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SEISMIC BEHAVIOR OF REGULAR BASE-ISOLATED BRIDGES WITH LOW REINFORCEMENT RATIOS

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ABSTRACT:

The main objective of this study is to assess the effect of lowering the minimum longitudinal reinforcement ratio on the seismic performance and the security of isolated bridges under the design and maximum credible earthquakes. This study investigates the bidirectional behavior of a regular base-isolated bridge, with minimum longitudinal reinforcement ratios, under severe seismic loadings expected in eastern and western Canada. A two span bridge with a multi-column central pier and a precast superstructure is considered as a case study. A 3D refined structural model where the seismic isolators were modelled by a coupled bidirectional nonlinear hysteresis. Columns were modelled by fiber elements to investigate the damage extent levels and ductility demands. Time history analyses and incremental dynamic analyses were performed on bridge models under simultaneous bidirectional components of seismic ground motion records, scaled to the National Building code 2005 spectra. Rectangular bridge columns with 0.3%, 0.5% and 0.8% of longitudinal reinforcement ratios were considered. Results focus on the effect of lowering the minimum longitudinal steel ratio on the damage extent and ductility demand within columns critical sections. Results are also compared to earlier findings on conventional fixed base bridge multi-column piers with low steel ratios and the effect of the differences between east and west Canadian sites and seismic zones are pointed out.

1. Introduction

In the context of seismic design, bridge structures are usually classified according to their importance as lifeline, emergency or other bridges. The Canadian Highway Bridge Design Code (CSA, 2006) requires that lifeline bridges remain in service after the design earthquake, with no or very limited damage. Emergency bridges should allow the passage of emergency vehicles while other bridges should be designed to prevent failure accepting to expose them to moderate to severe damage levels. In the CSA-S6 code, the accepted level of damage is modulated through the simultaneous use of the Response Modification Factor, R, and the Importance Factor, I. The two factors are applied to the elastic seismic force to obtain design seismic force for conventional fixed base bridges. The R factor allows reducing design forces, with consideration of the ductility and redundancy of the structural system while the I factor is intended to limit the damage extent for important bridges by increasing the design force. Furthermore, the seismic specifications in the CSA-S6 code aim to ensure that the input seismic energy is dissipated through a ductile and stable hysteretic behaviour by specifying a set of requirements relative to the amount and configuration of the longitudinal and confinement reinforcement at critical sections of Reinforced Concrete (RC) elements.

Innovative seismic protection technologies and particularly the seismic base isolation technique are gaining increasing interest in application for bridges in Canada since the early 1990's. Seismic base isolation constitutes an efficient and well-established alternative to the fixed base strategy design in both western and eastern Canada seismic areas (Ghobarah 1988, Guizani 2003,Guizani et Chaallal 2010, Dion, 2010)). This is mainly due to its efficiency in reducing the seismic forces and increasing the seismic performance level of structures and to the more stringent seismic requirements of the last 2000 and 2006 CSA-S6 editions. Seismic base isolated lifeline and emergency bridges are designed with no reduction of design forces (R=1), implying that the lateral resistance system such as piers and abutments remain elastic under earthquake ground motions. This performance level is easily achieved as seismic base isolation allows for an important reduction in force demand, typically 3 to 10 times, such that the design of these elements is generally governed by the minimum steel ratio required for shrinkage control and non-seismic loads.

Bridge design codes and regulations specify limits to the longitudinal reinforcement ratio for concrete bridge columns. In Canada, the CSA-S6-06 limits the minimum longitudinal reinforcement ratio to 0.8%. This lower limit is intended essentially to avoid brittle flexural failure modes and varies significantly between codes. While lower limits of 1.0 and 0.8% are specified in the United State (AASHTO, 2010) and New Zealand, respectively, limits as low as 0.3 and 0.5% are prescribed in Europe and Japan, respectively (Priestley et al. 1996). Previous studies have shown that bridge columns with low longitudinal reinforcement ratio exhibit excellent ductile behavior. Large-scale RC circular and rectangular bridge columns specimens with longitudinal reinforcement ratio of 0.5% sustained displacement ductility in excess of 10.0 when subjected to unidirectional and bidirectional cyclic inelastic lateral displacements (Priestley et al. 1996, Khaled et al. 2011).

The lower limit of reinforcement ratio should affect more the base isolated bridges than fixed base bridges, especially in moderate to high seismic zones such as in eastern and western Canada. In these zones particularly, the minimum longitudinal reinforcement ratio specified by the Canadian code is of a practical interest as the design of base isolated bridges is more than often governed by such a limit. Earlier studies on this topic are limited to fixed base designed bridges. In this paper, the seismic behavior of isolated bridges, designed for eastern (Montreal, Quebec) and western (Vancouver, British Columbia) Canadian sites, is investigated through bidirectional nonlinear time history analyses for a common two span bridge. The objective of this study is to evaluate numerically the effect of lowering the minimum longitudinal reinforcement ratio on the seismic performance and the security of isolated and fixed base bridges under the design and maximum credible earthquakes within the Canadian seismicity context. Column longitudinal reinforcement ratios varying from 0.3% to 0.8% were considered for the bridge models located in Montreal and Vancouver, respectively. Results are examined to evaluate the trend in performance level, stability and damage extent as a function of the longitudinal reinforcement ratio in the bridge columns.

2. Ground motion time histories

2.1. Selection of the ground motion records

Two ensembles of historical ground motions recorded in eastern Canada and in western North America were selected based on the seismic hazard at Montreal and Vancouver sites, respectively. Each earthquake record includes two orthogonal horizontal components. The properties of the selected ground motion earthquakes used in this study are presented in Tables 1 and 2 for Montreal and Vancouver sites, respectively, along with the orientation of the recorded historical ground motion horizontal components.

Table 1 – Properties of the unscaled ground motion records for eastern Canada (Montreal site)

Record No.	Date, Event	M_w	R (km)	Component, Orientation	PGA (g)
NHN_BC1	1985 Dec. 23, Nahanni	6.5	24	Bettlement Creek- S3, N270	0.186
NHN_BC2				Bettlement Creek- S3, N360	0.194
OTT_R1	2010 Juin. 23, Ottawa	5.0	58.7	Val-des-Bois- NS	0.034
OTT_R2				Val-des-Bois- EW	0.033
SAG_CN1	1988 Nov. 25, Saguenay	5.7	43	Chicoutimi Nord, N124	0.131
SAG_CN2				Chicoutimi Nord, N214	0.106
SAG_EB1	1988 Nov. 25, Saguenay	5.7	90	Les Eboulements, NS 0	0.125
SAG_EB2				Les Eboulements, EW 270	0.102
SAG_SA1	1988 Nov. 25, Saguenay	5.7	64	Saint-Andre, NS 0	0.156
SAG_SA2				Saint-Andre, EW 270	0.091

Table 2 – Properties of the unscaled ground motion records for western Canada (Vancouver site)

Record No	Date, Event	M_w	R (km)	Component, Orientation	PGA (g)
LP_SFP1	1989 Sept. 17, Loma Prieta	7.0	98	San-Francisco-Presidio, EW 90	0.199
LP_SFP2				San-Francisco-Presidio, NS 0	0.100
MH_SYGA1	1984 Apr. 24, Morgan Hill	6.2	36	San Ysidro Gilroy #6, EW 90	0.286
MH_SYGA2				San Ysidro Gilroy #6, NS 360	0.219
N_CORR1	1994 Jan. 17, Northridge	6.7	41	Castaic-Old Rte, EW 90	0.568
N_CORR2				Castaic-Old Rte, NS 360	0.514
N_SPPV1	1994 Jan. 17, Northridge	6.7	58	San Pedro Palos Verdes, EW 90	0.095
N_SPPV2				San Pedro Palos Verdes, NS 0	0.101
WN_PKC1	1987 Oct. 01,Wattier-Narrows	6.1	38	Pacoima-Kagel Can, EW 90	0.158
WN_PKC2				Pacoima-Kagel Can, NS 0	0.155

2.2. Transformation and scaling of ground motions

The two horizontal orthogonal components of the selected historical earthquakes were first transformed into their principal directions (Minor and Major), following the approach proposed by Penzien and Watabe (1975). The resulting minor and major principal components of each ground motion were then scaled to 2%/50 years uniform hazard spectra (USH), for a site condition representative of site Class C, using a spectral matching technique in the time domain (Hancock et al. 2006). The two resultant spectra were finally separated by scaling up and down the response spectra to achieve a spectral ratio of the minor to the major component of 0.75, as suggested by (López et al. 2006). The 5% damped absolute acceleration response spectra of the major and minor ground motion components at the two sites are presented in Fig. 1.

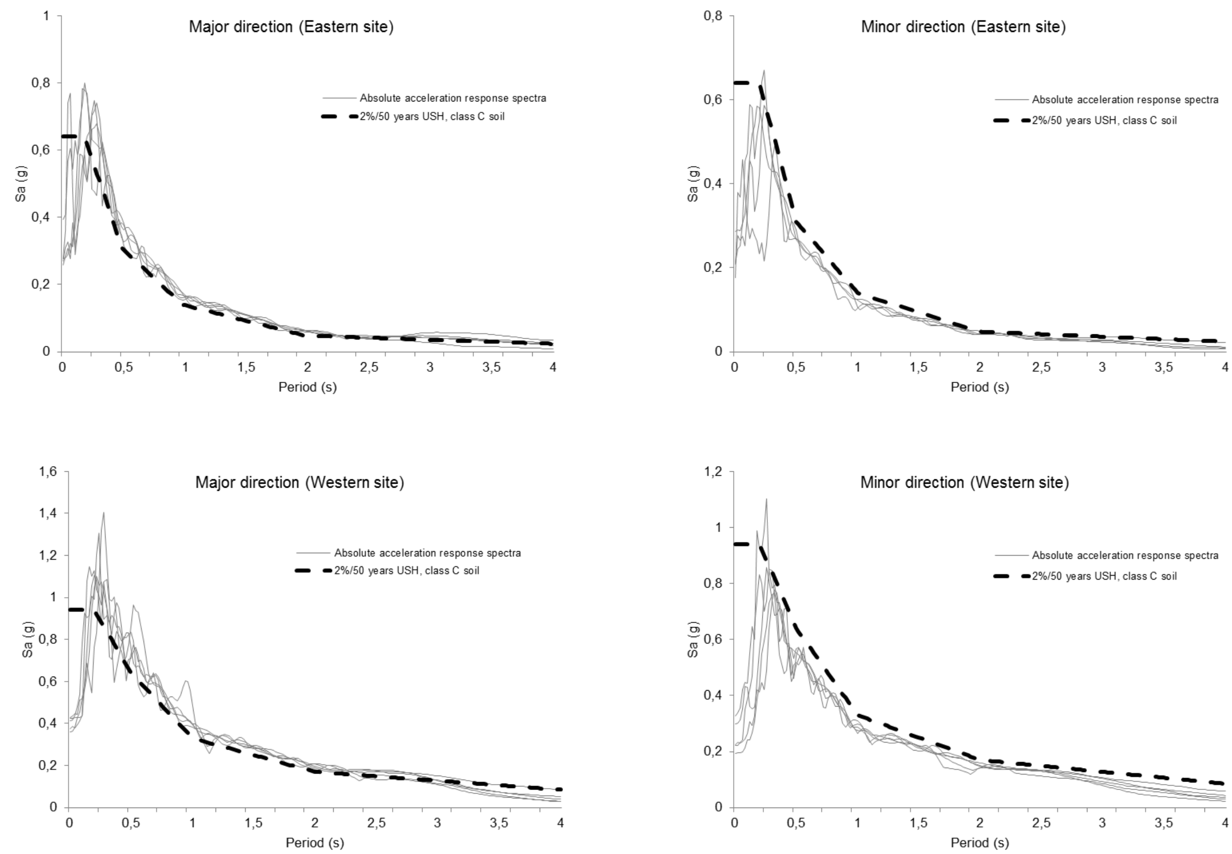

Fig. 1 – 5% damped absolute acceleration response spectra of the selected earthquake records

3. Bridge model

A common slab-on-girder concrete bridge was considered in this study, as it is representative of a large number of bridge structures encountered in North American highways. The bridge model has a total length of 72 m and an overall width of 12.9 m and consists of two-equal-span continuous straight bridge with a superstructure supported on two abutments and a central three columns bent having a rectangular cross section of 900×1800 mm and 6 m tall (Fig. 3-a). The superstructure consists of 200 mm thick cast-in-place concrete slab on six NEBT1600 prestressed concrete girders. The total superstructure weight is 12293 kN. At the abutment and pier locations, the girders are supported on rubber bearing isolators, as shown in Figure 2. The bridge is assumed to be founded on soft rock or very dense soil site. This corresponds to a Class C site, according to the NBCC 2010 soil classification. The specified strength f'_c

and the modulus of elasticity E_c for concrete were set to 35 MPa and 26000 MPa, respectively; whereas a yield stress f_y = 400 MPa and a modulus of elasticity E_s = 200000 MPa were assumed for the steel reinforcement.

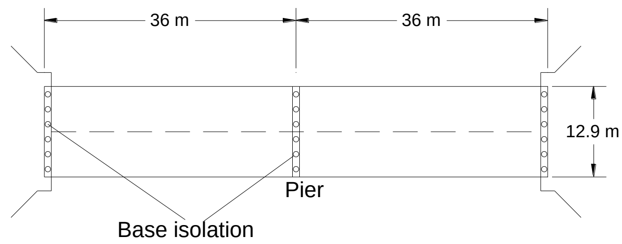

Fig. 2 – Abutment and pier locations

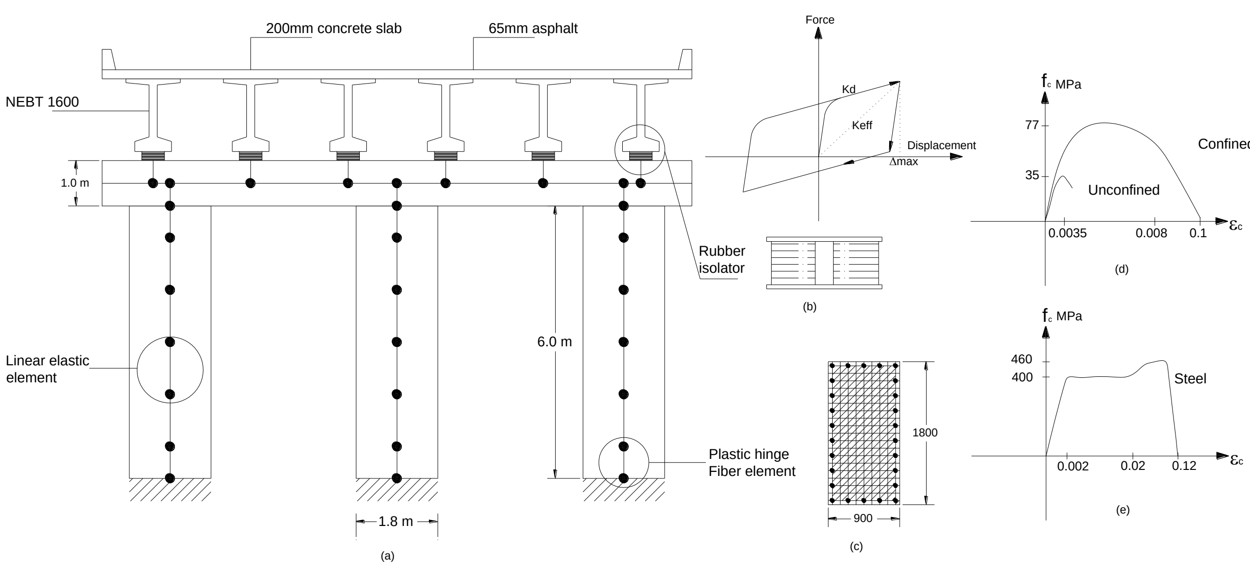

Fig. 3 – Base isolated bridge model

A 3D model of the bridge was built using the computer program SAP2000-v15 (CSI 2011). The bridge columns were modelled using nonlinear beam-column elements with fiber discretization of the cross section over the plastic hinge length at the top and bottom of columns whereas elastic beam-column elements were used outside the plastic hinge regions, as illustrated in (Fig. 3-c). The length of the plastic hinge was estimated using Equation 1 (Priestley et al. 1996).

$$L_p = 0.08\,L + 0.022\ f_y\ d_{bl} \geq 0.044\ f_y\ d_{bl} \qquad (f_y\ in\ MPa) \qquad (1)$$

In equation 1, L is the distance from the critical section of the plastic hinge to the point of contraflexure, d_{bl} and f_y are the diameter and the yield stress of the longitudinal reinforcement, respectively. The stress-strain relationships for normal strength concrete by (Mander et al. 1988) were adopted for the unconfined cover and the confined concrete core, with the loading and unloading rules described by Takeda et al. (1970). The stress-strain relationship of the steel reinforcement was described using the Park et al. (1986) model with kinematic behavior. (Fig. 3-d and 3-e).

4. Base isolation system and modeling

The base isolation system consists of 18 seismic isolation bearings, six at each abutment and six at the pier location. As these bearings serve also to support vertical loads, their hysteretic features in the horizontal plan were scaled in proportion to their vertical tributary area. Thus, the bearings at the central pier have twice the lateral stiffness (k_d) and characteristic strength (Q_d) of those at the abutments, and are consequently set equal to $1/12^{th}$ of the global characteristics of the isolation system. Bearings are supposed to have the same hysteretic characteristics for all the horizontal directions. The hysteretic characteristics were chosen so that the fundamental period of vibration of the isolated bridge and the equivalent effective damping, using the CSA-S6-06 design spectrum, are respectively 2.0 sec and 18%. This is in the range of typical values used for the design of seismic isolation systems. The global isolation system properties are shown in table 3.

Table 3 – Global isolation system hysteretic properties

Hysteretic properties			
$K_{effective}$	K_d	Q_d	K_u
12353 kN/m	8300 kN/m	243 kN	83000 kN/m

The isolation bearings were modeled by link/support elements with the hysteretic (rubber) isolator properties implemented in SAP2000 (CSI 2011). This model is a biaxial hysteretic isolator with coupled nonlinear plasticity properties, for the two shear deformations, based on Wen (1976) plasticity model (Fig. 3-b). The yield surface is defined by a circle equation (constant vector amplitude) using internal hysteretic variables monitoring the deformation state in both directions.

5. Analysis and design procedures

5.1. Response spectrum analysis

The bridge models were designed according to the CSA-S6-06 specifications for a lifeline bridge (I=3.0). The seismic design forces were derived from multi-modal spectral analyses results where columns effective inertia were set equal to 70% of their gross inertia ((I_e=0.7I_g) to reflect their cracked sections. Modal responses were combined according to the complete quadratic combination (CQC) method and the 30% rule was used to combine maximal responses obtained from the separate analyses in the two principal directions of the bridge.

For conventional bridge columns, response modification factors R of 3.0 and 5.0 were applied respectively for the longitudinal and transversal directions, while for the isolated bridge a modification factor R equal to 1.0 was considered for both directions. Dead load forces were combined to seismic design forces and the columns required longitudinal reinforcement steel ratios (Fig.4) were determined using the SPColumn software (StructurePoint 2012). The transverse reinforcement for confinement in the plastic hinge regions of the columns consists in #15 rebar's spaced at 75 mm c/c.

Table 4 – Required longitudinal reinforcement ratios for conventional and isolated bridges

East site (Montreal)		West site (Vancouver)	
Conventional bridge	Isolated bridge	Conventional bridge	Isolated bridge
1.6 %	0.02%	4.6%	0.6%

To assess the effect of lowering the minimum steel ratio, this study considered four types of columns for each site, with different longitudinal steel ratios. Columns C4 are for fixed base bridges and have steel ratios equal to the required steel ratios, i.e. 4.6% and 1.6% for western and eastern sites, respectively.

Columns C1 to C3 are for base isolated bridges in both sites. Columns C3 have a steel ratio equal to the minimum steel ratio specified by the CSA-S6-06 that is 0.8%. For both sites, this ratio is higher than the required steel ratios. Columns C2 have steel ratio of 0.6%, lower than the minimum specified steel ratio. However, it is equal to the required steel ratio for the isolated bridge in western site but it is much higher than what is required for eastern site (0.02%). Columns C1 have steel ratios of 0.5% and 0.3% for western and eastern site, respectively. These low values are thought to represent the minimum practical lower limits for longitudinal steel. For western site, the ratio of 0.5% is slightly lower than the required steel ratio while for eastern site the 0.3% is still much higher than what is required for seismic resistance purpose only.

5.2. Time history analyses

Nonlinear time-history analyses were carried out using the two orthogonal principal components of the selected earthquakes as input to simulate the nonlinear seismic response of the models under the scaled earthquakes for both sites. Two series of analyses were performed: (1) analyses under the ground motions scaled to the design spectra and, (2) analyses under the same ground motions multiplied by a factor of 1.5. This second series of analyses aimed to assess the performance and reserves of strength under an extreme event representing a maximum credible earthquake.

6. Results and discussion

Tables 5 to 8 present the maximum strains obtained for the unconfined concrete cover, confined concrete core and for the corner longitudinal rebar, within the columns hinge regions of the studied bridge models under all the ground motions for the design and the maximum credible earthquakes at both sites. Maximum fiber strains represent an excellent indicator and comparison parameter of the damage level associated with each case. For example, they are directly compared with the performance criteria presented in the new edition of the Canadian highway bridge design code (CSA 2014)) allowing to classify the damage severity level for each studied bridge column and to assess thereby the effect of the steel ratio content.

Table 5 – Maximum strains in columns under the design ground motions in western site

Fiber	Isolated bridge			Conventional bridge
	Model C1 (ρ = 0.5%)	Model C2 (ρ = 0.6%)	Model C3 (ρ = 0.8%)	Model C4 (ρ = 4.6%)
Unconfined Concrete Cover	-0.00148	-0.001376	-0.001246	-0.00165
Confined Concrete Core	-0.00127	-0.001185	-0.00108	-0.00136
Corner Longitudinal Rebar	0.001359	0.00106	0.000838	0.001615

Table 6 – Maximum strains under the maximum credible earthquakes in western site

Fiber	Isolated bridge			Conventional bridge
	Model C1 (ρ = 0.5%)	Model C2 (ρ = 0.6%)	Model C3 (ρ = 0.8%)	Model C4 (ρ = 4.6%)
Unconfined Concrete Cover	-0.00315	-0.003367	-0.003015	-0.00254
Confined Concrete Core	-0.00231	-0.001825	-0.001623	-0.00204
Corner Longitudinal Rebar	0.0085	0.008287	0.002178	0.003094

Table 5 shows that for western site, under the design earthquakes, all the bridges/columns perform within the elastic range with no damage. The maximum strains for the unconfined concrete cover are smaller than the plastic limit (-0.002) and cover spalling is not expected as these strains are lower than the crushing limit strain (0.003 to 0.004). Strains in the confined concrete core are much less critical as they are lower than those taking place in unconfined concrete cover and as the crushing limit is much higher. Furthermore, the strain in corner longitudinal rebar is still smaller than the yield strain (0.002).

Table 6 shows that when subject to the maximum credible earthquake, all the bridges responded beyond the elastic range as the corner longitudinal steel strains are larger than the yield strain (0.002). Columns C1 and C2 , with the lowest steel ratios, show the largest steel strain values while steel strains in columns C3 and C4 are slightly or moderately beyond the elastic limit. However, for all bridges, the maximum strain in steel is well below the limit of repairable damage range (0.015) defined in the CSA-S6-14. Furthermore, spalling of the unconfined concrete core should not take place for the columns C3 and C4 and should be not significant for columns C1 and C2. Confined concrete core should be still with no significant damage as the maximum strains are less than the plastic limit, except for column C1 where they are slightly over this limit. For all cases, concrete strains remain within the range of minimal damage (less than 0.004) defined in CSA-S6-14, for all bridges.

Table 7 – Maximum strains under the design ground motions in eastern site

Fiber	**Isolated bridge**			**Conventional bridge**
	Model C1 (**ρ = 0.3%**)	**Model C2** (**ρ = 0.5%**)	**Model C3** (**ρ = 0.8%**)	**Model C4** (**ρ = 1.6%**)
Unconfined Concrete Cover	-0.000460	-0.000440	-0.000410	-0.001304
Confined Concrete Core	-0.000425	-0.000408	-0.000379	-0.001119
Corner Longitudinal Rebar	0.000053	0.000050	0.000045	0.000961

Table 8 – Maximum strains under the maximum credible earthquake in eastern site

Fiber	**Isolated bridge**			**Conventional bridge**
	Model C1 (**ρ = 0.3%**)	**Model C2** (**ρ = 0.5%**)	**Model C3** (**ρ = 0.80%**)	**Model C4** (**ρ = 1.6%**)
Unconfined Concrete Cover	-0.00056	-0.00053	-0.00049	-0.001845
Confined Concrete Core	-0.000505	-0.000483	-0.000455	-0.001547
Corner Longitudinal Rebar	0.000119	0.000112	0.000102	0.001794

Table 7 shows that, for eastern site and under the design ground motions, all the bridges are within the elastic limit with no damage in concrete or in steel, even for columns with steel ratio as low as 0.3%. For the conventional bridge, (fixed base) strains are smaller but comparable to correspondent strains obtained for the western site. For the base isolated bridges, because the seismic demand is very low (0.02%), the obtained strains are much lower (about 3 times for concrete and 30 times for steel) than their correspondent strains for isolated bridges in western site. Isolated bridge models show much lower strains than conventional bridge models (3 to 5 times). No damage should take place under the design earthquakes in eastern site for all the bridge models.

Table 8 shows that, under the maximum credible earthquake, all the bridge models located in eastern site are still well below their elastic limit and show no damage. The strain values for the conventional bridge models are higher than those for the isolated bridge models (about 1.5 times for steel strains and about 3 to 4 times for concrete strains) but are still lower than the elastic limits for concrete (about 50%) and steel (about 80%).

7. Concluding remarks

All the studied bridge models, located in western and eastern sites, performed according to the design philosophy of the CSA-S6-06 that is in the elastic range under the design earthquake. This is the case also for the bridge models with reinforcement steel ratios less than the actual minimum required of 0.8%.

For the bridge Model C1, located in western site, with a steel ratio of 0.5%, the performance under design earthquake remains within the elastic range. This is because the required reinforcement steel ratio is calculated with the factored resistance while the performance is evaluated with the nominal resistance, which is 15 to 50% more.

The base isolated bridge models located in the eastern site have a very low seismic demand. Therefore, even with a steel ratio as low as 0.3%, they show no damage under both the design and the maximum credible earthquakes.

Lowering the steel ratio resulted in an increase of the maximum strains and damage indicator in concrete and in reinforcement steel. For western site bridges, higher strains and damage levels were observed under the maximum credible earthquake, however they remain within the repairable damage limit. Using the probable resistance values of materials would result in lower strains and damage levels.

Further case studies are required for the eastern site to assess the performance of base isolated bridges with a more important seismic demand and low steel ratios. Use of probable resistance of materials should also be considered to assess the range of the expected damage. Nevertheless, the preliminary results presented in this paper indicate that lowering the minimum longitudinal reinforcement ratio to 0.5% for isolated bridge columns is of a very important practical importance, especially for eastern site, and should not jeopardize the seismic performance of these bridges.

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