

# Effect of foundation rocking on the seismic response of shear walls<sup>1</sup>

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**Abstract:** Some designers have long known that elastically responding shear-wall or core-wall type high-rise structures will not overturn if the footing size is smaller than that required to resist the elastic forces. Most shear walls are designed and built with a yield hinge mechanism at the base using a relatively high value of the force reduction factor  $R$ , and the foundation should be stronger than the yield hinge strength if the wall is to perform as designed. Many walls, however, built with  $R = 2$  are stronger than they need to be because of reasons such as architectural sizing and minimum reinforcement requirements. If for these cases the foundation is to be stronger than the wall, then it will in effect be designed for forces corresponding to an  $R$  value of  $<2$ . This study looks at the effect on the displacement of a shear-wall type structure if the footing is allowed to rock. The structure is kept elastic and the footing is sized to correspond to  $R$  values ranging from 1.0 to 3.5. The analysis uses gap elements to model the foundation soil response so that the footing can lift off the soil. Soil stiffness and strength are modelled for a rock and a firm clay site. The response of 7-, 15-, and 30-storey structures to 11 different acceleration records, modified to match a spectrum given in the 1995 National Building Code of Canada (NBCC) for Vancouver, is determined for the different footing dimensions. The results indicate that a footing sized for an  $R$  value of 2 does not result in a significant increase in displacement when compared with the fixed base elastic case. In the next version of the NBCC it is suggested that footings need not be designed for forces corresponding to  $R < 2$ .

*Key words:* seismic shear walls, overturning, liftoff, rocking footings.

**Résumé :** Certains concepteurs savent depuis longtemps que les tours avec murs de refends répondant de manière élastique au cisaillement ne se renverseront pas si les dimensions de la fondation sont plus petites que celles requises pour résister aux forces élastiques. La plupart des murs de refends sont conçus et construits avec un mécanisme de rotule d'écoulement à la base, utilisant une valeur relativement élevée du facteur de réduction de force  $R$ , et la fondation devrait être plus résistante que la rotule d'écoulement pour que le mur se comporte comme escompté. Cependant, beaucoup de murs, construits avec, disons,  $R = 2$ , sont plus résistants que ce qu'il leur suffirait d'être, du fait de raisons qui ont rapport notamment aux exigences de taille architecturale et de renforcement minimum. Dans ce cas, si la fondation doit effectivement être plus résistante que le mur, alors, en pratique, il sera construit pour des forces correspondant à une valeur de  $R$  inférieure à 2. Cette étude considère l'effet d'un éventuel balancement de la fondation sur le déplacement d'une structure type mur de refends. La structure reste élastique et les dimensions de la fondation correspondent à des valeurs de  $R$  allant de 1,0 à 3,5. L'analyse utilise la méthode des écarts pour modéliser la réponse de la fondation de façon à ce que la fondation superficielle puisse s'arracher du sol. La rigidité et la résistance du sol sont modélisées pour un roc et un site d'argile solide. La réponse des structures de 7, 15 et 30 étages à onze accélérations enregistrées, modifiées afin de correspondre au spectre donné par le Code National du Bâtiment du Canada 1995 (NBCC) pour Vancouver, est déterminée pour chacune des dimensions de la fondation. Les résultats indiquent que, si la taille de la fondation correspond à une valeur de  $R$  égale à 2, aucune augmentation significative du déplacement s'ensuit par rapport au cas élastique de base fixe. Dans la prochaine version du NBCC, il est suggéré que la fondation n'ait pas besoin d'être construite pour des forces correspondant à  $R < 2$ .

*Mots clés :* murs de refends sismiques, renversement, arrachement, fondations basculantes.

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## Introduction

In the design of core walls as the lateral load resisting element in tall structures, it is quite common, because of architectural and minimum steel requirements, that the moment capacity of the core is greater than that required by the design code loads. The National Building Code of Canada (NBCC 1995) requires that the flexural capacity of the foundation be greater than the capacity of the walls. This is to ensure that damage does not occur in the footing, which could be difficult to assess or repair. This has resulted in very large footings, and many designers have questioned whether footings could be made smaller and allowed to rock on the soil.

Housner (1956) derived period and energy loss estimates for a block rocking on a rigid foundation; Priestley et al. (1978) extended this to look at the response under seismic loading; Byrne (1980) looked at the response of buildings on soft foundations; Yim and Chopra (1985) considered foundation uplift on elastic soils; Psycharis (1991) considered the response of single degree of freedom structures with uplift and gave some parametric rules to predict deformations; and Filiatrault et al. (1992) looked at a particular tall structure with a weak foundation. The present paper extends the analysis to consider how a reduction in footing size influences the displacement for buildings with different heights and designed with different footing sizes.

This paper presents the results of nonlinear numerical analyses of vertical cantilevers (core walls) where the footing is allowed to lift off and rock on the soil. The main response of interest is the displacement, or drift ratio, of the structure. The buildings investigated are 7, 15, and 30 storeys tall, situated in Vancouver, British Columbia. The structure is assumed to remain elastic, but footing sizes corresponding to code forces for values of the force reduction factor  $R = 1.0, 1.5, 2.0,$  and  $3.5$  have been investigated. A second parameter of importance is the ratio of the vertical load supported by the core to the total weight of the structure, termed the mass ratio (MR). This has a large influence on the footing size and the subsequent response. Mass ratios of 0.2, 0.4, and 0.6 have been considered. Two different soil conditions corresponding to sites with rock and with stiff clay have been investigated.

## Code proposal

According to the proposed 2005 edition of the NBCC if the footings are larger than what would be required for forces using an  $R$  value of 2.0, then they can be reduced in size to the  $R = 2.0$  value. This is the same as proposed in the draft Australia/New Zealand Standard DR00902 (Standards Australia 2002). This paper describes some of the work supporting this proposal.

## Background

Although the code design loads are treated like static loads, they represent the maximum inertial loads that occur during cyclic response to the ground motions. If the structure, while remaining elastic, cannot resist these inertial loads, then it deforms inelastically until the cyclic nature of

the motion reverses the forces. This is the reason we can use  $R$  factors and design for forces smaller than the elastic forces. The same philosophy applies to rocking footings; if the footing cannot resist the overturning moments, then it will lift off and rotate until the motion reverses. The response of a wall structure that has a rocking footing or a flexural hinge at the base is conceptually very much the same, the difference mainly being that the flexural hinge will dissipate more energy than the rocking footing. Thus, the displacements of the structure with the rocking footing may be greater than those of the structure with a flexural hinge, and this is the issue investigated in this paper.

## Analysis

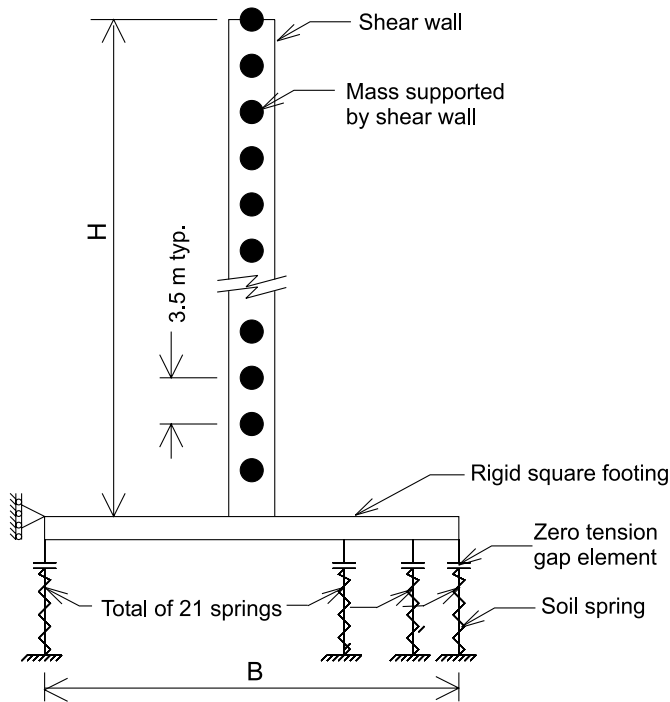
Figure 1 shows the analysis model used to represent a shear-wall structure. The wall was assumed uniform and had lumped masses representing the proportion of storey mass supported vertically by the wall, and located at 3.5 m storey intervals. A dummy column, not shown in the figure, supported the remaining part of the storey mass and was slaved to have the same lateral displacements as the wall so as to provide the correct lateral inertial forces to be carried by the shear wall. This dummy column provided no lateral resistance but was included for  $P$ -delta or second order purposes. The ratio of the mass supported vertically by the shear wall to the total mass is termed the mass ratio (MR). MR values of 0.2, 0.4, and 0.6 are thought to be representative of the practical range found in high-rise structures.

The mass assigned to each floor represents a typical Vancouver high-rise with a 30 m square floor plate with a weight  $W = 6480$  kN/floor. The stiffness of the wall was adjusted so as to give a first mode period, assuming the base of the wall was fixed against rotation, of  $T = 0.1N$ , where  $N$  is the number of storeys. This was selected so that the study would cover a reasonable range of periods and building heights.

The wall was supported by a square footing that was taken to be very stiff. In practice, the walls of the core are sometimes not that far removed from the edges of the footing, and so the footing itself may have little deformation. Some initial analyses were done with a flexible footing but with little difference to the results, so the decision was made to assume a very stiff footing, as this reduced the number of parameters in the study.

The footing has been designed for the overturning moment as calculated using the static procedure in NBCC (1995) for  $R$  values of 1.0, 1.5, 2.0, and 3.5. Two different soil conditions have been considered, one a rock or very stiff till, and the other a softer and weaker clay soil. The size of the footing is influenced by the vertical load (which is affected by the mass ratio), the overturning moment, and the soil strength. The vertical load is assumed to be supported on the toe of the footing using a bearing pressure equal to the ultimate bearing capacity of the soil, and then the size is based on the required eccentricity, or overturning moment divided by the vertical load. The smaller the mass ratio, the larger the footing, as the eccentricity is larger. This method of determining the footing size implicitly assumes liftoff but is the approach usually taken, even if the footing is not

**Fig. 1.** Model of shear wall and foundation spring system.  $B$ , footing size;  $H$ , height of structure (storey interval typically 3.5 m).



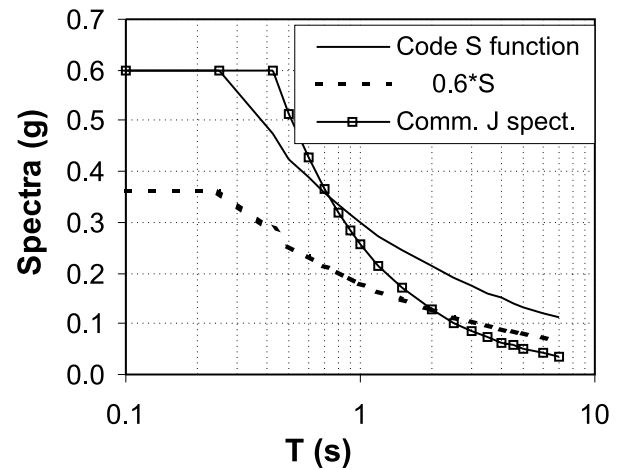
thought of as a rocking footing. No load factors were used in this calculation. The footing size was also checked that it would satisfy the factored gravity dead and live loads of a typical residential building, but this load case never governed the footing size. For this check it was assumed that the long-term or service strength of the soil was one third of the short-duration or ultimate bearing capacity used in the seismic analysis.

The stiffness of the soil springs has been selected to match the rotational stiffness of the soil supporting the footing as given by Veletsos and Wei (1971). Matching the rotational stiffness with equally spaced springs results in a total vertical stiffness that does not agree with the vertical soil foundation stiffness. For this analysis, however, it is considered that the rotational stiffness is more important than the vertical stiffness.

Viscous damping was maintained at 5% critical in the first two modes of vibration for the structure when supported on the soil springs. Stiffness-proportional damping was not included in the foundation springs or gap elements, as inclusion in these elements may produce excessive damping when the springs yield or the gaps open. No other foundation damping was considered, which might be conservative, as there would be some foundation radiation damping. Because the structure is designed not to yield, however, the 5% damping used in the structure may be high, somewhat compensating for the neglect of radiation damping. Yielding of the foundation springs was accounted for, but this would not increase the damping much because the yielding is not cyclic.

The earthquake records used in the nonlinear overturning analyses have all been modified using the program SYNTH (Naumoski 1985) to match the spectrum suggested in Com-

**Fig. 2.** Comparison of code S function and Commentary J spectrum (Comm. J spect.).  $T$ , period.



mentary J to the NBCC (1995). Figure 2 shows the code static S function and the spectrum. Although the code S function, which is used to determine the footing size, is greater than the spectrum for the periods of interest here, 0.7 s and greater, applying the code calibration factor  $U = 0.6$  factor to the S function brings it closer to the spectrum. Eleven different records from a wide variety of earthquake sources were used as seed records and modified by SYNTH. Although they were matched to the same spectrum, there was about a 40% difference in the elastic base shear value among the results.

The nonlinear dynamic analysis was carried out using the program DRAIN-2DX (Prakash et al. 1993).

## Results

For three different size buildings on rock and one 30-storey building on clay, Table 1 lists the size of the footing, the rotational stiffness of the soil springs, and the resulting first mode period when foundation flexibility is considered (with no liftoff). The period has been calculated using an approximate relation but was checked in several cases and found to be in good agreement with the results from a full analysis. The table also gives the foundation properties used and the relations for determining rotational stiffness and the fundamental period of vibration.

Note that for high mass ratios and large  $R$  values, the footings are very small and the period change from the fixed base case is large. Many of these footings would fall outside the range of practical consideration. At low mass ratios many of the footings are very large and in many cases they are larger than the footprint of the tower and thus would be carrying additional dead load, so they are not representative of a real case. The low mass ratios show that footings can become large, however, thus the urge by designers to reduce footing size.

The main result of interest is the structure displacement, represented here by the average drift ratio, defined as the top displacement divided by the height, produced by the 11 different earthquake records. Figures 3a–3d show the average drift ratio as a function of the  $R$  value used in determining the footing size. Figure 3 shows that, aside from the 7-storey

**Table 1.** Footing size ( $B$ ), foundation rotational stiffness ( $K_o$ ), and fundamental period ( $T$ ).

	$R$	Mass ratio = 0.2			Mass ratio = 0.4			Mass ratio = 0.6		
		$B$	$K_o$	$T$	$B$	$K_o$	$T$	$B$	$K_o$	$T$
Structure		(m)	(kN·m)	(s)	(m)	(kN·m)	(s)	(m)	(kN·m)	(s)
7R	1.0	36.9	$6.7 \times 10^{10}$	0.70	18.5	$8.4 \times 10^9$	0.70	12.4	$2.5 \times 10^9$	0.71
	1.5	24.6	$2.0 \times 10^{10}$	0.70	12.3	$2.5 \times 10^9$	0.71	8.3	$7.6 \times 10^8$	0.74
	2.0	18.4	$8.9 \times 10^9$	0.70	9.3	$1.1 \times 10^9$	0.73	6.3	$3.3 \times 10^8$	0.79
	3.5	10.6	$1.6 \times 10^9$	0.72	5.4	$2.1 \times 10^8$	0.84	3.8	$7.0 \times 10^7$	1.06
15R	1.0	44.6	$1.2 \times 10^{11}$	1.50	22.3	$1.5 \times 10^{10}$	1.51	15.0	$4.5 \times 10^9$	1.53
	1.5	29.7	$3.5 \times 10^{10}$	1.50	14.9	$4.4 \times 10^9$	1.53	10.1	$1.4 \times 10^9$	1.59
	2.0	22.3	$1.5 \times 10^{10}$	1.51	11.3	$1.9 \times 10^9$	1.57	7.7	$6.0 \times 10^8$	1.71
	3.5	12.8	$2.8 \times 10^8$	1.55	6.6	$3.8 \times 10^8$	1.82	4.7	$1.3 \times 10^8$	2.28
30R	1.0	65.1	$3.7 \times 10^{11}$	3.00	32.6	$4.6 \times 10^{10}$	3.01	21.9	$1.4 \times 10^{10}$	3.04
	1.5	43.4	$1.1 \times 10^{11}$	3.00	21.8	$1.4 \times 10^{10}$	3.04	14.7	$4.3 \times 10^9$	3.12
	2.0	32.6	$4.6 \times 10^{10}$	3.01	16.4	$5.9 \times 10^9$	3.08	11.2	$1.9 \times 10^9$	3.26
	3.5	18.7	$8.6 \times 10^9$	3.06	9.6	$1.2 \times 10^9$	3.40	6.8	$4.1 \times 10^8$	4.04
30C	1.0	65.2	$2.2 \times 10^{10}$	3.02	33.2	$2.9 \times 10^9$	3.17	23.1	$9.8 \times 10^9$	3.47
	1.5	43.6	$6.6 \times 10^9$	3.07	22.7	$9.3 \times 10^8$	3.50	16.4	$3.5 \times 10^8$	4.19
	2.0	32.9	$2.8 \times 10^9$	3.17	17.5	$4.3 \times 10^8$	4.01	13.3	$1.9 \times 10^9$	5.01
	3.5	19.2	$5.6 \times 10^8$	3.79	11.2	$1.1 \times 10^8$	5.98	9.9	$7.6 \times 10^7$	6.96

**Note:** Typical structure type notation: 7R, 15R, and 30R denote 7-, 15-, and 30-storey structures on rock; 30C denotes a 30-storey structure on a clay foundation. Rock foundation: ultimate bearing pressure capacity = 10 000 kPa and modulus of elasticity  $E = 1.0 \times 10^7$  kPa. Clay foundation: ultimate bearing pressure capacity = 3600 kPa and  $E = 6.0 \times 10^4$  kPa.  $K_o = 0.133EB^3$  is the rotational stiffness of the foundation;  $T = (T_o^2 + T_f^2)^{0.5}$  is the estimated structure fundamental period, where  $T_o$  is the period of a rigid structure on a flexible foundation, and  $T_f$  is the period of a structure assuming a fixed base =  $0.1N$  in this analysis.

structure on rock, the drift ratio increases modestly up to  $R = 2$ . It was on the basis of these results that the code proposal was made to limit the size of the footing to be not smaller than that for  $R = 2$ . The drift of a 7-storey structure with a mass ratio (MR) of 0.6 increases more rapidly with an increase in  $R$  than those of the other structures; however, the drifts are still small for this case, which represents one of the highest seismic hazards in Canada. It is clear from Fig. 3 that the taller structures with lower mass ratios could have smaller footings than those for  $R = 2$  and still not have an appreciable increase in the drift ratio.

The soil properties chosen for this study are representative of very good foundation conditions. This choice was made because strong soil results in the use of small footings and may have a large effect when rocking is considered; it was also felt that 30-storey buildings would not be built on poor soils. It was somewhat surprising to see that the 30-storey structure on the more flexible clay had slightly smaller drift than the structure on rock, especially since the structures on clay have longer periods than those supported on rock. The clay-supported structure has a larger footing with the same resulting moment capacity as that of the structure on rock, and this may have more influence on the maximum drift than the initial elastic period.

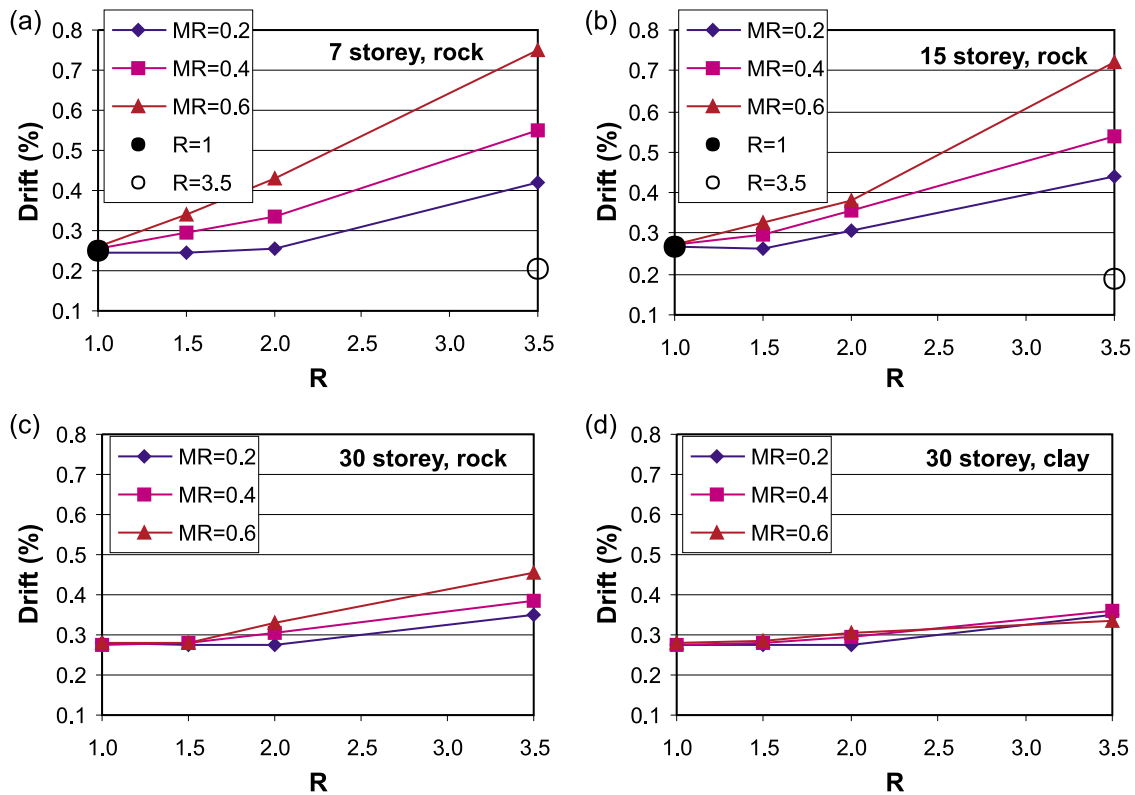
At the time the study was carried out the intent was to check the footing sizes given by the existing code (NBCC 1995) static procedure and see if they could be reduced. In carrying out the nonlinear dynamic analyses the spectral shape given in Commentary J of the NBCC was used. It was not scaled to give the same base shear as the code static procedure, however, as is recommended in the code for design, because this does not guarantee that the overturning moment from the static procedure and the dynamic procedure would be the same.

For this study it may have been more sensible if the spectra had been scaled so that, for the fixed base elastic case, the elastic overturning moment from the dynamic analysis matched the overturning moment from the static procedure. A modal analysis shows that for this to be the case the spectra should be multiplied by 0.78 for the 7-storey structure, 1.01 for the 15-storey structure, and 1.36 for the 30-storey structure. This trend is not unexpected, considering the relative shape of the spectrum and the S function shown in Fig. 2.

Thus the footing designed with the static procedure for the 7-storey structure is smaller than it should be for comparison with the dynamic analysis, and by the same token the footing for the 30-storey structure is larger than it should be. Figure 3 could be corrected to account for this by dividing the  $R$  factors by the ratios given in the previous paragraph. Although this would improve the performance of the 7-storey structure and reduce the performance of the 30-storey structure, it would not change the outcome appreciably. The 7-storey structure would still have increased displacements with an increase in  $R$ , and the 30-storey structure would remain much less sensitive.

Also shown in Figs. 3a and 3b for the 7- and 15-storey structures on rock, is the average drift ratio for two fixed base but yielding structures. The large circles in the figures are the drift ratios for the cases where the base is assumed fixed but the wall is allowed to yield at the base with an elastic – perfectly plastic yield hinge. The  $R = 1$  fixed base drift ratio is essentially the same as the  $R = 1$  case where liftoff is allowed. The  $R = 3.5$  fixed base case, however, has a much smaller drift ratio than the corresponding rocking footing case. The decreased drift for the yielding case is attributed to increased energy dissipation from the hysteretic yielding.

**Fig. 3.** Drift ratio versus  $R$  value: (a) 7-storey structure on rock foundation, (b) 15-storey structure on rock foundation, (c) 30-storey structure on rock foundation, and (d) 30-storey structure on clay foundation. MR, mass ratio.

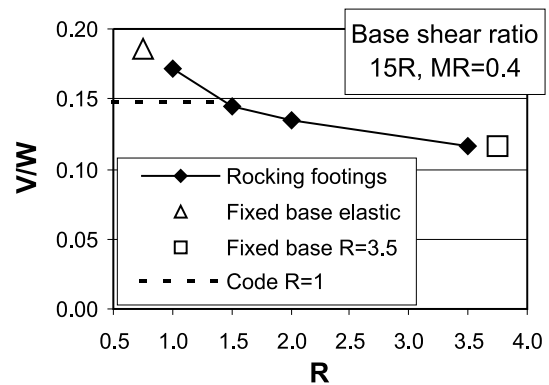


Also of interest is the base shear for the rocking footing cases. Figure 4 shows the average value (from the different earthquake records) of the maximum base shear for different  $R$  values for the 15-storey building on rock with a mass ratio of 0.4. The scatter in the results from the 11 different earthquake records was slightly less than  $\pm 20\%$ , and plots for other building sizes and mass ratios showed the same trends. The code base shear for  $R = 1$  is shown by the broken line at the left side of the plot. Also shown, for two different assumptions on wall strength, is the calculated base shear assuming the structure is fixed at the base. In the first case the wall has been assumed to remain elastic and the base shear, shown by the open triangle to the left of  $R = 1$ , is very close to but slightly larger than the base shear when the footing is sized for  $R = 1$  moments but allowed to lift off. In the second case the wall was designed to have a base yield moment capacity corresponding to  $R = 3.5$ , and the base shear is shown by the open square to the right of the  $R = 3.5$  line. Here the result is nearly identical to that found for the case where the footing is allowed to lift off. Note that the calculated  $R = 1$  base shear is greater than code base shear, and that the reduction in base shear for higher values of  $R$  is not proportional to  $R$ .

**Discussion**

The results confirm what many designers know, namely that footings can be made smaller than that required to resist the moment capacity of the shear wall and the building will not fall over. What this study shows is that the footing size can be reduced to a size consistent with  $R = 2$  forces without

**Fig. 4.** Base shear ratio for different  $R$  values.  $V$ , base shear;  $W$ , structure weight.



resulting in appreciably larger displacements of the structure, especially in those cases where the shear wall does not support much of the gravity load. It is in these light axial load cases that footings become very large, and so a reduction in size becomes important.

The present NBCC and Canadian Standards Association (CSA) material codes allow  $R = 1.3$  to be used as a cutoff for designing footings, i.e., footings do not have to be designed for  $R < 1.3$ . The proposal for the new code increases this to  $R = 2$ , an increase by a factor of 1.5. For Vancouver the 1.5 factor is not too different from the proposed increase in seismic hazard in the new code for the period range of interest here. Thus the  $R = 2$  cutoff would result in foundation sizes designed by the new code to be comparable with the  $R = 1.3$  cutoff in the old code.

Some limited comparisons show that for some structures with footing sizes determined using a high value of  $R$ , and allowed to lift off, the drifts are considerably greater than the displacements for a fixed base wall with a flexural hinge at the base, designed using the same value of  $R$ . For these cases it is important that the foundation moment capacity is greater than that of the wall so that the flexural hinge forms and the wall acts as designed.

The bearing capacity of the soil used in this analysis was high and the maximum soil pressure never exceeded the ultimate strength. Weaker soils will result in larger footings, and so the soil pressure may also not reach the capacity, but this should be checked, especially for structures with a high mass ratio. Byrne (1980) showed that if the ultimate bearing capacity was at least three times the service level strength, then the base rotation was not severe.

## Acknowledgments

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