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SEISMIC PERFORMANCE EVALUATION OF STEEL MOMENT RESISTING FRAMES

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ABSTRACT: Nonlinear dynamic analysis in time-domain is necessary to capture the response of the structure to severe ground motion and to obtain an accurate estimate of the engineering demand parameters for a structure. The information obtained from such analysis of a new or an existing structure can be used to evaluate its performance for the purpose of performance-based design or retrofit. While the building codes in Canada and elsewhere is moving towards performance-based design, there are ongoing efforts to develop simple and reliable methods for estimation of seismic performance of a structure, which is required to implement such a design approach. However, such methods need to be verified using inelastic dynamic analysis. The results of inelastic dynamic time history analysis are sensitive to the selected earthquake records, their characteristics, and modeling uncertainties. Therefore, it is usually carried out with a carefully selected suite of ground motion records. In this study, a set of buildings with steel moment resisting frames of five, ten, fifteen, and twenty stories in height and located in Vancouver area of Canada, have been considered for evaluating their seismic performance and determining the effects of the types of ground motion and scaling techniques. The key engineering damage parameters such as, interstorey drift, roof displacement, shear demand, and plastic hinge rotation have been used for the estimation of seismic performance and a suite of thirty earthquake records have been used in the analysis. Different techniques of scaling of ground motion have been considered for the selected earthquake records. In addition, a suite of eight spectrum compatible artificial records have been used in the analysis. Based on the results of the study a set of recommendations on the scaling techniques and types of ground motion.

1. Introduction

The seismic design of buildings in Canada is required to be performed according to the provisions of the National Building Code of Canada (NBCC). NBCC 2010 allows for the use of Equivalent Static Load Method (ESLM) for estimating the lateral forces due to seismic hazard for buildings with simple and regular shape and geometric configurations, and of a limited height. While dynamic analysis is recommended for all buildings, it is mandatory for structures of irregular, complex geometry and buildings of higher than the recommended level. While NBCC 2010 is not a performance-based design code, it is said to be an objective-based code that allows the use of new materials or design processes based on acceptable solutions to achieve the stated objectives in the design. In the context of performance-based design procedure, it is necessary to determine if a building designed according to the current provisions of the code actually is capable of achieving the given performance objectives assumed in the design and determine possible modifications to incorporate multiple levels of performance corresponding to various levels of seismic hazard. The research presented here looks at a number of buildings designed according to the current seismic provisions in Canada in the context of their performance achievements under the design level of seismic hazard utilizing various methods of response prediction and performance evaluation.

The objectives are outlined to determine the effect of ground motion scaling techniques on the estimation seismic demand parameters of the selected steel moment resisting frames designed based on the National Building Code of Canada (NBCC, 2010), and identify the appropriate scaling techniques. For that purpose, a set of buildings with steel moment resisting frames of 5, 10, 15, 20 stories in height and located in Vancouver area of Canada have been considered. A suite of 30 earthquake real and 8 artificial records have been used in the dynamic time history analysis. The mean values and the statistical dispersion in the key engineering damage parameters such as interstorey drift, energy dissipation and plastic hinge rotation are utilized to determine the performance of a scaling method and its practical use.

2. Seismic performance of buildings

Performance-based seismic design is a two-step process which involves performance evaluation and structural design. The main purpose of performance evaluation is to ascertain that if structure achieves the desired level performance under a specific level of seismic hazard. In that case, the capacity and the seismic response should be determined accurately to estimate the level of damage and corresponding performance of the structure. Damage parameters such as the inter-storey drift, roof-drift, joint rotation etc. are among the most widely used parameters to determine the level of seismic performance (Yousuf and Bagchi, 2009). These damage parameters can be determined using static and dynamic analyses of a structure. Usually the nonlinear time history analysis method of a structure subjected to seismic ground acceleration is considered to be more appropriate than the linear dynamic analysis to determine the response parameters accurately. During the last decade, elastic and inelastic dynamic analyses in the time domain have been made feasible for complex structures because of the rapidly increasing computational power and the evolution of engineering software.

The ground motion records can be obtained from natural earthquake records, or can be generated synthetically and artificially. However, the selection of seismic ground motion and scaling them appropriately for the use in the nonlinear time history analysis are important issues which still require further research (Naumoski et al. 2004). In order to achieve the required performance level by design, a performance objective is predefined and consists of specifications of performance level of a structure and a corresponding probability that this performance level may exceed (Yun et al 2002).

The Structural Engineers Association of California have laid down guidelines for Performance objectives under different levels of seismic hazard and performance objectives in the Vision 2000 document (SEAOC, 1995). NBCC 2010 addresses the overall building performance in a broader perspective, by considering the parameters of ground motions, site soil effects, analysis and design methodologies (De Vall, 2003). The important features for seismic design under NBCC 2010 are as follows: (a) it provides the Uniform Hazard Spectra (UHS) for the specific site to be used for seismic design purpose, the UHS has 2% probability of exceedance in 50 years with a recurrence interval of 2500 years (Humar and Mahgoub, 2003); (b) it has a broader objective to achieve the required performance and safety of the structure, hence it allows for use for alternate methods of analysis and design to meet the acceptable levels of performance, which may not be specified in the code; and (c) it also provides a description and guidelines for structural irregularity. It should be noted that NBCC 2010 does not provide explicit guidelines for seismic performance objectives, but specifies the maximum allowable interstorey drift as 2.5% for achieving life safety as a design criteria.

The buildings chosen for the performance evaluation and the research presented here are of steel moment resisting frames as the lateral load-resisting systems. They are assumed to be located in Vancouver, Canada. The Vancouver region in Canada is classified as high seismic zone as compared to the other parts of the country. Four buildings of five, ten, fifteen and twenty storey height, symmetrical steel frames are designed according to the seismic provisions of NBCC 2010 and CSA-S16-09 (CSA, 2009; CISC, 2010). Each building has a series of frames in the east-west (E-W) direction and three bays in the north-south (N-S) direction. The center to center spacing of the frames in the E-W direction is 6 meters whereas in the N-S direction the two exterior bays are of 9 meters and the interior bay is 6 meters.

The first storey height in the buildings is 4.85 meter and the remaining floors are spaced at 3.65 meter each. A typical layout plan and elevation of a five-storey building are shown in Figure 1. The other buildings (ten, fifteen and twenty storey) share the same plan and storey heights. The building frames along the north-south direction have been chosen for the design and a nonlinear dynamic analysis software, DRAIN-2DX (Prakash *et al.*, 1993) has been used for the modelling and analysis of the two-dimensional building frames. The dead load at the roof and interior level are computed to be 3.4 kPa and 4.05 kPa, respectively. Live loads on the roof, interior floor and corridor are assumed to be 2.32 kPa, 2.4 kPa, and 4.8 kPa, respectively. The design sections for beams and columns are shown in Tables 1 and 2, respectively. The fundamental period computed from the modal analysis for the 5, 10, 15 and twenty storey frames is found to be 1.41 s, 2.53 s, 3.57 s and 4.79 s, respectively. On the other hand, the period obtained from the empirical formula of the building code is 0.79 s, 1.29 s, 1.74 s and 2.15 s, respectively. Appropriate adjustment is made to the design base-shear based on fundamental period. The design base shear for the five, ten, fifteen and twenty storey frames are found to be 178.8 kN, 306.2 kN, 366.4 kN, and 401.0 kN, respectively. The weight associated with these frames are estimated to be 3580.3 kN, 7528.0 kN, 11454.1 kN, and 15104.4 kN, respectively. A response spectrum analysis has been performed to estimate the dynamic base shear to compare with the equivalent static base shear and appropriate adjustment is made and Tables 1 and 2 show the final sections.

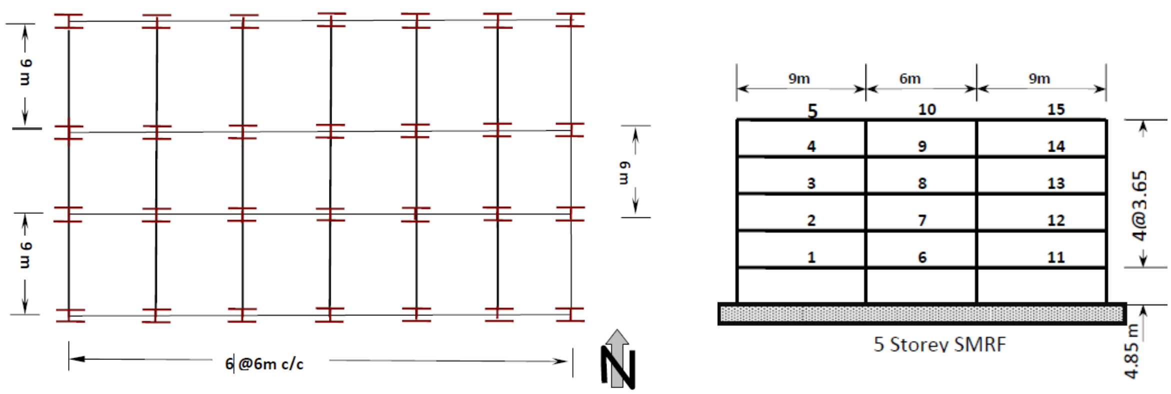


Fig. 1: Plan and elevation of the five-storey frame building

Table 1: Beam sections

Story Level	Building Height			
	5 Story	10 Story	15 Story	20 Story
Top Story	W310x79	W310x79	W310x107	W310x107
Other Story	W310x86	W310x107	W310x129	W310x129

Table 2: Column sections

Building Height	Column Row	Story 1 to 5	Story 6 to10	Story 11 to 15	Story 16 to 20
5 Story	External	W310x179			
	Internal	W310x253			
10 Story	External	W310x283	W310x158		
	Internal	W310x314	W310x202		
15 Story	External	W310x283	W310x253	W310x179	
	Internal	W360x314	W360x260	W310x283	
20 Story	External	W310x283	W310x253	W310x202	W310x179
	Internal	W360x314	W360x287	W360x262	W360x262

The pushover analysis of the frames shows the sequence of hinge formation in the structures follow a desirable pattern according to the capacity design principle (strong column-weak beam).

3. Selection of Seismic Ground Motion

Ground Motion Records (GMR) from past earthquakes around the world are often chosen to represent the seismicity of a given site. However, selection of such records is quite challenging as they may not match the sesimic hazard of a location fully, and may contain a wide variation in their characteristics. Nonlinear dynamic analysis requires the ground motion acceleration time histories whose spectrum should match the design response spectrum as far as possible. In this scenario, the ground motion records are obtained using one of the following procedures.

1) Selection of a real accelerogram from a GMR database with site specific conditions and characteristics (e.g., magnitude [M], distance [R], duration [D], soil condition [SSI]);
2) Simulate GMR from seismological model of fault rupture mechanisms; or
3) Artificial or synthetic ground motions generated from filtered noise.

Table 3 shows a set of ground motion records or input accelerograms selected for the present study. The records are chosen based on the seismicity and soil condition corresponding to Vancouver, and the peak acceleration to velocity ration (a/v) is close to 1 as expected for that location (Naumoski et al, 2004; PEER, 2014).

Table 3: Summary of Selected Ground Motion Records

Record No.	Location and year	PGA (*g*)	Peak Velocity (m/sec)	*a/v*
1	Imperial Valley (1940)	0.348	0.334	1.04
2	Kern Country (1952)	0.179	0.177	1.01
3	Kern Country(1952)	0.156	0.157	0.99
4	Borrego Country (1968)	0.046	0.042	1.09
5	San Fernando (1971)	0.150	0.149	1.01
6	San Fernando (1971)	0.211	0.211	1.00
7	San Fernando(1971)	0.165	0.166	0.99
8	San Fernando (1971)	0.180	0.205	0.88
9	San Fernando (1971)	0.199	0.167	1.19
10	Record No.S-882 Gazli USSR	0.07	0.07	1.00
11	Record No.S-634 Coalinga	0.078	0.068	1.15
12	Monte Negro-2 (1979)	0.171	0.194	0.88
13	Report Del Archivo: SUCH850919AL.T	0.105	0.112	0.94
14	Report del Archivo: VILE850919AT.T	0.123	0.105	1.17
15	Kobe, Japan (1995)	0.061	0.049	1.24
16	Kobe, Japan (1995)	0.694	0.758	0.92
17	Kobe, Japan (1995)	0.707	0.758	0.93
18	Kobe, Japan (1995)	0.144	0.150	0.96
19	Northridge, CA (1994)	0.469	0.571	0.82
20	Northridge, CA (1994)	0.510	0.493	1.03
21	Northridge, CA (1994)	0.088	0.072	1.22
22	Northridge, CA (1994)	0.080	0.082	0.98

Nonlinear dynamic analysis requires scaling of the real accelerograms for a GMR to that of the target spectrum, which can be done by scaling spectral ordinates without altering the spectral shape or scaling the spectral ordinates and modifying the spectral shape to match the target spectrum. Ideally, the analysis requires scaled real accelerograms without altering the spectral shape. This is because nonlinear displacement and ductility demands are sensitive to the details of the ground motions containing sequences of peaks and valleys as well as long duration pulses. The scaling of spectral ordinates and modification of the spectral shape could however be done in frequency domain or in time domain. From the structural damage assessment point of view, the effect of spectral matching and scaling techniques used to obtain the site-specific ground motion characteristics and the related damage potential needs to be studied.

4. Scaling of the selected GMRs

For time history analysis, a ground motion record is selected such that the record is compatible to the design spectrum. There are a number of methods available for scaling a ground motion record in order to obtain a record that would represent the seismicity of a location as expressed in the design response spectrum. Some of the most commonly used methods for scaling or deriving a design spectrum compatible ground motion records are listed below with a brief description.

5.1 Peak Ground Acceleration (PGA) Scaling

In this scaling technique the input accelerogram is multiplied by a scalar quantity to match the peak ground acceleration as that of the site specific target spectrum (Eq. 1). In Eq. 1, PGA_{ds}, and PGA_{gmr} indicate the peak ground acceleration corresponding to the expected or code specified value (target) and that of the selected ground motion.

$$\text{Scale Factor} = \frac{\text{PGA}_{\text{ds}}}{\text{PGA}_{\text{gmr}}} \qquad \text{Eq 1}$$

5.2 Ordinate Scaling Method

In this scaling technique the input accelerogram is multiplied by a scalar quantity to match the spectral ordinate at the fundamental period of vibration (T_1) of the structure as that of the target spectrum (Eq. 2, Figs. 2-3). Figures 2 and 3 show the response spectrum of an input accelerogram and the design response spectrum provided in the building code for a give site, respectively. The scale factor is obtained as the ratio of the ordinates of the design and input response spectra.

$$\text{Scale Factor} = \frac{\text{T1 Ordinate}_{\text{ds}}}{\text{T1 Ordinate}_{\text{gmr}}} \qquad \text{Eq 2}$$

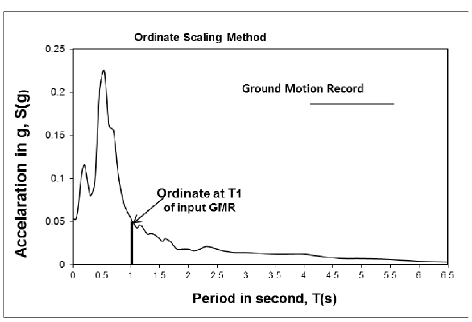


Fig. 2 Ordinate at T_1 on the input spectrum

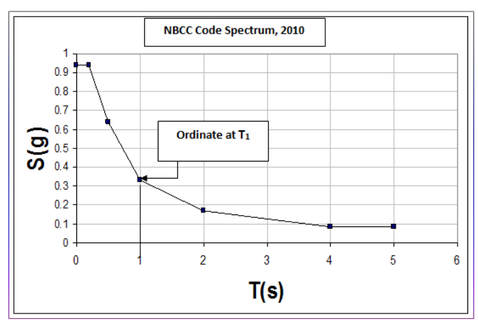
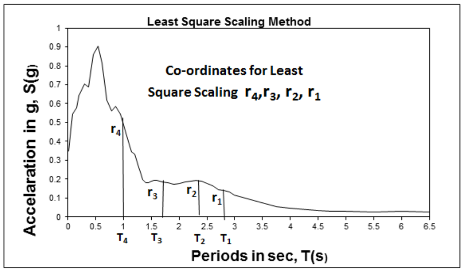


Fig. 3 Ordinate at T_1 on the NBCC 2010 Spectrum

5.3 Least Square Method

This scaling technique was proposed by Somerville et al., (1997a, b), where the input accelerogram is multiplied by a scalar that minimizes the weighted sum of the errors (differences) between the design response spectrum and the input spectrum. The weights used are 0.3, 1.0, 2.0, 4.0 at the period corresponding to the first, second, third and fourth modes (i.e., T_1, T_2, T_3, T_4), respectively (Figs.4-5).



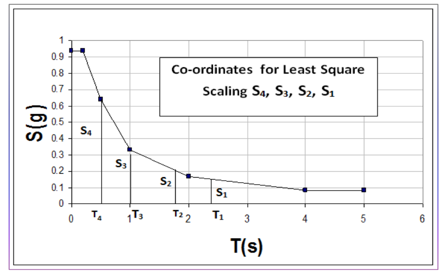
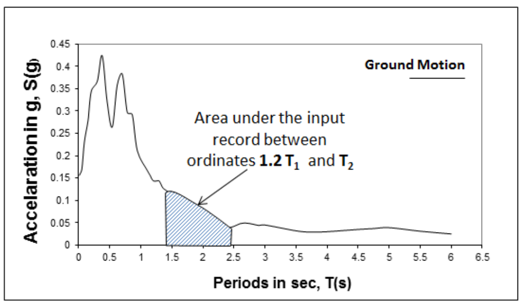


Fig. 4 Ordinates on the input GMR spectrum

Fig. 5 Ordinates on the NBCC Code Spectrum

5.4 Partial Area Method

In this scaling technique the area under the input response spectrum between the second mode period, *T2* and 1.2 times the first mode period, T_1 be the same as that of the target spectrum (Naumoski et al., 2004). The scale factor is obtained by dividing the partial area under the target spectrum by the corresponding partial area under the spectrum of the selected ground motion (Figs. 6-7).



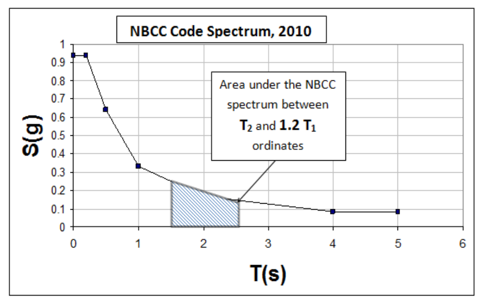


Fig. 6 Partial Area (P.A.) under the input spectrum

Fig. 7 P.A. under the NBCC 2010 Spectrum

5.5 Full Area Scaling Method (PSa).

This scaling technique requires the area under the input spectrum to be equal to that under the design spectrum within a wider range of periods, typically between range 0-2 s (Naumoski et al., 2004). This method is referred here as PSa method.

5.6 ASCE – 7 Scaling Method

This technique (ASCE, 2005) is similar to the Partial Area Method described above. However, it requires that the average value of spectral ordinates are not smaller than those of the target spectra for the period range 0.2T_1 and 1.5T_1 where T_1 is the fundamental vibration of the structure.

5.7 Spectrum Matching Technique

In this method spectrum matching is done by modifying the frequency contents of the input accelerogram to match its response spectrum to the target spectrum. There are different software programs such as SeismoMatch (Abrahamson 1992; and Hancock et al. 2006) or Synth (Naumoski et al. 2004) available for matching the frequency of input spectrum to that of the target spectrum and generating the corresponding time history signal for the ground acceleration.

5.8 Spectrum Compatible Artificial Earthquake Records (Atkinson 2009)

In this technique the input accelerogram which is pre matched with the site specific target spectrum are generated through simulation considering the geological and seismological conditions of a given site (e.g. Atkinson, 2009). Hence, these records are directly used as input accelerograms in time history analysis, and no scaling is required. In this study a set of 8 such records, 4 for short duration and 8 for long duration earthquakes, have been selected (Fig. 8).

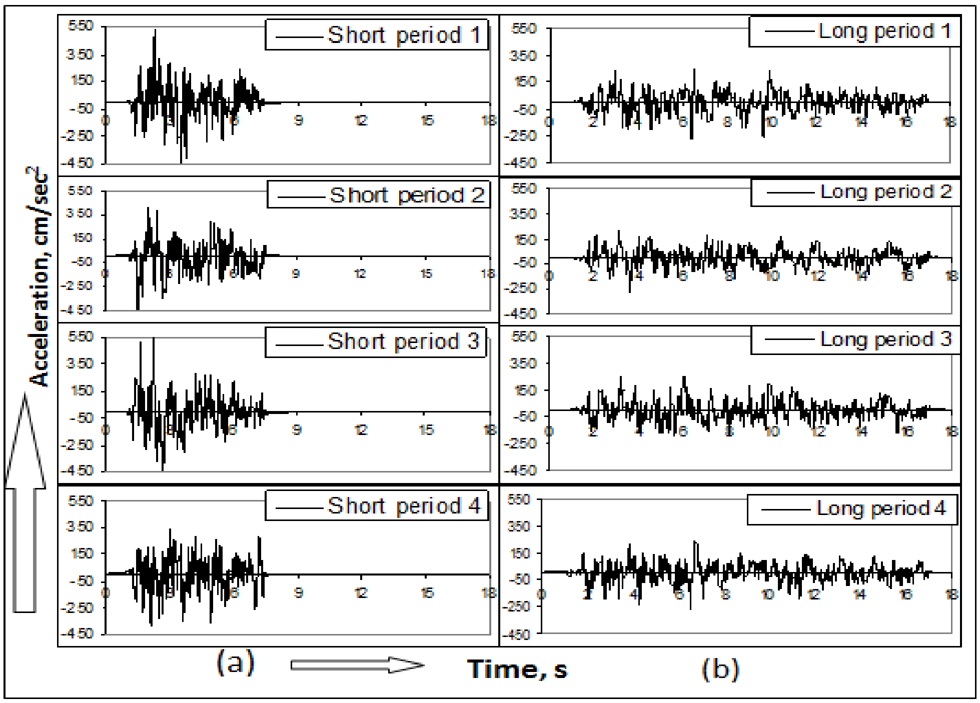


Figure 8: Time History of Atkinson's synthesized GMRs (a) Short period ground motions (b) Long period ground motions.

5. Results and discussion

Tables 5-7 show the summary of dynamic response parameters (interstorey drift, ISD; and base shear, BS) for the buildings. It is clear from these tables that the results of the dynamic time history analysis involves a significant uncertainty irrespective of the method used for scaling the ground motion records. Most methods produce large dispersion which is indicated here by the standard deviation as a percentage of the mean value in majority of the cases (i.e., more than 30%). Even the artificial records produce 33% to 57% dispersion of the interstorey drift values. Among all the methods used for scaling or matching the ground motion records to the design level of seismic hazard, the ASC-7 method and the frequency domain matching of the spectral shape as performed here by Seismo Match seem to produce the best results by limiting the level of dispersion to less than 30% except in the case of the 5-storey frame. In case of the 5-storey frame, the ordinate method of scaling produces the best result (10.5% dispersion). In cases of the 10, 15 and 20 storey buildings, the spectral matching of the ground motion records by SeismoMatch produces the best results with the following levels of dispersion: 20.6%, 21.2% and 12.0%, respectively.

Table 4: Summary of interstorey drift and base shear for the 5-storey frame

Scaling method	ISD, %h	SD, %h	Dispersion, %ISD	ISD+SD, %h	Min BS, kN	Max BS, kN
PGA	1.24	0.36	29.2	1.61	150	178
PSa	1.48	0.25	17.1	1.74	124	162
Ordinate	1.37	0.14	10.5	1.51	123	173
Partial Area	1.27	0.32	25.2	1.59	122	158
ASCE-7	1.22	0.61	50.0	1.83	210	255
Least Square	1.06	0.80	75.5	1.86	221	257
Spectrum Match	0.99	0.47	47.5	1.46	224	254
Atkinson	2.61	0.87	33.3	3.48	121	145

Table 5: Summary of interstorey drift and base shear for the 10-storey frame

Scaling method	ISD, %h	SD, %h	Dispersion, %ISD	ISD+SD, %h	Min BS, kN	Max BS, kN
PGA	0.88	0.30	33.5	1.18	165	203
PSa	1.13	0.26	22.9	1.38	173	215
Ordinate	1.30	0.41	31.6	1.71	183	220
Partial Area	1.15	0.24	21.1	1.40	178	214
ASCE-7	1.08	0.22	20.4	1.30	264	283
Least Square	1.10	0.39	35.5	1.49	261	288
Spectrum Match	0.68	0.14	20.6	0.82	272	318
Atkinson	1.92	0.79	41.1	2.71	185	218

Table 6: Summary of interstorey drift and base shear for the 15-storey frame

Scaling method	ISD, %h	SD, %h	Dispersion, %ISD	ISD+SD, %h	Min BS, kN	Max BS, kN
PGA	0.67	0.27	40.1	0.94	269	305
PSa	0.86	0.28	32.7	1.14	275	330
Ordinate	1.41	0.46	32.5	1.87	274	322
Partial Area	0.92	0.31	33.6	1.23	311	362
ASCE-7	0.89	0.18	20.2	1.07	332	357
Least Square	0.81	0.65	80.2	1.46	340	379
Spectrum Match	0.52	0.11	21.2	0.63	333	376
Atkinson	1.32	0.50	37.9	1.82	271	335

Table 7: Summary of interstorey drift and base shear for the 20-storey frame

Scaling method	ISD, %h	SD, %h	Dispersion, %ISD	ISD+SD, %h	Min BS, kN	Max BS, kN
PGA	0.67	0.16	23.6	0.83	389	470
PSa	0.84	0.22	26.8	1.06	385	423
Ordinate	1.10	0.49	44.8	1.59	379	443
Partial Area	0.84	0.31	37.4	1.15	511	552
ASCE-7	1.00	0.18	18.0	1.18	512	555
Least Square	0.54	0.48	88.9	1.02	522	545
Spectrum Match	0.50	0.06	12.0	0.56	485	547
Atkinson	2.40	1.36	56.7	3.76	396	432

Table 8: Roof displacement (% H) at Maximum M+SD of interstorey drift.

SMRF	Roof displacement (% H) (Dynamic)	Roof displacement (% H) (Pushover, at failure)
5 Storey	1.183	1.495
10 Storey	1.220	1.690
15 Storey	1.144	2.095
20 Storey	1.483	1.750

M=Mean value, SD=Standard deviation.

The ranges of the base shear (BS) obtained from the dynamic analysis using different methods of scaling of the ground motion records show a similar level of variability. However, in comparison to the design base shear as reported Section 2, the base shear from the nonlinear dynamic analysis are found to be in a similar range. For example, the design base shear from the 5-storey building frame is 178 kN, while the minimum and maximum values of the base shear obtained from the time history analysis are found to be 121 kN and 254 kN, respectively. Similar observation can be made for the other buildings. There is a wide range of variability in the response quantities irrespective of the ground motion scaling techniques used. However, the interstorey drift obtained from the time history analysis using different scaling methods show a uniform and consistent pattern of deformation in low rise to medium rise frames, whereas a greater dispersion of the results has been observed in taller buildings. The dynamic displacement demand is also consistent with that obtained from the pushover analysis at the limiting level (e.g. at failure or 2.5% interstorey drift) as shown in Table 8. Although a similar level of variability is observed in the base shear obtained from the dynamic time history analysis, they are consistent with the design base shear from the corresponding buildings.

6. Acknowledgement

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7. References

Abrahamson, N.A. (1992). Software program – RSPMatch, Non-Stationary Spectral Matching. Seismological Research Letters, 63:1, 30.

ASCE, 2005, ASCE Standard ASCE/SEI 7-05. Minimum Design Loads for Buildings and Other Structures, Including Supplement No.1, American Society of Civil Engineers, Reston, VA.

Atkinson, G.M. 2009. Earthquake time histories compatible with the 2005 National Building Code of Canada uniform hazard spectrum. Canadian Journal of Civil Engineering, 36: 991-1000.

CISC, 2010. *Handbook of Steel Construction*, Canadian Institute of Steel Construction, Willowdale, Ontario, Canada.

CSA. 2009. Design of Steel Structures, CSA-S16-09, Canadian Standards Association, Toronto, ON.

Hancock J, Bommer JJ. A state-of-knowledge review of the influence of strong-motion duration on structural damage. Earthquake Spectra 2006;22(3):827–45.

Humar, J. and Mahgoub M. A., 2003, *Determination of seismic design forces by equivalent static load method*, Canadian Journal of Civil Engineering, Volume 30 287-307.

Naumoski, N. 2001. Program SYNTH – Generation of artificial accelerograms compatible with a target spectrum. User's manual, Department of Civil Engineering, University of Ottawa, Ottawa, Ont., Canada.

Naumoski N., Saatcioglu M., and Amiri-Hormozaki K., 2004, *Effects of Scaling of Earthquake Excitations on the Dynamic Response of Reinforced Concrete Frame Buildings*, 13th World Conference on Earthquake Engineering, Vancouver, B.C., Canada, Paper No. 2917.

NBCC 2010, *National Building Code of Canada, 2005*, Canadian Commission on Building and Fire Codes, National Research Council of Canada, Ottawa

PEER, 2014, *Pacific Earthquake Engineering Research Center: NGA Database*, http://peer.berkeley.edu (cited October 2014).

Prakash V., Powell, Graham H, Campbell, Scott D., 1993, *Drain-2DX: Static and Dynamic analysis of Inelastic Plane Structure*, Department of Civil Engineering, University of California, Berkeley.

SEAOC, 1995. Performance-based seismic engineering, Vision 2000 Committee, Report prepared by Structural Engineers Association of California, Sacramento, CA

Somerville., P., N. C. Smith, and R. W. Graves, 1997a. Modification of Empirical Strong Ground Motion Attenuation Relations to Include the Amplitude and Duration Effects of Rupture Directivity, Seism. Res.Lett. 68, no. 1, 199–221.

Somerville, P., Smith, N., Punyamurthula, S. and Sun, J., 1997b. Development of ground motion time histories for phase 2 of the FEMA/SAC steel project. SAC Joint Venture, Report No.SAC/BD-97/04, California.

Yousuf, M. and Bagchi, A., 2009, "Seismic Design and Performance Evaluation of Steel-Frame Buildings Designed using NBCC-2005", *Canadian J of Civil Engineering*, 36: 280-294.

Yun S.Y., Hamburger R.O., Cornell C. A., and Foutch D.A., 2002, *Seismic Performance Evaluation for Steel Moment Frames,* Journal of Structural Engineering, Vol. 128, No. 4, pp. 534-545