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EFFECTIVE STIFFNESS FOR LINEAR DYNAMIC ANALYSIS OF CONCRETE SHEAR WALL BUILDINGS: CSA A23.3 – 2014

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ABSTRACT: The 2014 edition of Canadian Standard CSA A23.3 *Design of Concrete Structures* contains new displacement-based provisions that require the designer to estimate the top-wall displacement of shear walls due to the design earthquake. CSA A23.3 specifies the effective flexural stiffness – specifically the effective flexural rigidity $E_c I_e$ – that must be used for concrete shear walls in a linear dynamic (response spectrum) analysis. This paper summarizes the research that led to the development of the new expression for $E_c I_e$. Traditionally $E_c I_e$ has been assumed to equal the slope of the elastic portion of an equivalent elastic – plastic (bi-linear) bending moment – curvature relationship and this approach leads to the idea that axial compression increases effective stiffness. The 2004 edition of CSA A23.3 contains an expression based on this approach. Nonlinear response history analysis (NLRHA) of shear wall buildings using a rigorous model for concrete shear walls has revealed that the most important parameter influencing $E_c I_e$ is the ratio of elastic bending moment demand to the flexural strength of the shear walls. The more the flexural resistance is due to eccentric axial compression rather than tension in the vertical reinforcement, the lower the effective stiffness because of the reduced hysteretic damping.

1. Introduction

The seismic design provisions for concrete shear wall buildings in the 2014 edition of Canadian Standard A23.3 include a number of new displacement-based requirements. In order to use these design provisions, a designer must first make an estimate of the displacement of the shear wall building due to the design earthquake. This estimate is normally done by designers using a three-dimensional linear dynamic (response spectrum) analysis (RSA).

When a concrete shear wall building is subjected to the design earthquake motions, the concrete will crack over a significant portion of the shear walls and the reinforcement will yield in concentrated locations, e.g., at the base of the wall. Because of concrete cracking and reinforcement yielding, the stiffness of the concrete shear walls will be reduced from the initial uncracked concrete (gross section) stiffness. The reduction in stiffness, which is very important for making a good prediction of the shear wall displacement, is the subjected of this paper.

The important property of concrete cantilever shear walls that controls the displacement of shear wall buildings is the flexural stiffness. Shear deformations may influence the interstory drift of the building at certain levels (Bazargani and Adebar, 2015); but generally have a small influence on the top-wall displacement of the building. The flexural stiffness of a shear wall depends on the sectional stiffness, called the flexural rigidity *EI*, and the height of the wall. While the damage due to cracking will not be uniform over the height of a shear wall and the damage due to steel yielding will be even more concentrated, the approach normally taken is to apply a constant reduction factor on the flexural rigidity over the full height of the wall.

One reason a constant reduction factor has typically been applied to concrete shear walls is because the effective flexural rigidity $E_c I_e$ has traditionally been assumed to be the slope of the elastic portion of an equivalent elastic – plastic (bi-linear) moment – curvature relationship. If the moment – curvature response is bi-linear, the single effective stiffness $E_c I_e$ is appropriate over the full height.

Paulay (1986) proposed the following well-known expression:

$$E_c I_e = \left(0.6 + P \big/ f_c^{'} A_g\right) E_c I_g \qquad (1)$$

where P is the axial compression applied to the wall. A typical value of P = 0.10 $f_c'A_g$, thus the commentary to the 1994 Canadian concrete code (CPCA 1995) recommended, for simplicity, that I_e = 0.7I_g regardless of axial force level and this value continues to be used by many designers in Canada.

An alternate definition that has been suggested as a safe lower-bound estimate of effective flexural rigidity $E_c I_e$ of a concrete shear wall is the secant stiffness to the point where reinforcement yields (Paulay, 2001). This approach has the advantage that it can be easily estimated since the bending moment to cause the reinforcement to yield and the curvature when reinforcement yields can both be easily estimated. The result however is a significantly lower effective stiffness, often less than half the value given by Eq. (1). Consistent with this approach, the commentary to the seismic design provisions of the 1995 New Zealand concrete code recommended I_e = 0.25I_g for a wall with no axial compression and I_e = 0.35I_g for a wall with an axial compression force equal to 10% of $f_c'A_g$.

The FEMA 356 Prestandard for Seismic Rehabilitation of Buildings (ASCE, 2000) recommended only a 20% reduction in stiffness for uncracked concrete shear walls, and a 50% reduction in stiffness for previously cracked concrete shear walls. These reductions, which can be summarized as I_e = 0.8I_g and I_e = 0.5I_g , continue to be commonly used in the US.

2. Trilinear Bending Moment – Curvature Relationships for Shear Walls

Based on the results of large-scale reinforced concrete elements subjected to cyclic axial tension, Adebar and Ibrahim (2002) developed a rational model for the bending moment – curvature response of concrete walls. Rather than deal with an unlimited number of bending moment – curvature relationships depending on the maximum curvature reached in previous load cycles, the concept of an upper-bound and lower-bound response was used. The upper-bound response corresponds to a wall that is loaded monotonically to yield without having been previously cracked, while the lower-bound response corresponds to a wall that is reloaded after having been severely damaged.

Adebar and Ibrahim (2002) proposed that both bending moment – curvature relationships can be approximated by simple trilinear relationships. The slope of the first linear segment is assumed to equal the uncracked-section stiffness $E_c I_g$, while the slope of the second segment is assumed to equal the cracked-section stiffness $E_c I_{cr}$ when the section is subjected to zero axial load. The bending moment defining the intersection of the two linear segments depends primarily on the applied axial compression.

In order to validate the proposed trilinear bending moment - curvature model, Adebar et al. (2007) conducted a large-scale test on a slender concrete wall that is very typical of Canadian design practice. A 12.5 m "high" concrete wall with a height-to-length ratio of 7.6 was constructed and tested in a horizontal position supported on sliding bearings. The wall had a flanged cross section, low percentages of vertical reinforcement (0.65% in the flanges and 0.25% in the web), and was subjected to a uniform axial compression that corresponds to 0.10 $f_c'A_g$ in addition to a reverse cyclic lateral point load applied at 11.33 m from the critical flexural section.

Strain measurements from the test confirmed the proposed tri-linear shape of the bending moment – curvature response, and the measure load – deformation response compared well with the response predicted from the tri-linear relationship. The tri-linear bending moment – curvature model was used to develop the load – deformation response used in the push-over analyses done to study the influence of the shape of the load – deformation response (Section 3), as well as the nonlinear response history analysis described in Sections 4 and 5.

3. Influence of Load – Deformation Response on Effective Stiffness

Ibrahim and Adebar (2004) used the tri-linear bending moment – curvature relationship to develop a general method for determining the effective flexural rigidity $E_c I_e$ for concrete shear walls. With a tri-linear bending moment – curvature relationship, $E_c I_e$ cannot be determined directly from the bending moment – curvature relationship as the distribution of curvatures over the height of the wall influences the result.

To develop a general solution, the axial compression in the wall due to gravity loads was assumed to vary linearly over the height from zero at the top to a maximum value at the base. Two different lateral force distributions were used and the reference deflection was taken at two different heights. In the end however, the slope of the elastic portion of an equivalent elastic-plastic curve, i.e., the effective stiffness, as a ratio of the initial stiffness, was found to be essentially independent of these factors.

A general solution was presented for the upper-bound and lower-bound effective flexural stiffness of concrete walls accounting for the axial compression, amount and distribution of vertical reinforcement, concrete geometry, and all parameters that affect the flexural capacity of the wall. Axial compression was found to have the largest influence on the effective stiffness since it results in less cracking and therefore a more linear loading curve.

Simple expressions were also proposed for estimating the upper-bound and lower-bound effective flexural stiffness of concrete walls based only on the level of axial compression at the base of the wall. Eq. (1) was found to give a good estimate of the upper-bound effective stiffness of walls. The FEMA 356 recommendation of $I_e = 0.5I_g$ for cracked walls was found to be a reasonable average value for the lower-bound effective stiffness, but does not account for the influence of axial compression, which was found to have a very significant influence on the lower-bound effective stiffness. The approach of using the secant stiffness to the yield point fit well to the lower-bound only at low levels of axial compression.

CSA technical committee A23.3 decided to err on the side of not making the new displacement-based requirements too difficult to achieve. Thus Eq. (1) with the axial load calculated at the base of the shear wall was adopted into Clause 21.2.5 of the 2004 edition of CSA A23.3 as the expression that must be used to determine the effective stiffness of concrete shear walls. It was expected that future research would result in a refined approach with lower effective stiffnesses.

4. SDOF Nonlinear Response History Analysis of Concrete Walls

In all of the previous work summarized above, effective stiffness was determined from the shape of the load – deformation curves, and the conclusion was that effective stiffness can vary dramatically depending on whether the upper-bound (initial response) or lower-bound (after severe damage) is used. A completely different approach was used in the next phase to determine the effective stiffness of a concrete shear wall. Nonlinear response history analysis (NLRHA) of the wall was done and the effective stiffness was taken as that which gives the same maximum displacement demand from a linear dynamic (response spectrum) analysis. In the first step, the first mode response of the shear wall was modelled as a single degree of freedom (SDOF) system, while in the subsequent phase (Section 5) a full multi-degree of freedom model was used.

The force – displacement relationship for the first-mode response of concrete cantilever walls was developed based on the experimentally calibrated tri-linear bending moment – curvature model (Section 2). The load – deformation response included: a backbone curve with an initial linear range (prior to cracking) and a nonlinear segment that accounted for the degradation of the response due to previous cycles, rules for stiffness degradation, unloading, and mid-cycle reloading. The loading curve varied depending on how far the structure was loaded in the previous cycles. The hysteretic model was implemented in OpenSees by Korchinski (2007).

Considerable effort was put into ensuring the range of force – displacement curves that were included in the study represented a realistic range of typical shear walls in Canada. Wall geometry, axial load level, and amount of vertical reinforcement all influence the shape of the force – displacement curves. The nonlinear load – deformation response of a typical concrete shear wall can be described by a tri-linear curve similar to the bending moment – curvature response. The initial slope is the uncracked stiffness; the secondary slope depends primarily on the amount of reinforcement, while the load at which the curve transitions from the first to the second slope is primarily a function of axial compression applied to the wall.

13 different load – deformation curves were included in the study to represent the full range of typical shapes. The ratio of the load at the end of the initial linear portion to strength ranged from 20% to 80%, while the ratio of secondary slope to initial slope varied from about 10% to 40%.

In addition to investigating the shape of the load – deformation response, 8 different initial fundamental periods T_i ranging from 0.5 to 4 seconds, and 10 different elastic bending moment demand to flexural strength ratios R ranging from 0.5 to 5.0 were investigated. Thus in total 13 (load-deformation shapes) x 8 (initial periods) x 10 (strength ratios) = 1040 different walls were investigated.

Each wall was subjected to forty ground motions modified to fit a target spectrum similar to the 2005 NBCC design spectrum for Vancouver – Soil Class C, except that the spectrum decreased proportional to *1/T* for periods between 2 and 6 seconds. Damping was assumed to be 3% of critical.

Fig. 1 shows the results for one example T_i = 2.0 s. Each data point was determined from the mean value of the maximum displacements from 40 ground motions. This figure clearly shows that the shape of the load – deformation curve, which differs from line-to-line is less significant than the strength ratio R. Note also that wall W-L8-R2, which had the highest level of axial compression and lowest amount of reinforcement, has the lowest effective stiffness. The initial period of the wall T_i had a small influence except when the period became large enough so that the wall was responding in the constant displacement range of the spectrum.

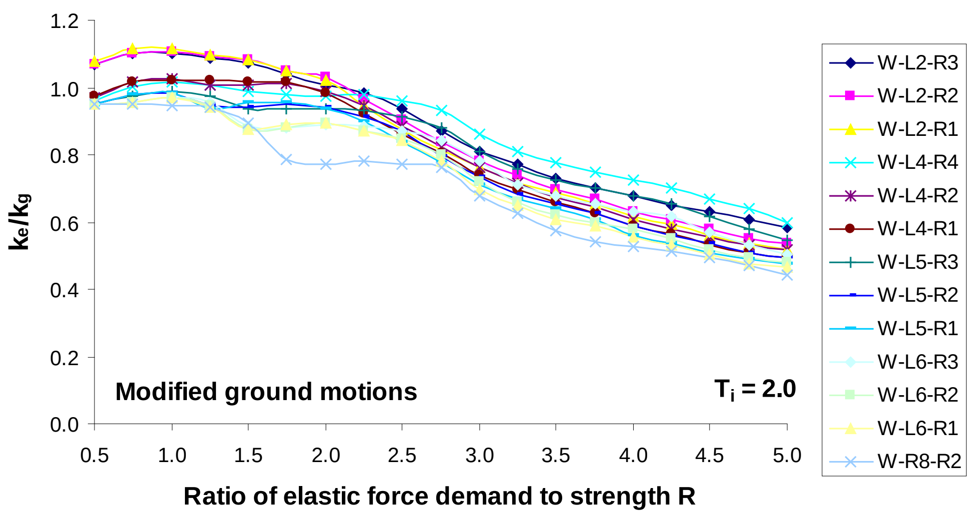

Fig. 1 – Stiffness reduction factors for concrete shear walls with an initial period of 2 seconds and 13 different load – deformation curves; determined from SDOF study.

The results from the SDOF study demonstrated that if two walls have the same bending strength, the wall with increased axial compression actually has a lower effective stiffness (increased displacements) because it has a reduced amount of hysteretic damping. This is because the axial compression closes the flexural cracks when the lateral load is reduced. Walls with the same bending strength provided by vertical tension reinforcement rather than axial compression have much fuller hysteretic loops and hence significantly reduced displacements. It is important to note that while axial compression reduces the energy dissipated (per unit strength), it does reduce the residual displacement after the earthquake.

5. MDOF Nonlinear Response History Analysis of Concrete Walls

In the final phase of the project to determine the effective stiffness of concrete shear wall buildings, nonlinear response history analysis (NLRHA) was used to determine the displacement demand in 13 different shear wall buildings modelled as multi-degree of freedom systems with uniform mass at each floor level. The main differences between the buildings were the number of stories (10 to 50) and the flexural strength of the shear walls. The ratio of the elastic bending demand (calculated using $E_c I_g$) to the flexural strength of the shear walls ranged from 1.4 to 4.4. The corresponding ratios for elastic bending moment demand calculated using $E_c I_e$ (lower stiffness, longer period, lower forces) ranged from 1.4 to 3.7.

The strength of the walls was adjusted by the quantity of vertical reinforcement that was provided, as well as the proximity of gravity-load columns to the core, which influences the axial compression applied to walls.

The ground motions used for the NLRHA were scaled to the UHS over the range of $0.2T_1$ to $1.5T_1$ consistent with the recommendations of ASCE standard 7-10 (ASCE 2010). The UHS used is the design spectrum for Site Class C (average shear wave velocity V_s between 360 and 760 m/s) in Vancouver BC, which is very similar to ASCE7-10 design spectrum for Site Class B in Seattle WA. De-aggregation of the UHS indicates the hazard representing the 2% probability of exceedance in 50 years has a mean magnitude M = 7.0 and mean source-to-site distance D = 50 km for periods greater than 1.0 s. Further information about the selection and scaling of the ground motions used in this study are given by Dezhdar and Adebar (2015a).

Similar to the study where the first mode response of shear walls were simulated as a single degree of freedom system, NLRHA was performed in order to determine the "true" top-wall displacement of the shear walls, and the appropriate stiffness reduction factor was determined so that the same displacement demand is determined from linear dynamic (response spectrum) analysis (RSA).

The input parameters to RSA are the design spectrum, damping (5% was assumed), and the structural characteristics of the building. The displacements from different modes were combined using Complete Quadratic Combination (CQC) method. The first four modes were included although the first mode is usually the governing mode for determining top-wall displacement. It is important to note that although RSA can be used to estimate the top-wall displacement, it cannot be used to estimate the displacement profile over the height of the building (Dezhdar and Adebar, 2015b).

Figure 2 summarizes the stiffness reduction factors needed to be applied in the RSA so that the top-wall displacement is identical to the results from the NLRHA of the 13 different shear wall buildings. The strength ratio is the elastic bending moment demand calculated using the effective stiffness that gives the correct displacement to the nominal flexural capacity of the test specimen.

All of the data points except four lie on a well-defined line. The four data points that are well above the line are all from buildings that had lower values of axial compression applied to the shear walls due to the gravity-load columns being located closer to the core walls. The walls had an axial compression equal to about 6% of $f_c'A_g$, where as the other wall all had and axial compression force equal to between 9 and 13% of $f_c'A_g$.

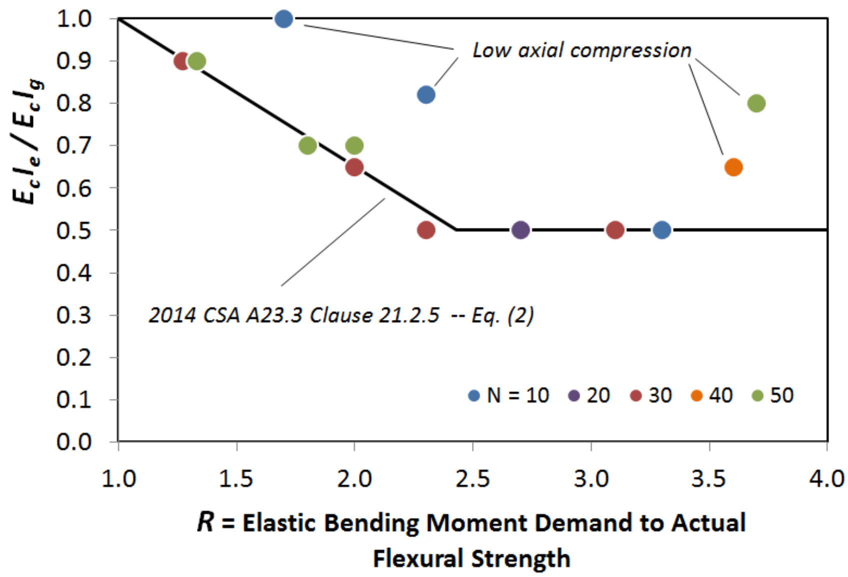

Fig. 2 – Results from a NLRHA study of 13 different cantilever shear wall buildings modelled as MDOF systems; comparison of results with equation developed for 2014 edition of CSA A23.3.

6. Effective Stiffness of Shear Walls in 2014 edition of CSA A23.3

Based on the work presented in this paper, the following expression was developed for the 2014 edition of CSA A23.3:

$$E_c\, I_e = \{1.0 - 0.35\,(R-1)\}\, E_c\, I_g \tag{2}$$

Where R is the ratio of the elastic bending moment demand to the actual flexural strength of the walls.

CSA A23.3 defines R as follows:

$$R = {R_d R_o}/{\gamma_w} \tag{3}$$

Where R_d and R_o are the ductility-related and overstrength-related force modification factors specified in the National Building Code of Canada (NBCC), and γ_w is the wall overstrength factor equal to the ratio of the load corresponding to nominal moment resistance of the wall system to the factored load on the wall system, but need not be taken as less than 1.3.

7. Verification with Large-scale Shaketable Tests

A full-scale 7-story shear wall test conducted at UC San Diego (Panagiotou 2008) provides an opportunity to verify the proposed expression (Eq. 2) for effective stiffness of concrete shear walls. Linear dynamic (response spectrum) analysis (RSA) was used to predict the maximum displacements observed during test.

The test structure was 19.2 m high, while the main wall was a 3.66 m long rectangular shear wall, thus h_w / l_w = 5.25. Two perpendicular walls, one precast segmental wall and one cast-in-place wall, provided lateral and torsional stability. Cast-in-place slabs at every story were supported on steel pinned-ended columns and had two large slots to reduce the coupling between the main shear wall and the perpendicular cast-in-place wall. Measured maximum bending moments indicated considerable coupling did occur.

The test consisted of four separate earthquakes applied in the direction of the main shear wall. White noise was used to measure the natural frequency of the system prior and after each earthquake. The initial fundamental period of the system was 0.51 s. This increased to 0.59 after 25 white-noise tests prior to the first simulated earthquake EQ1. The fundamental period shifted to 0.65, 0.82, 0.88, and 1.16 s after earthquakes EQ1, EQ2, EQ3, and EQ4, respectively.

A 3-dimensional linear model of the test specimen was developed using SAP (Computers & Structures Inc. 2010). The flexibility of the foundation was modelled using rotational springs at the base of the web wall and flange wall. Using $1.0E_c I_g$ for the three concrete walls and $0.2EI_g$ for the concrete slab resulted in a correct prediction of measured fundamental period, Wong (2010), Dezhdar (2012).

Each earthquake was treated separately with the initial stiffness of the wall defined by the results of white noise tests prior to strong motion shaking. For each case, the stiffness reduction factor that needs to be applied to the initial stiffness was determined so that the calculated top-wall displacement from RSA, using a smoothed spectrum fit to the ground motion, matched the observed maximum displacement. The calculated stiffness reduction factors from the test are compared with the proposed expression for stiffness reduction in Fig. (3). The strength ratio is the elastic bending moment demand calculated using the effective stiffness that gives the correct displacement to the nominal flexural capacity of the test specimen.

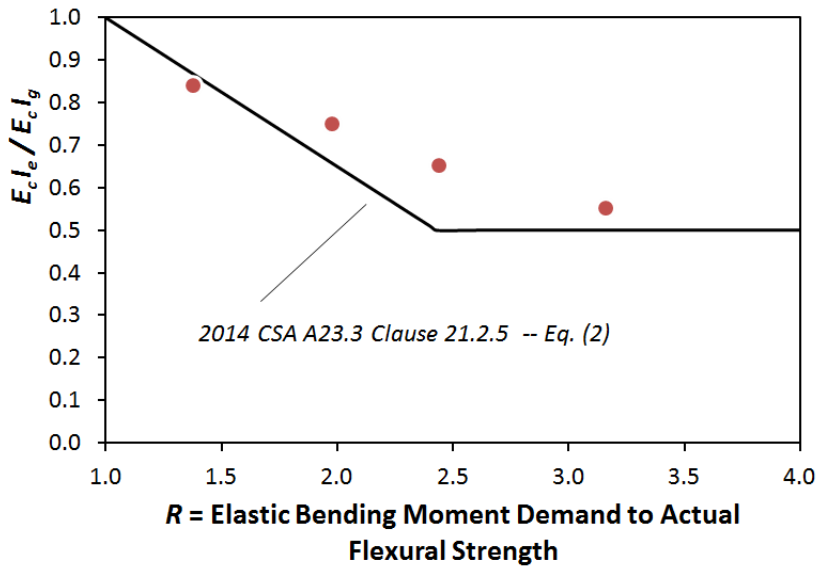

Fig. 3 – Comparison of equation for effective stiffness developed for 2014 edition of CSA A23.3 with the results from UCSD full-size shaketable tests on a shear wall.

8. Summary

This paper summarizes the research that led to the development of the new expression for $E_c I_e$ in the 2014 edition of Canadian Standard CSA A23.3 *Design of Concrete Structures*.

Traditionally $E_c I_e$ has been assumed to equal the slope of the elastic portion of an equivalent elastic – plastic (bi-linear) bending moment – curvature relationship. This approach leads to the idea that axial compression increases effective stiffness because axial compression causes the bending moment – curvature relationship (and the load – deformation relationship) to remain linear up to higher load levels. Eq. (1), which indicates that axial compression increases effective stiffness, is the expression for $E_c I_e$ in the current (2004) edition of CSA A23.3.

Nonlinear response history analysis (NLRHA) of shear wall buildings using a rigorous model for concrete shear walls has revealed that the most important parameter influencing $E_c I_e$ is the ratio of elastic bending moment demand to the flexural strength of the shear walls. In addition, the more the flexural resistance is due to axial compression applied to the wall rather than yielding of vertical reinforcement, the lower the effective stiffness, contrary to what was previously thought. Considerable hysteretic energy is dissipated when vertical reinforcement is cycled in the inelastic range. On the other hand, very little energy is dissipated by axial compression applied to a shear wall without vertical reinforcement – the mechanism is essentially "rocking."

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