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Site-City interactions modelling for earthquake scenario by hybrid BEM-FEM

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ABSTRACT

Earthquake-resistant buildings are vital in building earthquake-resilient cities in active seismic regions worldwide. The traditional practice of the seismic analysis of buildings considers an individual building as a separate entity. The numerical model of the buildings often consists of a fixed boundary at the foundation. In such a structural model, the effect of dynamic behaviour due to the Soil-Structural Interaction (SSI) and the interaction due to multiple buildings is often neglected. To account for such an effect, A hybrid Boundary/Finite Element Method (BEM-FEM) model is developed to simulate the effect of the Site-City Interaction (SCI) between multiple buildings and SSI during an earthquake. The 3D model considers the ground motion due to a distant seismic source, the effect of the medium through which it propagates and the amplification or the reduction in ground motion due to multiple building interactions and SSI. The seismic excitation consists of the analytical expression for the time-harmonic free-field motion. The wave propagation in the far field is modelled using BEM analytical expression for free-field motion. The near-field geological domain is modelled in FEM, where multi-story buildings are modelled as blocks. The ground and the top story displacement for each building are plotted, and the site-city interaction is shown with colour contours. The model is scalable and could easily incorporate additional soil and structural elements.

Keywords: Site-City Interaction, Soil-Structure Interaction, hybrid FEM-BEM, City-Scale seismic model, seismic wave propagation

INTRODUCTION

The damage to the buildings in city environments is one of the most important aspects of earthquake damage, as it is directly concerned with the safety of public life. Earthquakes have always become a considerable challenge for the development of disaster-resilient cities. Recent studies have shown that the amplitudes of the seismic waves during an earthquake get amplified by the variation in soil properties, surface and subsurface relief and the existence of multiple structures. The local amplification of the seismic waves in today's ever-expanding urban environment is much more complex. Thus, the risk of damage in a building is highly dependent not only on its structural components but also on the dynamic properties and extent of the neighbouring buildings.

In recent years, various research has been done to incorporate the site-city interaction phenomenon in the numerical model. Site-city interaction as a global phenomenon on the interaction between the buildings in the city and its subsoil and its plausibility in numerical models is discussed in [1], [2]. A brief review of the literature on structure-soil-structure interaction is presented in [3]. The structure-soil-structure effects on the dynamic response of piled structure are studied in [4]. Dynamic interaction between soil and a structural cluster and hence the displacement in the individual building is studied in [5]. [6] studies the spatial variability of the urban ground motion in highly heterogeneous site-city configurations. An analytical study on site-city seismic interactions in Mexico city-like environments is presented in [7]. A 3D numerical simulation of the site-city interaction during the 2011 Christchurch earthquake is done in [8]. Regional-scale analyses on seismic site-city interaction of high-rise building clusters and shallow basin effects with a case study of Hong Kong city are studied in [9]–[11]. Theoretical, numerical and experimental crossed-analysis on site-city interactions is studied in [12]. [13] present a review of the experimental evidence, the state-of-the-art of numerical models and the idea of non-linear site-city interaction simulation in seismic events. The main governing phenomena of seismic site-city interaction are discussed through simplified numerical models in [14]. Numerical coupling schemes for nonlinear time history analysis of buildings on a regional scale considering site-city interaction are studied in [15].

Over the years, the numerical modelling of the site-city effect has been dominated by simplified structural analytical models based on lumped mass and stiffness models or finite element models. These models are valuable and accurate for modelling the structural or soil-structure interaction behaviour around individual buildings on a small scale. However, wave propagation in the event of an earthquake occurs on a global scale of the infinite domain and is affected by the neighbouring behaviour. The boundary element method, being a boundary-based method, poses a clear advantage for dimension reduction with a sizeable volume-to-surface ratio. Additionally, BEM solutions are based on the solutions of boundary integral equations, considered highly accurate numerical solutions for seismic wave propagation problems. Over the past, pure BEM and hybrid BEM-FEM methods have been extensively used to solve wave propagation problems on a large scale, including the influence of site-city interaction. Seismic wave propagation in alluvial basins and the influence of site-city interactions are studied using BEM in [16], [17]. Time domain 2D site response analysis of non-homogeneous topographic structures by a hybrid BEM-FEM method is studied in [18]. The strategies for implementing 3D FEM-BEM for ground-structure interaction with pile-soil interaction are studied in [19]. The BEM-FEM coupling model for the analysis of soil-pile-structure interaction is studied in [20], [21].

From this brief state-of-the-art review, it can be concluded that the earthquake risk assessment in an urban environment is of utter importance. For an effective prediction of the dynamic response of an individual building, the influence of neighbouring buildings and the structure-soil-structure needs to be considered. An efficient city scale should be able to consider a full range of phenomena from the source of dynamic excitation, the effect of the medium through which it propagates and the local amplification of the waves due to structure-soil-structure interaction in urban environments. Numerous efforts have been made to consider the global city-scale model to simulate the site-city interaction phenomenon. BEM-FEM coupled models simulate the dynamic behaviour of soil-structure interaction behaviour. However, there is still a gap in modelling global-scale 3D site-city interaction using the hybrid BEM-FEM method. This chapter focuses on the implementation, validation and parametric study of a site-city interaction phenomenon by the hybrid BEM-FEM method. The following sections present a brief mathematical formulation of the hybrid BEM-FEM approach and numerical examples for method validation, and a sample parametric study using the method.

MATHEMATICAL FORMULATION

Problem formulation

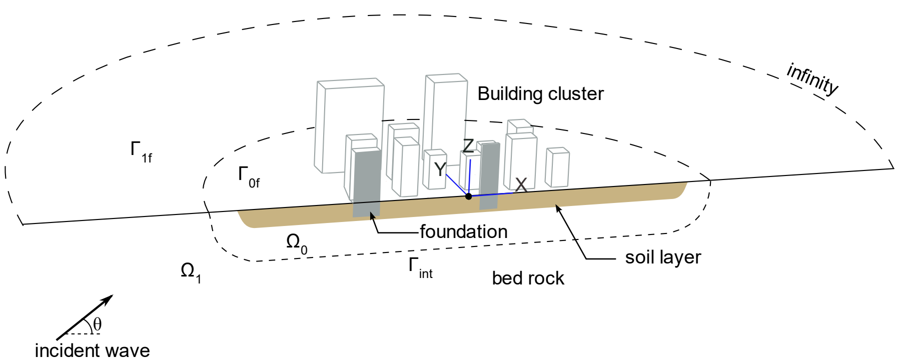


Figure 1. A representation of building stock in an urban environment with domain decomposition into near and far-field for seismic analysis using the FEM-BEM hybrid method.

Consider an urban environment with a group of buildings in a 3D half-plane with soil deposits on the bedrock, Figure 1. A representation of building clusters in an urban environment with domain decomposition into near and far-field for seismic analysis using the FEM-BEM hybrid method. The domain is broadly divided into near and far-field domains Ω0 and Ω1, respectively. The governing wave equation is derived from the constitutive law, kinematic relationship, and equation of motion. The elastodynamic wave equation in vector form is given by Eq. (1); see [22].

$$\mu \nabla^2 u + (\lambda + \mu) \nabla \nabla \cdot u + \rho b = \rho \ddot{u} \tag{1}$$

Where, u is the displacement field, (λ, μ) are Lame parameters of the medium, ρ is the mass density of the medium, b is the body forces due to the seismic source of the incident wave field, and $\ddot{u}$ is the acceleration field. In BEM-FEM coupled approach, the governing equation is formulated into the weak formulation of FEM and Boundary integral equations (BIEs) and for the domains Ω_0 and Ω_1, respectively.

BIEM formulation

Boundary integral equation describing the time-harmonic seismic field in the far-field semi-infinite region Ω_1 with boundary $\Gamma_1 \in \{\Gamma_{1f}, \Gamma_{int}\}$ is given by Eq. (2), see [23].

$$c_{lj}\left(u_l(x,\omega) - u_l^{ff}(x,\omega)\right) = \int_{\Gamma_1} U_{lj}^*(x,\xi,\omega)\ \left[\tau_j(x,\omega) - \tau_j^{ff}(x,\omega)\hat{f}(\omega)\right]d\Gamma$$
$$- \int_{\Gamma_1} \mathrm{T}_{lj}^*(x,\xi,\omega)\ \left[u_j(x,\omega) - u_j^{ff}(x,\omega)\,\hat{f}(\omega)\right]d\Gamma \ \text{ for } x \in \Gamma_1 \qquad (2)$$

Where, c_{lj} $(l = 1,2,3;\ j = 1,2,3)$ is the free-term depending on the geometry at the collocation nodes in BEM mesh; ω is the angular frequency; (x,ξ) are the positions of source-receiver couples; $U_{lj}^{*(1)}, \mathrm{T}_{lj}^{*(1)}$ are the 3D elastodynamic fundamental solution for displacement and traction respectively [22]; $u_{lj}^{ff(1)}, \mathrm{t}_{lj}^{ff(1)}$ are the free-field motion caused by the incident seismic wavefield from a distant source and $\hat{f}(\omega)$ $is\ the\ fourier\ amplitude\ of\ the\ source\ function.$ The formulation for the transient and the time-harmonic free-field motion in elastic isotropic continuum are given by analytical expressions, see [24]. By integration and transformation of Eq. (2) into a matrix form, the BEM formulation for the domain Ω_1 is represented by Eq. (3).

$$H(\omega)\,u(\omega) = G(\omega)\,\tau(\omega) + \zeta(\omega) \qquad (3)$$

Where, G and H are the BEM influence matrices; ζ is a load vector due to the incident seismic wave-field; u and τ are the displacement and traction vectors, respectively, and ω is the circular frequency.

FEM formulation

On the other hand, the FEM solution for domain Ω_0 is given by the weak formulation of the wave equation. The formulation after proper treatment transforms into the expression in Eq. (4), see [25].

$$M\ddot{u}(\omega) + C\dot{u}(\omega) + Ku(\omega) = F(\omega) \qquad (4)$$

Where, M, C, K are frequency dependent mass, damping and stiffness matrices, respectively; $\ddot{u}, \dot{u}, u$ are the acceleration, velocity and displacement vectors respectively; and F is the force vector. In frequency domain the matrix in governing equation reduces to the force and displacement expressions as given in Eq. (5).

$$(-\omega^2 M + K + i\omega C)\,u(\omega) = F(\omega) \ \text{ where } \ u(\omega) = u_0 e^{i\omega t} \qquad (5)$$

BEM-FEM coupling

The direct coupling is carried out between the system of algebraic equations of BEM and FEM from Eq. (3) and Eq. (5), respectively. The direct coupling is carried out based on the interface condition along the interface Γ_{int} i.e., the displacement and traction in both domains along the interface are given by $u^{\Omega_0} = u^{\Omega_1}$ and $\tau^{\Omega_0} = -\tau^{\Omega_1}$. The traction-based formulation in the system of BIEs in Eq. (3) are transformed to equivalent force terms to be compatible to the force displacement formulation in the FEM by integration along the element boundaries in the interface Γ_{int} as given in Eq. (6).

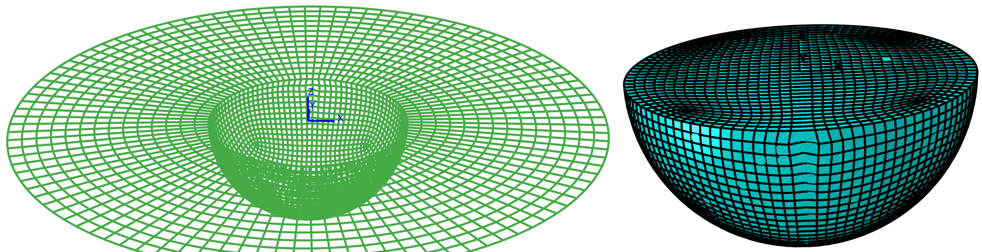
$$F_e = \int_{\Gamma_e} \tau_e\, d\,\Gamma \qquad (6)$$

In the FEM hosted coupling approach the system of equations in the BEM domain are inserted into the FEM equation as a Macro-Finite-Element, see [26]–[28].

NUMERICAL MODEL

The numerical simulations of seismic SCI model are performed by using the in-house 3D BEM codes developed by the first author and commercial finite element analysis software ABAQUS [29]. The system of BIEs is incorporated into ABAQUS as the macro-finite-element by substructure generation step and the coupled system is simulated in ABAQUS finite element environment. The process is first validated with simple models from literature, and parametric studies are conducted for an example of the SCI with a cluster of buildings.

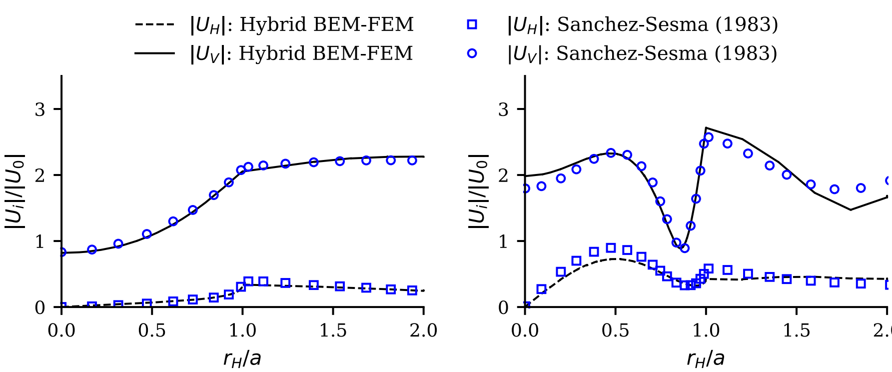
Validation example



(a) (b)

Figure 2. The numerical model of a semi-hemispherical basin in a half-space: (a) BEM model of semi-infinite region, (b) FEM model of alluvial basin.

Consider a homogeneous elastic isotropic semi-hemispherical alluvial basin with radius R in an elastic isotropic half-space. The alluvial basin is modelled using FEM, and the remaining semi-infinite portion of the half-space is modelled in BEM, see Figure 2. The medium parameters for the semi-infinite region are mass density $\rho^{(B)} = 1950 kg/m3$, shear wave velocity ${c_2}^{(B)} = 250\ m/s$, and Poisson's ratio $\nu^{(B)} = 0.25$. Three different cases are considered for the semi-hemispherical region: **(a)** no alluvial region and free-surface boundary condition along the hemi-spherical surface, i.e. the global model becomes a pure BEM model for semi-hemispherical cavity in half-space; **(b)** the alluvial region with same medium parameter as the semi-infinite half-space, i.e. the ground motion is free-field motion in a homogeneous elastic isotropic half-space; **(c)** alluvial regions with following medium parameters: mass density $\rho^{(F)} = \ 0.6\,\rho^{(B)}$shear wave velocity ${c_2}^{(F)} = \frac{1}{\sqrt{2}}{c_2}^{(B)}$, Poisson's ratio $\nu^{(F)} = 0.3$. The seismic source is considered as vertical incident time-harmonic P wave with normalized frequencies $\eta = 0.25,\ 0.75$. The normalized frequency is given by $\eta = \frac{qR}{\pi}$, where q is the wave number of the incident wavefield in half-space, and R is the radius of the hemispherical cavity/interface. The solutions for cases (a) and (c) are compared to the solutions in [30], where an approach based on multipolar wave expansion in spherical coordinates. The solutions for case (b) are compared to the analytical solutions of free-field motion. The hybrid numerical model is modelled with 3484 3D 4-node linear boundary elements and 16900 8-node 3D finite elements. The resulting displacement magnitudes normalized by the amplitude of incident wavefields show excellent validation with the existing numerical and analytical solutions in the literature, see Figure 3 and Figure 4.



(a) (b)

Figure 3. The normalized surface amplitudes of the horizontal and vertical displacements by semi-hemispherical cavity in half-space with vertical incident P wave: (a) normalized frequency $\eta = 0.25$; (b) normalized frequency $\eta = 0.75$.

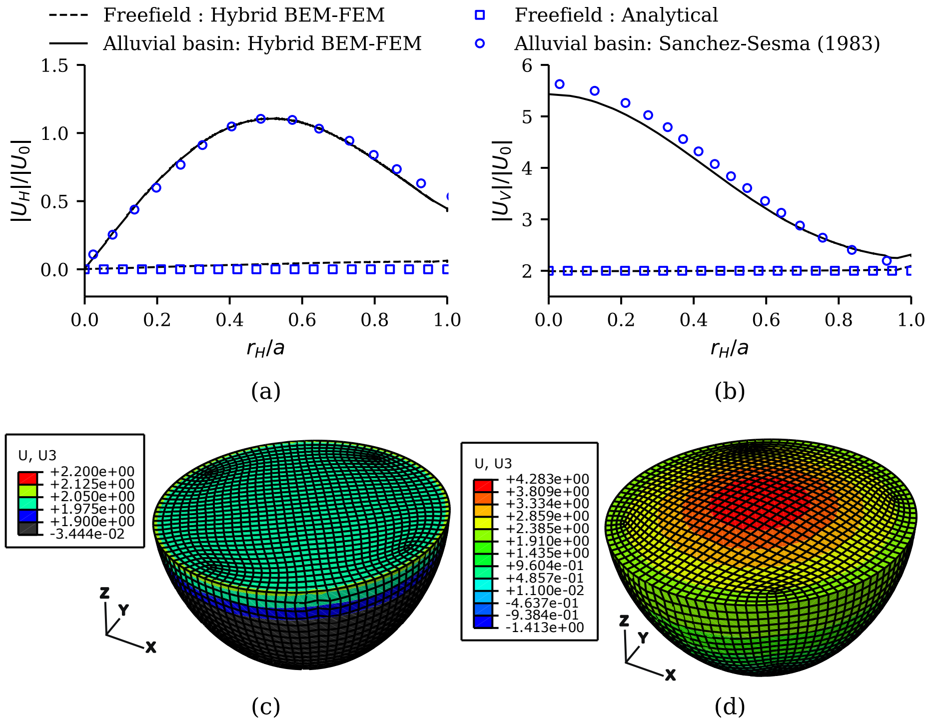


Figure 4. The normalized displacement amplitudes by a hemispherical near-field region in half-space with vertical incident P wave with normalized frequency $\eta = 0.5$: (a) horizontal displacements on the surface, (b) vertical displacements on the surface, (c) vertical displacements in hemispherical region in free-field, (d) vertical displacements in hemispherical alluvial basin.

Parametric study

A building cluster with a random arrangement in $0.5km$ diameter is considered for the supplication of the hybrid FEM-BEM model. The geological formation consists of a $20m$ deep soil layer extending to a diameter of $0.6km$ resting on bedrock in a homogeneous half-space, see Figure 5. The dynamic properties are shear wave velocity ($c_2 = 600m/s,\ 2600m/s$), mass density ($\rho = 1800kg/m^3,\ 2200kg/m^3$) , Poisson's ratio ($\nu = 0.3,\ 0.25$) for the soil layer and bedrock, respectively. The area is subjected to an incident S-wave at 45 degree angle to the xz plane. The properties of the buildings are shown in Table 1.

Table 1. Properties of the building considered for numerical simulation of site-city interaction.

SN	Building footprint	Building Height	Depth of foundation	First 5 fundamental periods	Number of buildings
1	20m × 20m	40m	10m	3.41s, 3.41s, 1.73s, 0.94s, 0.92s	6
2	20m × 30m	60m	15m	7.06s, 5.15s, 2.75s, 1.58s, 1.42s	11
3	30m × 30m	80m	15m	8.57s, 8.57s, 3.47s, 2.01s, 2.01s	4
4	30m × 40m	100m	20m	12.94s, 10.20s, 4.47s, 2.77s, 2.46s	1
5	30m × 70m	60m	20m	12.78s, 6.89s, 5.42s, 2.76s, 2.36s	1
6	40m × 20m	40m	20m	2.08s, 1.54s, 1.46s, 0.70s, 0.69s	1
7	50m × 30m	100m	20m	12.87s, 8.60s, 4.74s, 2.76s, 2.36s	1

The hybrid numerical model consists of 2541 3D 4-node linear boundary elements and 92637 8-node 3D finite elements. A Ricker wavelet of unit amplitude at central frequency $5Hz$ is considered, see Figure 7. The signal is bandpass filtered in the range of (0.05Hz, 10Hz) and a steady state dynamic analysis is performed at a discrete uniform interval of 0.05Hz. The representative displacements amplitudes normalized by the amplitudes of the incident wave at the respective frequencies are shown in Figure 6.

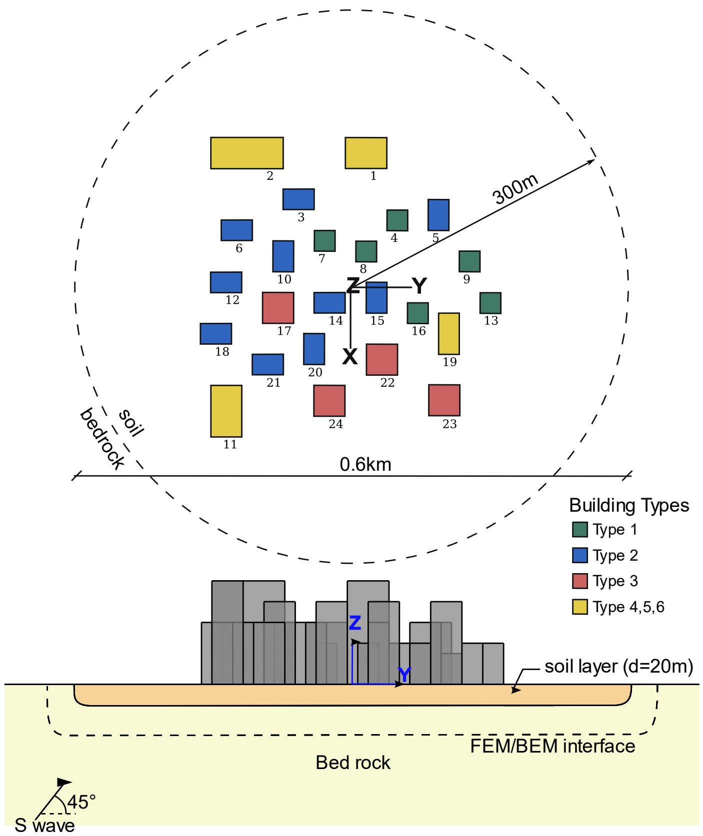


Figure 5. The plan and the section of a building stock on soil deposit.

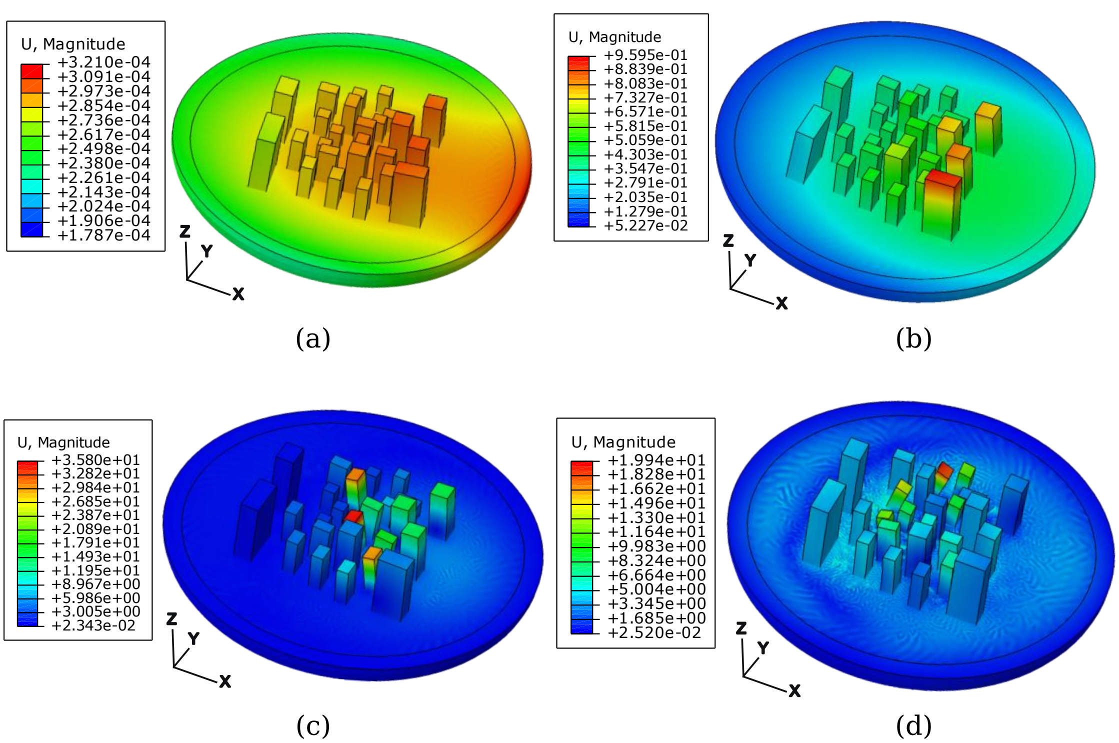


Figure 6. The normalized displacement magnitudes in the building cluster for incident S-wave at 45° angle to the vertical in xz-plane at different frequencies: (a) 0.05Hz, (b) 2.00Hz, (c) 4.00Hz, (d) 6.00Hz.

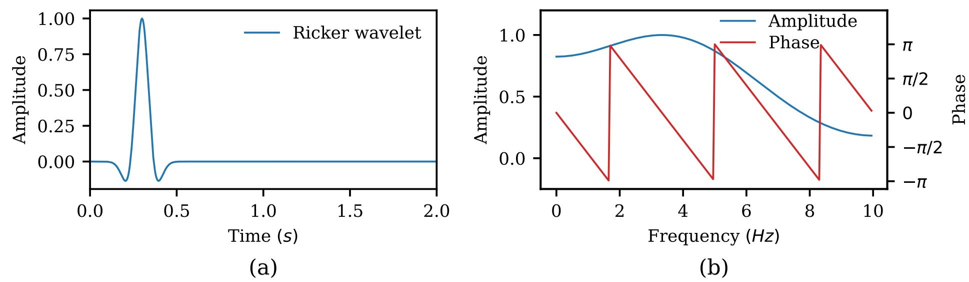


Figure 7. Amplitude of S-wave incident at 45° to the vertical axis in xz-plane: (a) Time domain, (b) Frequency domain.

RESULTS AND DISCUSSION

It can be seen from the outputs of the SCI model that with the wave incident at an angle from the negative x-axis in the xz-plane, the response of the buildings on the opposite side of incidence has a higher amplitude of displacements at the lower frequency. This phenomenon is observed in the ground shaking as well, that the building cluster alters the response on the ground. At higher frequencies, the peak amplitudes are higher at the central positions within the cluster. Similarly, secondary response peaks are observed in the time histories of some of the buildings. It is worth noting that this response is from the uniform excitation of the Ricker wavelet. From the observed phenomenon, it can be deduced that the complex phenomenon of site-city interaction occurring from the interaction of multiple buildings and ground layers can be simulated with the hybrid BEM-FEM models.

CONCLUSIONS

A hybrid BEM-FEM model for the simulation of seismic wave propagation in a Site-City Interaction (SCI) environment is developed and verified, and the simulation examples are presented. Simulations are carried out for the wave incident from a distant source, like an earthquake event. The simulation results show that the ground motion is highly modified in the city environment, altering the response of the individual buildings. In general, the existence of the neighbouring buildings affects the seismic response of the buildings in the urban environment. It is directly associated with the damage susceptibility and the resilience of the cities. With city-scale hybrid BEM-FEM models, prominent cities and critical infrastructure in risk zones can be modelled more precisely to develop highly accurate city-scale earthquake response models. The underground lifeline structures, such as tunnels, pipelines, valley effect and heterogenous laying, could be studied in detail. Such an approach provides a way forward for the risk analysis and development of earthquake-resilient cities. A detailed study considering above mentioned variables will be reported elsewhere.

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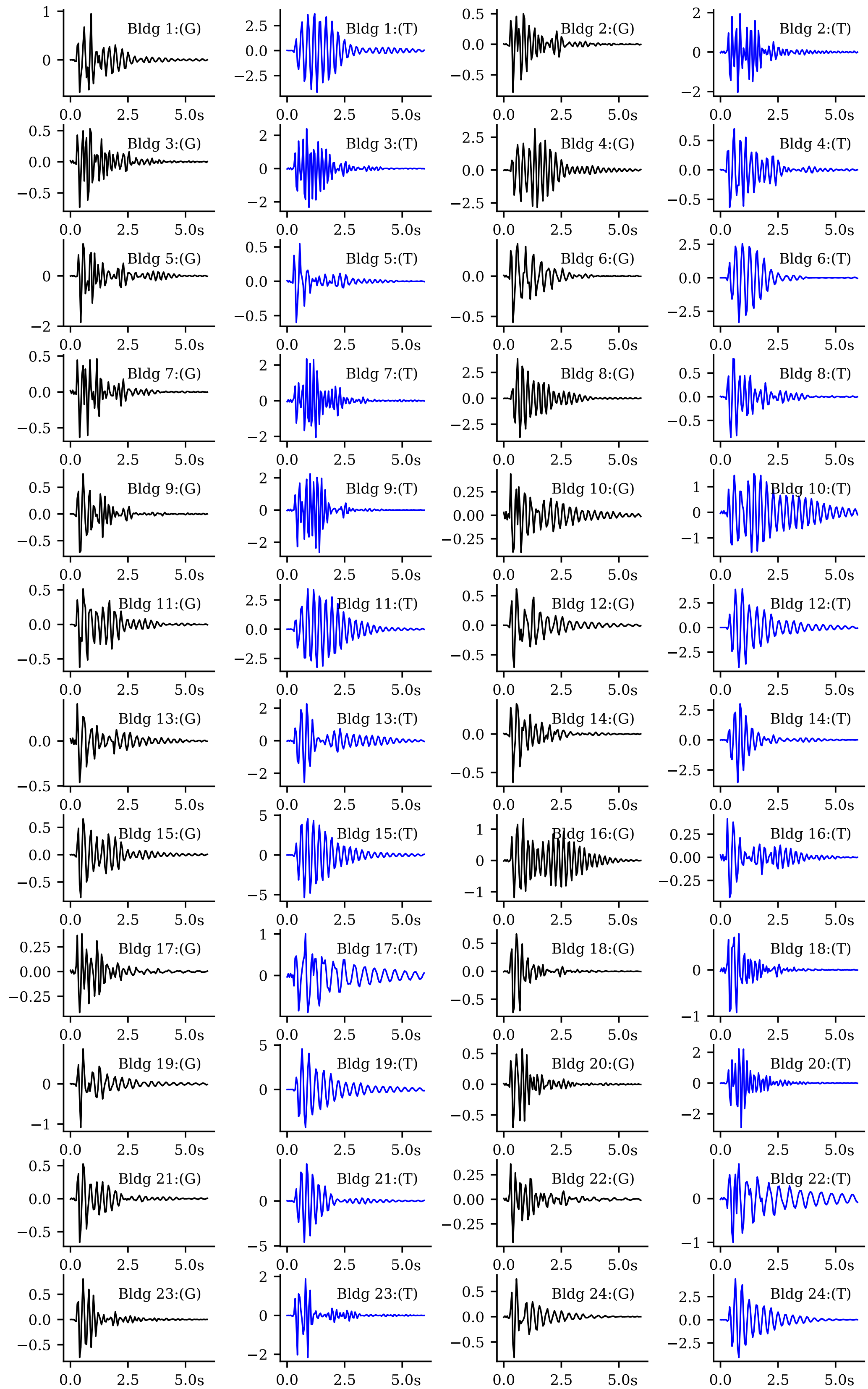

Figure 8. The horizontal displacement time history in the x-direction normalized by the amplitude of the incident S-wave. The responses are taken in a corner at the ground level (G) and top of the buildings (T).

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