



Mitigating P-delta Effects on the Seismic Response of Mid- and High-rise Eccentrically and Buckling-restrained Steel Braced Frames in the Vancouver Region

Bashar Hariri^{1*} and Robert Tremblay²

¹PhD Student, Department of Civil, Geological and Mining Engineering, Polytechnique Montreal, QC, Canada

²Professor, Department of Civil, Geological and Mining Engineering, Polytechnique Montreal, QC, Canada

bashar.hariri@polymtl.ca (Corresponding Author)

ABSTRACT

This article numerically examines the seismic stability of mid- and high-rise steel eccentrically braced frames (EBFs) and buckling-restrained braced frames (BRBFs) when subjected to ground motions from earthquake events contributing to the seismic hazard in Vancouver, BC. The study is conducted for 10-, 20-, 30-, and 40-storey prototype office buildings designed with two distinguished approaches to mitigate P-delta effects: 1) Lateral strength amplification as currently specified in the CSA-S16 standard 2) Braced frames coupled with a secondary single eccentric brace-beam assembly designed to remain elastic up to the design storey drift and provide a positive storey shear stiffness that overcome the negative storey shear stiffness due to P-Delta effects and ensure stable seismic response upon yielding of the main braced frame system. Nonlinear response history analyses (NLRHA) are conducted on the prototype buildings to obtain peak and residual and inter-storey drifts resulting from both design approaches. For reference, NLRHA is also performed on the same frame without considering P-Delta effects in the analyses. The study shows that the current strength amplification approach of CSA S16 is not sufficient to mitigate global instability, as it leads to the several occurrences of structural collapse or excessive drift, especially under the long-duration ground motions from interface subduction earthquakes. This behaviour was observed for both the EBF and BRBF buckling systems. On the contrary, all EBFs and BRBFs designed with the single eccentric-beam brace secondary system sustained peak inter-storey drifts identical to those obtained for the reference frames for which NLRHA without P-delta effects was performed. Furthermore, the residual storey drifts obtained for all frames were within repairability limits.

Keywords: Seismic stability, P-delta effects, EBF, BRBF, Secondary storey shear stiffness.

INTRODUCTION

Eccentric steel braced frames (EBFs) and Buckling-restrained steel braced frames (BRBFs) rely on ductile inelastic storey shear deformation to dissipate seismic energy. Considering the seismic component only, low-rise EBFs and BRBFs designed respecting the elastic modal properties demonstrate a favourable close-to-uniform inelastic storey drift distribution along the building heights, given the stable and close-to-symmetric response of their ductile elements under cyclic loading that involves no brace buckling [1-6]. However, incorporating gravity loads introduces additional second-order storey shear force demand (i.e., P-delta) that reduces the energy dissipation capacity and disturbs the uniformity of drift distribution, leading to excessive drifting or global instability [7-11]. Taller buildings are more vulnerable to P-delta effects as they carry more substantial gravity loads and undergo more complex lateral deformations in the inelastic range that involves higher modes. The characteristic of the seismic excitation also affects the seismic stability of steel-braced frames. Recent studies [12-19] have shown a higher tendency of P-delta caused drift concentration under the long-duration Subduction Interface earthquakes, a dominating seismic source contributor in the long-period range to the Western Canada Uniform Hazard Spectrum [20].

Two distinguished approaches are proposed in the literature to mitigate P-delta effects: the Lateral storey shear-strength amplification approach and the Lateral storey shear-stiffness incorporation approach. In the first approach, the ductile elements of EBF and BRBF are sized to develop lateral storey shear beyond the Uniform Hazard Spectrum design requirements and yield at amplified strengths that account for the reduction in the lateral storey shear strength capacity imposed by the P-delta effects. Figure 1a illustrates the approach utilizing an elastic perfectly-plastic SDOF system. Researchers [21-27] developed methods to quantify the strength amplification factor based on SDOF oscillators; however, the challenge was estimating the

inelastic deformation. Although this estimation might be sufficiently accurate in single-mode SDOF systems, Adam, Ibarra and Krawinkler [28] reported that it is meaningless to estimate the storey drift of MDOF systems under seismic excitations, given the complexity of the inelastic multi-mode deformed configuration.

Contrarily, in the Lateral storey shear-stiffness incorporation approach, the ductile elements of the EBF and BRBF are designed respecting the Uniform Hazard Spectrum; however, incorporating a secondary lateral storey shear stiffness (k'_s) tuned at every storey to develop positive post-yielding storey stiffnesses to cancel the negative slope of P-delta (i.e., P/h_s), and thereby annihilating P-delta effects. Furthermore, providing secondary stiffness that exceeds the negative slope of the P-delta introduces self-centring capabilities, as illustrated in Figure 1b.

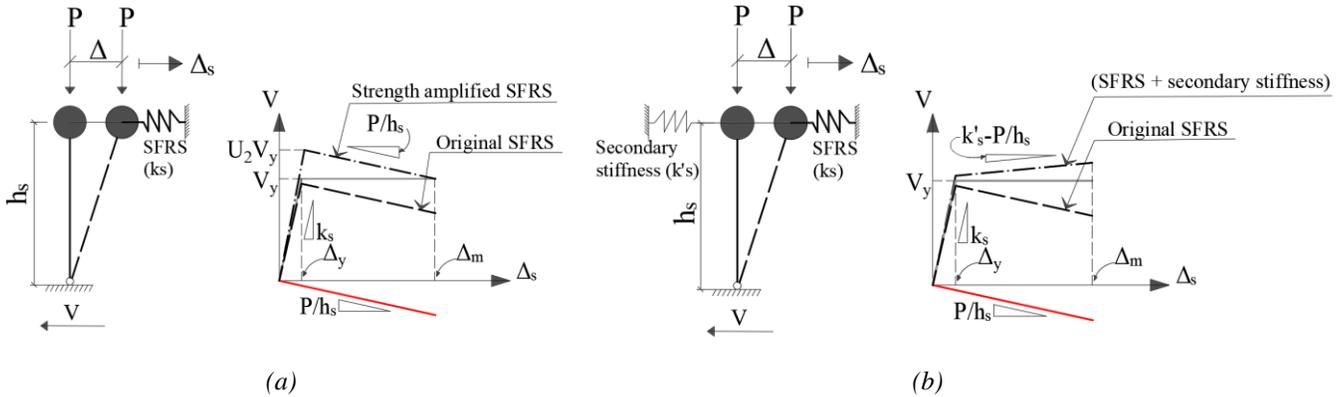


Figure 1. P-delta mitigation approaches: (a) strength amplification approach, (b) secondary stiffness approach.

Hariri and Tremblay [18] validated this approach utilizing a fictitious source of secondary storey shear stiffness using 12 prototype buildings. The validation involved three distinguished seismic force-resisting systems, EBFs, BRBFs, and bolted-end friction-braced systems (FBFs). The total height of the buildings ranged from 40 m to 160 m. The study concluded that incorporating a source of positive secondary storey shear stiffness equal to the negative stiffness of P-delta on a storey-by-storey bases annihilates P-delta effects, providing that the secondary stiffness source maintains the stiffness for storey drifts adequate for the energy dissipation process. The study outlined this drift by 2.5% of storey height for the 10- and 20-storey buildings and 2% for the 30- and 40-storey prototype buildings. It is critical to mention that incorporating secondary storey shear stiffness varies from the strongback systems. The SFRS in the latter system involves adding a substantially lateral-stiff mast to prevent storey drift concentration by distributing non-first modal drifts to adjacent storeys. However, this substantial lateral stiffness triggers unfavourable higher mode that limits the application of strongback systems in tall buildings [13]. Contrarily, the lateral secondary storey shear stiffness in the former systems is relatively small as it only intends to cancel the second-order P-delta storey shear forces. Moreover, Hariri and Tremblay [29] reported that the secondary lateral storey shear stiffness within the range of P-delta annihilation imposes a neglectable change in SFRS base shear.

The challenging task in this approach, however, is replacing the fictitious source of secondary storey shear stiffness with a genuine one applicable in steel-braced frames. Tremblay [17] proposed a system referred to herein as E-FBF that utilizes the flexural stiffness of the braced frames' beams as a source of post-yielding secondary storey shear stiffness. E-FBF uses the classical inverted-V bracing configuration after replacing one energy-dissipative brace with a conventional one designed to remain elastic. Upon energy dissipative brace yielding, the elastic brace applies an unbalanced load on the beam and creates the intended positive stiffness. Tremblay validated the proposed system using bolted-end friction energy dissipation braces, then extended its application to buckling restrained braced frames (i.e., E-BRBF) [14]. Figure 2 illustrates the secondary stiffness development mechanism in E-BRBFs by comparing its ideal monotonic pushover response with a conventional BRBF.

This article numerically assesses the adequacy of the two P-delta effects mitigation approaches utilizing eight prototype buildings. The study involves examining the seismic stability of 10-, 20-, 30-, and 40-storey steel eccentrically braced frames (EBF) and buckling-restrained braced frames (BRBF). The seismic force resisting systems (i.e., SFRSs) in this article are designed per the two distinguished P-delta effects mitigation approaches. The Lateral strength amplification per currently specified in the CSA-S16 standard (Design A) and the Lateral storey shear-stiffness incorporation approach (Design B). The secondary storey shear stiffness in the latter is applied using a single eccentric brace-beam assembly. The assembly's configuration is derived from the E-FBF and E-BRBF systems. The assessment involves monitoring the peak inter-storey drifts and residual storey drifts using nonlinear response history analyses (NLRHA) conducted under seismic excitations from earthquake events contributing to the seismic hazard in Vancouver, BC. (i.e., Crustal, In Slab, and Subduction Interface). For reference, NLRHA is also performed on the same SFRS-only frames without considering P-Delta effects in the analyses (Design C).

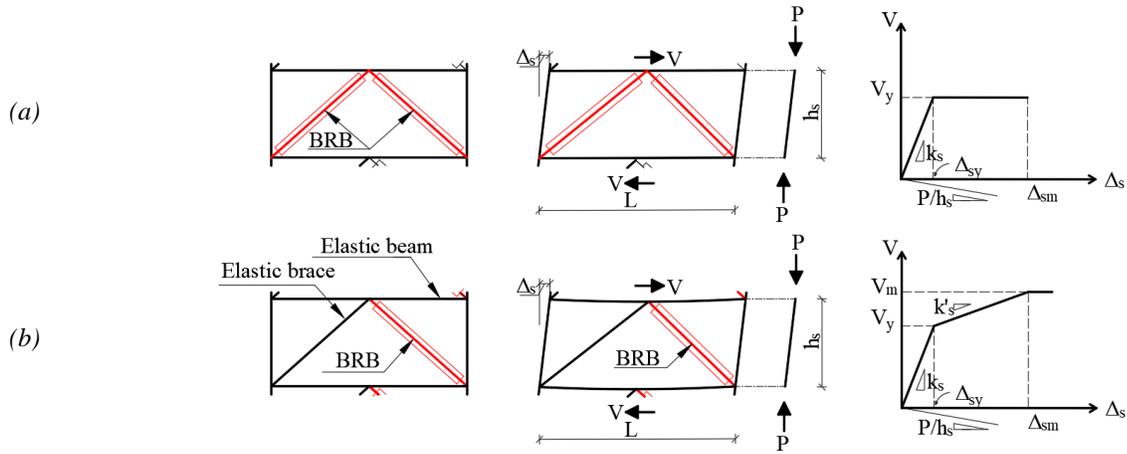


Figure 2. (a) Conventional BRBF, (b) E-BRBF system proposed by Tremblay [17].

P-DELTA EFFECTS MITIGATION APPROACHES

Lateral strength amplification approach

This article adopts the CSA-S16 [30] strength amplification approach, where lateral storey shear strengths are amplified using the U_2 factor. Eq. (1) illustrates the amplification factor, where V is the original (non-amplified) storey shear calculated per the design spectrum. Δ is the first-order elastic storey drift under storey shear V . R_d is the ductility adjustment factor, and h_s is storey height. It is important to mention that CSA-S16 requires the amplification to be applied using Eq. (1) if it exceeds 1.10, and requires modifying the design if the amplification factor exceeds 1.4.

$$U_2 = 1 + \frac{R_d \Delta}{V h_s} P \quad (1)$$

Lateral secondary storey shear stiffness approach

SFRSs (i.e., EBFs and BRBFs) in this article incorporate the secondary lateral storey shear stiffness using a single eccentric brace-beam assembly constructed in a bay separate from the SFRS. The system utilizes the beams' flexural stiffness as a secondary storey shear stiffness source. During storey drifting, the single bracing configuration imposes an unbalanced load on the beam creating the intended stiffness (Fig. 3). This mechanism is derived from the E-FBF and E-BRBF proposed by Tremblay [17]; however, the single eccentric brace-beam assembly has a broader application, where it is not limited to BRBFs or FBFs and can be implemented to mitigate P-delta effects in most steel seismic force resisting systems providing that a rigid diaphragm is designed to transfer the load between the main SFRS and the eccentric beam-beam assembly.

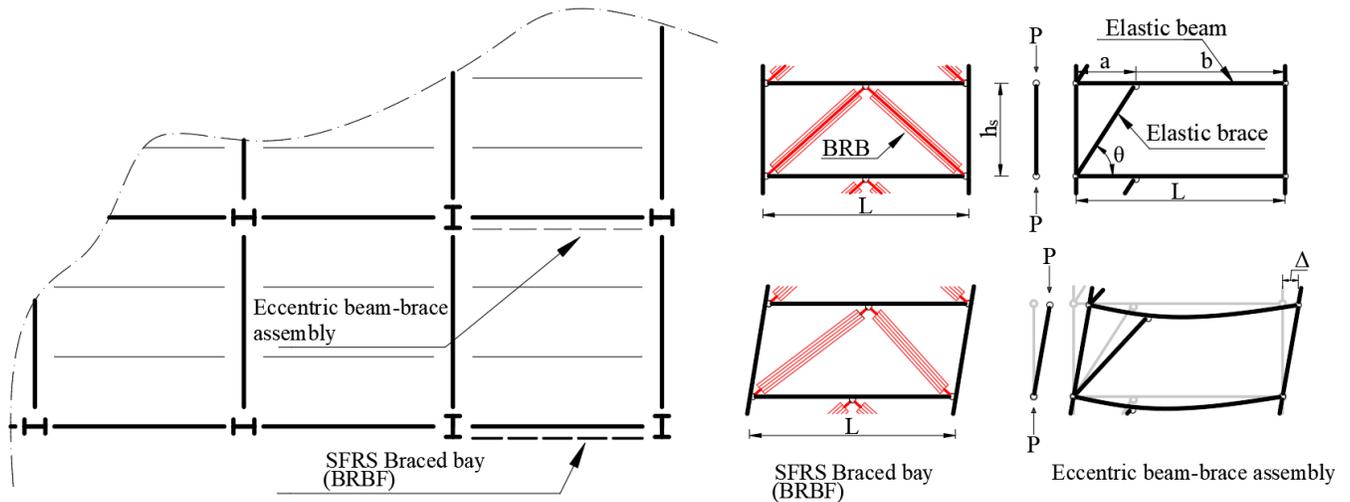


Figure 3. BRBF incorporating the single eccentric brace-beam assembly.

DESIGN OF PROTOTYPE BUILDINGS

Description of studies buildings

This article considers eight prototype office buildings. 10-, 20-, 30-, and 40-storey, located on soil Class C in Vancouver, BC., and designed using two distinguished SFRS, EBF and BRBFs. The buildings are five bays at 9 m with a symmetrical layout. The storey height is constant (4 m) except for the first storey (4.5 m). The SFRSs in the 10-storey and 20-storey EBF utilize frames with single-braced bays, while SFRS in the 30-storey and 20-storey BRBF utilize frames with double-braced bays. The 40-storey EBF and BRBF utilize triple- and four-braced bays, respectively. Eccentrically braced frames use 650 mm continuous beam link elements for the 10-, 20-, and 30-storey and bolted-ends modular link elements with the same length for the 40-storey. A classical inverted-V Chevronned bracing scheme is used for the BRB frames with low-strain hardening steel-restrained buckling-restrained braces (i.e., stability-critical braces [31]). Figure 4 illustrates the building layout, and SFRSs bracing configurations.

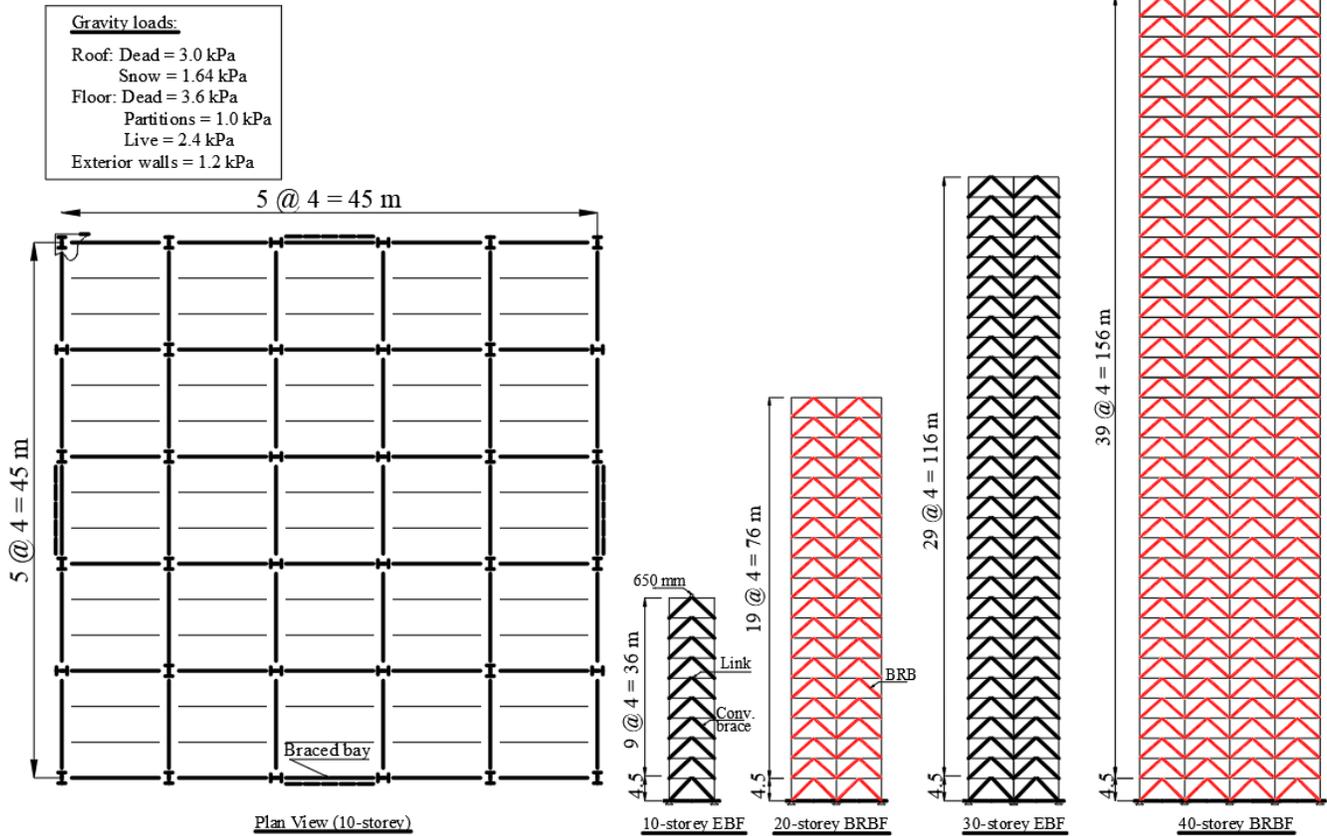


Figure 4. Samples of studied prototype buildings.

Seismic design requirements

The National Building Code of Canada (NBC) [32] stipulates calculating the base shear (V) as a fraction of the total seismic weight (W) as per Eq. (2). Where S represents the spectral acceleration calculated using the 2% in 50 years Uniform Hazard Spectrum at T_a period. T_a represents the minimum of the dynamic period and $0.05h_n$, where h_n is the total building height. R_o , R_d , I_E , and M_v are, respectively, the overstrength, ductility, importance, and higher-mode adjustment factors. A minimum base shear cut-off (V_{\min}) is stipulated by the NBC for frames with fundamental periods exceeding 2 sec. The base shear calculated per Eq. (2) can be reduced by 20% for regular structures when comparing with the base shear calculated using the Response Spectrum analysis to choose the greater as the design base shear.

$$V = \frac{S(T_a)I_E M_v W}{R_d R_o} \geq V_{\min} \quad (2)$$

SFRSs in this article are designed respecting the 2% in 50-year Uniform Hazard Spectrum of Vancouver, BC. using ductility, importance, and higher mode factors of 4, 1, and 1, respectively. The overstrength adjustment factors are 1.5 in EBFs and 1.2 in BRBFs. The design is carried out using Response Spectrum analysis. The structural non-ductile members of the braced

frames are designed respecting the capacity design principles stipulated by NBC with probable to nominal adjustment factors of 1.3 in EBFs and 1.4, 1.1 for tension and compression in BRBF, respectively. Figure 5 presents the design spectrum and plots the design accelerations of the studied buildings in both Designs (i.e., A and B).

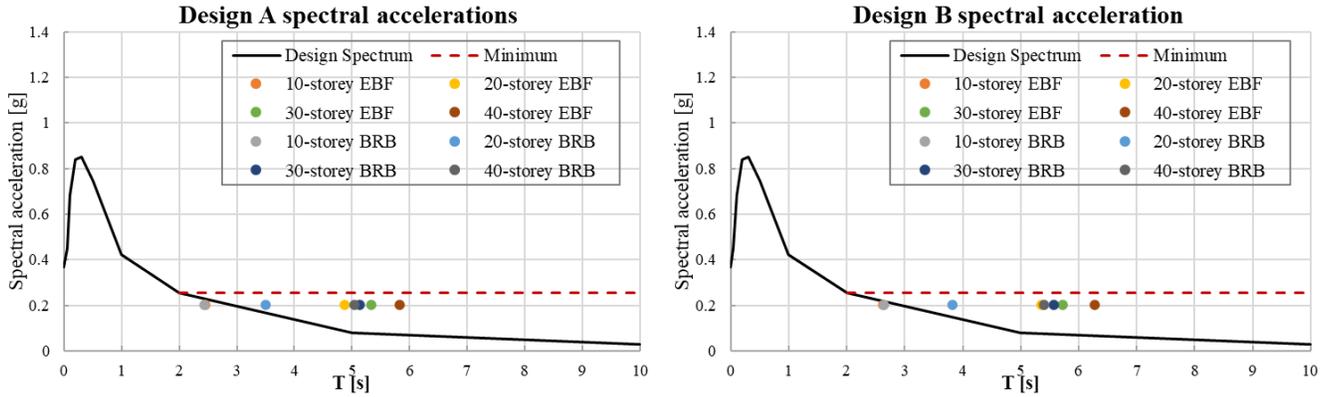


Figure 5. Design spectrum and buildings' modal periods and design accelerations.

P-delta effects mitigation

This article considers two designs for each set of eight prototype buildings to assess the two distinguished P-delta mitigation approaches. Design A employs the CSA-S16 [30] lateral storey shear strength amplification approach, where yielding strengths of the SFRS ductile elements are amplified using the U_2 factor described in Eq. (1). SFRS in Design B incorporates the secondary lateral storey shear stiffness utilizing the single eccentric brace-beam assembly while maintaining the original (un-amplified) storey yielding strengths. The beams in the secondary stiffness assembly of Design B are tuned along buildings' heights to develop positive storey shear stiffness equal to the storey negative stiffness of P-delta (i.e., $\Sigma P/h_s$), where ΣP is the total storey gravity load, and h_s is storey height. As recommended by Hariri and Tremblay [29], the source of secondary stiffness is designed to maintain the secondary storey shear stiffness for storey drift limits in the range of 2.5% h_s in the 10- and 20-storey buildings and in the range of 2.0% h_s in the 30- and 40-storey buildings. To fulfil these requirements, a double-bay with a single eccentric brace-beam assembly is used for the 10- and 20-storey buildings and a single concentric one is used for the 30- and 40-storey buildings. Equations that govern the design are illustrated in Hariri and Tremblay [29]. Figure 6 demonstrates the first storey of the secondary storey shear stiffness design configurations. It is critical to mention that this study ignores the CSA S16 stability-related BRB height limitation (i.e., 40 m).

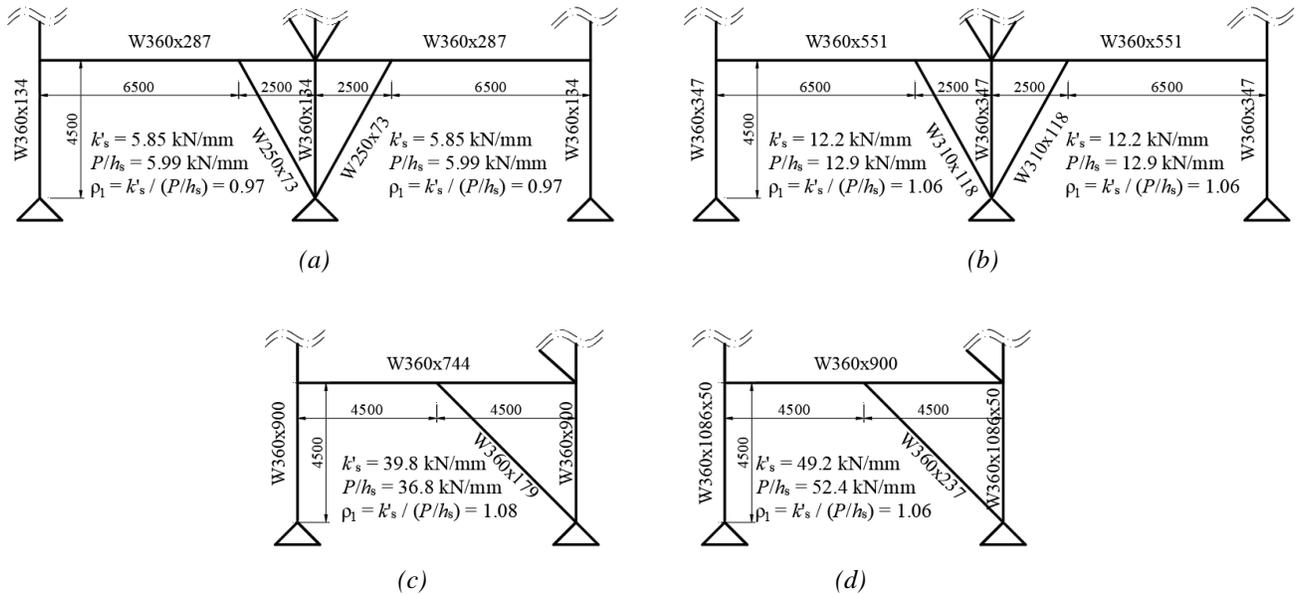


Figure 6. Eccentric beam-brace assembly: (a) 10-storey, (b) 20-storey, (c) 30-storey, (d) 40-storey.

MODELLING OF PROTOTYPE BUILDINGS

SFRSs in Designs A, B, and C are modelled in the OpenSees platform [33]. The developed models represent half portions of the prototype buildings given the symmetric layouts. The ductile elements of the EBF frames are modelled using link elements with axial, flexural, and shear stiffnesses, as illustrated by Hariri and Tremblay [18]. BRB elements in BRBFs are modelled using truss elements with a 1.5 amplified axial stiffness to represent the stiff end connections of the BRBs [34]. Calibrated Giuffr -Menegotto-Pinto material (i.e., steel02) is assigned to EBF link elements, and the calibrated Zsarn czay [35] (i.e., steel4) material is assigned to BRBs. SFRSs' beams are pin-ended, and columns are spliced continuously every two storeys. Beams, columns, and bracing members are modelled using multi-element nonlinear beam columns utilizing fibre sections with four integration points. Initial imperfection per CSA-S16 and the residual stresses per Galambos and Ketter [36] are modelled for all SFRS W-section members. Inelastic steel02 material with default parameters is assigned to the SFRSs' non-ductile structural members. Gravity loads in Designs A and B are simulated using an axially-stiff leaning column constrained with the SFRS braced bays at every storey using single-node pinned constraints. Masses are lumped at storey levels and assigned to leaning columns. Structural members of the secondary storey shear stiffness source in Design B (i.e., single-eccentric braced bays) are modelled using the same assumptions; however, columns are pinned-spliced every storey. Current-step stiffness- and mass-proportional damping is considered using a 3% damping coefficient. Figure 7 illustrates the 10-storey BRBF model.

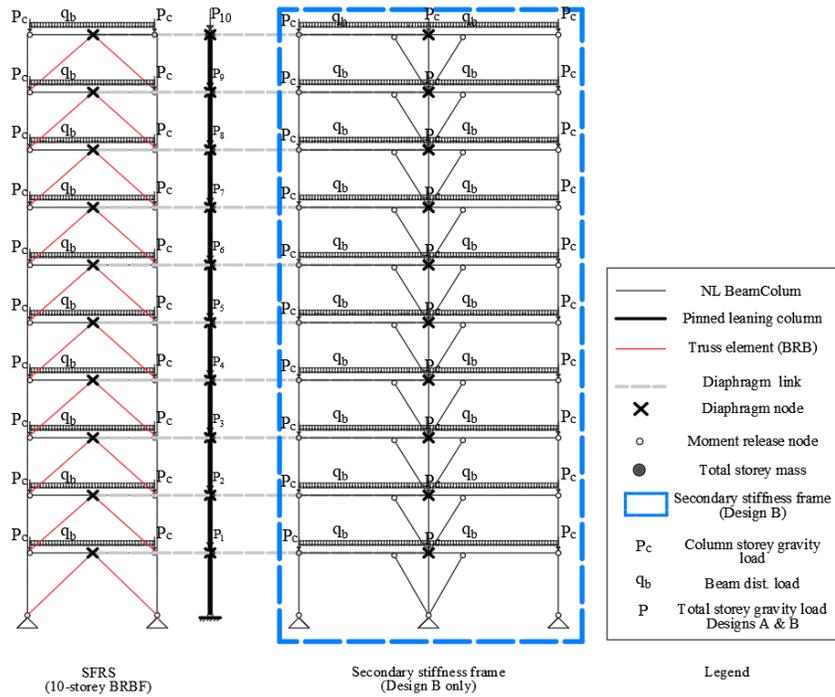


Figure 7. 10-storey BRBF modelling layout.

Ensembles of 33 historical ground motions are selected and scaled for each building utilizing an automated tool [37] to match the Uniform Hazard Spectrum of Vancouver, BC., as per NBC [38]. The scaling is carried out using scenario-specific period ranges utilizing three suites of 11 ground motions representative of the seismic sources of Vancouver (i.e., Crustal, In Slab, and Subduction Interface) as disaggregated [20]. Figure 8 illustrates a sample ensemble of scaled records. NBC defines the Seismic Demand curve (SD) as the largest mean response of the three suites' means, and the maximum acceptable peak inter-storey drift to be 2.5% of storey height. This study considers a termed repair-limit residual drift of 0.5% of storey height.

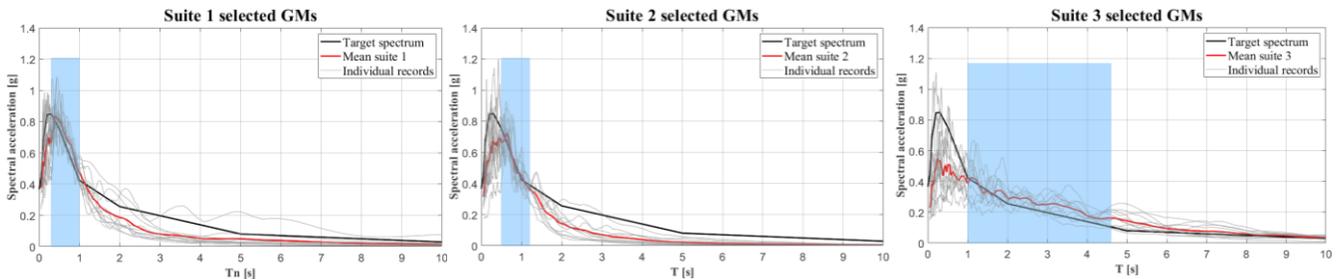


Figure 8. 10-storey EBF ensemble of scaled motions.

RESPONSE HISTORY ANALYSIS

Figures 9 and 10 present the nonlinear response history analysis peak inter-storey drift ratios as a percentage of storey height in Designs A, B, and the reference design (i.e., Design C) for EBF and BRBF prototype buildings, respectively. The figures reveals that the conventionally-designed SFRSs (i.e., Design A) were insufficient to mitigate P-delta effects where the SD drift curves (i.e., Seismic Demand curve) exceeded the maximum drift allowed by the design code (i.e., $2.5\%h_s$) in most of the studied buildings. More specifically, Peak inter-storey drift mean-curves under the Crustal (i.e., Suite 1) and the Inslab (i.e., Suite 2) motions were within the code-imposed acceptable criteria; however, collapse or excessive drifts occurred explicitly under the Subduction Interface suite of ground motions (i.e., Suite 3). It is noticeable that the SD curve in the 30- and 40-storey prototype buildings demonstrate improved performance, this however, is attributed to the 2 s base shear cut-off imposed by the design code. Contrarily, peak inter-storey drifts in Design B demonstrated more uniform drift distribution and respected the maximum drift limitation. Compared with the reference no P-delta analysis, peak inter-storey drifts in Design A diverged significantly while close-to-identical responses were obtained in Design B.

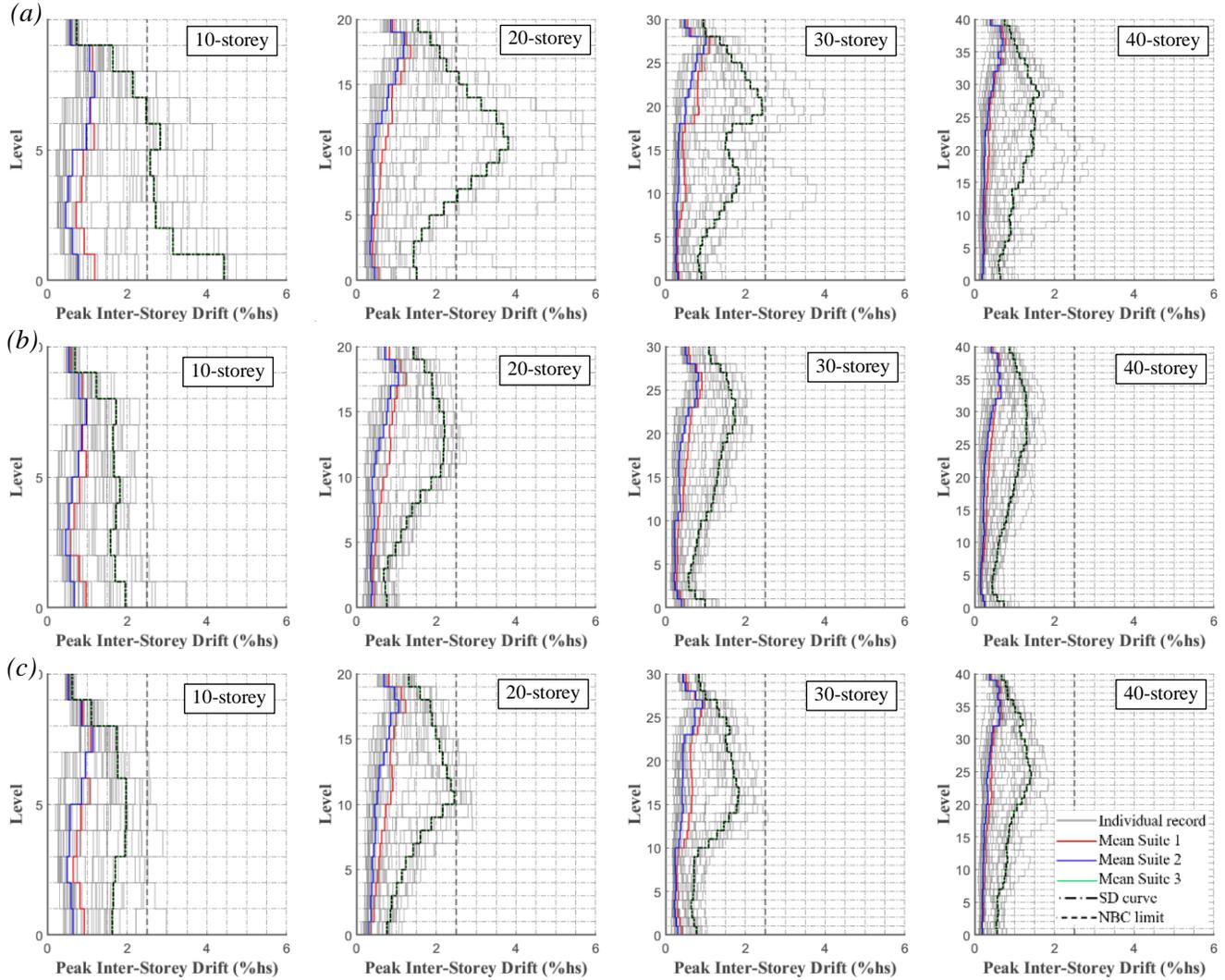


Figure 9. EBF peak inter-storey drift ratios: (a) Design A, (b) Design B, (c) Design C.

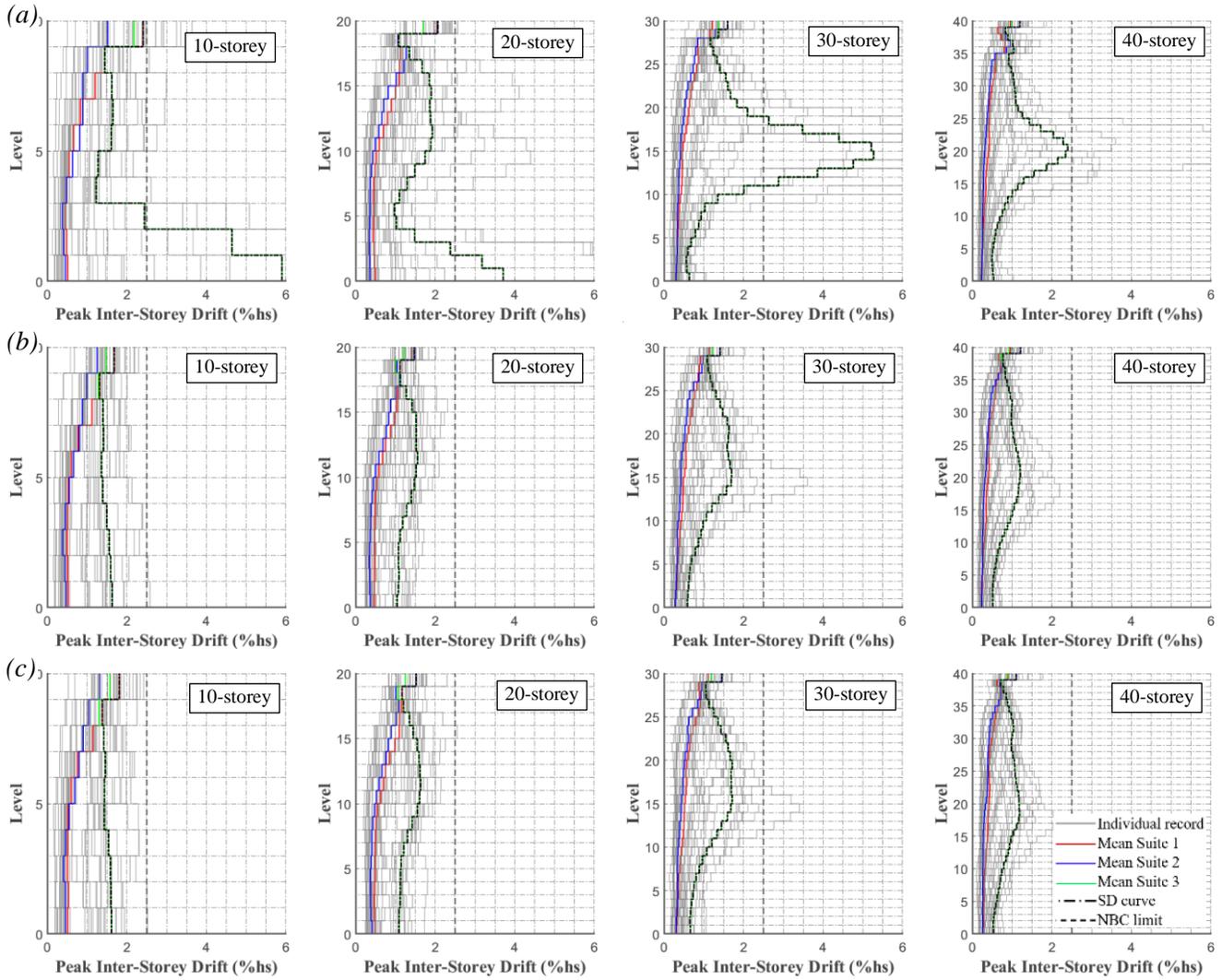


Figure 10. BRBF peak inter-storey drift ratios: (a) Design A, (b) Design B, (c) Design C.

Figures 11 and 12 present the residual inter-storey drifts ratios for EBF and BRBF prototype buildings, respectively. The drifts are measured at the end of 10 s free vibration response following each seismic excitation. The figures demonstrate that conventionally designed SFRS (i.e., Design A) developed unreparable residual mean drifts despite the type of seismic excitation (i.e., Crustal, In Slab, or Subduction Interface). Contrarily, prototype buildings that utilize the secondary storey shear stiffness (i.e., Design B) resulted in repair limit residual mean drifts (i.e., $<0.5\%h_s$). Similar to peak-inter storey drifts, maximum residual drifts in Design A significantly diverged from the reference no P-delta analysis case (i.e., Design C). Contrarily, incorporating secondary storey shear stiffness resulted in a close-to-identical maximum residual storey drifts to the case of no P-delta analysis (i.e., Design C).

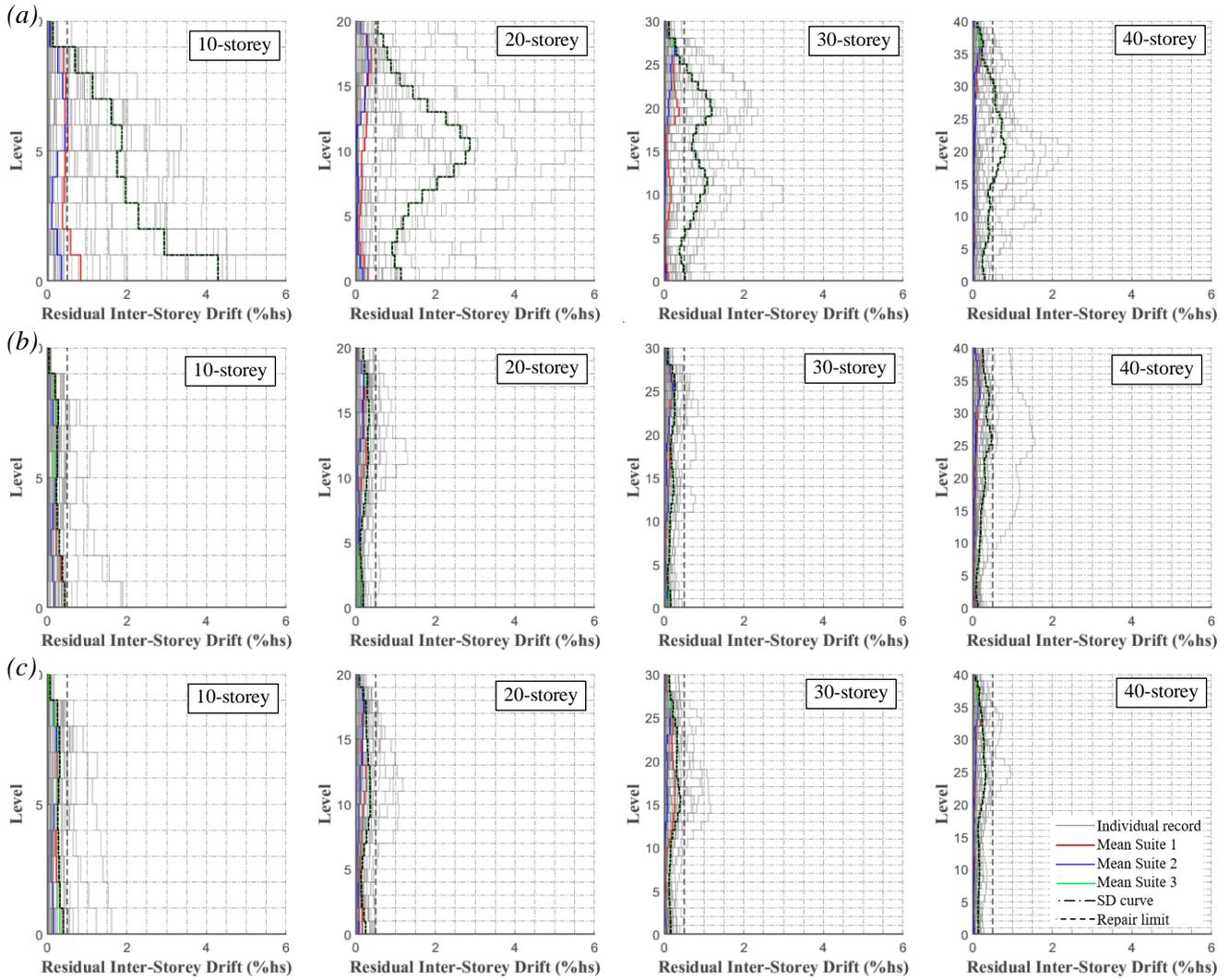


Figure 11. EBF residual drifts: (a) Design A, (b) Design B, (c) Design C.

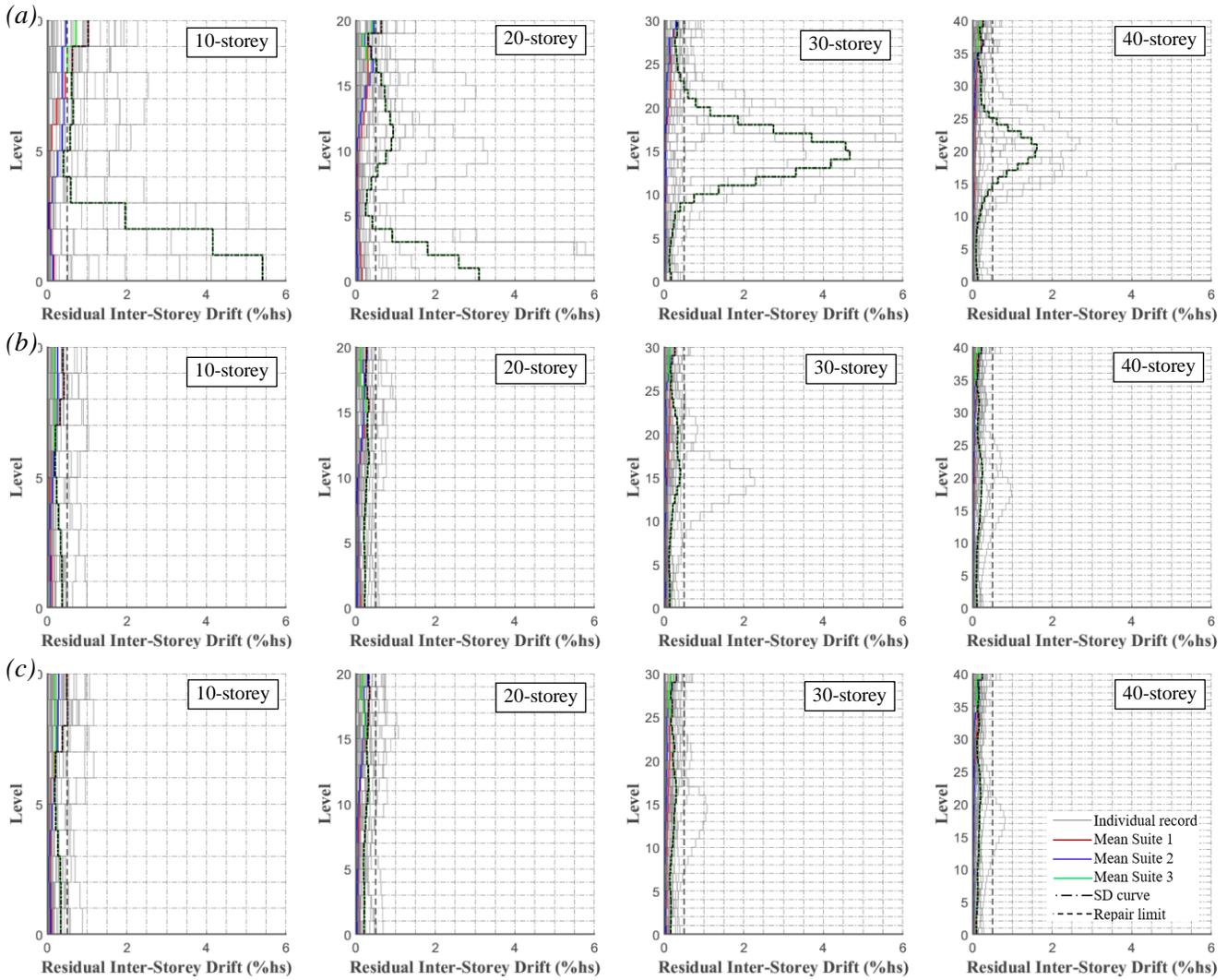


Figure 12. BRBF residual drifts: (a) Design A, (b) Design B, (c) Design C.

CONCLUSIONS

This article assessed the adequacy of the lateral strength amplification approach and incorporating the secondary shear stiffness approach in mitigating P-delta effects utilizing eight prototype buildings. The study examined the seismic stability of 10-, 20-, 30-, and 40-storey steel eccentrically braced frames (EBF) and buckling-restrained braced frames (BRBF) using nonlinear response history analysis under seismic excitations representative of the Uniform Hazard Spectrum of Vancouver, BC. (i.e., Crustal, In Slab, and Subduction Interface). To fulfill the requirements of the first approach, the yielding strengths of the EBFs and BRBFs ductile elements were amplified as currently specified by the CSA S16 standard. In the second approach, un-amplified yielding strengths were used; however, after incorporating lateral secondary storey shear stiffness utilizing single eccentric brace-beam assembly. The assembly is designed to develop positive lateral storey shear stiffness equal to the negative storey shear stiffness due to P-Delta effects and maintains this stiffness for drifts adequate to dissipate the seismic energy to ensure stable seismic response upon yielding of the SFRS system. The article found that code-imposed seismic design provisions were adequate to ensure stable performance and uniform drift distribution, as well as repair-limit residual drifts considering the seismic component only. However, introducing the second-order P-delta effects revealed that the lateral strength amplification approach adopted by the design code was insufficient to mitigate P-delta effects where excessive drifts and global instability occurred, especially under the Subduction Interface seismic excitations, in addition to resulting in significant exceedance of the repair limits residual drifts under the considered three seismic recourses. Contrarily, incorporating lateral secondary storey shear stiffness developed mitigated P-delta effects, where responses were comparable to those obtained using the reference no P-delta analysis.

ACKNOWLEDGMENTS

Financial support was provided by the Québec Aide financière aux études (AFE), the Natural Sciences and Engineering Research Council of Canada (NSERC), Fonds de recherche du Québec – Nature et technologies (FRQNT), and the Canadian Institute of Steel Construction (CISC).

REFERENCES

- [1] Black, C.J., Makris, N. and Aiken, I. D. (2004). "Component testing, seismic evaluation and characterization of buckling-restrained braces". *Journal of Structural Engineering*, 130(6), 880-94.
- [2] Uang, C.-M., Nakashima, M. and Tsai, K.-C. (2004). "Research and application of buckling-restrained braced frames". *Steel Structures*, 4, 301-13.
- [3] Kersting, R.A., Fahnestock, L.A. and López, W.A. (2015). "Seismic design of steel buckling-restrained braced frames". *NEHRP Seismic Design Technical Brief No. 11, NIST GCR*, 15-917-34.
- [4] Roeder, C.W., and Popov, E.P. (1978). "Eccentrically braced steel frames for earthquakes". *Journal of the Structural Division*, 104(3), 391-412.
- [5] Hjelmstad, K.D., and Popov, E.P. (1984). "Characteristics of eccentrically braced frames". *Journal of Structural Engineering*, 110(2), 340-53.
- [6] Kasai, K., and Popov, E.P. (1986). "General behavior of WF steel shear link beams". *Journal of Structural Engineering*, 112(2), 362-82.
- [7] Fahnestock, L.A., Sause, R. and Ricles, J.M. (2003). *Analytical and experimental studies on buckling restrained braced composite frames: Proceedings of the international workshop on steel and concrete composite construction* National Center for Research on Earthquake Engineering.
- [8] Kiggins, S., and Uang, C.-M. (2006). "Reducing residual drift of buckling-restrained braced frames as a dual system". *Engineering Structures*, 28(11), 1525-32.
- [9] Tremblay, R., and Poncet, L. (2007). "Improving the seismic stability of concentrically braced steel frames". *Engineering Journal-American Institute of Steel Construction*, 44(2), 103.
- [10] Foutch, D.A. (1989). "Seismic behavior of eccentrically braced steel building". *Journal of Structural Engineering*, 115(8), 1857-76.
- [11] Erochko, J., Christopoulos, C., Tremblay, R. and Choi, H. (2011). "Residual drift response of SMRFs and BRB frames in steel buildings designed according to asce 7-05". *Journal of Structural Engineering*, 137(5), 589-99.
- [12] Bosco, M., and Rossi, P. (2009). "Seismic performance of eccentrically steel braced frames". *Engineering Structures*, 31, 664-79.
- [13] Chen, L., Tremblay, R. and Tirca, L. (2019). "Practical seismic design procedure for steel braced frames with segmental elastic spines". *Journal of Constructional Steel Research*, 153, 395-415.
- [14] Hariri, B., and Tremblay, R. (2022). "Effective steel braced frames for tall building applications in high seismic regions". In *10th International Conference On The Behaviour Of Steel Structures In Seismic Areas*, Timisoara, Romania.
- [15] Li, S., Tian, J.-b., and Liu, Y.-h. (2017). "Performance-based seismic design of eccentrically braced steel frames using target drift and failure mode". *Earthquakes and Structures*, 13(5), 443-54.
- [16] Tapia-Hernández, E., and García-Carrera, S. (2019). "Inelastic response of ductile eccentrically braced frames". *Journal of Building Engineering*, 26, 100903.
- [17] Tremblay, R. (2018). "An inverted V-braced frame system exhibiting bilinear response for seismic stability under long duration subduction earthquakes". In *9th International Conference on the Behaviour of Steel Structures in Seismic Areas*, Christchurch, New Zealand.
- [18] Hariri, B., and Tremblay, R. (2023). "Adequacy of secondary storey shear stiffness in annihilating the inelastic seismic P-delta effects in mid- and tall-rise steel braced frames". Paper submitted to *Journal of Constructional Steel Research*.
- [19] Hariri, B., and Tremblay, R. (2023). "Optimum post-yielding stiffness design guidelines for comprehensive annihilation of seismic p-delta effects in single- and multi-storey steel buildings". Paper submitted to *Earthquake Engineering and Structural Dynamics*.

- [20] Halchuk, S., Adams, J., Kolaj, M. and Allen, T. (2019). "Deaggregation of NBCC 2015 seismic hazard for selected Canadian cities". In *12th Canadian Conference on Earthquake Engineering*, Quebec, QC.
- [21] Rosenblueth, E. (1965). "Slenderness effects in buildings". *Journal of the Structural Division*, 91(1), 229-52.
- [22] Bernal, D. (1987). "Amplification factors for inelastic dynamic P- δ effects in earthquake analysis". *Earthquake Engineering & Structural Dynamics*, 15(5), 635-51.
- [23] Paulay, T. (1978). "A consideration of p-delta effects in ductile reinforced concrete frames". *Bulletin of the New Zealand society for earthquake engineering*, 11(3), 151-60.
- [24] Calvi, G., Priestley, M. and Kowalsky, M. (2007). "Displacement-based seismic design of structures". In *New Zealand Conference On Earthquake Engineering*, New Zealand.
- [25] MacRae, G.A. (1994). "P- δ effects on single-degree-of-freedom structures in earthquakes". *Earthquake Spectra*, 10(3), 539-68.
- [26] Aschheim, M., and Montes, E.H. (2003). "The representation of P- δ effects using yield point spectra". *Engineering Structures*, 25(11), 1387-96.
- [27] Wei, B., Xu, Y. and Li, J. (2012). "Treatment of P- δ effects in displacement-based seismic design for SDOF systems". *Journal of Bridge Engineering*, 17(3), 509-18.
- [28] Adam, C., Ibarra, L.F. and Krawinkler, H. (2004). "Evaluation of P-delta effects in non-deteriorating MDOF structures from equivalent SDOF systems".
- [29] Hariri, B., and Tremblay, R. (2023). "Innovative secondary storey shear stiffness frame for seismic P-delta annihilation in tall steel buildings". Paper submitted to *Journal of Constructional Steel Research*.
- [30] Canadian Standard Association - CSA (2016). *CAN/CSA-S16 Design of steel structures*. Prepared by the CSA, Toronto, ON.
- [31] Hariri, B., and Tremblay, R. (2021). "Influence of brace modelling on the seismic stability response of tall buckling restrained braced frame building structures". In *17th World Conference On Earthquake Engineering*, Sendai, Japan.
- [32] National Research Council of Canada NRCC (2015). *NRCC/NBC National building code of Canada*. Prepared by NRCC, Ottawa, ON.
- [33] Mazzoni, S., McKenna, F., Scott, M.H. and Fenves, G.L. (2006). "Opensees command language manual". *Pacific Earthquake Engineering Research (PEER) Center*, 264.
- [34] CoreBrace LLC. (2021). Bolted brace design guide [Available from: <https://corebrace.com/wp-content/uploads/2021/04/Bolted-Connection-11-20.pdf>].
- [35] Zsarnóczay, Á. (2013). *Experimental and numerical investigation of buckling restrained braced frames for Eurocode conform design procedure development*. Ph.D. dissertation Budapest University of Technology and Economics. Budapest.
- [36] Galambos, T.V. and Ketter, R.L. (1959). "Columns under combined bending and thrust". *Journal of the Engineering Mechanics Division*, 85(2), 1-30.
- [37] Hariri, B. (2023). *Innovative steel bracing systems for tall building application in high seismic regions*. Ph.D. dissertation Polytechnique Montreal. Montreal, Canada.
- [38] National Research Council of Canada NRCC (2016). *Commentary J - design for seismic effects*. Prepared by the NRCC, Ottawa, ON.