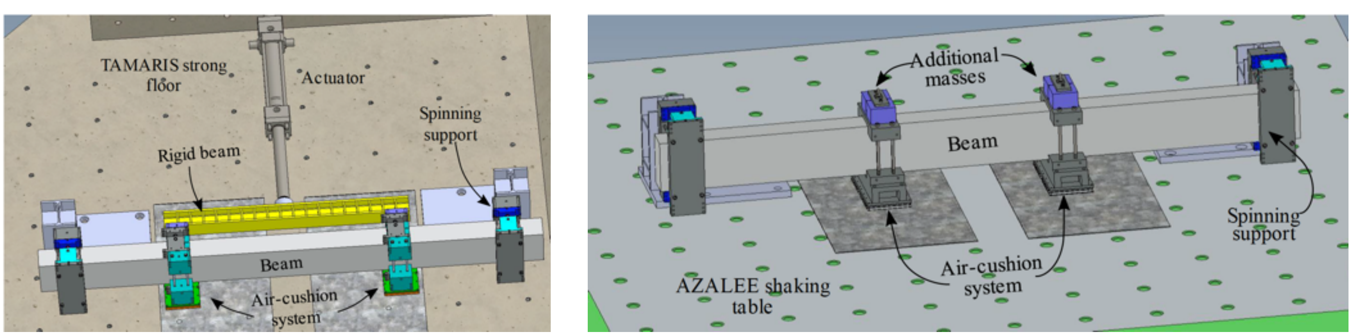
The measurements of the steel cross-section diameter distribution have been realized by the means of a laser scan along the corroded rebar, after extraction from concrete and mechanical cleaning (Figure 1). Measurements are registered every 1mm along the beam.



(a) *(b)*

Figure 1. Corroded steel cross-section measurement [1]. (a) Profilometer, (b) Measured diameter distribution for the 9.4% corrosion rate.

Quasi-static and dynamic tests



(a) *(b)*

Figure 2. Experimental set-up for the corroded RC beam tests [1]. (a) Quasi-static test, (b) Dynamic test

As regards the quasi-static tests, the beams are subjected to a classical four-point alternate bending. The loading is applied by means of a long-stroke actuator. The applied loading is composed of blocks of 3 identical cycles during which displacement is prescribed, with an increasing amplitude between two consecutive blocks. The amplitude range varies from 0.4 mm up to 200 mm, with a constant loading velocity.

The dynamic tests are performed on the AZALEE shaking table. The dynamic loading consists in a synthetic signal able to excite only the first natural mode of the beam. It is a bandlimited signal between 2 and 13Hz. Five acceleration levels are used: 0.125 g, 0.5 g, 0.8 g, 1.25 g and 2 g.

The results of the experimental tests will be used as a validation reference for the numerical model developed to address the behavior of corroded RC beams.

INGREDIENTS OF THE NUMERICAL MODEL ACCOUNTING FOR CORROSION

The fiber beam element method

The fiber beam element method has been chosen as a basis to model the behavior of corroded RC structural elements. Indeed, the method has proven to be efficient to model the seismic response of reinforce concrete structures [3, 4]. The principle of the fiber beam element model is described in Figure 3. Based on beam elements, the interesting feature of fiber beam elements is the addition of the cross-section scale instead of using a generalized constitutive law. It is thus possible to describe concrete and steel separately. Concrete and corroded steel constitutive laws used here are described in the following. The prescribed kinematics of the cross-section and the multi-scale formulation make the fiber-beam elements still suitable for computations at the structural scale.

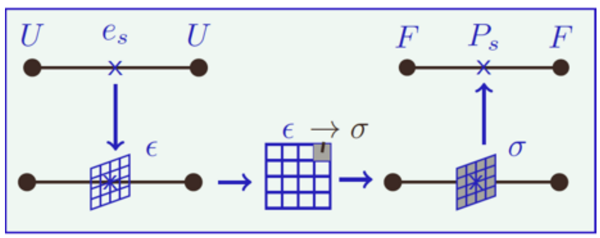


Figure 3. Principle of fiber-beam elements [5]

Material constitutive laws

For the computation of the behavior of the RC beams with corroded rebars, constitutive models for concrete and steel need to be chosen. The damage model of [6], able to account for the unilateral behavior, is chosen for concrete. The local response of concrete in compression and tension is displayed in Figure 4. An elasto-plastic model accounting for the effect of corrosion is chosen for steel [7]. The strain threshold of sound steel has been modified to 15% to correspond to the rebars used in the experimental campaign. Figure 5 displays the corroded steel behavior, for the corrosion rates obtained in the experimental campaign. The more fragile behavior of corroded steel is thus taken into account though the dedicated constitutive law.

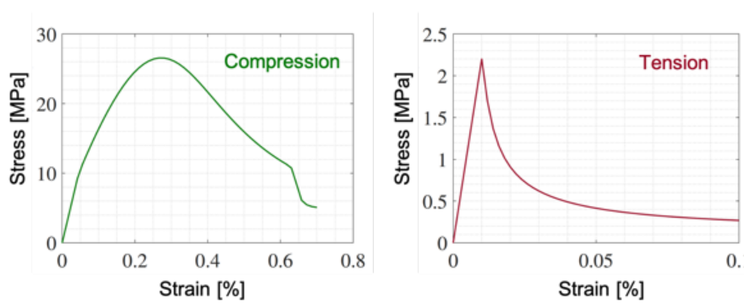


Figure 4. Concrete damage model used for the numerical model of the beams under quasi-static cyclic loading and dynamic loading [6]

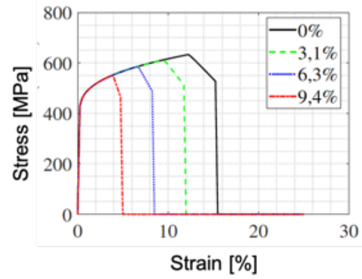


Figure 5. Corroded steel constitutive law, as a function of the corrosion rate

In addition to the reduction of the ultimate strain of steel, corrosion also has the direct effect of a reduction of the resisting cross-section. This is directly taken into account in the definition of the beam's cross-section mesh, either uniformly along the beam, or following a distribution of the corrosion pits.

Concrete damage due to the expansion of the corroded rebars

Steel corrosion of the rebars leads to a swelling, which induces cracking in the concrete. This is a long-term process, which develops before the occurrence of a seismic event. To investigate the influence of a pre-cracked concrete cross-section on the dynamic behavior of the beam, the effect of swelling of the corroded rebars on the concrete damage has been represented in the numerical model. A two-step modelling strategy has been developed.

First, the RC cross section is modeled in 2D using a fine mesh, comprised of 5232 linear triangles. The steel volume increase due to corrosion is represented by an expansion factor α, related to the mass loss, and depending on the corrosion products. This expansion factor is used to compute the increase in radius of the rebar [8]. The interface between steel and concrete is modelled using joint elements [9]. The expansion of the rebars is represented by a thermal expansion model. Concrete is modeled by the Mazars damage model [10]. The spatial variability of concrete is taken into account through a random distribution of the Young modulus around a mean of 22 GPa. The resulting damage field is displayed in Figure 6, for a corrosion rate corresponding to a 9.4% mass loss and to a mean expansion coefficient $\alpha = 3.6$.

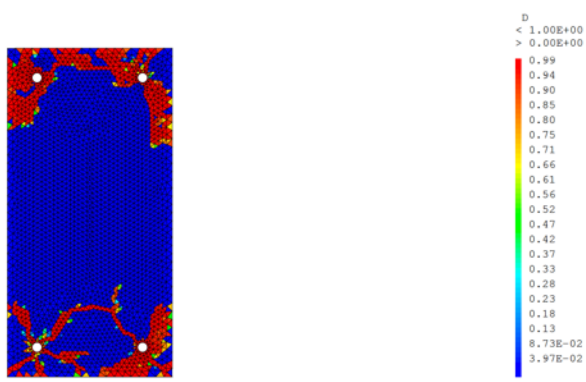


Figure 6. Damage field obtained in the RC cross-section by thermal-like expansion of the rebars, for a 9.4% corrosion rate.

For the purpose of a computation at the structural scale, it is not relevant to use such a fine mesh of the cross-section. Thus, a projection technique has been used to obtain an equivalent damage field on a coarser mesh, based on both mesh shape functions. The final mesh is comprised of 152 triangular elements (Figure 7). Since the damage field shown in Figure 6 is stored in the cross-section integration points, damage values can be computed at the nodes of the fine mesh, then at the nodes of the coarse mesh, using the fine mesh shape functions. Then, using the shape functions of the coarser mesh, the damage field can be computed at the integration points.

After this projection process, the cross-section stiffness matrix is modified. To address this issue, a correction based on an optimization procedure is proposed. It consists in multiplying the initially projected damage field $\boldsymbol{D_0}$ by a scalar a to obtain the new projected damage field $\boldsymbol{D} = a\,\boldsymbol{D_0}$, with the aim of minimizing $\| K_{s,ref} - K_{s,coarse}(\boldsymbol{D}) \|$. $K_{s,ref}$ is the stiffness matrix of the reference cross-section, meshed by the very fine mesh. $K_{s,coarse}(\boldsymbol{D})$ is the stiffness matrix of the coarse cross-section, computed with the damage field $\boldsymbol{D}$ [8].

The damage field obtained after projection and optimization for the 9,4% corrosion rate is displayed in Figure 7. This damage field will be used as an initial state for the quasi-static and the dynamic computations of the beams, with the model accounting for corrosion.

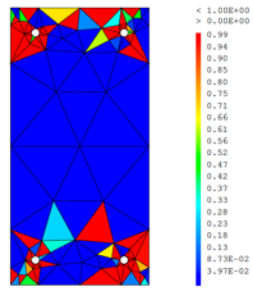


Figure 7. Damage field obtained for the 9,4% corrosion with the coarse cross-section mesh.

Non-uniform corrosion modelling: probabilistic analysis and distribution of the corrosion pits

As stated in the first section, the diameter of the corroded longitudinal rebar is not uniform along the beam. To study this effect on the failure of the corroded RC beams, non-uniform corrosion needed to be represented in the numerical model. From the measurements carried out during the experimental campaign, a statistical analysis of the corrosion pits has been performed. The empirical cumulative distribution function (cdf) has been derived for each corrosion rate, as displayed in Figure 8.

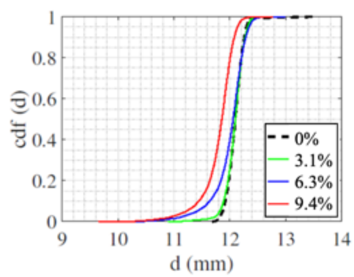


Figure 8. Empirical cumulative distribution function of the diameters along the corroded rebar, as a function of the experimental corrosion rate.

From the obtained distribution functions, it is possible to generate a sample composed of *m* diameter measurements characterized by the same distribution function. In this work, the Karhunen–Loeve method [11] has been used. The method is based on a modal decomposition approach, and significantly reduces the computation times. With this method, hundreds of distributions could be generated, which can be useful for probabilistic mechanical analyses. One realization for each corrosion rate is displayed in Figure 9.

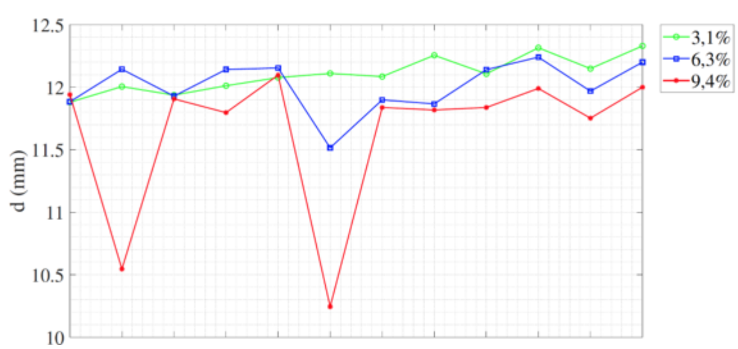


Figure 9. Computed diameter for the steel longitudinal rebar along the beam discretized in 12 elements.

To sum up, the numerical model accounting for corrosion consists in three enhancements of fiber beam elements:

- A constitutive model accounting for steel corrosion at the local level.
- An initially damaged state of concrete, due to the expansion of the corroded rebars.
- A random distribution of corrosion pits, to account for non-uniform corrosion.

In the next section, the numerical representation of the experimentally tested beams is described. The effect of each enhancement in the model is then analyzed.

CALIBRATION OF THE NUMERICAL MODEL

Mesh and boundary conditions

Figure 10 shows the beam longitudinal discretization. There is a beam element node for each experimental sensor or beam support. 12 fiber beam elements have been used to discretize the tested beams. The additional masses (360 kg) used for the experimental setting (see Figure 2) have been affected on nodes *P1L* et *P1R*. The gravity center of these masses being close to the beam neutral fiber, the rotational inertia has been neglected in the simulations. Boundary conditions are applied on the nodes *P2L* and *P2R* (see Figure 10). Only the rotation around the z-axis is allowed for node *P2L*. The rotation around the z-axis and the translation along the x-axis are allowed for node *P2R*. All the other degrees of freedom have been blocked for both nodes.

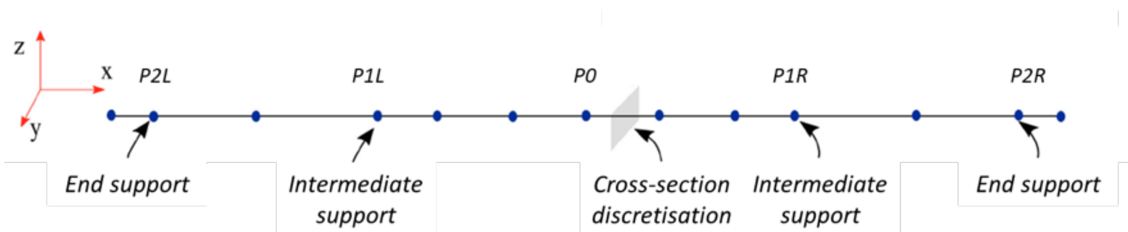


Figure 10. Longitudinal finite-element modelling for the tested beams

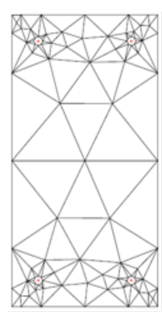


Figure 11. Mesh of the cross-section

A detailed view of the final cross-section (coarse) mesh used in the fiber beam elements is displayed in Figure 11. 152 linear triangular elements are used for concrete. Each steel rebar is modeled as a "point element" affected by its diameter and moment of inertia properties. The bond between steel and concrete is considered as perfect in the numerical simulations.

All the beam computations have been performed using the CAST3M Software [12].

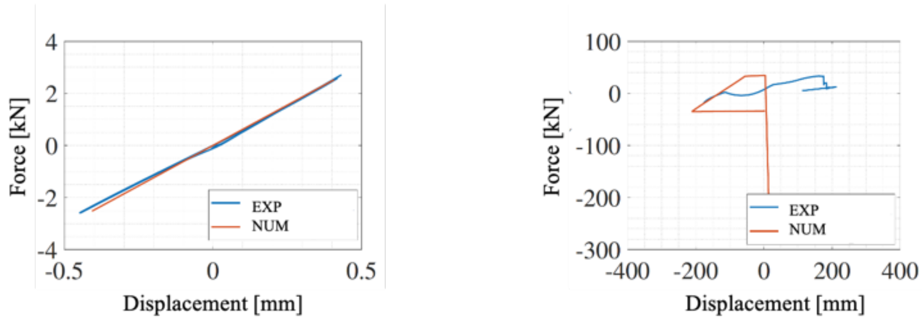
Loading

The loading sequences used during the experimental tests have been imposed to the numerically tested beams. Regarding the dynamic test simulations, an additional viscous damping (Rayleigh) of 2% has been considered. The two extreme levels of loading are presented here to analyze the numerical results:

- The **low loading level** corresponds to the 1st imposed displacement (±0.8 mm) in the QS case, and to the 1st applied seismic signal (0.125 g) in the DYN case.
- The **high loading level** corresponds to the last imposed displacement in the QS case (leading to failure), and to the last applied seismic signal in the DYN case.

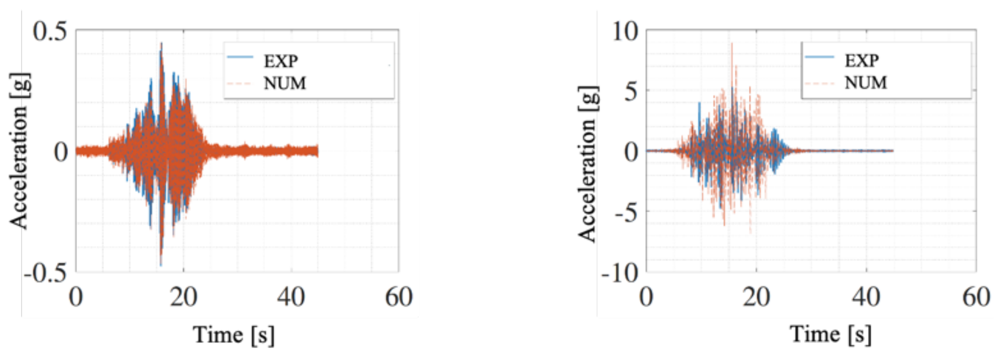
Calibration and test of the material constitutive laws on the non-corroded tested beams

The experimental tests on non-corroded beams have been used to calibrate some of the parameters of the constitutive laws. As previously described, the constitutive model of [6] has been used for concrete. The constitutive law of [7], modified by [13] has been used for steel (here with a 0% corrosion rate). The resulting behavior of point *P0* is displayed in Figure 12 (QS behavior), Figure 13, and Figure 14 (DYN behavior), for the two levels of loading considered.



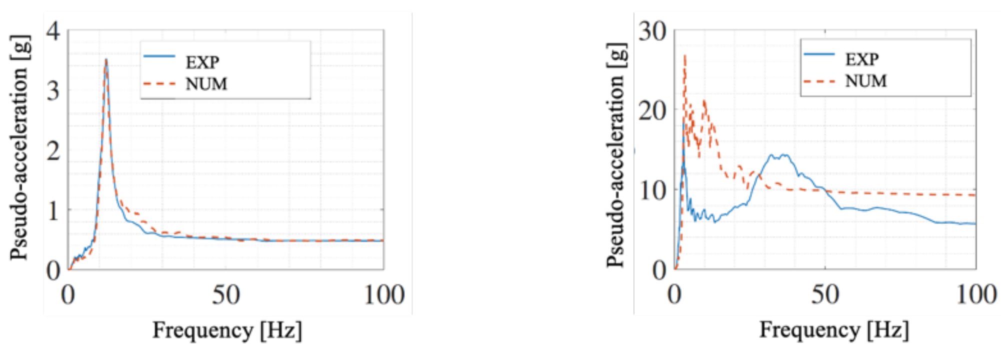
(a) Low loading level *(b) High loading level*

Figure 12. Comparison of the numerical and the experimental results for the quasi-static behavior of the non-corroded beam



(a) Low loading level *(b) High loading level*

Figure 13. Time history analyses of the numerical and experimental behavior of the non-corroded beam under dynamic loading



(a) Low loading level *(b) High loading level*

Figure 14. Comparison of the numerical and experimental spectral accelerations for the non-corroded beam under dynamic loading

Figure 12(a), Figure 13(a), and Figure 14(a) show that there is a relatively good matching between the experimental and numerical QS and DYN behaviors for a low level of loading. With the fiber beam element formulation, the failure (QS case) occurs when at least one steel fiber in at least one cross-section (fiber element) has exceeded the ultimate strain, which introduces a divergence at the end of the iterations within the concerned time step. This is why the force in Figure 12(b) seems to diverge. Figure 12(b) indicates however that the model correctly predicts the level of displacement leading to the ruin of the beam in the QS case. Some difference between numerical and experimental results can be observed for the dynamic behavior at the highest level of loading (Figure 14(b)). This can be explained by the longitudinal localization of crack growth that can not be predicted by the numerical model.

PREDICTION OF THE BEHAVIOR OF BEAMS WITH CORRODED REBARS

Modelling strategy

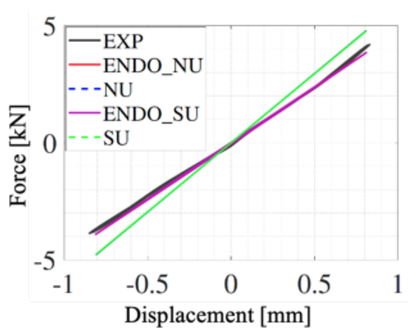
The overall modelling strategy to account for the corrosion effects in the seismic response of corroded reinforced-concrete beams has been detailed. It is applied to the experimental beams with corroded longitudinal rebars, for the three measured corrosion rates: 3.1%, 6.3%, and 9.4%.

A comparison between the experimental and numerical results for the different simulated beams has been performed, in terms of the QS cyclic (force-displacement curve) and dynamic (acceleration and response spectra) behaviors. The sensitivity of the structural responses to the ingredients of the numerical model has been evaluated. Therefore, four numerical models are tested for each corrosion rate of the longitudinal rebars:

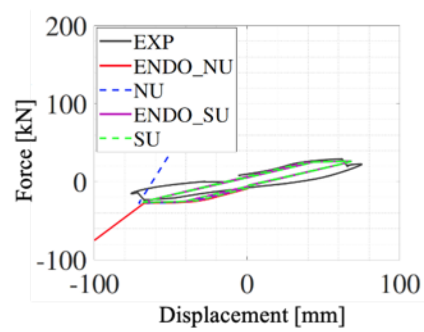
- **SU**: A model with a uniform steel cross-section (although reduced by corrosion). In this model, neither the initial damage of concrete nor the distribution of the corrosion pits is taken into account.
- **ENDO_SU**: A model accounting for the initial state of concrete damage. This state is computed according to section *"Concrete damage due to the expansion of the corroded rebars"*.
- **NU**: A model with a non-uniform cross section. For this numerical model, a random distribution of the corrosion pits is taken into account, computed according to section *"Non-uniform corrosion modelling: probabilistic analysis and distribution of the corrosion pits"*0. A single diameter distribution has been assumed for the four longitudinal rebars of a given corrosion rate.
- **ENDO_NU**: The complete model, accounting for the initial damage state of concrete in the cross-section, and for non-uniform corrosion of the rebars.

The results for the highest tested corrosion rate (9.4%) are displayed in the following.

Quasi-static cyclic behavior



(a) Low loading level

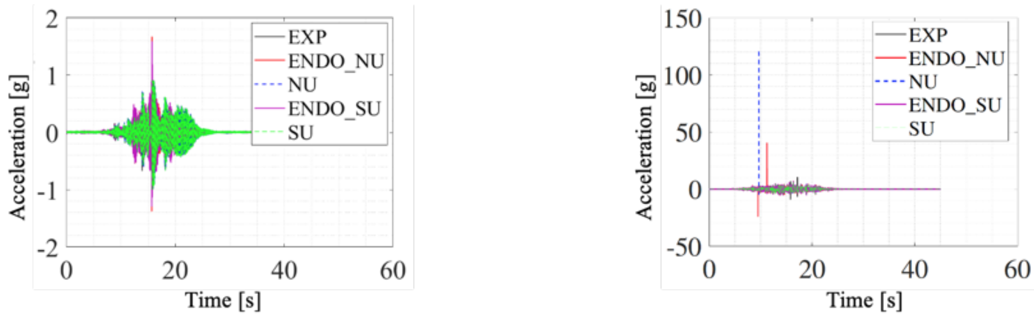


(b) High loading level

Figure 15. Quasi-static behavior of the corroded beam (corrosion rate 9.4%)

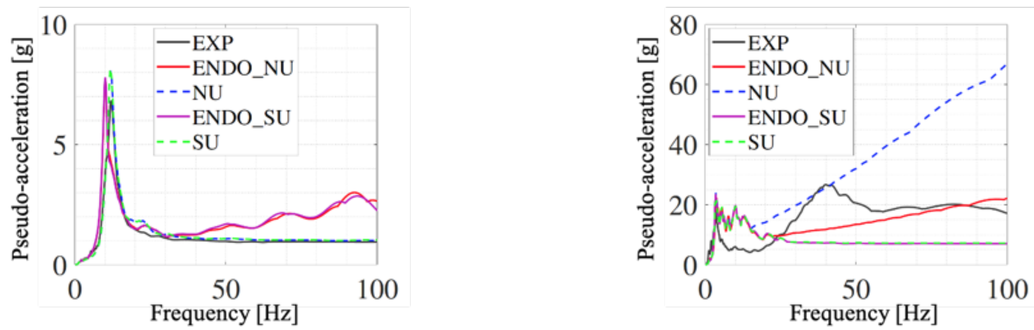
From the results of the quasi-static tests, it can be concluded that without initial damage, the behavior of the beam with corroded rebars at low loading level is not well-predicted (Figure 15(a)). At the failure stage, only the models with non-uniformly reduced rebars are able to predict the rupture.

Dynamic behavior



(a) Low loading level *(b) High loading level*

Figure 16. Time histories of the corroded beam (corrosion rate 9.4%) under dynamic loading



(a) Low loading level *(b) High loading level*

Figure 17. Response spectra of the corroded beam (corrosion rate 9.4%) under dynamic loading

Similarly to the QS case, it can be seen in Figure 16(b) and Figure 17(b) that only the models with a non-uniform cross-section seem to be able to predict the failure. However, it seems in Figure 17(a) that the model with an initially damaged cross-section overestimates the level of damage compared to the other models for which the initial frequency is closer to the experimental one.

It must be noted that the initial damage state of the cross-section depends on the expansion coefficient of the corrosion products, which is difficult to evaluate. For this study, a mean coefficient has been considered for each level of corrosion, and has not been fitted on the structural behavior.

CONCLUSIONS

A numerical model to account for corrosion of the longitudinal rebars on the quasi-static and dynamic response of reinforced concrete beams has been presented. In addition to a specific steel constitutive law and the loss of the resisting cross-section of the corroded rebars, two effects of corrosion have been represented in the model.

First, the concrete cracks due to the expansion of the corroded steel rebars has been represented by an initial damage state of the concrete part of the beam cross-section. This initial damage state enables to get the right initial stiffness of the beam at low levels of loading. But a good knowledge of the expansion coefficient, depending on the corrosion products is needed to accurately compute this state of damage.

Secondly, non-uniform corrosion has been represented through a statistical analysis of corrosion pits of a longitudinal rebar. The non-uniform corroded cross-section is necessary to predict failure of the tested beams, under quasi-static or dynamic loading. All the numerical simulations presented here have been carried out by considering a single diameter distribution per corrosion rate. However, the random nature of the corrosion pathology would require to perform several simulations with several diameter distributions. A different distribution for each rebar in the cross-section could also be tested.

It is important to note that the numerical model has been developed for the relatively low corrosion rates (3% to 10%) targeted by the DYSBAC experimental campaign. Therefore, a perfect adhesion has been considered between steel and concrete in the cross-section. For higher corrosion rates, a loss in steel-concrete bond should be added in the model.

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