

Dynamic analysis of buildings for earthquake-resistant design¹

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Abstract: The proposed 2005 edition of the National Building Code of Canada specifies dynamic analysis as the preferred method for computing seismic design forces and deflections, while maintaining the equivalent static force method for areas of low seismicity and for buildings with certain height limitations. Dynamic analysis procedures are categorized as either linear (elastic) dynamic analysis, consisting of the elastic modal response spectrum method or the numerical integration linear time history method, or nonlinear (inelastic) response history analysis. While both linear and nonlinear analyses require careful analytical modelling, the latter requires additional considerations for proper simulation of hysteretic response and necessitates a special study that involves detailed review of design and supporting analyses by an independent team of engineers. The paper provides an overview of dynamic analysis procedures for use in seismic design, with discussions on mathematical modelling of structures, structural elements, and hysteretic response. A discussion of the determination of structural period to be used in association with the equivalent static force method is presented.

Key words: dynamic analysis, earthquake engineering, elastic analysis, fundamental period, hysteretic modelling, inelastic analysis, National Building Code of Canada, seismic design, structural analysis, structural design.

Résumé : L'édition 2005 du Code National du Bâtiment du Canada spécifie l'analyse dynamique comme méthode de calcul de préférence afin de déterminer les forces de conception sismiques et les déflexions, tout en maintenant la méthode de force statique équivalente pour les régions à faible sismicité et pour les bâtiments ayant certaines limitations en hauteur. Les procédures d'analyse dynamique sont catégorisées comme étant linéaire (élastique) avec des méthodes de spectre de réponse modale élastique et des méthodes temporelles d'intégration numérique linéaire, ou des méthodes temporelles non-linéaires (inélastiques). Bien que les deux types d'analyse (linéaire et non-linéaire) requièrent une modélisation analytique soignée, cette dernière requière des considérations additionnelles afin de simuler correctement la réponse d'hystérésis et nécessite une étude spéciale, qui comporte une révision détaillée de la conception et des analyses secondes provenant d'équipes indépendantes d'ingénieurs. Cet article procure un survol des procédures d'analyse dynamique utilisées pour la conception sismique, avec des discussions sur les modèles mathématiques des structures, éléments structurels et réponse d'hystérésis. Une discussion sur la détermination de la période structurelle à être utilisée en association avec la méthode de force statique équivalente est présentée.

Mots clés : analyse dynamique, génie sismique, analyse élastique, période fondamentale, modèle d'hystérésis, analyse inélastique, Code National du Bâtiment du Canada, conception sismique, analyse structurale, conception structurale.

[Traduit par la Rédaction]

Introduction

Structural response to earthquakes is a dynamic phenomenon that depends on dynamic characteristics of structures

and the intensity, duration, and frequency content of the exciting ground motion. Although the seismic action is dynamic in nature, building codes often recommend equivalent static load analysis for design of earthquake-resistant buildings due to its simplicity. This is done by focusing on the predominant first mode response and developing equivalent static forces that produce the corresponding mode shape, with some empirical adjustments for higher mode effects. The use of static load analysis in establishing seismic design quantities is justified because of the complexities and difficulties associated with dynamic analysis. Dynamic analysis becomes even more complex and questionable when non-linearity in materials and geometry is considered. Therefore, the analytical tools used in earthquake engineering have been a subject for further development and refinement, with significant advances achieved in recent years.

The seismic provisions of the 1995 edition of the National Building Code of Canada (NBCC 1995) are based on the equivalent static load approach with dynamic analysis permitted for obtaining improved distribution of total static base

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shear over the height, and for special cases where a better assessment of building response is required near its ultimate state. This is especially beneficial for irregular buildings with significant plan and elevation irregularities, buildings with setbacks, significant stiffness tapers, and mass variations. It is also beneficial for buildings with significant torsional eccentricities. Dynamic analysis in the 1995 NBCC is not, however, intended for independent determination of the design base shear. The base shear obtained by the equivalent static force approach is specified as *minimum base shear*, with implications that those obtained by dynamic analyses should be scaled up to the static value when lower. It is possible to obtain substantially lower design base shear forces through dynamic analysis, depending on the assumptions made in structural and behavioural models and the ground motion records used. Engineers have been discouraged from using dynamic analysis as a means of establishing seismic design force levels because of its sensitivity to the characteristics of ground motions selected and engineering assumptions made, which in turn are dependent on the experience and judgment of the analyst. Furthermore, the restrictions of available computer software, sometimes very severe, are not always clear to the users, raising concerns over the accuracy of results. Studies in the past have shown that distinctly different results could be obtained from analyses of the same building conducted by different analysts. Therefore, dynamic analysis procedures were regarded as unsafe, unless conducted by experienced and knowledgeable engineers. Consequently, the 1995 NBCC limited the computation of design base shear to the equivalent static load approach, which had been calibrated based on previous experience to provide an acceptable level of protection.

Despite the aforementioned concerns over the use of dynamic analysis in seismic design, it is used in practice to carry out special studies of tall buildings and irregular structures because of its superiority in reflecting seismic response more accurately, when used properly. These studies often include a large number of analyses under different ground motion records and different structural parameters to provide insight into the structural behaviour.

With the advent of personal computers and the subsequent evolution in information technology, coupled with extensive research in nonlinear material modelling, more reliable computational tools have become available for use in design of buildings. The proposed 2005 edition of the National Building Code of Canada recognizes dynamic analysis as a reliable design tool. In fact, dynamic analysis is specified as the preferred procedure for structural analysis. The application of the technique and various types of dynamic analysis methodologies for seismic analysis of buildings and some aspects of mathematical modelling are discussed in this paper.

The 2005 NBCC permits the use of the equivalent static load method of analysis for buildings satisfying certain height and regularity restrictions and (or) located in areas of low seismicity. For such buildings the design base shear may be obtained from the uniform hazard spectrum (UHS) specified for the site (Adam and Atkinson 2003; Humar and Mahgoub 2003). Such a spectrum provides the spectral acceleration $S_a(T)$ for a reference ground condition. The spectral acceleration measured from the UHS, corresponding

to the fundamental period of building, is multiplied by the weight of building and then adjusted for higher mode effects to determine the base shear. The fundamental period can be determined by using the empirical expressions provided in the 2005 NBCC. Alternatively, it may be determined from standard methods of engineering mechanics, including the Rayleigh method. The empirical expressions for the calculations of fundamental period and the restrictions associated with the use of methods of engineering mechanics are discussed in the following sections.

Period determination

The fundamental period is an important design parameter that plays a significant role in the computation of design base shear. The 2005 NBCC provides approximate empirical expressions to estimate the fundamental period. Although the use of more accurate methods of mechanics is permitted in the code, it is specified that the value obtained by such methods must not exceed 1.5 times the value determined by the empirical expressions. This limit may be justifiable from the point of view of safety for three reasons: (i) uncertainties associated with the participation of nonstructural elements, whose effects may not have been considered in period determination and on the seismic response; (ii) possible inaccuracies in analytical modelling when applying the more accurate methods of mechanics; and (iii) potential differences between the design and as-built conditions, especially in terms of structural stiffness and mass. It may be noted that the limit of 1.5 times the value determined by the empirical expression may not apply to shear wall structures. As shown later in the paper, for such structures the period calculated by methods of mechanics is close to the measured value.

Frame buildings

The empirical expressions given in the 2005 NBCC for the calculation of fundamental period T_a of moment resisting frames are given as follows: for concrete frames,

$$[1] \quad T_a = 0.075(h_n)^{3/4}$$

for steel frames,

$$[2] \quad T_a = 0.085(h_n)^{3/4}$$

and for all other moment frames,

$$[3] \quad T_a = 0.1N$$

where N is the total number of stories above grade and h_n is the total height of building above the base in metres. The same expressions are also specified in the 1995 NBCC, except that eq. [3] can be used for any moment frames.

The periods of actual concrete buildings recorded during past earthquakes are compared in Fig. 1 with periods calculated using eq. [1]. The database for these buildings includes a total of 91 cases, consisting of 44 actual buildings in two orthogonal directions. In some cases the measured periods are two to three times larger than the code-computed values. The scatter of data indicates that the empirical expression given in the 2005 NBCC provides approximate estimates of actual building periods. A similar comparison is made in

Fig. 1. Comparison of measured periods with those calculated using eq. [1] for reinforced concrete moment resisting frame buildings.

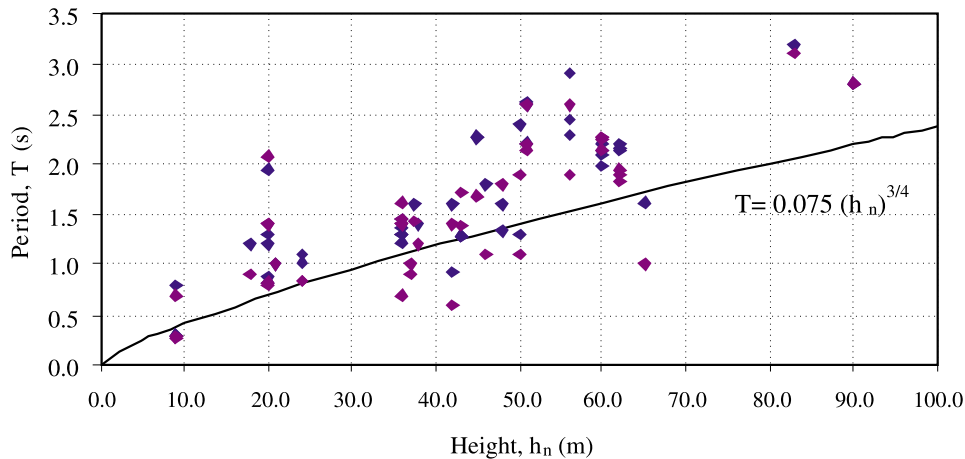


Fig. 2. Comparison of measured periods with those calculated using eq. [2] for steel moment resisting frame buildings.

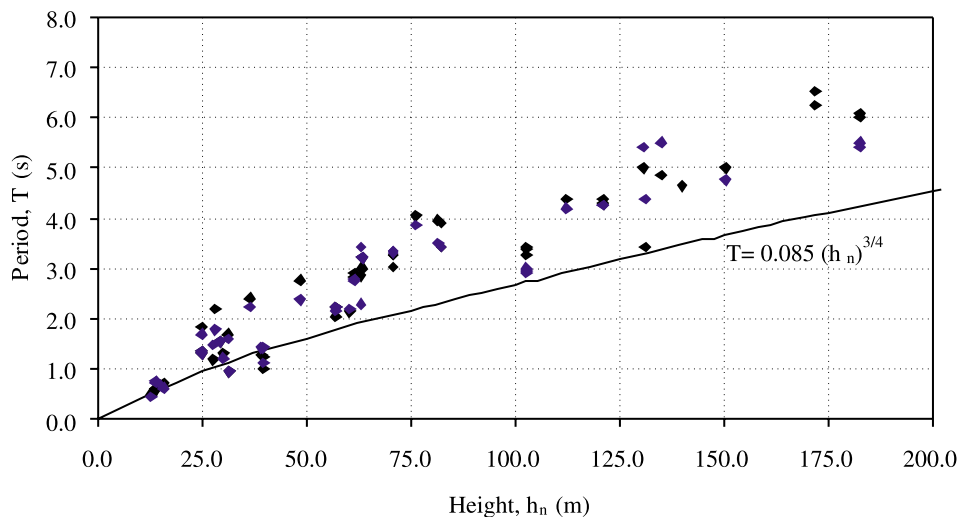


Fig. 2 for 53 steel frame buildings in two orthogonal directions, providing measured data for 103 cases.

It has been observed during recent earthquakes that non-structural elements in existing buildings are often not well separated from the structural framing system and they do participate in seismic response at varying degrees, even though they were not designed as seismic-resisting elements (Saatcioglu et al. 2001). Although the 2005 NBCC clearly calls for the separation of these nonstructural components from lateral force resisting systems and otherwise requires their consideration in seismic design, unintended participation of these elements may explain the scatter of data observed in Fig. 1. The effects of unintended participation of nonstructural elements were assessed by Morales and Saatcioglu (2002), who compared fundamental periods of reinforced concrete frame buildings with different heights and degrees of lateral bracing provided by nonstructural masonry walls. Figure 3 illustrates the variation of computed periods with the amount of lateral bracing expressed in terms of the ratio of wall cross-sectional area to floor area. When the presence of nonstructural masonry walls was ignored, the fundamental periods for 5-, 10-, and 15-storey bare frame buildings, with reduced stiffnesses due to cracking, were

computed as 1.7, 3.7, and 5.7 s, respectively. The same buildings were computed to have fundamental periods of 0.7, 1.2, and 1.6 s, respectively, based on eq. [1] (the 2005 NBCC equation). The analytically computed values were 2.4–3.6 times the values computed by the expression given in the 2005 NBCC, indicating that analytically computed periods could be significantly longer. The results also indicate that the computed values approach those empirically determined by eq. [1] when the bracing effects of nonstructural infill walls are considered. It was found that a few nonstructural masonry infill walls were sufficient to significantly shorten the fundamental period of low- to medium-rise frame buildings to approximately the levels given by eq. [1] when the walls were not well separated from the lateral force resisting system. Therefore, designers should exercise caution when using bare frame models for the computation of fundamental period and ensure that the structure is not braced by nonstructural components that were not considered in structural design, after it is built.

Braced frame and shear wall buildings

The overall stiffness and period of a structure are often dominated by the characteristics of braced frames and shear

Fig. 3. Effects of nonstructural masonry infills on fundamental periods of 5-, 10-, and 15-storey reinforced concrete frame buildings when not properly isolated from the frames.

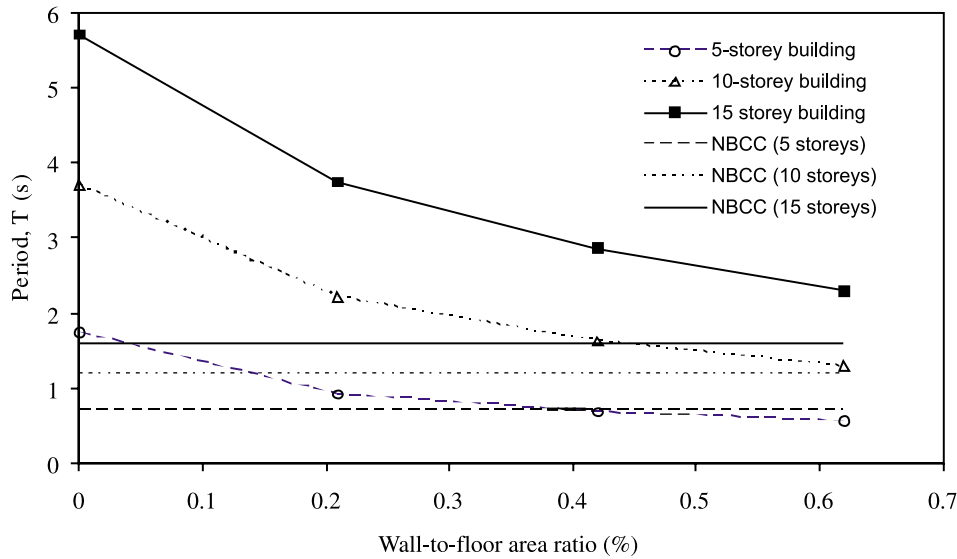
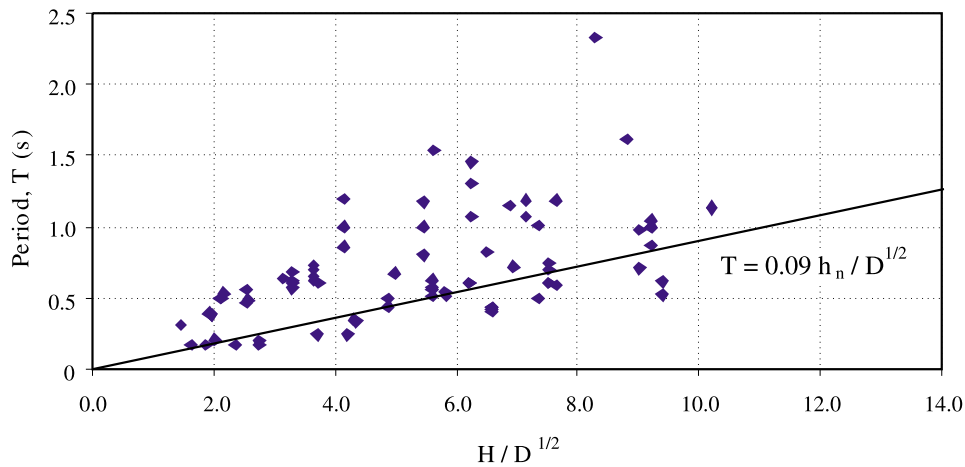


Fig. 4. Comparison of measured periods with those calculated using eq. [4] for reinforced concrete shear wall buildings.



walls in the structure. The 1995 NBCC specifies the fundamental period of such structures as a function of the length of braced frames or shear walls and building height as follows:

$$[4] \quad T = \frac{0.09h_n}{\sqrt{D_s}}$$

where h_n and D_s are building height and wall length in metres, respectively. D_s is defined as the length of the primary lateral load resisting wall or braced frame, although this is often not very clear when more than one bracing element is used in the structure, as is typically the case in most practical applications. When D_s cannot be defined clearly, the entire building length in the direction of analysis (D) is used instead. This results in a conservative estimate of period.

Data on measured periods of shear wall buildings, obtained during previous earthquakes, are plotted in Fig. 4. The data show a scatter similar to that shown in Fig. 1 for frame buildings, with eq. [4] providing conservative values

when D is substituted in place of D_s . Hence, it may be argued that the implied accuracy of eq. [4] may not be justifiable. Instead, an expression similar to those recommended for frame buildings may be used for shear wall and braced frame buildings as a function of building height only (Goel and Chopra 1997; Morales and Saatcioglu 2002). Figure 5 shows the correlation of measured periods with the following equation, which has been adopted by UBC (1997), IBC (2000), and the 2005 NBCC:

$$[5] \quad T = 0.05(h_n)^{3/4}$$

Among the instrumented buildings, 10 buildings had shear walls in two orthogonal directions with well-identified geometry. These buildings were modelled and analyzed using computer software SAP2000 to determine their periods of vibration analytically. The flexural rigidities were reduced to account for cracking in concrete. Accordingly, 70 and 35% of uncracked rigidities were used for vertical elements (columns and walls) and beams, respectively. The results presented in Fig. 6 indicate that reasonably accurate values of

Fig. 5. Comparison of measured periods with those calculated using eq. [5] for reinforced concrete shear wall buildings.

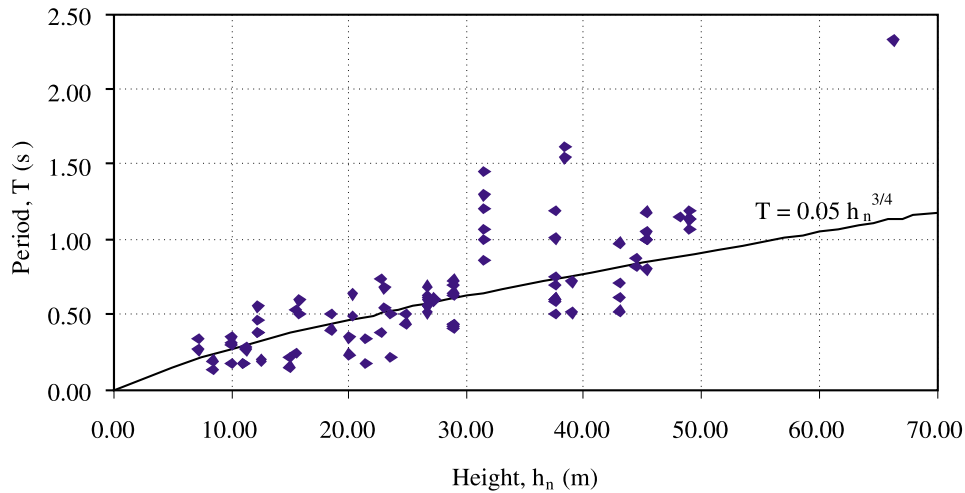
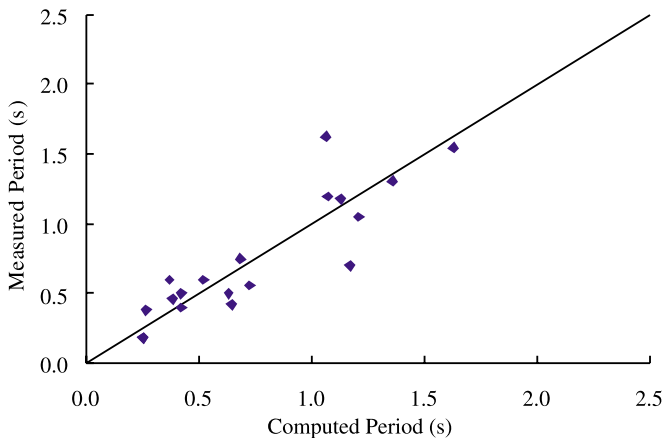


Fig. 6. Comparison of computed periods (by SAP2000) and measured periods for shear wall buildings.



fundamental period can be computed for shear wall buildings analytically. The 2005 NBCC allows the computation of fundamental periods of shear wall buildings by accepted methods of mechanics without an upper limit based on the empirical expression.

Methods of dynamic analysis

Dynamic analysis is specified in the 2005 NBCC as a general method of analysis to compute design earthquake actions. The equivalent static force procedure is permitted for buildings in low seismic regions, regular buildings below a certain height limit, and short buildings with certain irregularities. It may be preferred by designers because of its simplicity when dynamic analysis is not mandatory. In the 2005 NBCC, dynamic analysis is mandatory for the following classifications of buildings: (i) regular structures that are 60 m or taller or have fundamental period greater than or equal to 2.0 s and are located in areas of high seismicity with $I_E F_a S_a(0.2) \geq 0.35$, where I_E is the moment of inertia, F_a is an acceleration-based site coefficient, and $S_a(0.2)$ is the spectral response acceleration for a period of 0.2 s; (ii) irregular buildings that are 20 m or taller or have a fundamental period of 0.5 s or longer and are located in areas of high

seismicity with $I_E F_a S_a(0.2) \geq 0.35$; and (iii) all buildings that have rigid diaphragms and are torsionally sensitive.

Dynamic analysis is conducted to obtain either a *linear (elastic)* or a *nonlinear (inelastic)* structural response. When elastic analysis is conducted, an empirical assessment of inelastic response is made, since the design philosophy is based on nonlinear behaviour of buildings under strong earthquakes. This does not, however, deter engineers from preferring elastic dynamic analysis because of its simplicity and direct correspondence to the design response spectra provided in building codes.

The first step in dynamic analysis is to devise a mathematical model of the building, through which estimates of strength, stiffness, mass, and inelastic member properties (if applicable) are assigned. Detailed discussion of mathematical modelling is presented later in the paper in the section titled Mathematical modelling.

Linear (elastic) dynamic analysis

The modal response spectrum method or the numerical integration linear time history method is used to conduct linear analysis. Spectral analysis is intended for the computation of maximum structural response. The spectral analysis is conducted for a single-degree-of-freedom structure or for a building that can be approximated to behave in its first mode response by selecting the value corresponding to its period, directly from the design response spectrum. Correct assessment of the fundamental period becomes important in obtaining spectral values, as discussed in the section Period determination. In the 2005 NBCC, the design response spectrum is based on the UHS for the site as adjusted for the ground condition (Humar and Mahgoub 2003). A UHS provides the maximum spectral acceleration that a single-degree-of-freedom system with 5% damping is likely to experience with a given probability of exceedance and reflects the seismicity of the region. Such spectra have been developed for use with the 2005 NBCC (Adam and Atkinson 2003).

A water tank supported by a single column (or a truss system) with a concentrated mass at the top represents a typical single-degree-of-freedom structure with a corresponding mode shape and a period. On the other hand, a typical frame

building with rigid floors develops as many modes as the number of floors (with concentrated mass at floor levels). The treatment of a multistory building as a single-degree-of-freedom structure may be possible on the basis of its dominant first mode response but provides only an approximation of its complete structural response.

In general, for a multistory building it is necessary to take into account contributions from more than one mode. Each mode has its own particular pattern of deformation. For building applications, the dominant first mode shape resembles the flexural deformation of a cantilever beam. The contribution of higher modes diminishes very quickly, and it is nearly always sufficient to consider the first three modes of vibration to obtain reasonably accurate results for most short- to medium-rise buildings. For high-rise buildings, it may be necessary to consider more than three modes. The significant modes that contribute to response may be determined by selecting the number of modes such that their combined participating mass is at least 90% of the total effective mass in the structure.

Once the number of significant modes is established, the response is computed as the superposition of responses of all the contributing modes. This implies that the response of the entire structure can be modelled as a number of single-degree-of-freedom responses with their respective modal properties and contributions to overall response. This approach, known as modal analysis, is a useful design tool where spectral values for each of the participating modes can be obtained separately from UHS and superimposed with due considerations given to the participation of each mode. The use of UHS as the design response spectrum for a multi-degree-of-freedom system is conservative but provides reasonably accurate results (Humar and Mahgoub 2003).

The method of modal analysis is described in standard texts (Humar 1990; Clough and Penzien 1993; Chopra 2001) and will not be discussed in detail. It provides the elastic base shear V_e and the elastic storey shears, storey forces, member forces, and deflections. The design base shear V_d is determined by dividing the elastic base shear V_e by $R_o R_d$ to allow for overstrength and ductility and multiplying by the importance factor I_E :

$$[6] \quad V_d = \frac{V_e}{R_o R_d} I_E$$

where R_d is a ductility-related force modification factor that reflects the capability of a structure to dissipate energy through inelastic behaviour, and R_o is an overstrength-related force modification factor that accounts for the dependable portion of reserve strength in a structure designed in accordance with the 2005 NBCC. If the base shear V_d obtained from eq. [6] is less than 80% of the lateral earthquake design force, V , established by the equivalent static force procedure, V_d is replaced by $0.8V$, except for irregular structures requiring dynamic analysis as previously specified, in which case V_d is taken as the larger of V_d determined by eq. [6] and 100% of V .

The elastic quantities obtained from modal analysis are multiplied by V_d/V_e to obtain design elastic storey shears, storey forces, member forces, and deflections.

As stated earlier, the restrictions imposed on the design values obtained from a dynamic analysis are justified by the uncertainties involved in the modelling, so it is not considered prudent to deviate too far from the design values obtained by the equivalent static load procedure. Compared with the 1995 NBCC, however, the limits prescribed in the 2005 NBCC are more liberal. Thus under the provisions of the 2005 NBCC, if the period determined from a dynamic analysis is longer than that obtained from the empirical expressions, such a longer period, up to a limit of 50% longer than the empirical period, may be used in calculating the static base shear V . The limit of 80% (or 100%) is then related to the shear calculated from the longer period.

The linear time history method is employed when the entire time history of the elastic response is required during the ground motion of interest. In this method, the dynamic response of the structure is computed at each time increment under a specific ground motion. The structure is first modelled with due considerations given to stiffness and mass distributions and other factors as outlined in the section Mathematical modelling. It is then subjected to pairs of ground motion time history components that are compatible with design response spectra specified in building codes. It is usually a requirement to conduct time history analysis for an ensemble of ground motion records that represent magnitudes, fault distances, and source mechanisms that are consistent with those of the design earthquakes used to generate design response spectra. This is done either by using previously recorded ground motions, scaled to match the design response spectrum, or by using artificially generated records with similar properties. Depending on the number of ground motion records considered, either the maximum or the average of each design parameter obtained from different analyses is used in design. When the analysis conforms to the 2005 NBCC, the ground motion records should be compatible with the response spectrum constructed from the specified design spectral acceleration values, $S(T)$, which are the product of spectral acceleration $S_a(T)$ and site coefficients F_a or F_v .

The base shear obtained from a linear time history analysis has to be divided by R_o and R_d to allow for the reductions associated with overstrength and ductility in the system and multiplied by the importance factor I_E to arrive at the design base shear level V_d . If the base shear V_d obtained is less than 80% of the static base shear V established by the equivalent static force procedure, V_d is taken as $0.8V$, except for irregular structures requiring dynamic analysis as previously specified, in which case V_d is taken as the larger of V_d determined by linear dynamic analysis and 100% of V . If the design base shear is taken to be larger than that computed by the elastic time history analysis, all other design quantities, including storey forces, storey shears, member forces, and deflections, should be proportionately increased before they are used in design.

Nonlinear (inelastic) response history analysis

Nonlinear time history analysis involves the computation of dynamic response at each time increment with due consideration given to the inelasticity in members. Nonlinear analysis allows for flexural yielding (or other inelastic actions) and accounts for subsequent changes in strength and

stiffness. Hysteretic behaviour under cyclic loading is evaluated. Softening caused by inelasticity of deformations during loading, unloading, and reloading is computed. This implies that the interaction between changing dynamic characteristics of structures due to inelasticity, such as lengthening of period, and the exiting ground motion is accounted for, improving the accuracy of dynamic response.

The most important distinction between linear and nonlinear time history analyses is the inelastic hysteretic behaviour of elements that make up the structure. This behaviour is incorporated into analysis computer software through *hysteretic models*. Therefore, mathematical modelling of structures gains a new dimension, with new difficulties and challenges introduced. The hysteretic behaviour depends on the characteristics of structural materials, design details, and a large number of design parameters, as well as the history of loading. It is a rather complex behavioural phenomenon that has to be understood clearly by the analyst before such an analysis is attempted. A section is presented later in the paper under Mathematical modelling to highlight salient features of selected hysteretic models.

A nonlinear time history analysis provides the maximum ductility demands in members and the maximum deflections experienced by the structure. If the ductility demands are less than the ductility capacities and the deflections are within acceptable limits, the design is satisfactory. The results of nonlinear time history analysis directly account for reduction in elastic forces associated with inelasticity. The structural overstrength can also be accounted for directly through appropriate modelling assumptions. The analysis results need not therefore be modified by R_d and R_o . The importance factor I_E can be accounted for either by scaling up the design ground motion histories or by reducing the acceptable deflection and ductility capacities.

Inelastic time history analysis has been adopted by the 2005 NBCC with a *special study* clause. Accordingly, when nonlinear time history analysis is used to justify a structural design, a special study is required, consisting of a complete design review by a qualified independent engineering team. The review is to include ground motion time histories and the entire design of the building, with emphasis placed on the design of lateral force resisting system and all the supporting analyses.

Mathematical modelling

Mathematical modelling of a structure, for the purpose of structural analysis, can be done in a variety of different ways depending on the method of analysis adopted. In a general sense, modelling involves the representation of a structure by elements to which physical and material characteristics are assigned and the appropriate loading is applied or transmitted. These elements involve different degrees of discretization of the structure (or structural component). Perhaps the most common forms of structural analysis involve modelling by finite elements or by line elements. When dynamic inelastic response history analysis is considered, it is usually more convenient and reliable to represent the behaviour of an entire structural element or component mathematically. Furthermore, the analysis results should be

in an easily interpretable form that can be related to the design parameters used in practice. Therefore, in this section a common form of mathematical modelling involving line elements is used to illustrate important aspects of modelling for dynamic analysis, while also highlighting some of the pitfalls that should be avoided to improve the accuracy of results.

There are three stages of modelling that an analyst has to go through prior to performing a dynamic analysis: (i) structural modelling, (ii) member modelling, and (iii) hysteretic modelling (for nonlinear analysis).

Structural modelling

The first stage in mathematical modelling involves the representation of the entire structure by elements to which physical and material properties can be assigned. Figure 7 illustrates the modelling of common forms of building structures with line elements. In frame and frame-wall interactive systems the vertical elements such as columns and flexure-dominant structural walls are represented by vertical line elements, and the horizontal elements, such as beams, and slab diaphragms are represented by horizontal line elements (Figs. 7a, 7e). Sometimes shear panels, like shear walls and infill panels, as well as bracing elements are modelled by diagonal struts and ties (Figs. 7b–7d).

An important aspect of structural modelling is the selection of correct boundary conditions that represent supports and connections with appropriate end restraints. Another important aspect is the consideration of finite widths of members and corresponding stiffness variations. This becomes especially important in modelling wide columns and structural walls for which the segments of elements that are integral with the adjoining members can be represented with infinite stiffness. Figures 7f and 7g illustrate the member-end regions, which are sometimes referred to as member-end eccentricities in certain computer programs. The analyst must be aware of the procedure used for considering finite widths of members in the computer software employed and incorporate these regions properly.

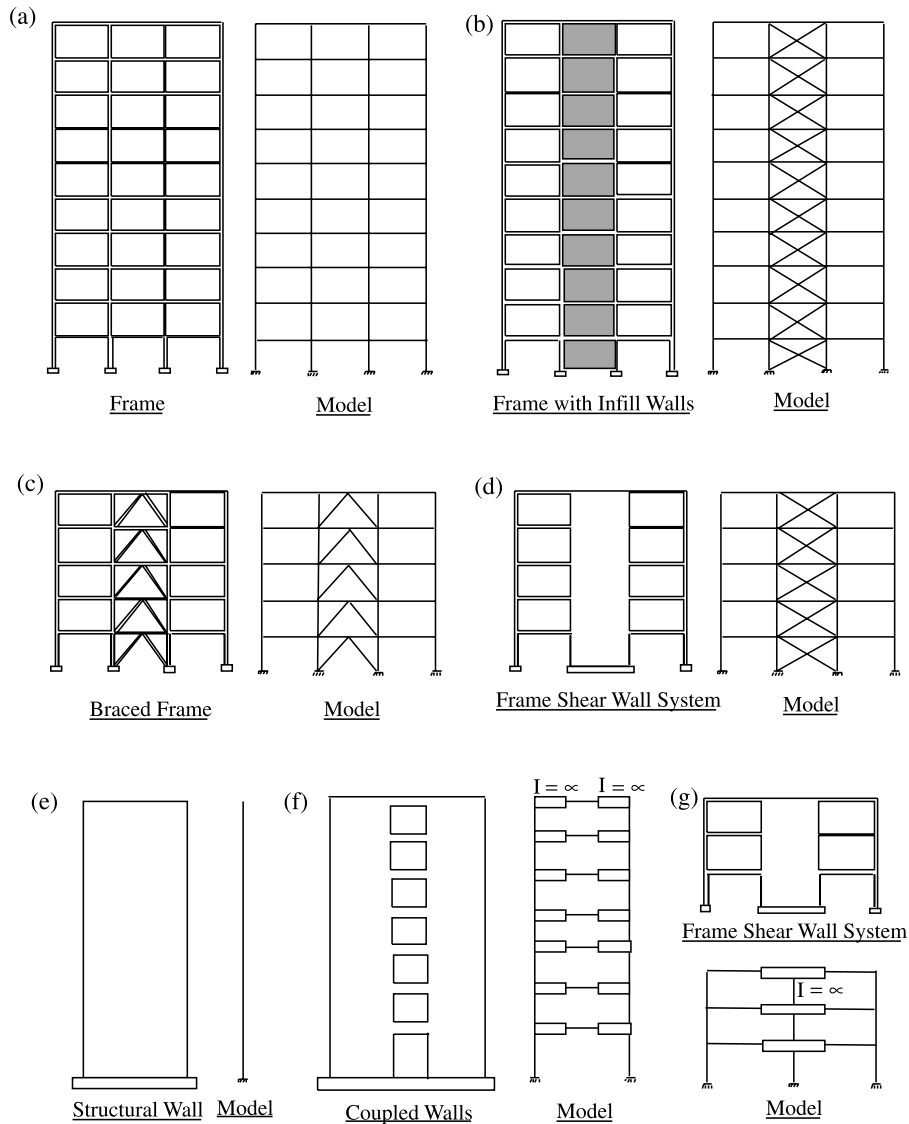
Seismic analysis is conducted in two orthogonal directions, separately and independently. When a building with rigid diaphragms is torsionally irregular, a three-dimensional (3-D) analysis must be carried out. Even when the building is determined to have no torsional irregularity, accidental torsional eccentricity will still cause torsional response, and again a 3-D analysis is required.

The 2005 NBCC defines a torsional sensitivity index B as a measure of the susceptibility of a building to large torsional motion during an earthquake (Humar et al. 2003). The index B is determined by calculating the ratio B_x for each level x according to the following equation for each orthogonal direction determined independently:

$$[7] \quad B_x = \frac{\delta_{\max}}{\delta_{\text{ave}}}$$

where δ_{\max} is the maximum storey displacement at the extreme points of the structure at level x in the direction of the earthquake, and δ_{ave} is the average of the displacements at the extreme points. Displacements δ_{\max} and δ_{ave} are automatically obtained when a 3-D dynamic analysis is carried out.

Fig. 7. Structural modelling.



The index B is taken as the maximum of all values of B_x in both orthogonal directions.

Buildings for which B is greater than 1.7 are considered to be torsionally sensitive and must be designed with extra care. For such buildings the effect of accidental torsion is accounted for by combining the static effects of torsional moments produced by the application of a force of $\pm 0.1D_{nx}F_x$ at each level x with the effects determined by dynamic analysis, where the forces F_x may be taken as those determined from a dynamic analysis or the equivalent static analysis.

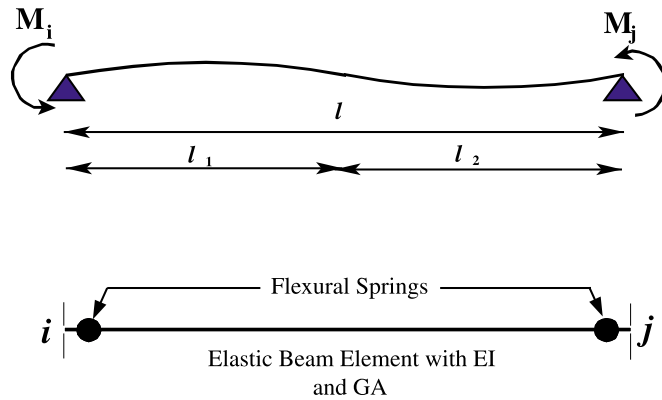
When B is less than 1.7, accidental torsion can be accounted for by carrying out a set of 3-D dynamic analyses with the centres of mass shifted by distances of $-0.05D_{nx}$ and $+0.05D_{nx}$.

Buildings with rigid diaphragms have three degrees of freedom at each floor, two translational and one rotational. The inertia masses at the floor levels are assigned to these degrees of freedom. The inertia mass may be computed as being equal to the entire dead load and a portion of the live load, depending on the occupancy of the building.

Member modelling

Force–deformation characteristics of individual members, essential for structural analysis, are specified through member models. In a linear dynamic analysis, members are modelled by elastic line elements. In a nonlinear analysis, member modelling can be done in a number of different ways. A convenient procedure is to idealize each member as an elastic line element with inelastic springs at the ends. The springs account for potential plastic hinges at member ends. This model, known as single-component model, was developed by Giberson (1967). According to the single-component model, inelastic member-end deformation at one end is directly related to the member-end force at the same end, making it a simple and convenient model for structural analysis. Inelasticity along the member is lumped at the springs whose characteristics are determined by assuming deformed shapes for members. Double curvature can be assumed for columns and beams with a fixed point of contraflexure, as illustrated in Fig. 8. Single curvature can be assumed for walls, with some moment gradient along the

Fig. 8. Single-component element model.



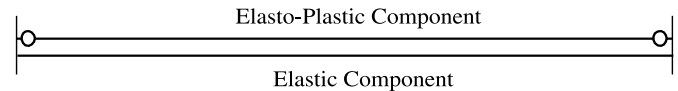
member. The single-component model can also have more than one spring at each end, where each spring represents a separate deformation component, as in the case of flexure and shear. Similarly, a third spring may be added to simulate potential softening of fixity at member ends, as in the case of anchorage slip deformations in reinforced concrete elements. The main disadvantage of the single-component model is the assumption that fixes the location of the point of contraflexure for the purpose of inelastic deformation computations. During response, the points of contraflexure in structural members move continually, and the member-end rotations become a function of member-end forces at the two ends. The single-component model is therefore regarded as an approximate and yet reasonably accurate member model.

A dual-component model was suggested for elastoplastic analysis of structures (Clough et al. 1965). In this model, two parallel line elements, one representing perfectly elasto-plastic behaviour to mark the yield point and the other to introduce the post-yield stiffness of the member, are connected at their ends so that the two parallel elements have the same member-end deformation. This simplifies the formulation of the stiffness matrix. Member-end deformations are related to member-end forces at both ends. The dual-component model, however, is applicable to structural members that exhibit elastoplastic behaviour without any stiffness degradation. This model can be extended into a multicomponent model where each parallel element represents different characteristic features of hysteretic response. Figure 9 illustrates the dual-component model.

Another form of member modelling is to divide the member into segments. This provides a convenient approach to account for the spread of inelasticity more accurately. In this case, the behaviour of each subelement can be represented by a separate inelastic spring. The resultant model is termed a multiple-spring model (Takayanagi and Schnobrich 1976). In other similar approaches, a variable length of inelastic zones was considered in segmenting the member (Roufaei and Meyer 1983; Chung et al. 1988; Keshavarzian and Schnobrich 1984). The variation of rigidity along the member length can also be represented by a continuous function (Umemura et al. 1974).

Members can be modelled by dividing into segments not only along the length but also across the cross section. Sections of a member can be divided into layers, and sectional response is computed using material constitutive models.

Fig. 9. Dual-component element model.

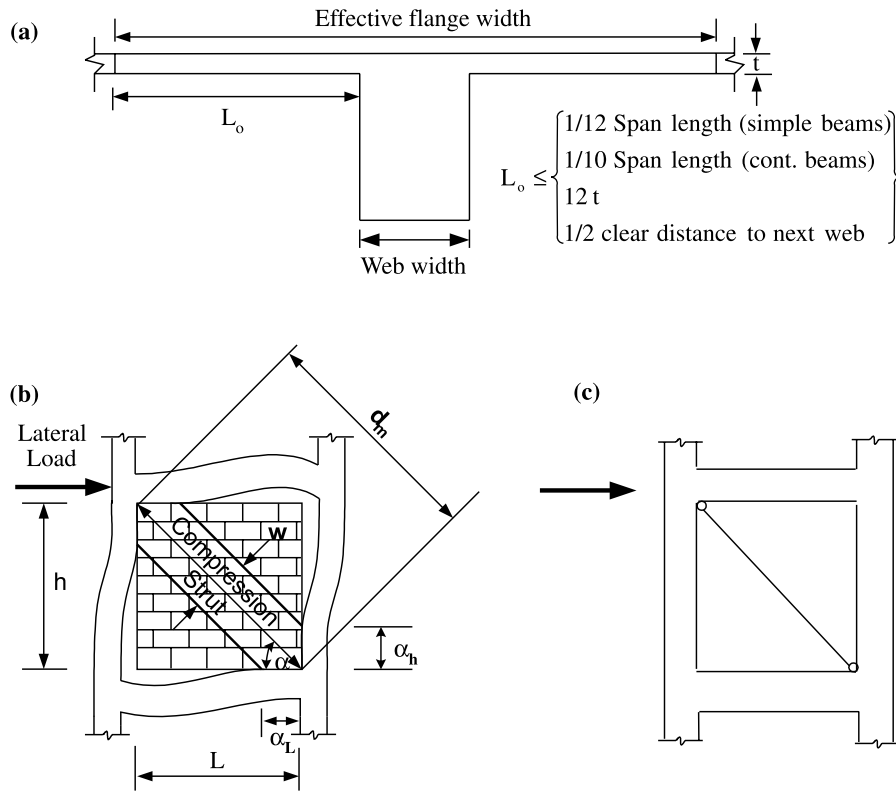


This type of model is known as a layered model. Although the approach followed in a layered model is more rational, the computational effort involved makes the approach prohibitive for general-purpose use.

Keshavarzian and Schnobrich (1985a) have presented a comprehensive review of member models. The first task in any member modelling is the computation of elastic member properties. This can be done using the principles of mechanics with due considerations given to member geometry and material behaviour. Areas and moments of inertia for elements with well-defined cross-sectional dimensions are easy to compute. Sometimes, however, the members represent portions of 3-D elements where concentrations of internal flow of forces occur. Portions of concrete slabs in monolithic construction that are effective as beam flanges and diagonal compression struts and tension ties in walls can be given as examples of this type of member for which the analyst has to select an “effective member width” and define the sectional geometry. The effective width is defined in Canadian Standards Association Standard A23.3 (CSA 1994) for reinforced concrete slabs as illustrated in Fig. 10a. When a column – flat plate system is used, the column strip of slab may be used as the effective width of a beam element. Similarly, various approximate methods have been proposed to compute the geometric properties of struts in wall panels. One such method proposed by Stafford-Smith (1966) on the basis of contact lengths is illustrated in Fig. 10b. Holmes (1963) recommended that the diagonal width could be taken as one third of the diagonal panel length. The New Zealand Code (Standards Association of New Zealand 1990) specifies the effective wall width as one quarter of the wall length.

Once the sectional geometry is established, element properties are computed analytically. Flexural rigidity EI , shear rigidity GA , and axial rigidity AE can be determined from cross-sectional properties (A and I , where A is the cross-sectional area and I is the moment of inertia) and material moduli (E and G , where E is the modulus of elasticity and G is the shear modulus). This is especially true for steel structures where material characteristics are well defined. Although elastic rigidities of steel structures can be computed on the basis of elastic material properties, inelastic behaviour leads to analysis techniques that may require different levels of sophistication. Figure 11 illustrates typical behaviour of a cantilever steel beam with a plastic end region. The sectional behaviour, incorporating post-yield response, can be computed by establishing the plastic hinge length and the variation of curvatures within the hinging region. For engineering applications, however, this level of sophistication is often not warranted and a rigid-plastic hinge model can be employed with zero plastic hinge length and bilinear moment-curvature idealization (Bruneau et al. 1998). For reinforced concrete structures, where significant cracking is expected beyond a relatively low initial resistance, the analyst may have to conduct a moment-curvature analysis to obtain the flexural rigidity. While a standard plane-section

Fig. 10. Effective widths of members: (a) effective flange width in beams, (b) formation of diagonal struts, and (c) equivalent braced frame.

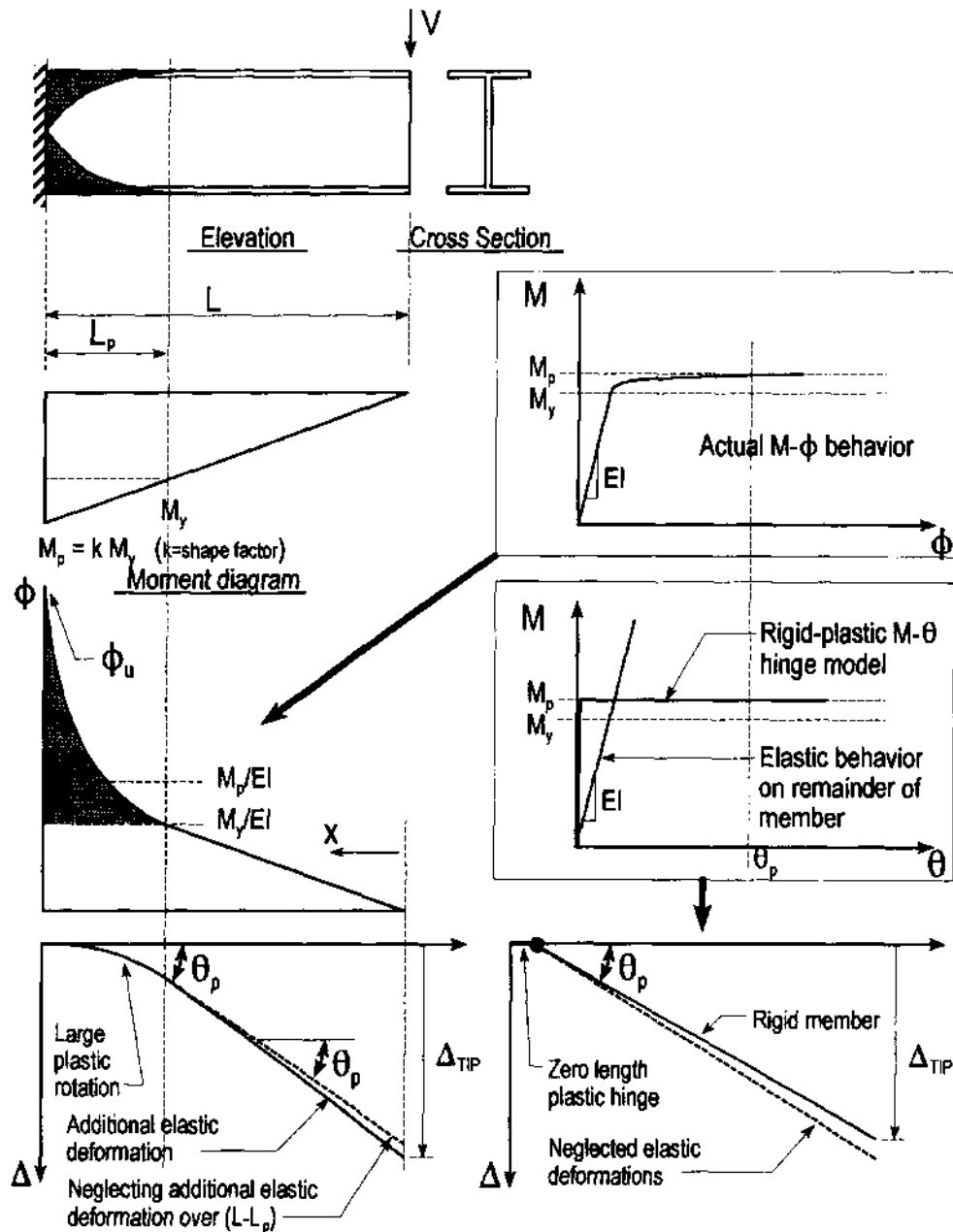


analysis is sufficient for this purpose, it is important to incorporate strain hardening in steel and confinement in concrete for improved accuracy of results in assessing the post-yield rigidity. Figure 12 depicts a typical moment–curvature relationship for a reinforced concrete section. The relationship consists of three segments: (i) the elastic region prior to cracking, (ii) the post-cracking segment between the cracking and yield points, and (iii) the post-yield segment beyond yielding. These regions can be idealized by a trilinear relationship. Bilinear relationships are often used in practice for convenience, however, with the first segment representing “the effective elastic rigidity” including the effects of cracking. Therefore, the elastic rigidity used in elastic dynamic analysis must include the effects of cracking. Post-cracking and post-yield rigidities depend on many parameters, including cross-sectional shape, amount and arrangement of reinforcement, material properties, and level of accompanying axial force. The effective elastic rigidity varies between approximately 30% and 50% of the rigidity based on gross, uncracked section properties (EI_g , where I_g is the moment of inertia based on gross uncracked properties), for most beams. Wall sections with boundary elements show similar behaviour, although the rigidity can be substantially lower for walls with relatively low percentage of distributed reinforcement, especially under a low level of axial compression. In the latter case the yielding of low-percentage steel occurs one row at a time, as the depth of the neutral axis becomes smaller very quickly, resulting in a rapid degradation of flexural rigidity upon cracking. It has been the practice to use 50% EI_g for beams and 100% EI_g for columns in static gravity load analysis. This was justified because the beams crack even under their own weight, whereas the columns un-

der axial compression remain mostly uncracked. It was believed that the 1:2 ratio used in the reduction of column and beam stiffnesses, respectively, would result in a reasonable distribution of forces at beam column connections. Furthermore, the overestimation of column rigidities by ignoring cracking would result in higher design forces for columns, which are the critical elements. The same justification cannot be used for lateral seismic analysis where the columns are subjected to significant bending and cracking. Both the ACI Committee 318 (2002) and the CSA (1994) recommend effective elastic flexural rigidities to be 35% and 70% of their gross, uncracked values for beams and vertical members (columns and walls), respectively. For shear wall structures, if moments exceed the modulus of rupture, then the analysis should be repeated with wall stiffnesses further reduced to 35% of elastic rigidities. The use of elastic rigidities, based on gross sectional properties may result in substantial overestimation of member stiffnesses and the corresponding shortening in fundamental period.

Different approaches may be used to idealize moment–curvature relationships depending on their application. For elastic analysis, the only parameter required is the initial slope of the relationship, which defines the effective elastic rigidity. This can be achieved by drawing a horizontal line parallel to the curvature axis at nominal flexural capacity. Another line can be drawn to connect the origin and a point on the ascending branch of the curve corresponding to 75% of the nominal capacity (Park and Paulay 1975). The intersection of the two lines gives the yield point, with the slope of the ascending line representing the effective elastic rigidity. This is illustrated in Fig. 12b. In this approach, the analyst need not consider the strain hardening of steel and the

Fig. 11. Formation of plastic hinge in a cantilever steel beam and associated deformations (adopted from Bruneau et al. 1998).

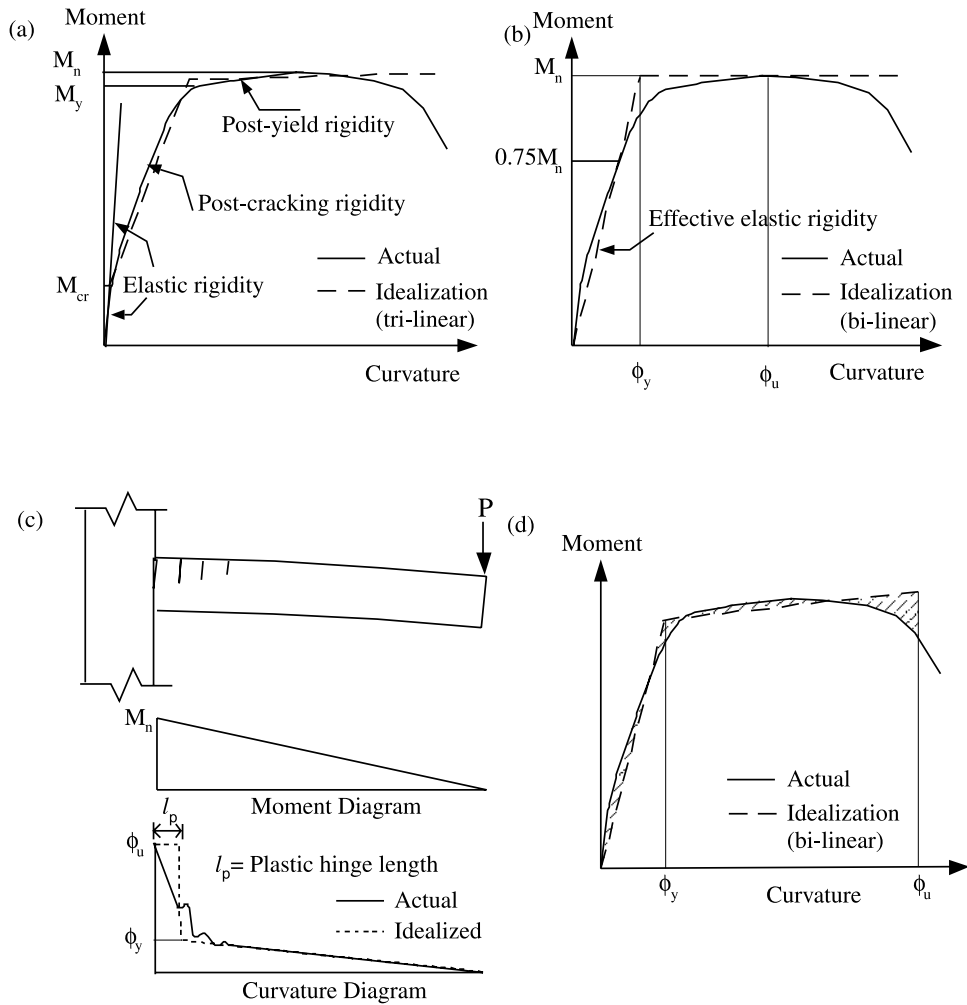


confinement of concrete in computing the actual moment–curvature relationship, and the accuracy of the post-yield branch is of little interest. For nonlinear dynamic analysis, the post-yield slope becomes important. In such analysis it is usually required to compute the member behaviour rather than the sectional behaviour. Post-yield slope of the moment–rotation or moment–drift (chord angle) relationship may have to be specified to define the hysteretic model. This requires the construction of curvature distribution along the length of the member and modelling of the potential plastic hinge region. It is convenient to model a plastic hinge at ultimate load by a fully developed hinge with constant curvature. In this case, an empirically suggested plastic hinge length can be used with constant curvature equal to the ultimate curvature, as illustrated in Fig. 12c. The area under the

curvature diagram provides member rotation, with the moment of the area giving member displacement. The ultimate member deformation obtained in this manner can be used along with yield deformation to establish the second slope of the force–deformation relationship. It is also possible to compute the entire post-yield region by an incremental moment–rotation analysis. In this case two line segments can idealize the moment–curvature relationship such that the area between the actual and idealized curves is approximately equal, as illustrated in Fig. 12d. The resulting idealization may have a nonzero post-yield slope and can be used in integrating curvatures within the plastic hinge region.

The aforementioned approaches result in the ascending slope of the force–deformation relationship that ignores gradual strength degradation with increased inelasticity. Al-

Fig. 12. Moment–curvature idealizations for reinforced concrete elements: (a) trilinear idealization, (b) bilinear idealization, (c) idealized plastic hinge length, and (d) idealization based on equal areas.



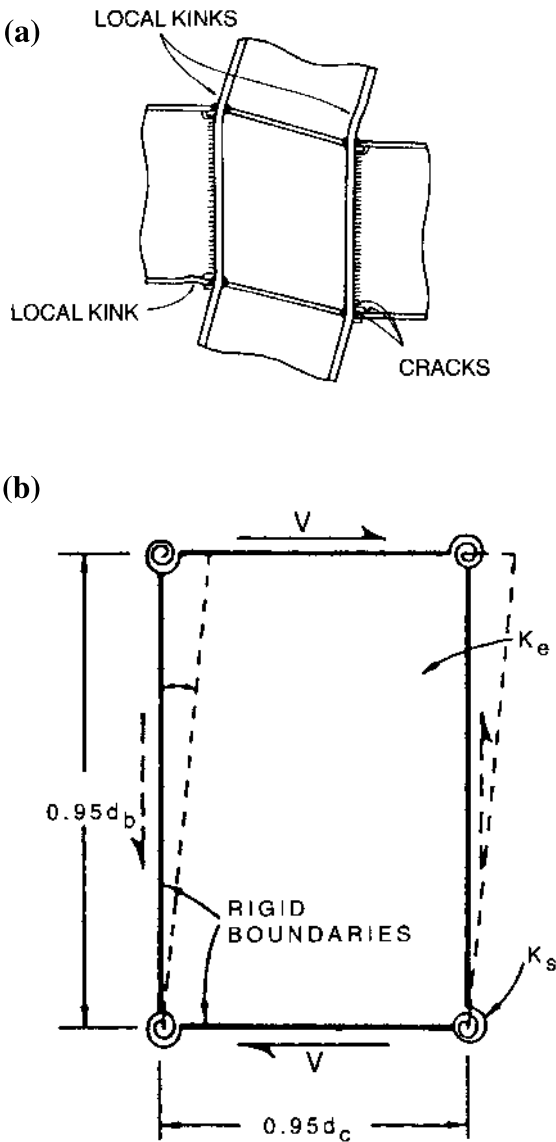
though they can be used until the onset of strength decay, computer programs equipped with such models do not flag this limit. The analyst should be aware of this hidden restriction and should ignore the analysis results beyond a realistic level of inelastic capacity. If, however, the analysis program is equipped with a hysteretic model that allows strength degradation, it is possible to compute the progression of plastic hinging and the accompanying strength degradation by an incremental force–deformation analysis (Razvi and Saatcioglu 1999).

Sound seismic design practice calls for proper design and detailing of beam–column joints. This does not, however, ensure full rigidity at joints during seismic response. Additional deformations may occur due to the softening in joints, connections, and anchorage regions. In steel-frame buildings, distortions of the beam–column panel zones can contribute to overall softening of the structure, even if they are well designed to prevent column web yielding and crippling, flange distortions, and the panel zone failure. Tests of steel beam – column subassemblages by Krawinkler et al. (1971, 1975) showed that panel zones can develop significant shear distortions with the ability to dissipate seismic-induced energy. Figure 13 shows shear distortions of a panel zone and

an analytical model proposed by Krawinkler et al. (1971), consisting of an elastoplastic column segment enclosed by rigid elements connected by inelastic springs.

In reinforced concrete, flexural reinforcement anchored in adjoining members may develop significant extensions due to the penetration of yielding into the adjacent members. These deformations are not accounted for in flexural analysis. Although anchorage failure can be prevented by proper design, the extension of properly anchored reinforcement in tension cannot be avoided if the critical section is located near the interface of two adjacent members. The extension and (or) slippage of reinforcement produce a rigid-body rotation at member end. This type of deformation may be negligible prior to yielding of longitudinal reinforcement and hence may be conveniently ignored. The penetration of yielding into the anchorage zone, however, especially beyond the strain hardening of reinforcement, can be significant and lead to inelastic deformations whose magnitudes approach that caused by flexure. Sometimes the force–deformation relationship for flexure is softened by the analyst to account for the combined effects of bar extension and flexure. Figure 14a illustrates the strain profile in an embedded reinforcing bar, the integration of which gives the exten-

Fig. 13. (a) Panel zone deformations (Krawinkler et al. 1971). (b) Modelling panel zones in steel beam – column joints (Krawinkler et al. 1978).



sion of reinforcement due to yield penetration. Figure 14b shows displacement caused by the extension of reinforcement in an adjoining member (Alsiwat and Saatcioglu 1992).

In most practical applications it is sufficient to consider deformations associated with flexure alone. Short and stubby members that are subjected to significant shear stresses develop additional deformations due to shear. It may be sufficient to use cross-sectional area and shear modulus to establish elastic shear rigidity for elastic dynamic analysis. Computation of inelastic shear effects may become necessary for certain members, with a complete shear force – shear deformation relationship assigned to shear springs of member models, if available.

Another type of commonly encountered structural element to model is steel bracing. Lateral bracing in steel structures results in concentrically braced frames (CBF) or eccentrically braced frames (EBF), as illustrated in Fig. 15. These

braces are axially loaded members and are modelled by specifying their elastic and post-yield properties under axial force. The elastic force – deformation relationship is specified in terms of axial rigidity AE . The tensile strength is governed by axial yielding and the compressive strength is governed by buckling and post-buckling residual compression force, which may be taken as approximately 20% of the buckling load (NEHRP 1997). The EBF is a hybrid framing system that may possess both increased lateral stiffness and good ductility characteristics, both achieved through the link beams. It becomes important to assess the flexural and shear rigidities of link beams before they can be modelled. Short links dissipate seismic energy primarily through inelastic shearing, whereas long beams develop flexural hinging at the ends. Figure 16 shows a bilinear idealization of the force–deformation relationship of a link beam. The plastic rotation capacity of an adequately stiffened short link may be taken to be approximately equal to 0.12 rad (NEHRP 1997).

Hysteretic modelling

Dynamic inelastic response history analysis of reinforced concrete structures requires realistic conceptual models that can simulate strength, stiffness, and energy-dissipation characteristics of members. The current state of knowledge may not be sufficient to model every aspect of hysteretic response. Significant advances have been made in recent years, however, that enable analysts to obtain reasonably accurate results from nonlinear dynamic analyses. A large number of hysteretic models have been proposed for seismic evaluation of structures. These models are often based on specific test data. Therefore, their applicability to other cases requires careful examination of their features and limitations.

Structural members exhibit certain hysteretic features that are common to members of similar properties, which are associated with well-established design parameters. Some of the unfavourable features of hysteretic response, for example, early strength decay caused by lack of proper design and detailing practices, can be prevented and need not be considered in hysteretic modelling. Other features of hysteretic response, however, some of which are outlined in the following paragraphs, may have to be considered depending on specific applications. Special care should be exercised to make sure that the computed response could be attained with the design and detailing techniques employed.

Most hysteretic models consist of a *primary curve* (backbone curve), which can be computed analytically by well-established procedures of mechanics, and a set of empirical rules that define the branches of loading, unloading, and reloading under reversed cyclic loading. The primary curve provides an envelope of hysteresis loops of force–deformation relationships. Therefore, it provides a convenient means of defining the strength boundary for modelling purposes. Primary curves used for hysteretic modelling are generally in the form of moment–curvature, moment–rotation, shear force – shear distortion, moment–slip, and force–displacement relationships. They can be computed as discussed earlier in the section Member modelling.

Hysteretic rules within the strength boundary (primary curve) simulate the actual member behaviour, often as ob-

Fig. 14. Deformations caused by anchorage slip in reinforced concrete structures (Alsawat and Saatcioglu 1992).

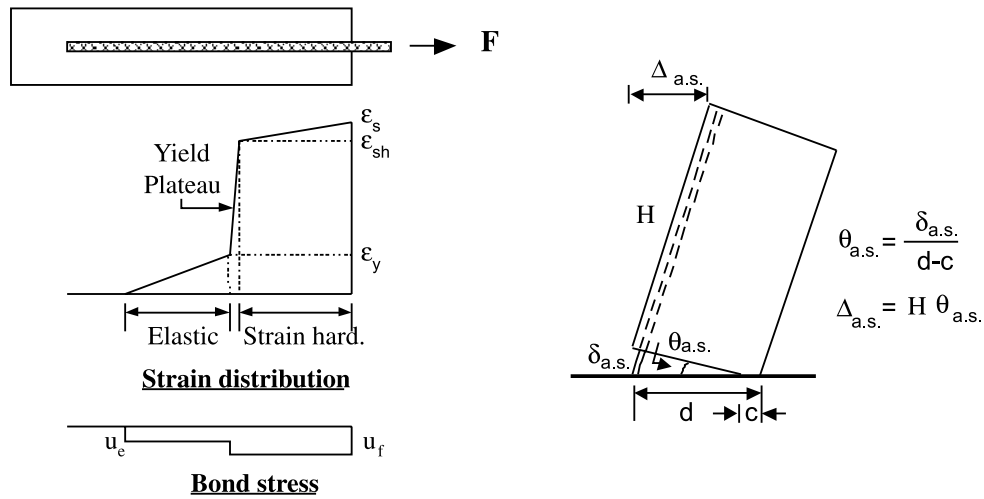
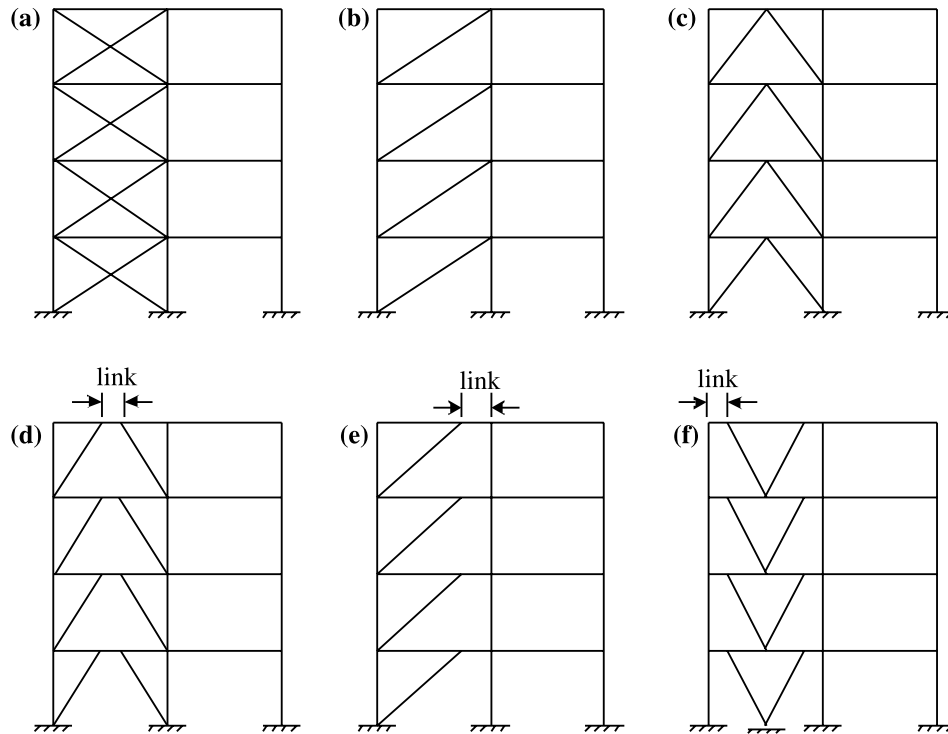


Fig. 15. (a) Cross-braced CBF; (b) diagonally braced CBF; (c) inverted, V-braced CBF; (d) split K-braced EBF; (e) D-braced EBF; and (f) V-braced EBF.



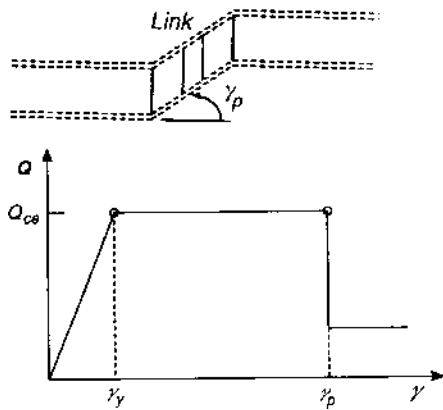
served during tests. Structural steel elements exhibit elastoplastic behaviour, with unloading and reloading branches of hysteresis loops parallel to the initial elastic branch. The post-yield slope of the primary curve may reflect steel strain hardening with a post-yield rigidity approximately equal to 0.5–5% of the elastic rigidity. The Bauschinger effect, in the form of rounded yield regions upon load reversals, as opposed to the sharp initial yield, can be considered as refinement. This is illustrated in Fig. 17, which shows different variations of the elastoplastic hysteretic model. Zero post-yield slope of the primary curve may also be used for simplicity, though this may occasionally lead to numerical problems in some computer software. Steel structures do not

exhibit significant stiffness degradation in flexure and hence are relatively simple to model by means of elastoplastic models.

Hysteretic response of steel bracing elements under axial force reversals is different from that exhibited by frame elements in flexure. Figure 18 illustrates a typical response of a steel brace under reversed cyclic loading and a hysteretic model proposed by Jain and Goel (1978). The model exhibits higher capacity in tension, as governed by yielding, and a lower capacity in compression, dictated by buckling and a subsequent residual compressive capacity.

Panel zones in beam–column joints of moment resisting steel frames show shear distortions with stable, well-rounded

Fig. 16. Typical force–deformation relationship of a link.

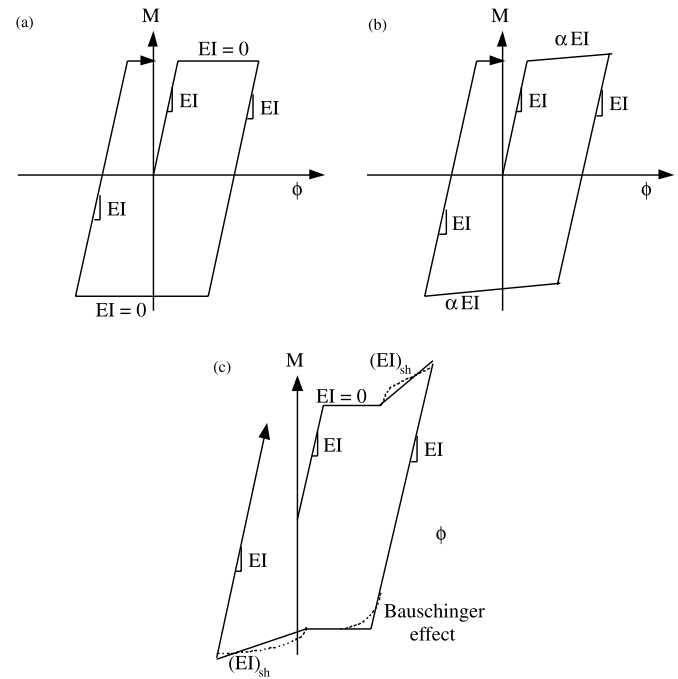


loops as illustrated in Fig. 19. If these regions are to be modelled in structural analysis, the elastoplastic hysteretic models shown in Fig. 17 may be employed with shear stiffness, for improved accuracy of overall frame response.

Unlike structural steel elements, reinforced concrete develops stiffness degradation under inelastic deformation reversals. The degradation of stiffness takes place during reloading and unloading as evidenced by reductions in the slopes of corresponding force–deformation hysteresis loops. The degradation in stiffness increases at a varying rate, usually as a function of the number of cycles and the level of peak deformations attained. Figure 20 shows the degradation of stiffness (reductions in the slopes of the force–deformation relationship) recorded during a wall test. A simple hysteretic model was developed by Clough (1966), incorporating stiffness degradation into a perfectly elastoplastic response. Accordingly, the reloading branches of hysteresis loops are aimed at previous maximum response points, thereby simulating stiffness degradation. The reloading slope is decreased with increasing maximum response deformation. Unloading slopes remain parallel to the effective elastic stiffness, which implies that the model does not recognize the stiffness degradation during unloading. A bilinear primary curve is used to set the limitations for strength, with a nonzero post-yield slope. Figure 21 illustrates the basic features of the Clough model. The model is simple to use and suitable for modelling stable hysteresis loops, typically observed in flexural response. It was reported (Saiidi 1982; Clough 1966) that the response waveforms of the Clough model were significantly different from those of the elastoplastic model and showed better correlation with data obtained from tests of reinforced concrete elements.

An improved degrading stiffness model was developed by Takeda et al. (1970) with a trilinear primary curve. The reloading points in the model are aimed at the response point at previous maximum deformations. Unloading slopes are reduced as a function of the previous maximum deformation, hence the stiffness degradation is introduced in a more refined manner as compared to the Clough model. Energy dissipation during small amplitudes is considered. Figure 22 illustrates the basic features of the Takeda et al. model, which is applicable to reinforced concrete members with stable hysteresis loops under constant axial load. The Takeda et

Fig. 17. Elastoplastic hysteretic model for steel structures: (a) zero post-yield rigidity; (b) with strain hardening; and (c) with optional strain hardening and Bauschinger effects.

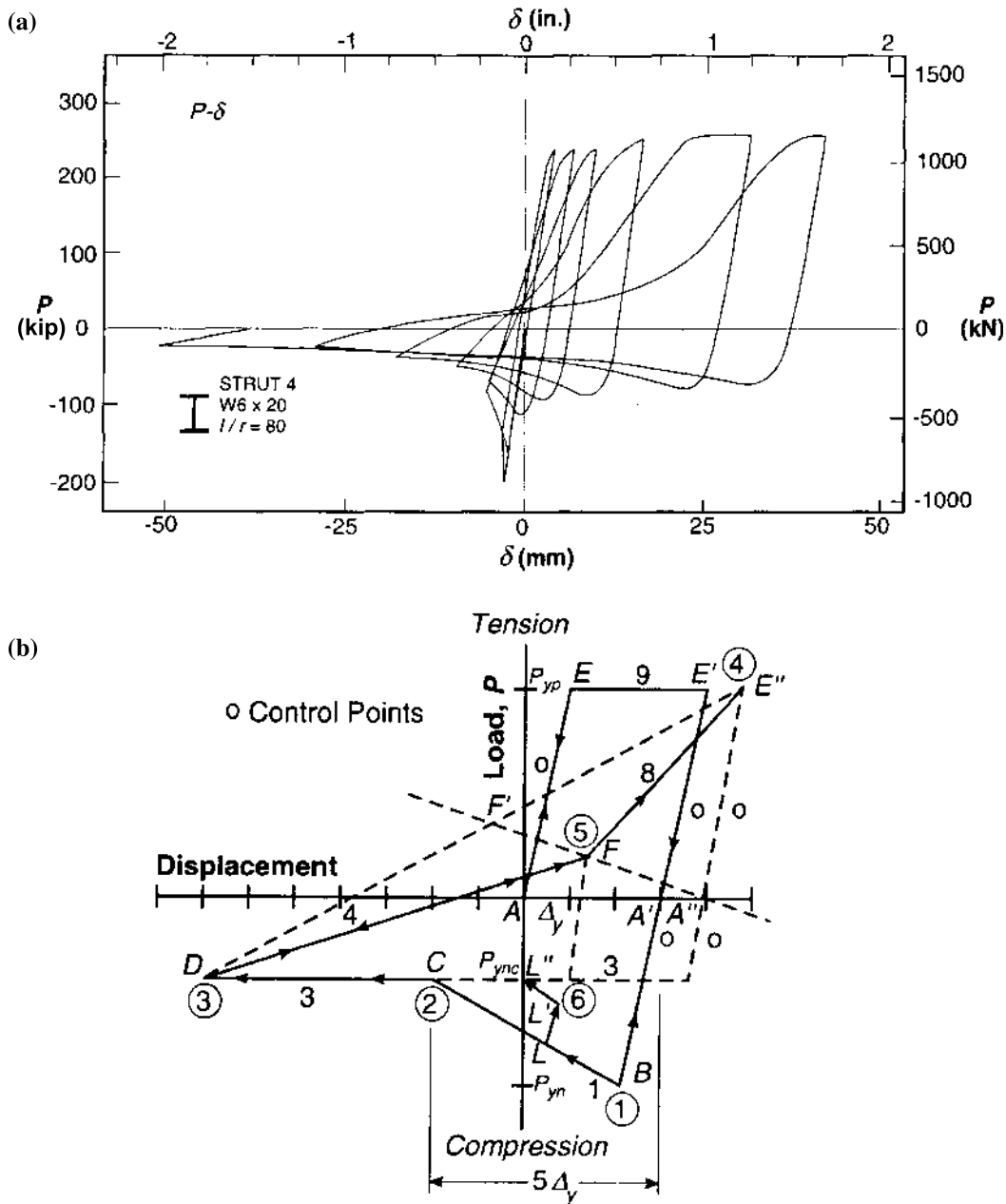


al. model was later simplified by Otani and Sozen (1972) and Powell (1975), who used a bilinear primary curve. Riddell and Newmark (1979) introduced improvements relative to small cycle reversals. The model was further simplified by Saiidi and Sozen (1979), who incorporated a bilinear primary curve and simpler rules for the unloading slope. Another degrading trilinear model was developed by Fukuda (1969) for a predominantly flexural response of reinforced concrete, consisting of a trilinear primary curve with unloading point considered as the new yield point.

An important feature of hysteretic response is *strength decay*. Structural members exhibit progressive loss of strength under relatively high levels of inelastic deformation cycles. Figure 23 illustrates a strength decay obtained in a concrete column under reversed cyclic loading. The degree of strength decay depends on many parameters, including the governing deformation mode, concrete confinement, shear strength, loading history, and level of axial load. The envelope of hysteresis loops in such a member cannot be obtained by bilinear or trilinear idealizations discussed earlier. Early and rapid strength decay can have a very significant effect on structural response and, if not considered in hysteretic modelling, can lead to significant errors in dynamic analysis.

Hysteresis loops of both steel and reinforced concrete members generally show a marked change in slope during reloading. In steel members this may be attributed to the slippage of a bolt or a rivet until the force is completely reversed. The change in slope in concrete elements is associated with opening and closing of cracks under cyclic loading. Following a crack caused by a load in one direction, reversed loading in the opposite direction meets with little

Fig. 18. (a) Typical hysteretic force–displacement relationship of a steel brace (NEHRP 1997) (1 kip = 4.45 kN). (b) Hysteretic model for steel brace members by Jain and Goel (1978).



initial resistance as the crack closes. Subsequently, the cracked surfaces come in full contact, increasing the load resistance. This phenomenon is reflected in the force–deformation relationship as a change in slope during reloading, and is known as *pinching* of hysteresis loops. If the cracks are inclined diagonal tension cracks, as in the case of shear response, some sliding occurs between the cracked surfaces before they come in full contact. Also, the slippage of reinforcing bar in the vicinity of a crack results in increased deformations with very low resistance in the opposite direction as the reinforcement slips back to its previous position before the crack is completely closed and full resistance is attained. Therefore, pinching is more prevalent in shear force – shear distortion and force – bar slip relation-

ships and may have to be considered in the hysteretic models used for analysis of structures where these deformation components are significant. Strength decay and pinching of hysteresis loops were modelled by Takayanagi and Schnobrich (1976), who modified the Takeda et al. model to incorporate features that are prevalent in shear and bar slip, as illustrated in Fig. 24. Ozcebe and Saatcioglu (1989) developed a hysteretic model for inelastic shear effects that considers the interaction between flexural and shear yielding and the pinching of hysteresis loops as shown in Fig. 25. Other hysteretic models exhibiting strength and stiffness degradation and pinching were developed by others for analysis of reinforced concrete structures (Roufaiel and Meyer 1983, 1987; Banon et al. 1981).

Fig. 19. Experimentally obtained steel panel zone hysteretic relationship (Krawinkler et al. 1971).

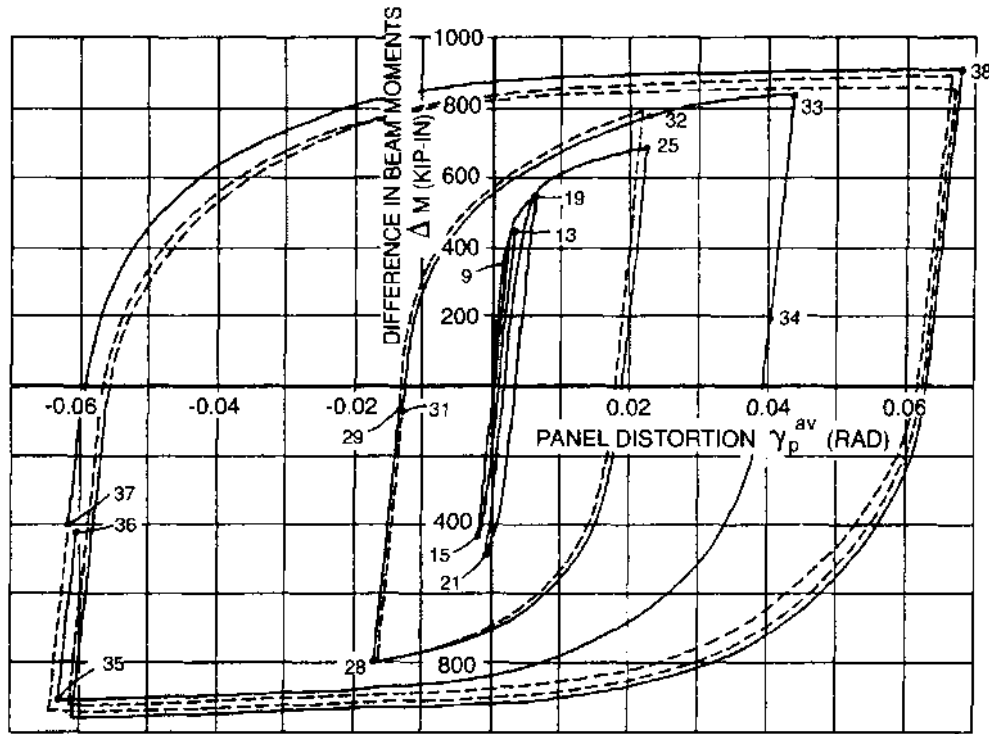


Fig. 20. Stiffness degradation recorded during a reinforced concrete wall test (Oesterle et al. 1979).

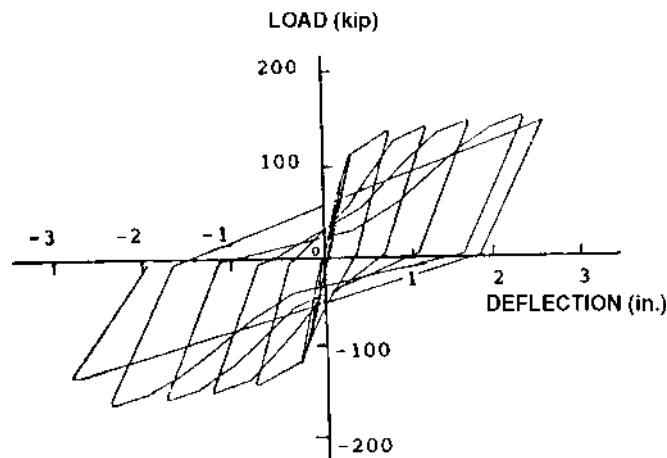
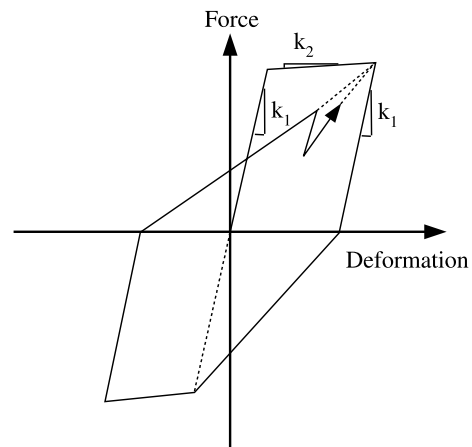


Fig. 21. Stiffness degrading model by Clough (1966).



The pinching of hysteresis loops is a dominant feature in reinforcement bar slippage. A number of hysteretic models were developed specifically to simulate this phenomenon. Fillipou et al. (1983a, 1983b) developed an analytical approach to describe the hysteretic bond-slip relationship. Bond deterioration and associated bar slip were evaluated by dividing the reinforcement into three segments. The method was later simplified (Fillipou 1985) and used to describe the response of an anchored bar to generalized excitation. Morita and Kaku (1983) proposed a hysteretic model for the moment-slip rotation relationship. It is based on linear stress and strain distributions along the reinforcement and uses an average bond stress observed experimentally. Pinching of hysteresis loops is introduced empirically. Saatcioglu et al. (1992) developed a hysteretic model for the moment -

anchorage slip rotation relationship. The primary curve is computed by establishing a nonlinear strain distribution along the bar, with pinching of hysteresis loops considered, as illustrated in Fig. 26. Others approximated bar-slip deformations through additional softening in response, without the pinching effect. Otani (1974) modified the Takeda et al. model to introduce the additional softening by assuming constant bond stress within the development length. Soleimani et al. (1979) modelled bar slip deformations by means of a rotational spring. The elastic rigidity of the moment - bar slip rotation relationship was taken as one third of the flexural rigidity. The hysteretic rules proposed by Clough (1966) were followed without the pinching of hysteresis loops.

Important changes may occur in the hysteretic response of vertical members due to the interaction of axial forces with

Fig. 22. Stiffness degrading model by Takeda et al. (1970).

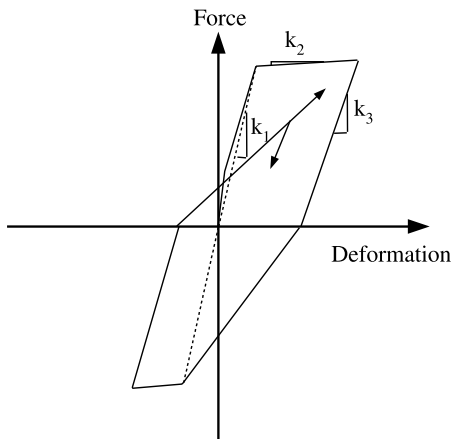


Fig. 23. Strength decay in a column (Ozcebe and Saatcioglu 1989).

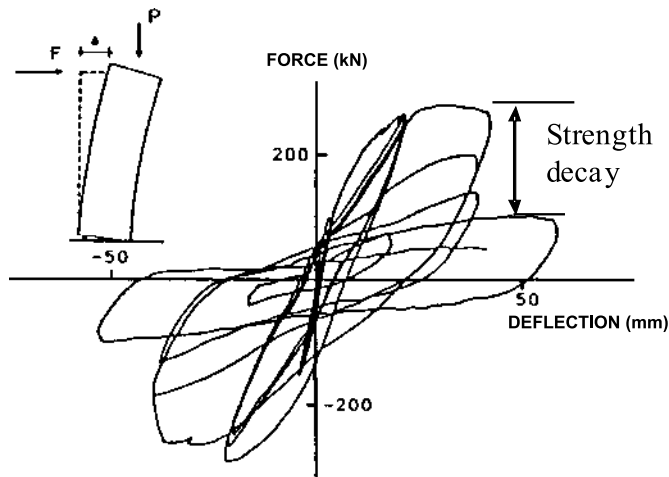
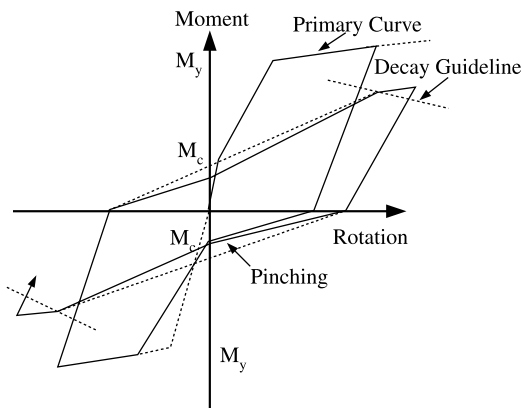


Fig. 24. Hysteretic model incorporating pinching and strength decay (Takayanagi and Schnobrich 1976).



flexure and shear during seismic response. Variable axial forces can be induced during seismic response in columns of frame structures and in coupled walls as a result of the coupling action of the linking beams (Saatcioglu et al. 1983;

Fig. 25. Hysteretic shear model (Ozcebe and Saatcioglu 1989).

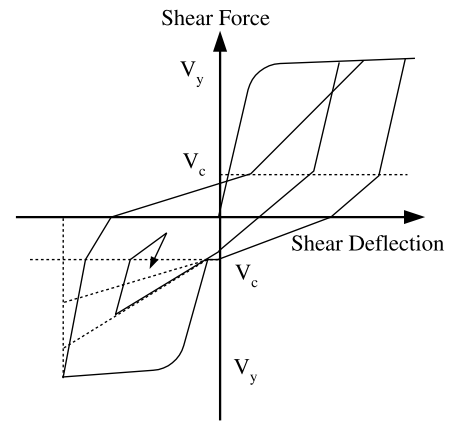
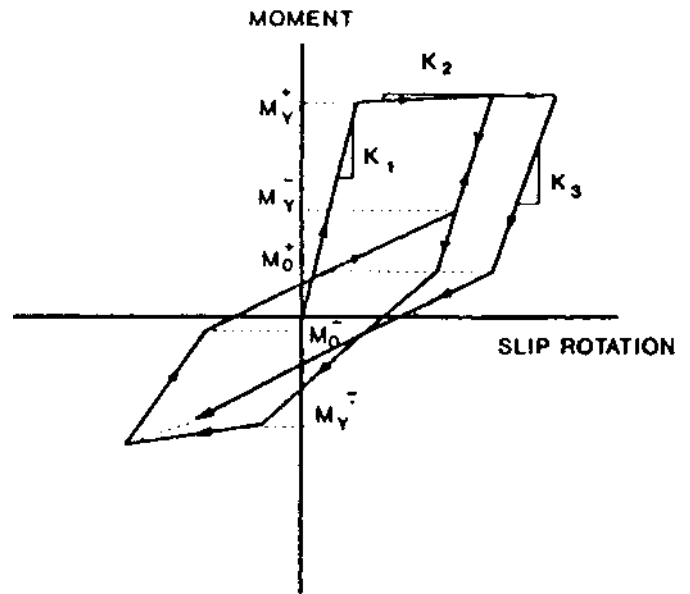


Fig. 26. Moment – anchorage slip model.



Abrams 1987). Figure 27 illustrates the behaviour of a column specimen tested under simultaneous variable axial force and lateral deformation reversals. The increase and decrease in strengths accompanied by axial compression and tension, respectively, can be observed in Fig. 27. It is also evident in the figure that the presence of axial compression reduces ductility and accelerates strength decay. The effect of variable axial force was shown to be especially significant on coupled walls (Takayanagi and Schnobrich 1976; Saatcioglu and Derecho 1980; Saatcioglu et al. 1983; Keshavarzian and Schnobrich 1985b). The effect of axial force on flexural yield level was considered by Mahin and Bertero (1976) and Aktan and Bertero (1982) by modifying the elastoplastic model. Takayanagi and Schnobrich (1976) modified the Takeda et al. model to include the effects of axial force flexure interaction. Saatcioglu et al. (1980, 1983) modified Powell's (1975) version of the Takeda et al. model to incorporate the axial force – flexure interaction during response. The model, shown in Fig. 28, is based on updating member stiffnesses for the subsequent time increment based on axial force computed during the current time step.

Fig. 27. Effect of variable axial force on hysteretic behaviour (Saatcioglu and Ozcebe 1989).

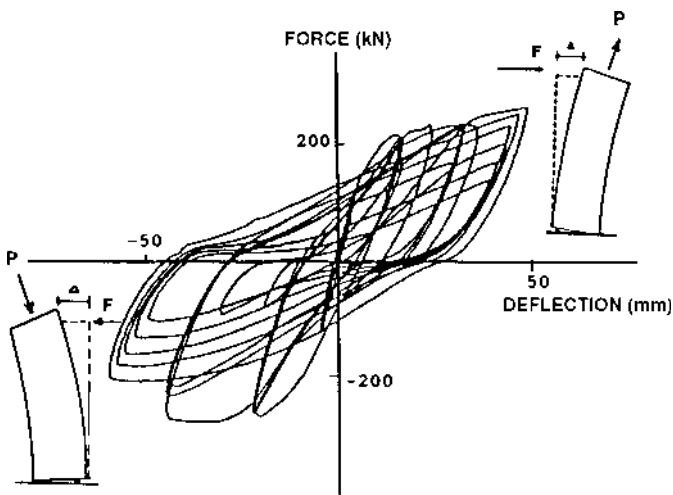
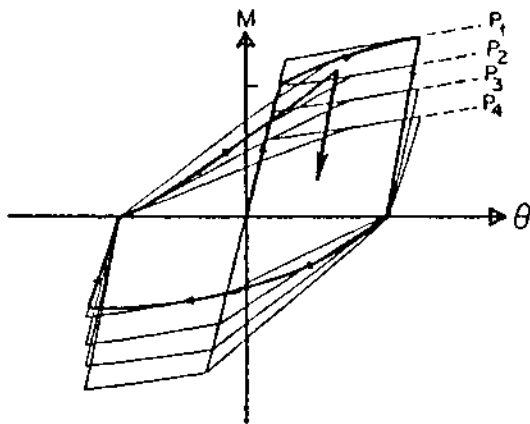


Fig. 28. Axial force – moment interaction model (Saatcioglu et al. 1983).



Conclusions

Dynamic analysis of buildings requires careful structural modelling, appropriate selection of ground motion records, and thorough knowledge and familiarity of the analyst with the procedures and computer software employed. Nonlinear time history analysis continues to provide challenges. Significant advances have been made in recent years, however, in developing reliable analysis tools. Therefore, it is possible to attain reasonably accurate assessment of inelastic seismic response of buildings through dynamic analysis, which can be used in earthquake-resistant design of buildings. Although the issues to be confronted appear to be numerous, often it is possible to eliminate many of the modelling parameters discussed in the paper with appropriate justifications and simplify the process considerably. Design and detailing provisions of current building codes often ensure the elimination of undesirable features of response that translate into premature strength decay, excessive pinching, and inelastic behaviour of brittle members or deformation modes. In such buildings it is possible to ignore features of hysteretic response intended to model these aspects of structural re-

sponse. This is especially true for flexure-dominant buildings where hysteretic response is dominated by well-rounded stable hysteresis loops, a feature that is addressed by the majority of available computer software. In most cases it is sufficient to analyze these buildings with the use of an elastoplastic model for steel structures and a stiffness degrading model for reinforced concrete structures. Special care should be exercised, however, to make sure that the computed response can be attained with the design and detailing practices employed.

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List of symbols

- A cross-sectional area
- B maximum value of B_x
- B_x ratio at level x used to determine torsional sensitivity and defined in eq. [7]
- c neutral axis depth, measured from the extreme compression fibre
- d effective depth, measured from the extreme compression fibre to the centroid of tension steel
- d_b depth of structural steel beam section
- d_c depth of structural steel column section
- d_m diagonal length of masonry strut
- D dimension of the building in a direction parallel to the applied forces
- D_{nx} plan dimension of the building at level x perpendicular to the direction of seismic loading being considered
- D_s dimension of wall or braced frame that constitutes the main lateral load resisting system in a direction parallel to the applied forces
- E modulus of elasticity of material
- $(EI)_{sh}$ flexural rigidity in steel strain hardening region
- F lateral force
- F_a acceleration-based coefficient, as defined in the 2005 NBCC
- F_x lateral force applied to level x
- G shear modulus of material
- h height of infill wall
- h_n total height of building above the base in metres
- H height of column
- I moment of inertia
- I_e earthquake importance factor of the structure
- I_g moment of inertia based on gross (uncracked) sectional properties
- i, j member ends of an element
- K, k stiffness (slope of force–deformation relationship)
- K_e elastic stiffness
- K_s spring stiffness
- l, l_1, l_2 member length
- L length of infill wall
- L_o length of overhang of an effective beam width
- L_p, l_p plastic hinge length
- M bending moment
- M_c, M_{cr} cracking moment at which the first cracking occurs
- M_i, M_j moment at ends i and j
- M_n nominal moment capacity
- M_o moment at which unloading slope changes
- M_p plastic moment
- M_y yield moment
- N total number of stories above grade
- P vertical force
- P_1, P_2, P_3, P_4 axial force levels where $P_1 > P_2 > P_3 > P_4$
- P_{yn}, P_{ync}, P_{yp} tension and compression yield force levels in a steel brace, as illustrated in Fig. 18b
- Q force in steel link
- Q_{ce} elastic limit of steel link force
- R_d ductility-related force modification factor that reflects the capability of a structure to dissipate energy through inelastic behaviour
- R_o overstrength-related force modification factor that accounts for the dependable portion of reserve strength in a structure designed in accordance with the 2005 NBCC
- $S(T)$ design spectral response acceleration for a period of T
- $S_a(T)$ 5% damped spectral response acceleration for a period of T as defined in the 2005 NBCC
- t thickness of infill wall
- T fundamental lateral period of vibration of the building or structure in seconds in the direction under consideration
- u_e elastic bond stress
- u_f frictional bond stress
- V lateral earthquake design force at the base of the structure, as determined by the equivalent static force procedure
- V_c shear force at which the first diagonal tension crack occurs
- V_d lateral earthquake design force at the base of the structure as determined by dynamic analysis
- V_e lateral earthquake elastic force at the base of the structure as determined by dynamic analysis
- V_y shear force at which shear yielding occurs
- w width of equivalent wall strut
- α angle of strut with the horizontal
- α_h length of the contact surface between equivalent strut and column
- α_L length of the contact surface between equivalent strut and beam
- δ axial displacement of a steel brace
- $\delta_{a.s.}$ extension of reinforcement in the adjoining member due to anchorage slip
- δ_{max} maximum storey displacement at the extreme points of the structure at level x in the direction of earthquake
- δ_{ave} average of the displacements at the extreme points at level x
- Δ displacement
- $\Delta_{a.s.}$ displacement caused by anchorage slip
- Δ_{TIP} tip deflection of a cantilever member
- Δ_y yield deflection

ϵ_s steel strain
 ϵ_{sh} strain at the onset of steel strain hardening
 ϵ_y yield strain of steel
 ϕ curvature
 ϕ_u ultimate curvature
 ϕ_y yield curvature
 γ link distortion

γ_p plastic link distortion
 γ_p^{av} average panel zone distortion
 γ_y yield link distortion
 θ member rotation
 $\theta_{a.s.}$ member-end rotation caused by anchorage slip
 θ_p plastic rotation