Design for forces induced by seismic torsion[1]

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Abstract: Eccentricities between the centres of rigidity and centres of mass in a building cause torsional motion during an earthquake. Seismic torsion leads to increased displacement at the extremes of the building and may cause distress in the lateral load-resisting elements located at the edges, particularly in buildings that are torsionally flexible. For an equivalent static load method of design against torsion, the 1995 National Building Code of Canada specifies values of the eccentricity of points through which the inertia forces of an earthquake should be applied. In general, the code requirements are quite conservative. They do not place any restriction on the torsional flexibility, however. New proposals for 2005 edition of the code which simplify the design eccentricity expressions and remove some of the unnecessary conservatism are described. The new proposals will require that a dynamic analysis method of design be used when the torsional flexibility of the building is large. Results of analytical studies, which show that the new proposals would lead to satisfactory design, are presented.

Key words: torsional response to earthquake, natural torsion, accidental torsion, design for torsion, National Building Code of Canada, interdependence of strength and stiffness.

Résumé : Les excentricités entre les centres de rigidité et les centres de masse d'un édifice produisent un mouvement de torsion lors d'un tremblement de terre. La torsion sismique conduit à un déplacement accru aux extrémités de l'édifice et peut causer un stress dans les éléments latéraux résistant aux charges et localizés aux extrémités, particulièrement avec les édifices qui sont flexibles en torsion. En tant que méthode de conception contre la torsion, équivalente à celle de charges statiques, le Code national du bâtiment du Canada de 1995 spécifie des valeurs d'excentricités pour des points sur lesquels les forces d'inertie d'un séisme devraient être appliquées. En général, les exigences du code sont vraiment conservatrices. Cependant, elles ne placent aucune restriction sur la flexibilité en torsion. Les nouvelles propositions pour la prochaine version du code, qui simplifient les expressions des excentricités de conception et retirent des parties conservatrices non-nécessaires, sont décrites dans cet article. Les nouvelles propositions vont demander qu'un méthode d'analyse dynamique pour la conception soit employée lorsque la flexibilité en torsion de l'édifice est large. Des résultats d'études analytiques sont présentés et montrent que les nouvelles propositions devraient conduire à une conception satisfaisante.

Mots clés : réponse en torsion à un séisme, torsion naturelle, torsion accidentelle, conception pour la torsion, Code national du bâtiment du Canada, inter-dépendance entre résistance et rigidité.

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Introduction

Observation of damage during earthquakes, including some recent earthquakes, has indicated that torsional oscillations often cause severe distress in buildings. In the elastic range of responses, torsional motion results when the centres of rigidity of the structural system do not coincide with the centres of mass. Structures with non-coincident centres of mass and rigidity are referred to as asymmetric structures or torsionally unbalanced structures, and the torsional motion induced by asymmetry or unbalance is referred to as natural torsion. Asymmetry may in fact exist even in a nominally symmetric structure because of uncertainty in the evaluation of the centres of mass and stiffness, inaccuracy in the measurement of the dimensions of structural elements, or lack of precise data on material properties, such as the modulus of elasticity. Torsional vibrations may also result from a rotational motion of the ground about the vertical axis. Torsions arising from undetermined asymmetry and ground rotational motion are together referred to as accidental torsion.

The 1995 National Building Code of Canada (NBCC 1995) contains specific provisions for design against torsion. Changes to the torsion design provisions of the 1995 NBCC are being proposed for the 2005 edition of the NBCC. The

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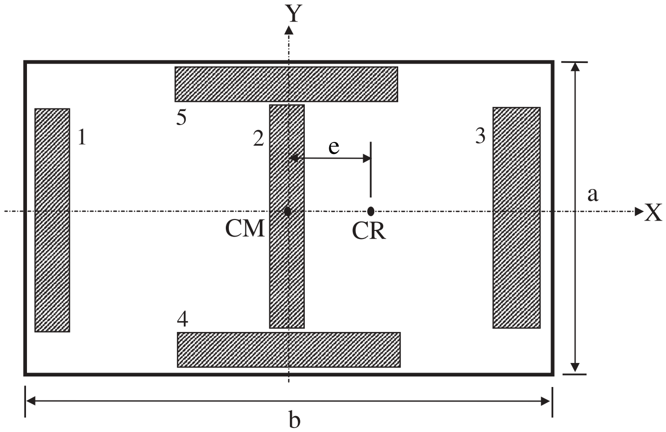
Fig. 1. Model of a single-storey torsionally unbalanced building. 1–5, resisting planes.



effectiveness of the 1995 NBCC provisions is examined in this paper through an analytical study of the elastic response of simple asymmetric building models to earthquake motions represented by idealized response spectra. The motivations for changes to the 1995 NBCC provisions are outlined, and revised provisions and their adequacy are discussed.

Torsion design provisions of the 1995 NBCC

The design provisions of the 1995 NBCC are, to a large extent, based on a study of the elastic response of an idealized structural model of a single-storey building, such as the one shown in Fig. 1. In this model the building floor is assumed to be infinitely rigid in its own plane, so it moves as a rigid body. The entire mass of the structure is distributed at the floor level. The origin of the coordinate axis is assumed to be at the mass centre, denoted by CM. Forces opposing the motion are provided by vertical structural elements, referred to as resisting planes, oriented along the two orthogonal axes. The resisting planes may comprise columns, shear walls, braced frames, or a combination thereof. The ith plane parallel to the x axis has a stiffness k_{xi}, and the ith plane in the y direction has a stiffness k_{yi}. The distribution of stiffness is assumed to be symmetrical about the x axis but is unsymmetrical about the y axis. Thus the centre of stiffness, which in the elastic range is also the centre of resisting forces CR, is eccentric with respect to CM and lies at a distance e from CM.

The inertia force produced by an earthquake acting in the y direction acts through the centre of mass and causes the model to twist about CR. As a result, the flexible side of the building, i.e., the edge farthest from CR, experiences increased displacement. In the 1995 NBCC the effect of torsional motion is accounted for by carrying out a static analysis of the building model for its response to the design earthquake force applied through a point that is eccentric with respect to CR. The distance between CR and the assumed point of action of the inertia force generated by the earthquake is referred to as the design eccentricity. To account for the possible amplification in torsion produced by the dynamic nature of response and for the accidental torsion, the design eccentricity e_d is assumed to vary over a range of values as given by the following expressions:

$$[1a]\quad e_{d1} = 1.5e + 0.1b$$

$$[1b]\quad e_{d2} = 0.5e - 0.1b$$

where b is the dimension of the building model perpendicular to the direction of the earthquake.

It is stipulated that the resisting structural elements be designed for the larger of the shears obtained when the total design shear acts at either e_{d1} or e_{d2} from the CR. In general, e_{d1} controls the design of elements on the flexible side, i.e., those farther from the CR, and e_{d2} controls the design of elements on the stiff side, i.e., those closer to the CR.

In eqs. [1a] and [1b], the term 0.1b is assumed to account for the accidental torsion. In fact, recent research (De La Llera and Chopra 1994) has shown that a design eccentricity of about 0.05b is enough to cover the effect of accidental torsion. Thus, the following components of the design eccentricities given by eqs. [1a] and [1b] may be considered as being available to take care of the natural torsion:

$$[2a]\quad \bar{e}_{d1} = 1.5e + 0.05b$$

$$[2b]\quad \bar{e}_{d2} = 1.5e - 0.05b$$

It may be noted that, although the 1995 NBCC provisions were essentially derived from studies on the single-storey building model shown in Fig. 1, the torsional response of a special class of multistoreyed buildings, in which the storey centres of resistance lie on one vertical line while the centres of mass lie on another vertical line, is similar to that of an associated single-storey model having the same eccentricity between the centre of resistance and the centre of mass.

Torsional response of single-storey building model

It is of interest to examine the elastic torsional response of the single-storey building model shown in Fig. 1 to an earthquake acting in the y direction. The response is calculated here by the standard mode superposition method of analysis, assuming that the earthquake can be represented by a response spectrum, which is hyperbolic in shape. The properties of the building model are defined in terms of the total stiffness in the y direction, K_y, the floor mass, m, the radius of gyration about CM, r, and the torsional stiffness about CR, $K_{\theta R}$. The last parameter is given by

$$[3]\quad K_{\theta R} = \sum_{i=1}^{N} k_{y_i}(x_i - e)^2 + \sum_{i=1}^{M} k_{x_j} y_i^{\,2}$$

where x_i is the distance from the origin of the ith resisting plane oriented in the y direction, y_j is the distance from the origin of the jth resisting plane oriented in the x direction, N is the number of planes in the y direction, and M is the number of planes in the x direction. It is also useful to define an uncoupled translational frequency ω_y and an uncoupled torsional frequency ω_θ given by

$$[4a]\quad \omega_\theta = \sqrt{\frac{K_y}{m}}$$

[4*b*] $$\omega_y = \sqrt{\frac{K_{\theta R}}{mr^2}}$$

The ratio of the two uncoupled frequencies is denoted by Ω_R, so

[5] $$\Omega_R = \frac{\omega_\theta}{\omega_y}$$

The maximum deflections of the flexible edge, Δ_f, and that of the stiff edge, Δ_s, are calculated by mode superposition analysis using a complete quadratic combination (CQC) of the modal values. These values are then normalized by Δ_0, the maximum displacement in the y direction produced by the same earthquake in an associated torsionally balanced building with K_y and m values similar to those of the asymmetric building but coincident centres of mass and rigidity. The normalized edge displacements are denoted by $\overline{\Delta}_f$ and $\overline{\Delta}_s$. In the elastic range these displacements are proportional to the normalized shears in the resisting planes on the flexible and stiff edges, respectively.

The normalized flexible edge displacement, $\overline{\Delta}_f = \Delta_f/\Delta_0$, is plotted as a function of Ω_R for four different values of eccentricity ratio e/b and a plan aspect ratio of $a/b = 1$ in Fig. 2. In all cases $\overline{\Delta}_f$ is greater than 1, implying that the flexible edge displacement in the unbalanced structure is greater than the displacement of the associated torsionally balanced structure. Of particular interest is the fact that there is a sharp increase in $\overline{\Delta}_f$ when Ω_R falls below about 1.0. It is also of interest to note that resonance between uncoupled translational and torsional frequencies, i.e., when $\Omega_R = 1$, does not cause any significant increase in response. Frequency resonance is not therefore a critical issue. Plots of normalized stiff edge displacement $\overline{\Delta}_s = \Delta_s/\Delta_0$ are shown in Fig. 3, again for four different values of e/b and a plan aspect ratio of $a/b = 1$. Displacement $\overline{\Delta}_s$ is less than 1 for $\Omega_R \geq 1$. For $\Omega_R < 1$, that is for torsionally flexible structures, $\overline{\Delta}_s$ starts to increase and can be substantially higher than 1. The results presented in Figs. 2 and 3 clearly suggest that buildings with low torsional stiffness may experience large deflections, causing distress in both structural and nonstructural components. Similar results, not presented here for the sake of brevity, are obtained for other aspect ratios and other spectral shapes.

To compare the analytical values of response with those implied in the code provisions, it is useful to define effective eccentricities e_f and e_s. Eccentricity e_f is the distance from CR at which application of base shear V produced in the associated torsionally balanced building by the design earthquake would cause a static flexible edge displacement Δ_f. Eccentricity e_s is the distance from CR at which the application of V would cause a static stiff edge displacement Δ_s. The flexible edge displacement caused by the application of V at a distance e_f from CR is given by

[6] $$\Delta_f = \frac{V}{K_y} + \frac{Ve_f}{K_{\theta R}}\left(\frac{b}{2} + e\right) = \frac{V}{K_y}\left[1 + \frac{K_y e_f b}{K_{\theta R}}\left(\frac{1}{2} + \frac{e}{b}\right)\right]$$

Fig. 2. Normalized displacement of the flexible edge of a torsionally unbalanced building; hyperbolic spectrum and aspect ratio = 1.

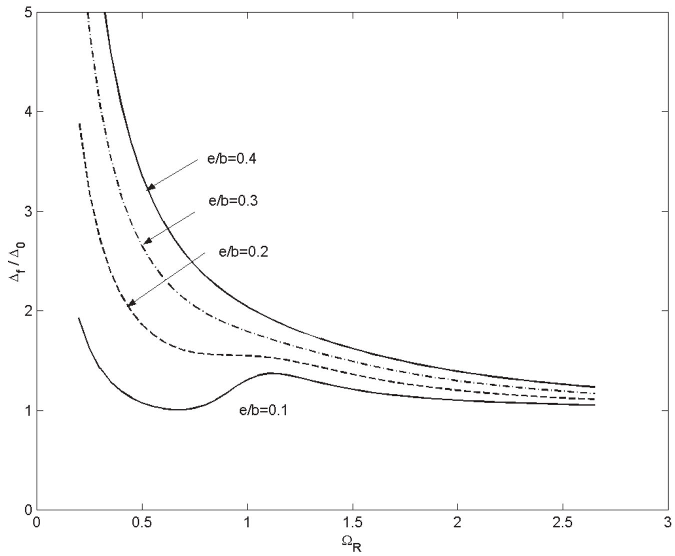
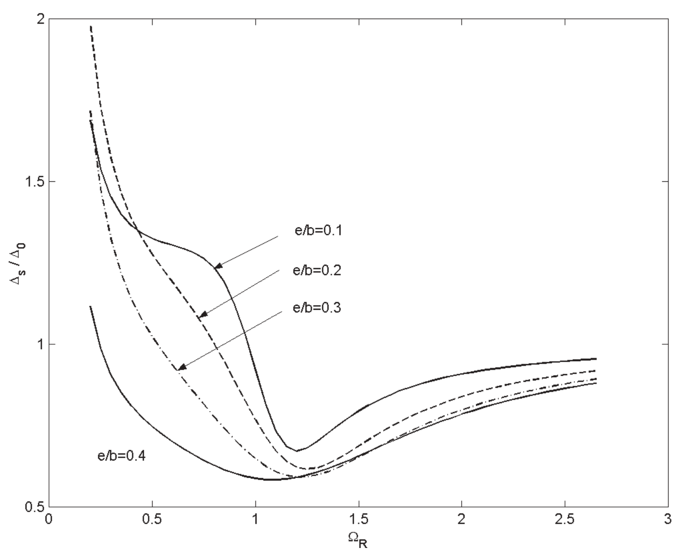


Fig. 3. Normalized displacement of the stiff edge of a torsionally unbalanced building; hyperbolic spectrum and aspect ratio = 1.



Noting that $\Delta_0 = V/K_y$ and $\overline{\Delta}_f = \Delta_f/\Delta_0$, eq. [6] reduces to

[7] $$\overline{\Delta}_f = 1 + \frac{K_y e_f b}{K_{\theta R}}\left(\frac{1}{2} + \frac{e}{b}\right)$$

Equations [4] and [5] give

[8] $$\frac{K_{\theta R}}{K_y} = \Omega_R^2 r^2$$

Substitution of eq. [8] in eq. [7] gives

[9] $$\overline{\Delta}_f = 1 + \frac{e_f}{b}\frac{b^2}{r^2}\frac{1}{\Omega_R^2}\left(\frac{1}{2} + \frac{e}{b}\right)$$

or

$$[10]\quad \frac{e_f}{b} = (\bar{\Delta}_f - 1)\frac{\Omega_R^2}{(b/r)^2(1/2 + e/b)}$$

In a similar manner it can be shown that

$$[11]\quad \frac{e_s}{b} = (1 - \bar{\Delta}_s)\frac{\Omega_R^2}{(b/r)^2(1/2 + e/b)}$$

The normalized flexible edge effective eccentricity e_f/b given by eq. [10] is plotted as a function of e/b for four different values of Ω_R and an aspect ratio of 1 in Fig. 4. For the purpose of comparison, the 1995 NBCC provision given by eq. [2*a*] is also plotted in Fig. 4. The NBCC expression is clearly quite conservative, particularly for large eccentricity values. The eccentricities obtained from eqs. [10] and [11] account only for natural torsion. They should therefore be compared with the natural torsion components of the NBCC design eccentricities as given by eqs. [2*a*] and [2*b*], and not with the eccentricities given by eqs. [1*a*] and [1*b*], which include a provision for accidental torsion.

The stiff edge effective eccentricity given by eq. [11] is plotted in Fig. 5 as a function of e/b and four values of Ω_R. Also plotted is the 1995 NBCC design eccentricity expression given by eq. [2*b*]. It should be noted that in this case a smaller effective eccentricity is more conservative. The NBCC design eccentricity is conservative for $\Omega_R \geq 1.0$ but inadequate for $\Omega_R = 0.75$ and a range of values of e/b.

Fig. 4. Effective flexible edge eccentricity; hyperbolic spectrum and aspect ratio = 1.

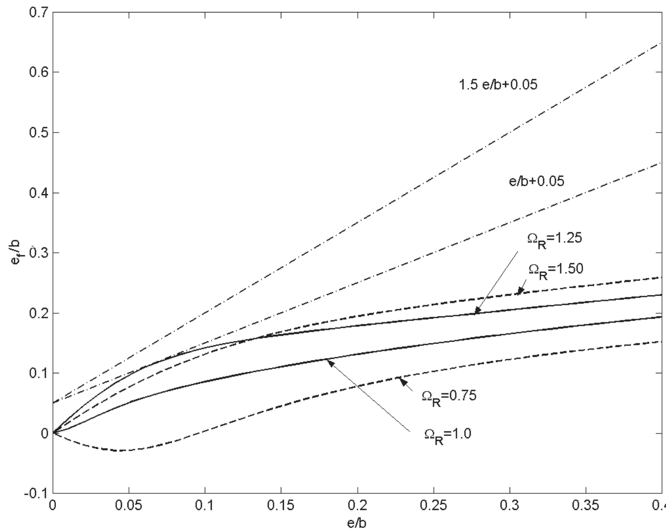
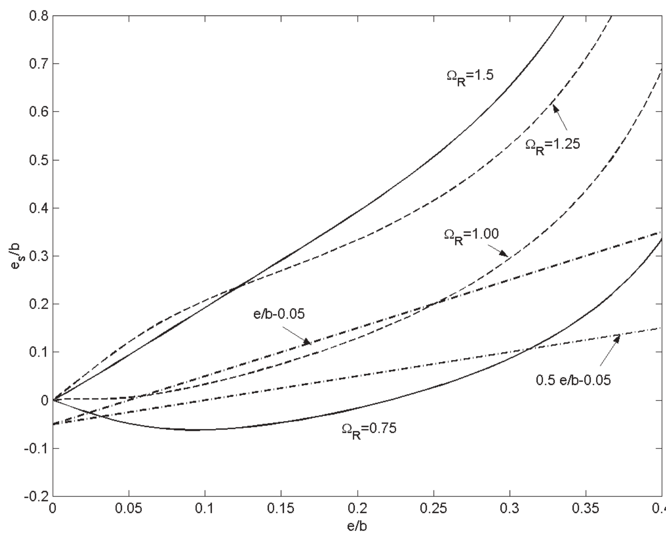


Fig. 5. Effective stiff edge eccentricity; hyperbolic spectrum and aspect ratio = 1.



Motivation for a change in design eccentricities

The results presented in Figs. 2 and 3 show that when the frequency ratio Ω_R is less than 1, torsional motion may lead to very large displacements. This is true for both the flexible and stiff edges and for all values of the eccentricity. A small value of Ω_R implies that the torsional stiffness of the structure is small and (or) that the lateral force resisting planes are located close to the centre of rigidity. The displacements in such a structure can be kept within reasonable limits only by making the resisting planes very stiff so that Δ_0 is small. In practice a design that gives low rotational stiffness should be avoided, and efforts should be made to achieve a frequency ratio greater than 1, whenever possible. The rotational stiffness of the structure can be improved by locating the lateral load resisting elements farther away from CR, for example, near the edges of the building.

The results presented in Figs. 4 and 5 and other similar results reported by Humar and Kumar (1998*a*) show that the 1995 NBCC torsion design expression for a flexible edge is overly conservative for the entire range of values of e and Ω_R. The NBCC design expression for a stiff edge is adequate, and in fact quite conservative, for $\Omega_R \geq 1.0$. For $\Omega_R <$ 1.0, however, it may lead to inadequate design for a range of values of e.

The 1995 NBCC eccentricity expressions for both the flexible and the stiff edge include a multiplier on the static eccentricity. This multiplier is 1.5 for the flexible edge and 0.5 for the stiff edge. An implication of these provisions is that e and hence the locations of the CRs must be determined. This requirement is difficult to comply with for all but the simplest of multistorey structures. This is so, first because there is no clear definition of the centre of rigidity for multistorey structures (Tso 1990), and second because, even when a definition for CR is agreed upon, the determination of such centres requires complicated analytical procedures. Methods have been devised to implement the provisions of NBCC without explicitly determining the locations of CRs, but even these require many analysis steps (Goel and Chopra 1993).

Torsion design provisions proposed for 2005 edition of the NBCC

In view of the observation that elements in torsionally flexible buildings may experience large displacement, the proposed 2005 edition of the NBCC will restrict the use of the equivalent static load method of design to buildings that are relatively stiff in torsion. The results presented in the previous sections show that the frequency ratio Ω_R provides a good measure of torsional stiffness and that, for the purpose of code provisions, buildings with $\Omega_R < 1.0$ may be considered torsionally flexible. In the case of multistorey buildings, calculation of the frequency ratio Ω_R may involve considerable computational effort and may be considered as being too complicated for routine design. An alternative measure of torsional stiffness is therefore being proposed.

In the 2005 NBCC, a building with a rigid diaphragm will be considered torsionally sensitive if a ratio B exceeds 1.7. Parameter B is determined by calculating the ratio B_x for each level x, and independently for each orthogonal direction, according to the following equation:

$$[12] \quad B_x = \frac{\delta_{max}}{\delta_{ave}}$$

where δ_{max} is the maximum storey displacement at the extreme points of the structure at level x in the direction of the earthquake induced by the equivalent static forces acting at a distance $\pm 0.1D_{nx}$ from the centres of mass at each floor, δ_{ave} is the average of the displacements of the extreme points of the structure at level x produced by the above forces, and D_{nx} is the dimension of floor x perpendicular to the direction of earthquakes. Ratio B is then taken as the maximum of all values of B_x in both orthogonal directions. In determining B, the B_x values for one-storey penthouses with a weight less than 10% of the level below need not be considered. Determination of δ_{max} and δ_{ave} requires that a three-dimensional (3D) static analysis of the structure be carried out for the equivalent static forces representing the earthquake. It is expected that such an analysis would have to be carried out anyway to determine the earthquake forces produced in individual elements of a structure, taking into account the effect of torsion. Determination of B does not therefore involve much extra computation. The 2005 NBCC will require that a dynamic analysis be carried out for determining the design forces whenever B exceeds 1.7.

To obtain a relationship between Ω_R and B, consider again the single-storey building model shown in Fig. 1. The maximum displacement produced by a shear V applied through a point whose distance is $e + 0.1b$ from CR ($0.1b$ from CM) is given by

$$[13] \quad \delta_{max} = \frac{V}{K_y} + \frac{V(e+0.1b)}{K_{\theta R}}(e+0.5b) = \frac{V}{K_y}\left[1 + \frac{K_y}{K_{\theta R}}(e+0.1b)(e+0.5b)\right]$$

Substitution of eq. [8] in eq. [13] gives

$$[14] \quad \delta_{max} = \frac{V}{K_y}\left[1 + \left(\frac{b}{r}\right)^2 \frac{1}{\Omega_R^2}\left(\frac{e}{b}+0.1\right)\left(0.5+\frac{e}{b}\right)\right]$$

The minimum displacement, derived in a similar manner, is

$$[15] \quad \delta_{min} = \frac{V}{K_y}\left[1 - \left(\frac{b}{r}\right)^2 \frac{1}{\Omega_R^2}\left(\frac{e}{b}+0.1\right)\left(0.5-\frac{e}{b}\right)\right]$$

Considering that $\delta_{ave} = (1/2)(\delta_{max} + \delta_{min})$, the ratio B is

$$[16] \quad B = \frac{1 + 1/\Omega_R^2(b/r)^2(e/b+0.1)(0.5+e/b)}{1 + 1/\Omega_R^2(b/r)^2(e/b+0.1)(e/b)}$$

The ratio b/r depends on the aspect ratio of the building and is given by

$$[17] \quad \frac{b}{r} = \sqrt{\frac{12}{1+\alpha^2}}$$

where $\alpha = a/b$ is the aspect ratio.

Figure 6 shows the variation of B with e/b for four different values of Ω_R and an aspect ratio of 1.0. It is clear from the plots that by restricting B to 1.7 as a condition for the use of the equivalent static method of design, the use of such a method of design will be precluded for buildings having $\Omega_R = 0.75$, except when the eccentricity ratio is smaller than about 0.04.

For buildings where the equivalent static load method of design is permitted, the 2005 NBCC will specify the following values for the design eccentricities:

$$[18a] \quad e_{d1} = e + 0.1b$$

$$[18b] \quad e_{d2} = e - 0.1b$$

As in the 1995 NBCC, eq. [18*a*] will govern the design of resisting elements on the flexible side, and eq. [18*b*] will usually govern the design of elements on the stiff side. Considering that a portion $0.05b$ of the design eccentricity is required to cover accidental torsion, the eccentricity values available to account for natural torsion are

$$[19a] \quad \bar{e}_{d1} = e + 0.05b$$

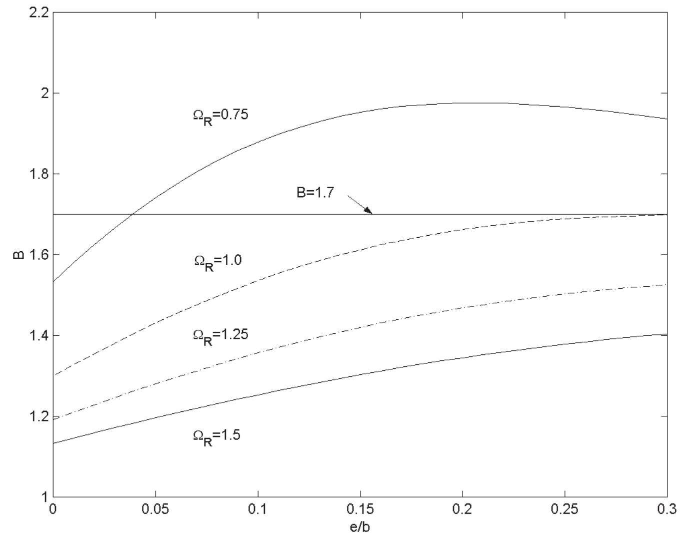
$$[19b] \quad \bar{e}_{d2} = e - 0.05b$$

For the purpose of comparison with the results obtained from a spectral analysis, eq. [19*a*] has also been plotted in Fig. 4. The design eccentricity value is appropriate for the entire range of eccentricities and is, in fact, quite conservative, particularly for larger values of e/b. Equation [19*b*] is plotted in Fig. 5 for comparison with the effective eccentricity obtained from spectral analysis. Eccentricity $\bar{e}_{d2}$ is less than the spectral effective eccentricity for the entire range of e/b, signifying that the design eccentricity expression given by eq. [19*b*] is adequate. When compared to the 1995 NBCC eqs. [1*a*] and [1*b*], the use of eqs. [19*a*] and [19*b*] offers several advantages. First, the latter lead to an adequate, but not overly conservative design. Second, as stated earlier, the application of eqs. [18*a*] and [18*b*] is easier because these equations do not include a multiplier on e, so the position of CR need not be determined. All that is required is to carry out two sets of 3D static analyses, one with the earthquake forces applied at $0.1b$ from the CMs, and another with the same forces applied at $-0.1b$ from the CMs.

Effect of torsion on inelastic response

The code provisions for design against torsion are based on studies of the elastic response of torsionally unbalanced buildings to earthquake motion. Since a majority of building structures are expected to become inelastic during the design earthquake, it is important to examine whether buildings de-

Fig. 6. Variation of torsion sensitivity parameter B with eccentricity and frequency ratio.



signed for torsion-induced forces derived from an elastic analysis perform adequately in the inelastic range.

In a building that remains elastic during the design earthquake, torsional motion would cause an increase in the lateral forces imposed on some of the planes. The application of earthquake forces through points that are eccentric with respect to the CR provides the increased forces resulting from torsion. The design then ensures that the yield strength of each plane is equal to or greater than the maximum force imposed on it. Of course, a check is also required to ensure that the maximum displacement is within acceptable limits.

In an inelastic system the requirement of design is to limit the ductility demand on the resisting planes so that it is no greater than that in the associated torsionally balanced system. For the design of a torsionally balanced structure that is expected to be strained into the inelastic range during the design earthquake, the 1995 NBCC specifies that the design shear V_0 be obtained from

$$[20] \quad V_0 = \frac{V}{R}$$

where V is the design shear when the structure remains elastic, and R is a modification factor related to the ductility capacity of the structure. It is expected that a satisfactory design for a torsionally unbalanced structure expected to be strained into the inelastic range could be achieved by designing each resisting plane to have a yield strength that is equal to or greater than the maximum force determined by applying V_0 through points whose distances from CR are equal to the design eccentricities. Humar and Kumar (1998*b*) have studied this with reference to the response of a single-storey building model shown in Fig. 1 to a set of different earthquakes. In their study the strengths f_1–f_3 assigned to the three resisting planes oriented in the y direction are obtained from the following expressions:

$$[21a] \quad f_1 = V_0 \frac{k_1}{K_y}\left[1 + \frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{d1}}{b}\left(\frac{e}{b} + 0.5\right)\right]$$

$$[21b] \quad f_2 = V_0 \frac{k_2}{K_y}\left[1 + \frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{d1}}{b}\frac{e}{b}\right]$$

$$[21c] \quad f_3 = V_0 \frac{k_3}{K_y}\left[1 + \frac{1}{\Omega_R^2}\left(\frac{b}{r}\right)^2 \frac{e_{d2}}{b}\left(0.5 - \frac{e}{b}\right)\right]$$

The results obtained by Humar and Kumar (1998*b*) show that in all cases the ductility demand in the flexible edge plane is lower than that in the associated torsionally balanced building. The same is true of the stiff edge plane, at least for buildings with $\Omega_R \geq 1.0$.

Studies performed by Humar and Kumar (1998*a*, 1998*b*) also show that the design provisions derived from response calculation for the single-storey model apply also to multistorey buildings, particularly those of a special class in which the centres of mass and centres of rigidity at the various floors lie on two vertical lines.

Interdependence of strength and stiffness

In determining the seismic forces induced by torsion the stiffness of the individual resisting planes must be known. These stiffnesses also enter into the strength equations, such as those given by eqs. [21*a*]–[21*c*]. The implication is that the stiffness is an independent parameter, which is known at the beginning of design. In reality, strength and stiffness are interdependent, so when a revised strength value is determined from one of eqs. [21*a*]–[21*c*], assignment of such strength will also result in a change in stiffness. Design calculations must then be repeated with the updated values of stiffness and the process will have to be continued until convergence is achieved.

It is apparent that, like many other design procedures, design for seismic torsion is iterative in nature because of the interdependence of strength and stiffness. The latter is easily seen to be the case in the design of steel structures, where each rolled section has a given yield strength and a given stiffness, so if a new section is selected to satisfy a strength requirement, the stiffness also changes. It is sometimes believed that in reinforced concrete sections, the stiffness is determined from the section geometry and the yield strength can be decreased or increased without changing the geometry, and hence without changing the stiffness. Recently, Paulay (1998, 2000) and Priestley and Kowalsky (1998) have emphasized the fact that the foregoing is not true, particularly when the reinforced concrete section is strained into the inelastic range. These authors have shown that the yield curvature of a reinforced concrete section depends only on the yield strain of steel and the depth (in case of walls, the length) of the section and is quite insensitive to the amount of steel or the axial load on the section. This implies that stiffness will vary almost in direct proportion to the strength.

The interdependence of strength and stiffness will always affect the design process, irrespective of the torsion design provisions used, whether the 1995 or 2005 NBCC. The fact that a study of such interdependence is being presented here in the context of revisions to the torsion design provisions does not imply that the impact of interdependence is specific only to the revised provisions. The motivation for presenting the study arises from the fact that the effect of a relationship between strength and stiffness has been recently highlighted in the literature, with particular reference to design for seismic torsion.

The need to carry out iteration in the design process, arising because the initial assumption for the stiffness of an element proves to be incorrect when a section is selected for that element to satisfy a strength criterion, is not unique to design for seismic torsion. Consider, for example, the design of a redundant steel structure for the forces of wind. To determine the forces produced by factored wind loads in individual elements of the structure, initial assumptions must be made for the stiffness of such elements. When the elements are sized for the calculated forces, the initial assumptions of stiffness may prove to be incorrect. The design process must then be repeated with modified values of element stiffness until convergence is achieved. As an illustration of the interdependence of strength and stiffness, consider a rectangular reinforced concrete shear wall. A section of the wall is shown in Fig. 7. The wall has a length l_w and thickness w. The reinforcing steel is placed in two layers and distributed uniformly along the length. The first and last sets of bars are placed at distance d_c from the nearest concrete face. The reinforcing bars are placed at an even spacing of s. The Canadian standard for design of concrete structures, CSA-A23.3-94 (CSA 1994), recommends an effective moment of inertia $I_{eff} = 0.7I_g$ for walls, where I_g is the gross moment of inertia of the shear wall cross section. Paulay and Priestley (1993) suggest values between $0.3I_g$ and $0.5I_g$. As shown here, however, the true value of the moment of inertia, or the flexural rigidity EI, depends on a number of parameters, including geometry, reinforcement ratio, layout of reinforcement, and axial load.

A representative moment–curvature relationship for the cross section shown in Fig. 7 is plotted in Fig. 8. It is derived on the assumption that in bending plane sections remain plane and using the stress–strain diagrams for concrete and reinforcing steel shown in Fig. 9. The flexural rigidity, i.e., the slope of the moment–curvature curve, is high as long as the concrete below the neutral axis, which is in tension, does not crack. Beyond the cracking moment, the concrete in tension becomes ineffective and the flexural rigidity decreases. The next significant event takes place when the reinforcing bars farthest from the neutral axis start to yield. The moment at this point is denoted by M'_y and the curvature by ϕ'_y. Beyond this point, the slope of the moment–curvature curve decreases progressively as additional layers of steel start to yield. Ultimately, the failure of section takes place either through fracture of steel layers or crushing of concrete in compression.

It has been shown that for the purpose of analysing a reinforced concrete structure for its dynamic response to earthquake motion, it is adequate to represent the moment–curvature relationship for individual members by an idealized bilinear relationship (Paulay 1998) (broken lines in Fig. 8). The first branch of the curve is obtained by drawing a straight line from the origin passing through the point (M'_y, ϕ'_y) and terminating at point (M_y, ϕ_y). A number of alternative methods have been proposed for defining the point

Fig. 7. Wall section with distributed steel: (*a*) wall cross section, (*b*) strain at section, and (*c*) stress at section.

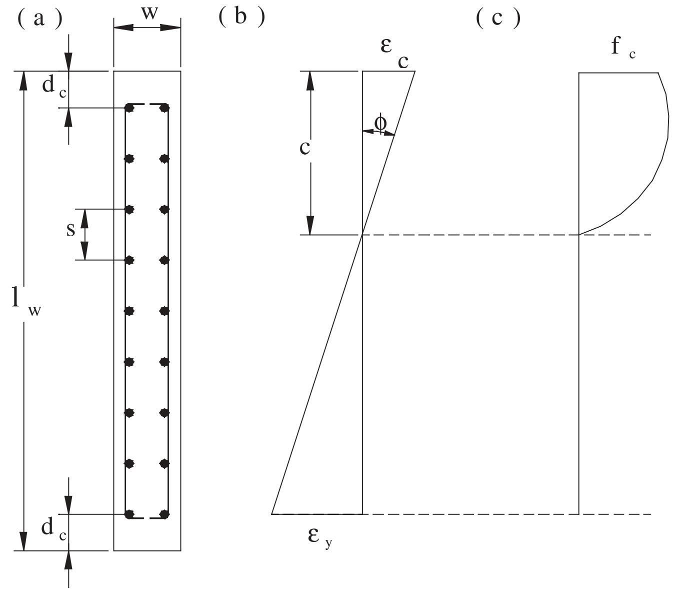
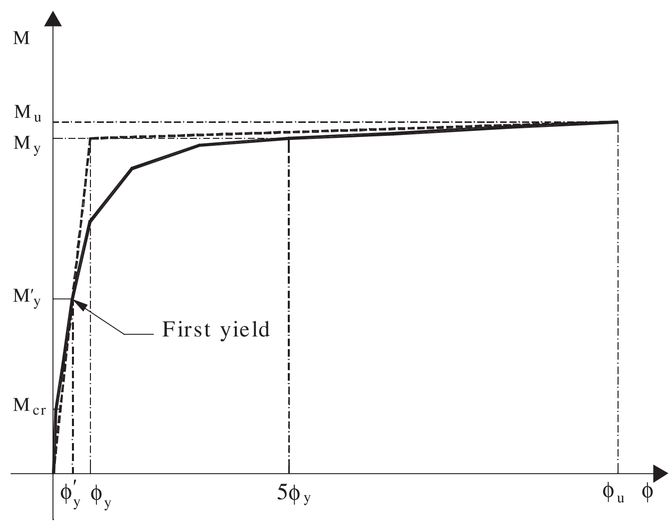


Fig. 8. Moment–curvature relationship for a rectangular concrete wall.



(M_y, ϕ_y). For example, Priestley and Kowalsky (1998) have proposed that point (M_y, ϕ_y) be selected so that a horizontal straight line through (M_y, ϕ_y) intersects the moment–curvature curve at $5\phi_y$.

Humar and Yavari (2002) recently studied the moment–curvature relationships for concrete shear walls. They obtained the effective flexural rigidity E_cI_{eff} based on the slope of the initial branch of the idealized bilinear moment–curvature relationship and compared it with E_cI_g, where I_g is the gross moment of inertia of the shear wall cross section. The variation of the ratio I_{eff}/I_g with the axial load and longitudinal steel ratio (ρ) is shown in Fig. 10, where the axial load ratio is defined as N/f'_cA_g, where N is the axial load, f'_c is the compressive strength of concrete, and A_g is the gross cross-sectional area. The ratio I_{eff}/I_g varies from 0.07 to 0.50 as the steel ratio varies from 0.25 to 3.00% of the gross

Fig. 9. Stress–strain relationships: (*a*) reinforced concrete, and (*b*) reinforcing steel bars.

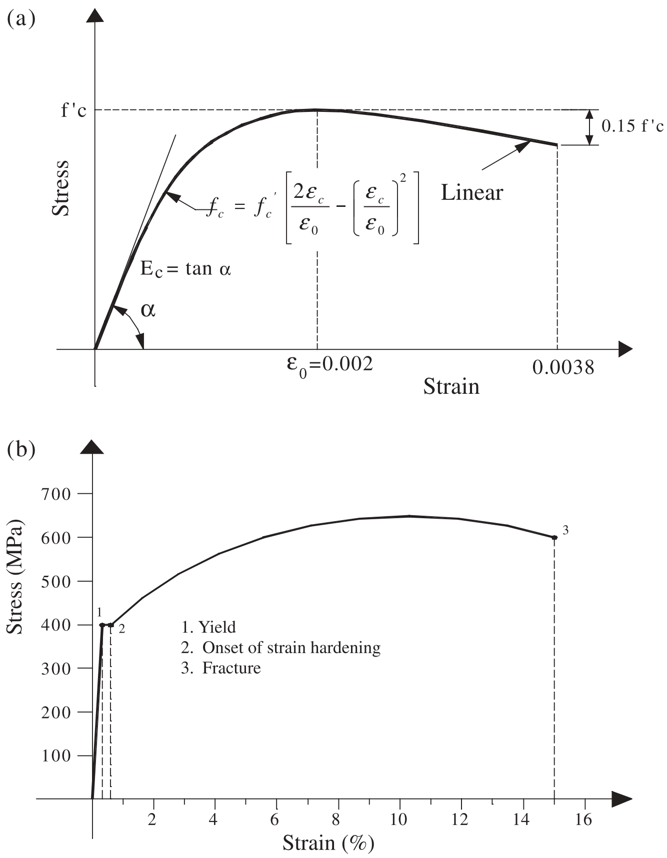


Fig. 10. Ratio of effective moment of inertia to gross moment of inertia of a rectangular wall with distributed reinforcement.

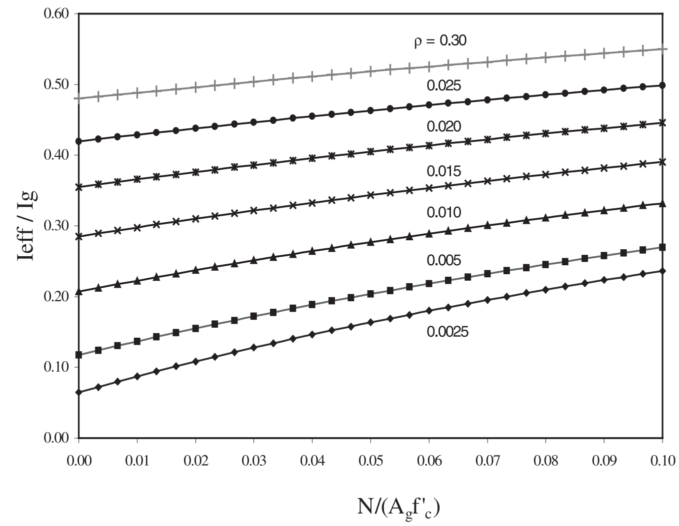
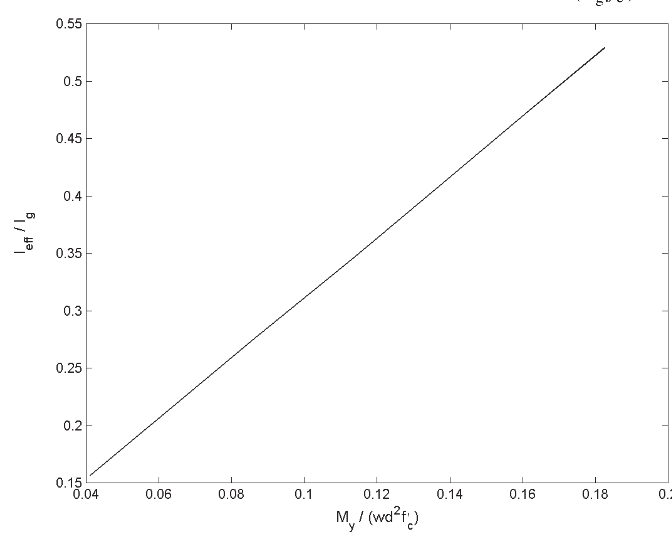


Fig. 11. Relationship between strength and the flexural rigidity of a shear wall cross section with an axial load ratio $N(A_g f_c')$.



cross-sectional area. The strength of the wall, of course, depends on the steel ratio and varies with the latter. To illustrate this further, consider again the wall in Fig. 7 subject to an axial load, the load ratio being 0.05. The relationship between the strength M_y and flexural rigidity ratio (I_{eff}/I_g), obtained by varying ρ from 0.0025 to 0.3, is plotted in Fig. 11 and indicates that the stiffness of the wall will vary in almost direct proportion to the strength. It is thus clear that strength and stiffness are not independent, but are related to each other.

Despite the interdependence between strength and stiffness, the torsion design provisions specified in eqs. [18*a*], [18*b*], and [21*a*]–[21*c*] are valid as long as correct values of the element stiffnesses are used in these equations. The design process becomes iterative, however, because of the interdependence of stiffness and strength. Examples of such an iterative design process have been presented by Humar and Yavari (2002). It will help in converging to a design solution if the initial assumptions of stiffness are made with some care. For example, in shear wall type elements, if the reinforcement ratio is expected to be near 1.5%, then on the basis of results presented in Fig. 10 an assumed value of I_{eff} between $0.30I_g$ and $0.35I_g$ would be considered reasonable.

Summary and conclusions

It is well known that asymmetric or torsionally unbalanced buildings are vulnerable to damage during an earthquake. Resisting elements in such buildings could experience large displacements and distress. The 1995 National Building Code of Canada (NBCC 1995) provides design eccentricity expressions for the design of building structures against earthquake-induced torsion. Studies have shown that the NBCC provisions may be too conservative for the design of elements on the flexible side. The provisions are also quite conservative for the design of elements on the stiff side, at least for buildings with significant torsional stiffness. The NBCC provisions apply to buildings that are fairly regular but place no special restrictions on the torsional stiffness of the structure. In addition, multipliers on the static eccentricity included in design eccentricity expressions imply that the positions of the centres of rigidity be explicitly determined, which often requires complicated computations.

The new torsion design provisions proposed for the 2005 edition of the NBCC overcome some of the difficulties associated with the 1995 NBCC provisions. The new provisions require that a dynamic analysis procedure be used for the design of torsionally flexible buildings. The conservatism associated with the 1995 NBCC design provisions is reduced. In addition, the multipliers on the static eccentricity values included in the design eccentricities are removed, so the static eccentricity and hence the locations of the centres of rigidity need not be explicitly determined.

The design procedures in both the 1995 NBCC and the 2005 NBCC require that the stiffness of the individual elements be known before the yield strength of resisting elements that takes into account torsion-induced forces can be determined. The stiffness and strength are interrelated parameters, however. This is known to be true of steel sections. Recent studies have highlighted the fact that this is also true of concrete sections. The interdependence of the strength and stiffness does not invalidate the torsion design provisions, however, as long as the correct stiffness is used in the associated equations. The design process, of course, becomes iterative in nature.

Acknowledgments

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References

CSA. 1994. Design of concrete structures. Standard CSA-A23.3-94, Canadian Standards Association, Rexdale, Ont.

De La Llera, J.C., and Chopra, A.K. 1994. Accidental and natural torsion in earthquake response and design of buildings. Report 94/07, Earthquake Engineering Research Centre, University of California, Berkeley, Calif.

Goel, R.K., and Chopra, A.K. 1993. Seismic code analysis of buildings without locating centres of rigidity. ASCE Journal of Structural Engineering, **119**: 3039–3055.

Humar, J.L., and Kumar, P. 1998*a*. Torsional motion of buildings during earthquakes. I. Elastic response. Canadian Journal of Civil Engineering, **25**: 898–916.

Humar, J.L., and Kumar, P. 1998*b*. Torsional motion of buildings during earthquakes. II. Inelastic response. Canadian Journal of Civil Engineering, **25**: 917–934.

Humar, J.L., and Yavari, S. 2002. Design of concrete shear wall buildings for earthquake induced torsion. *In* Proceedings of the 4th Structural Speciality Conference, 5–8 June 2002, Montréal, Que. Canadian Society for Civil Engineering, Montréal, Que.

NBCC. 1995. National Building Code of Canada 1995. Institute for Research in Construction, National Research Council of Canada, Ottawa, Ont.

Paulay, T. 1998. Torsional mechanisms in ductile building systems. Earthquake Engineering and Structural Dynamics, **27**: 1101–1121.

Paulay, T. 2000. Understanding torsional phenomena in ductile systems. Bulletin of the New Zealand Society for Earthquake Engineering, **33**: 403–420.

Paulay, T., and Priestley, M.J.N. 1993. Seismic design of concrete and masonry structures. John Wiley & Sons Inc., New York.

Priestley, M.J.N., and Kowalsky, M.J. 1998. Aspects of drift and ductility capacity of rectangular structural walls. Bulletin of the New Zealand Society for Earthquake Engineering, **31**: 73–85.

Tso, W.K. 1990. Static eccentricity concept for torsional moment estimation. ASCE Journal of Structural Engineering, **116**: 1199–1212.

List of symbols

a dimension of the building parallel to the direction of earthquake
A_g gross cross-sectional area of shear wall
b dimension of building perpendicular to the direction of earthquake
B torsional sensitivity ratio
B_x torsional sensitivity ratio for floor x
c distance to neutral axis from the compression face of the shear wall cross section
d effective depth of shear wall cross section
d_c distance of the first set of reinforcing bars from the nearest face of concrete
D_{nx} dimension of floor x perpendicular to direction of earthquake
e eccentricity between the CM and CR
e_d, e_{d1}, e_{d2} design torsional eccentricities
$\bar{e}_{d1}$, $\bar{e}_{d2}$ components of design eccentricities to cover natural torsion
e_f effective flexible edge eccentricity determined from spectral analysis
e_s effective stiff edge eccentricity determined from spectral analysis
E modulus of elasticity
E_c modulus of elasticity of concrete
f_c compressive stress in concrete
f_c' compressive strength of concrete
f_i strength of the ith plane
I moment of inertia
I_{eff} effective moment of inertia of shear wall cross section
I_g gross moment of inertia of shear wall cross section
k_{xi} stiffness of the ith lateral load resisting element parallel to x axis
k_{yi} stiffness of the ith lateral load resisting element parallel to y axis
K_y total lateral stiffness in the y direction
$K_{\theta R}$ torsional stiffness about CR
l_w length of shear wall
m floor mass
M_{cr} moment at first cracking in shear wall cross section
M_u moment at failure of shear wall cross section
M_y effective yield moment for defining the bilinear moment–curvature relationship
M_y' moment at the instant of first yield of a reinforcing bar
N axial load on the shear wall cross section; also defined as the number of planes in y direction
r radius of gyration about CM
R force modification factor related to ductility
s spacing of reinforcing bars
V seismic base shear in an elastic structure
V_0 seismic base shear in an inelastic structure
w thickness of shear wall

x_i distance from the origin of the *i*th resisting plane oriented in the *y* direction
y_j distance from the origin of the *j*th resisting plane oriented in the *x* direction
α plan aspect ratio (= *a*/*b*)
δ_{ave} average of δ_{max} and δ_{min}
δ_{max} maximum storey displacement at the extreme points of a structure at level *x*
δ_{min} minimum storey displacement at the extreme points of a structure at level *x*
Δ_f maximum deflection of the flexible edge
Δ_s maximum deflection of the stiff edge
Δ_0 maximum displacement in *y* direction of a torsionally balanced building
$\overline{\Delta}_f$ normalized flexible edge displacement (= Δ_f/Δ_0)
$\overline{\Delta}_s$ normalized stiff edge displacement (= Δ_s/Δ_0)
ε_0 strain corresponding to the compressive strength of concrete
ε_c compressive strain in concrete
ε_y strain in steel at tensile yielding
ϕ curvature
ϕ_u curvature at moment M_u
ϕ_y curvature at moment M_y
ϕ_y' curvature at moment M_y'
ρ steel reinforcement ratio
ω_y uncoupled translational frequency
ω_θ uncoupled rotational frequency
Ω_R ratio between ω_θ and ω_y