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MAGNIFICATION OF INTERNAL FORCES IN THE FLEXIBLE DIAPHRAGMS OF SINGLE-STORY BUILDINGS

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ABSTRACT: North America has a large inventory of single storey buildings with large footprints, generally used for recreational, commercial, educational and industrial purposes. The roof diaphragm of such buildings often consists of steel or wooden deck that is quite flexible in its plane. The response of these buildings to earthquake excitations is strongly influenced by the flexibility of the roof diaphragm. Diaphragm flexibility increases the period of the building, magnifies the ductility demand on the lateral load resisting system, and changes the manner in which the inertia forces are distributed along the length of the diaphragm. The last one leads to an increase in the in-plane shear force and bending moments along the length of the diaphragm. In particular, the shear force at quarter span and bending moment at mid-span are significantly magnified. While previous studies have made a note of such magnification, there is lack of literature on the extent of the magnification. The proposed 2015 National Building Code of Canada also recognizes this behaviour but does not provide a method for the prediction of such magnifications. An analytical study is carried out to examine the influence of diaphragm flexibility on the distribution of inertia forces along the length of the diaphragm in one-storey buildings subjected to seismic ground motions. Time history analyses are carried out on a large number of representative flexible diaphragm buildings for their response to synthetic ground motions. The magnifications of in-plane bending moments and shear forces are investigated and methods for the prediction of such magnifications are recommended.

1. Introduction

In one-storey buildings in which the gravity loads are comparatively small, the roofing system consists of un topped steel deck panels or wood structural panels, which exhibit low in-plane stiffness compared to that of reinforced concrete slabs or steel panels with concrete topping. The diaphragm is, in such cases, often classified as flexible. The flexibility of the diaphragm has a strong influence on the response of the building to earthquake excitation. An important effect of the flexibility is on the manner in which the inertia forces are distributed along the length of the diaphragm. The changes in the distribution of inertia forces lead to significant magnifications in the bending moment closer to the mid span and in the quarter span shear forces. Such magnifications may have an important bearing on the design. This has been recognized in many previous studies, but definitive recommendation on the determination of increases in the bending moments and shear forces are not available.

A number of research studies have been carried out on the impact of diaphragm flexibility on seismic response. Those that are closely related to the magnification of internal forces in the diaphragm include analytical studies by Humar and Popovski (2013), Trudel-Languedoc et al (2012), and Tremblay and Stiemer (1996). These studies have noted that the flexibility of the diaphragm increases the ductility demand on the lateral load resisting system (LLRS) and magnifies the bending moment and the shear force acting along the length of the diaphragm. The studies recommend the need for additional research to assess the magnitude of such amplification and the development of methods to account for it in design.

Massarelli et al. (2012), Paquette and Bruneau (2006) and, Tremblay et al. (2000) have carried out experimental studies on the seismic response of one-storey buildings with flexible diaphragms. These studies have corroborated the findings of analytical studies and have proved that the flexibility of the diaphragm increases the deformations and the shear force along the length of diaphragm.

The common approach to the design of one-storey buildings with flexible diaphragms is to design the diaphragm to remain elastic while the members of the LLRS, such as braces, shear walls etc. dissipate the seismic energy through inelastic deformation. Such behavior can be ensured by adopting a capacity design approach. The diaphragm, collectors, chord members, joists, struts and their connections are then designed not to yield under the seismic load. This requires reliable prediction of the bending moments and shear forces along the length of the diaphragm produced by seismic excitation.

In the current practice, the diaphragm members are designed for bending moments and shear forces obtained from a uniform distribution of the total seismic force along the length of the diaphragm. This fails to account for the magnification of mid-span moments and in-plane shears. The present study attempts to predict the amplification of bending moment at mid span as well as that of the shear force at quarter span in a single span diaphragm. For this purpose, response analyses are carried out on a set of 33 representative rectangular single-storey buildings with flexible diaphragms subjected to ten synthetic ground motions. Five of these motions are compatible with the uniform hazard spectrum (UHS) for Vancouver and five are compatible with the UHS for Montreal. The selected hazard spectra are specified in the 2010 National Building Code of Canada (NRCC 2010). For each building, two separate analyses are carried out, one for the earthquake acting parallel to the short side of the building and the other for the earthquake acting parallel to the long side of the building. In all of the response analyses, it is assumed that the diaphragm remains elastic while the LLRS is strained in to the inelastic range.

2. Analytical Model

Figure 1(a) shows a one-storey building in which the roof diaphragm consists of steel deck panels and the LLRS consists of concentric brace frames located along the perimeter of the building. The displacements of the diaphragm when it is subjected to a uniformly distributed lateral load along its span are shown in Figure 1(b). The mid span displacement consists of two parts: (i) the lateral displacement at the top of the braced frames, Δ_B and (ii) the deflection of the diaphragm at the centre of the span, Δ_D. The ratio Δ_D/Δ_B, termed the drift ratio, provides a measure of the diaphragm flexibility.

For the purpose of analysis, the diaphragm is modeled as a deep beam supported by springs that represent the LLRS. The beam is divided into 20 interconnected beam elements in which the shear deformations as well as the flexural deformations are taken into account. The mass of the diaphragm is lumped at the nodes located at the intersections of the beam elements. The steel deck and its connections resist the shear force in the diaphragm while the chords located along the boundaries of the diaphragm provide the entire bending moment resistance. The springs, representing the LLRS, provide resistance against lateral loads. The LLRS could consist of braced frames, concrete shear walls, masonry shear walls or wood panel walls. However, in the present study, the LLRS is assumed to consist of concentric braces. Figure 1(c) is a schematic representation of the analytical model in which the beam is divided into 6 elements rather than 20. Linear elastic behaviour is assigned to the diaphragm while a bilinear hysteretic behavior is assumed for the LLRS. Each spring has an elastic stiffness of K_B and a post-yield stiffness of αK_B. Four different values for α are used in the study, namely: 0.0, 0.02, 0.05, and 0.10.

3. Building Database

A total of 33 buildings are selected for the present study. The selected buildings are part of a set of buildings designed by Tremblay and Stiemer (1996). The buildings in the set were designed according to the provisions of the 1995 National Building Code of Canada (NRCC 1995), and for six different locations. For each location, six buildings were designed.

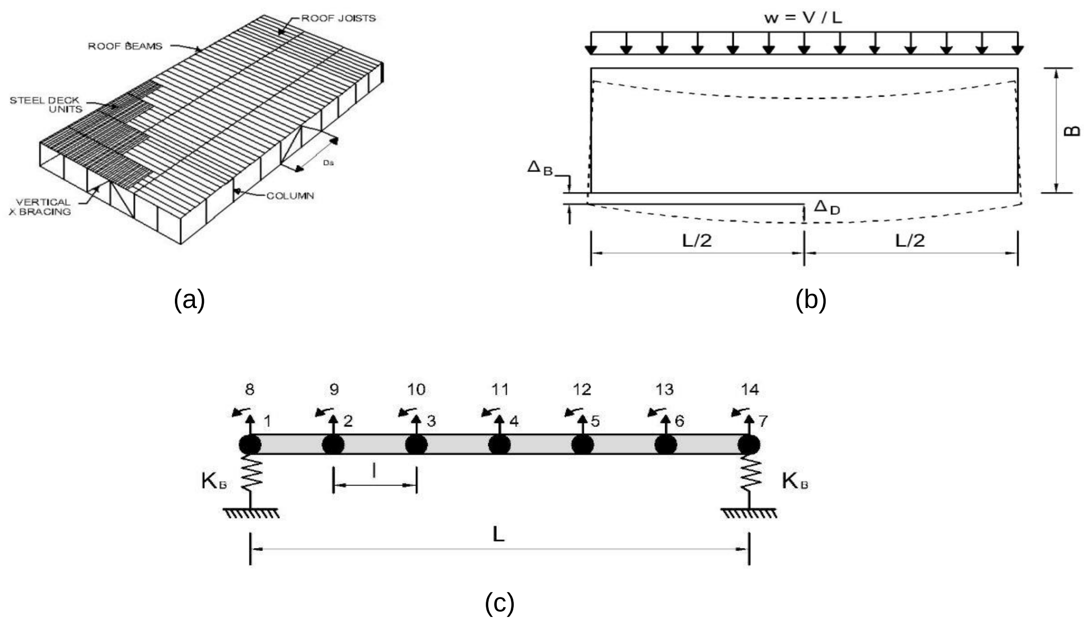

Figure 1- (a) One Storey Building Consisting of Steel Deck Panels Supported by Concentric Braces; (b) Displacement of the Diaphragm under Lateral Loads; (c) Analytical Model

The set of buildings included in the study are of three different sizes: small (15 × 30 × 5.4 m), medium (30 × 60 × 6.6 m), and large (60 × 120 × 9.0 m) and 2 types of roofing systems, a heavy roofing system and a light roofing system. All buildings had the length to width ratio of two in the plan. Details of the buildings are available in the paper by Tremblay and Stiemer (1996). Humar and Propovski (2013) provide the drift ratios and the elastic first mode period for the buildings. The drift ratios range from 1.04 to 10.3 and the periods range from 0.254 to 1.38 s

4. Spectrum Compatible Ground Motions

Using stochastic finite fault method, Atkinson (2009) has generated a series of ground motion time histories that are compatible with eastern (Montreal) and western (Vancouver) UHS. In fact, a set of time histories for a range of magnitudes, distances and site conditions were generated. With appropriate scaling such time-histories could be used to provide a response spectrum matching the UHS for any specified site. In the present study, records M6C1, M6C2, M6C26, M6C31 and M6C38, with scaling factors of 0.78, 0.87, 1.19, 0.99 and 1.43, respectively, are used to provide a match for the Vancouver UHS while records E6C1, E6C13, E6C15, E6C18 and E6C42, with scaling factors of 0.55, 0.74, 0.56, 0.61 and 1.01, respectively, are used to match the UHS for Montreal. The uniform hazard spectra for Vancouver and Montreal along with the spectra for the matching records are shown in Figures 2a and 2b.

5. Magnification of Internal Forces

Previous studies have shown that the flexibility of the diaphragm can cause an increase in the bending moments and shear forces acting along the length of the diaphragm. The reason for such behavior can be explained by the changed spatial distribution of the seismic force in flexible diaphragms and the increased contribution of higher modes.

In a flexible diaphragm the displacements along the length of the diaphragm are no longer uniform. The diaphragm experiences larger displacements near mid span. This suggests that larger accelerations and therefore larger inertia forces will be produced near mid span. In fact, the distribution of inertia forces along the length of the diaphragm follows a distribution similar to that of a parabola. In addition to increasing the inertia forces near mid span, flexibility of the diaphragm also increases the relative contribution of higher modes, particularly the third mode. This leads to greater bending moments near the mid span and increased in span shear forces, particularly that at the quarter span, which may even exceed the shear force at the end supports.

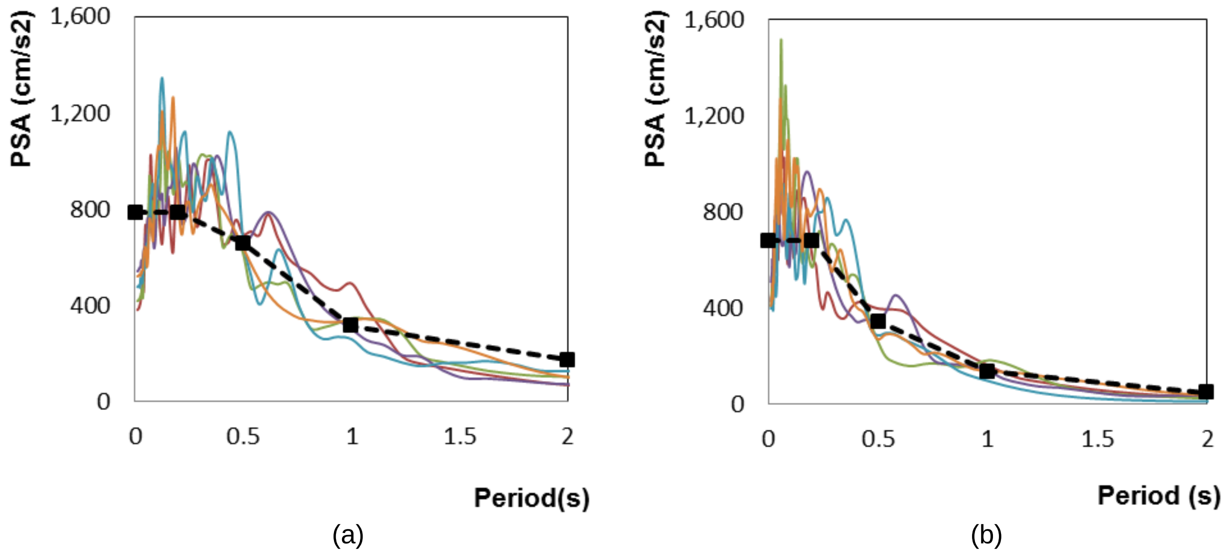

Figure 2 - Comparison of the Target UHS with the Spectra of Appropriately Scaled Spectrum Compatible Records; (a) Vancouver, (b) Montreal

Another effect of the flexibility of the diaphragm is the elongation of the period of the building. As a consequence, the total seismic base shear in the building becomes smaller. However, this reduced shear is accompanied by concentration of inertia forces near mid span. This results in larger bending moments at mid span and larger shear forces at quarter span that can exceed the corresponding values obtained from the rigid diaphragm case, which has a larger base shear, but a uniform distribution of inertia forces. Thus, even if a building with flexible diaphragm is treated as one with a rigid diaphragm, and designed for a larger base shear, the design may not be conservative in terms of the internal shear forces and bending moments. Figure 3 shows the bending moments and shear forces acting along the length of the diaphragm for buildings SL1, ML1 and LH1 (Tremblay and Stiemer 1996) when subjected to the record M6C1, first with no scale factor on the stiffness of the diaphragm and next with a scale factor of 200 to represent a rigid diaphragm. A force reduction factor of R_y =3.0 is used in these analyses and a post-yield stiffness value equal to zero is assigned to the LLRS. It can be observed that even though a greater base shear is associated with the response of buildings with rigid diaphragms, the bending moment at mid span and the shear force at quarter span for the flexible case still exceed those in the rigid diaphragm case.

Humar and Popovski (2013) have shown that when the total seismic base shear is distributed in the form of a parabola, as suggested by FEMA 356 (FEMA 2006), reasonable estimates are obtained for the bending moment at mid span when the structure is responding in the linear range. The parabolic distribution is given by

$$f_d = \frac{1.5F_d}{L_d}\left[1-\left(\frac{2x}{L_d}\right)^2\right] \tag{1}$$

where f_d is the inertia load intensity at a distance x from the centre line of the diaphragm, F_d is the total of inertia forces acting on the diaphragm, and L_d is the unsupported length of the diaphragm.

The present study suggests that with increasing diaphragm flexibility and nonlinearity in the supporting LLRS, the distribution of inertia forces departs more and more from a parabola, further increasing the inertia forces toward the mid-span and the contribution from the higher modes. Nevertheless, the parabolic distribution suggested by FEMA can serve as an appropriate basis for the development of methods that would account for the magnification of shear force at quarter span and bending moment at mid span.

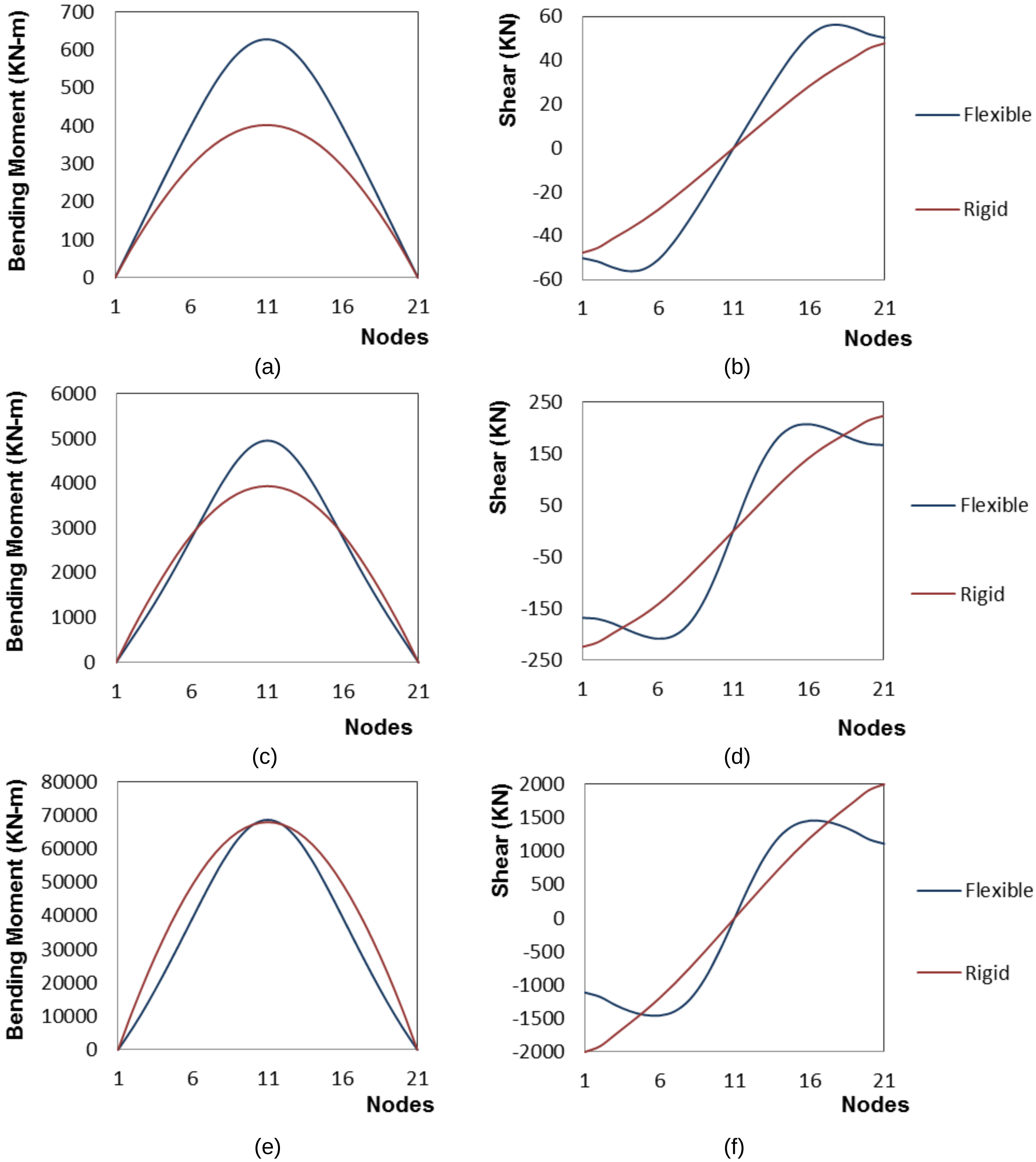

Figure 3 - The Distribution of Bending Moment and Shear Force along the Length of the Diaphragm: (a) Bending Moment for Building SL1, (b) Shear Force for Building SL1 (c) Bending Moment for Building ML1, (d) Shear Force for Building ML1 (e) Bending Moment for Building LH1, (f) Shear Force for Building LH1

6. Nonlinear Response to Earthquake Motions

The selected 66 buildings (33 in each direction) are subjected to the ten UHS compatible ground motions. Force reduction factors for the design of LLRS are those associated with target ductility demands of 2.0, 3.0 and 4.0; their values are available in Humar and Popovski (2013). Post-yield stiffness ratios of 0.00, 0.02, 0.05 and 0.10 are used for the bilinear force-displacement relationship in the brace, resulting in a total of 7920 analysis cases. The bending moment at mid span and the shear force at quarter span obtained from the time history analyses are compared to their corresponding values produced by the application of the seismic load distributed in the form of a parabola as suggested by FEMA 356. The values of the following parameters are determined:

a) M_f – The ratio of actual mid span moment, obtained from nonlinear time history analyses, to the mid span moment obtained from a parabolic distribution of the total seismic load.

b) V_f – The ratio of actual quarter span shear, obtained from nonlinear time history analyses, to the quarter span shear obtained from a parabolic distribution of the total seismic load.

The values for M_f and V_f obtained from the time history analyses, corresponding to α = 0.00 are presented in Figure 4, in the form of histograms. Similar histograms are obtained for other values of α, but are not presented here. All of the results suggest that correlation exists between the magnification and the extent of nonlinearity in the LLRS, and as the post yield stiffness increases, the magnifications decrease. For elastic response, corresponding to μ = 1, the mean value is 1, signifying that for the elastic case the FEMA parabolic distribution provides a reasonable estimate of the internal forces in the diaphragm. In all cases there is considerable scatter around the mean. However, given the uncertainty inherent in defining the seismic hazard, it would be reasonable to use the mean values in design. The mean values of M_f and V_f obtained from the nonlinear time history analyses are shown in Tables 1 and 2, respectively.

Table 1 – Mean Moment Magnification Ratios

	α = 0	**α = 0.02**	**α = 0.05**	**α = 0.10**
μ = 1.0	1.00	1.00	1.00	1.00
μ = 2.0	1.17	1.15	1.13	1.10
μ = 3.0	1.28	1.25	1.20	1.13
μ = 4.0	1.37	1.32	1.25	1.14

Table 2 – Mean Shear Magnification Ratios

	α = 0	**α = 0.02**	**α = 0.05**	**α = 0.10**
μ = 1.0	1.00	1.00	1.00	1.00
μ = 2.0	1.28	1.27	1.25	1.21
μ = 3.0	1.44	1.41	1.35	1.27
μ = 4.0	1.56	1.50	1.42	1.30

7. Development of Design Expressions

The magnification factors are functions of both α and μ. Two sets of alternative design expressions are obtained for these factors: a linear set and a quadratic set, as described in the following sections.

7.1. Linear Representation

Using a set of least squares fits, the following linear equations are obtained for the magnification factors:

$$M_f = 0.83 + 0.17\mu - 0.68\alpha(\mu - 1) \quad \text{for } \mu \le 2 \tag{2a}$$

$$M_f = a\mu + b \quad \text{for } \mu > 2 \tag{2b}$$

where

$$a = -0.80\alpha + 0.10 \tag{3}$$

$$b = 0.92\alpha + 0.97 \tag{4}$$

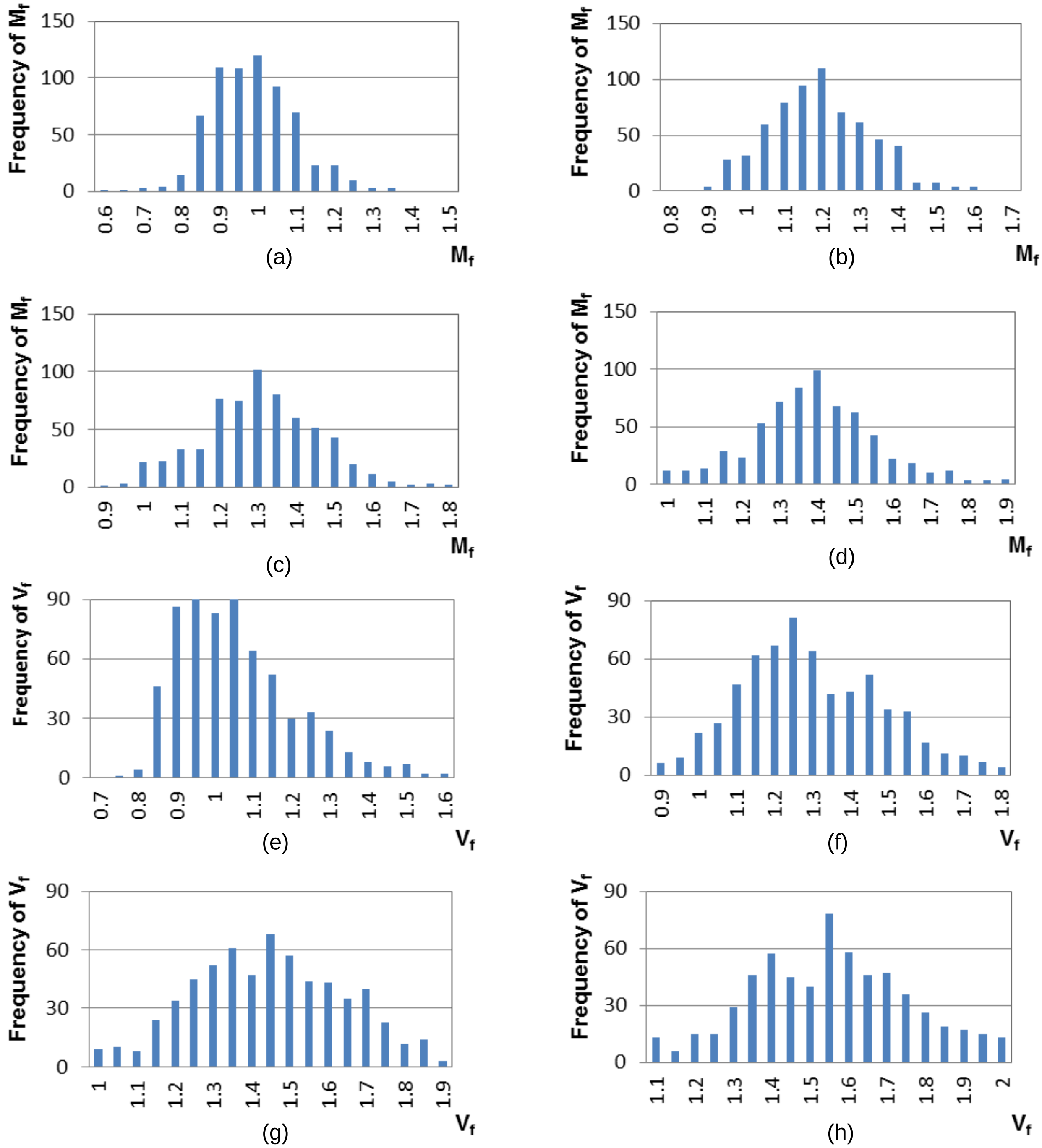

Figure 4 - Frequency of Occurrence for: (a) M_f, when μ = 1.0 and α=0.00, (b) M_f, when μ = 2.0 and α=0.00, (c) M_f, when μ = 3.0 and α=0.00, (d) M_f, when μ = 4.0 and α=0.00, (e) V_f, when μ = 1.0 and α=0.00, (f) V_f, when μ = 2.0 and α=0.00, (g) V_f, when μ = 3.0 and α=0.00, (h) V_f, when μ = 4.0 and α=0.00,

$$V_f = 0.72 + 0.28\mu - 0.70\alpha(\mu - 1) \quad \text{for } \mu \le 2 \tag{5a}$$

$$V_f = a\mu + b \quad \text{for } \mu > 2 \tag{5b}$$

where a and b are given by Equations 6 and 7, respectively.

$$a = -0.94\alpha + 0.14 \quad (6)$$
$$b = 1.14\alpha + 1.02 \quad (7)$$

Linear representations for M_f and V_f have been plotted against the ductility demand in Figures 5a and 5b, respectively, along with the mean values presented in Tables 1 and 2.

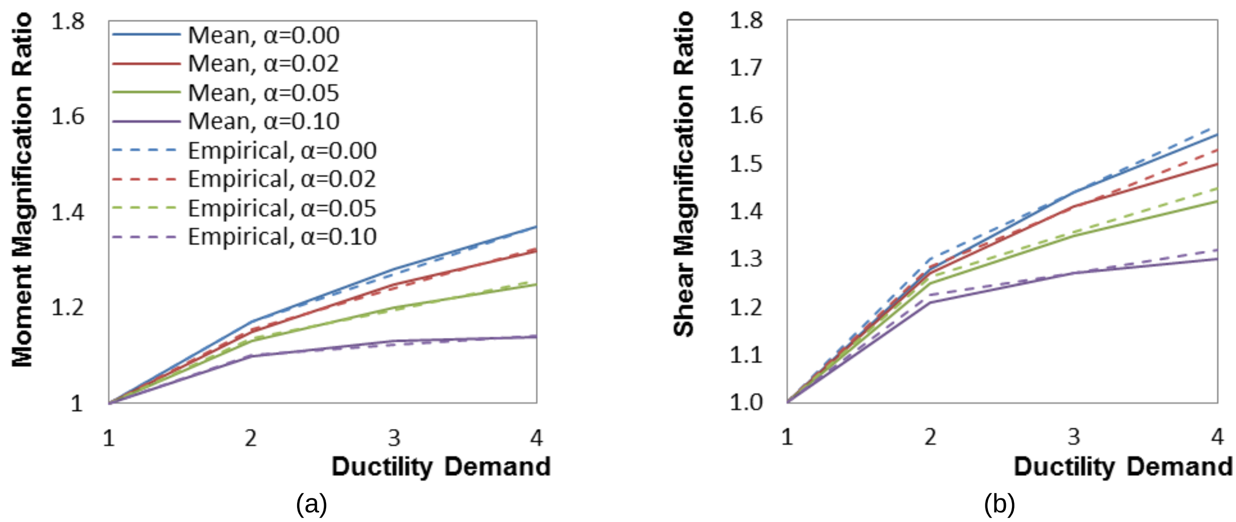

Figure 5 – Linear Representation of the Magnification Ratios Plotted with the Mean Values Obtained from the Time History Analyses - (a) M_f, and (b) V_f

7.2. Quadratic Representation

As an alternative to the linear equations the following set of quadratic equations are obtained, again using a series of least squares fits.

$$M_f = a\mu^2 + b\mu + c \quad (8)$$

where

$$a = -0.025\alpha - 0.02 \quad (9)$$
$$b = -0.64\alpha + 0.22 \quad (10)$$
$$c = 0.68\alpha + 0.80 \quad (11)$$

$$V_f = a\mu^2 + b\mu + c \quad (12)$$

where

$$a = -0.04\alpha - 0.04 \quad (13)$$
$$b = -0.68\alpha + 0.39 \quad (14)$$
$$c = 0.74\alpha + 0.65 \quad (15)$$

The moment and shear magnification curves obtained from Equation 8 and Equation 16, along with the mean values presented in Tables 1 and 2, are plotted against the ductility demand in Figure 6a and 6b, respectively.

8. Application of Expressions in Design

It must be noted that the magnification ratios, namely M_f and V_f, are developed based on the ultimate force reached in the brace and not the brace yield force. Therefore, in the design, the seismic force must be scaled up, based on the post-yield stiffness ratio and the target ductility demand, to match the ultimate capacity of the brace

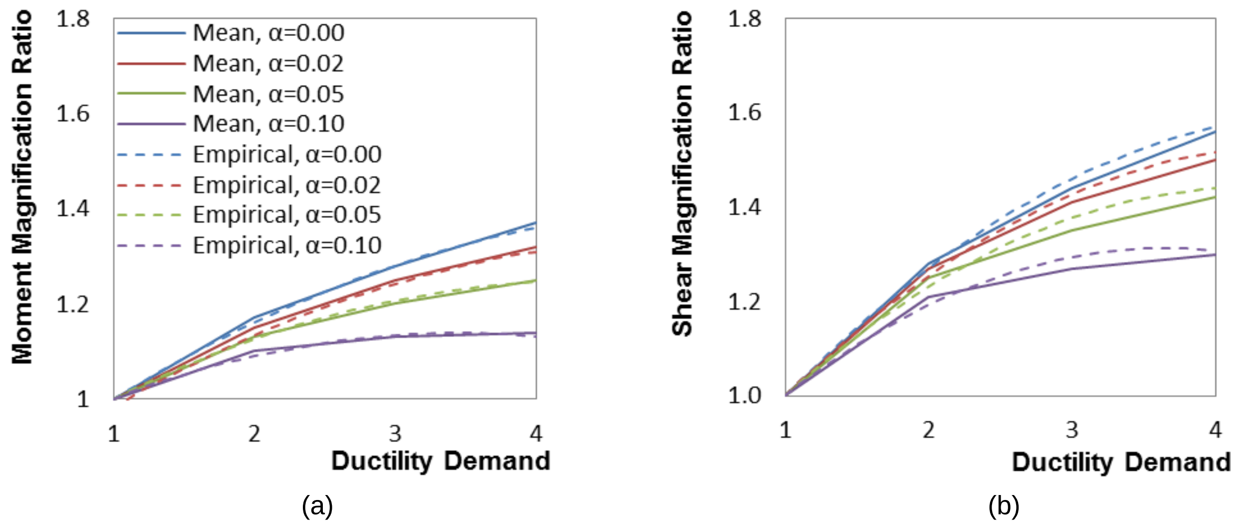

Figure 6 – Quadratic Representation of the Magnification Ratios Plotted with the Mean Values Obtained from the Time History Analyses - (a) M_f, and (b) V_f

The following steps outline the procedure for using the design expressions to predict the seismic shear forces and bending moments in the elastic roof diaphragm of one story buildings.

1. Determine the design base shear from the uniform hazard spectrum for the site and the appropriate force modification factors.

$$V = \frac{S(T_a)W}{R_d R_o} \tag{16}$$

   where $S(T_a)$ is the spectral acceleration for the fundamental period T_a, W is the total weight assigned to the level of the diaphragm, R_d is the ductility related force modification factor and R_o is the over-strength force modification factor.
2. Using Equation 17, scale up the design base shear to account for the post-yield strength reached in the brace.

$$V_u = V_d(1+\alpha(\mu-1)) \tag{17}$$

   where α is the brace post-yield stiffness ratio and μ is the target ductility demand for the design.
3. Distribute the total ultimate seismic load along the length of the roof diaphragm, using the parabolic distribution recommended by FEMA, given by Equation 1.
4. Determine the mid span bending moment. Using Equation 2 or Equation 8, scale up the mid span bending moment and use the obtained value in the design. Moments at other locations can be obtained by using the same magnification factor
5. Determine the shear force at quarter span and at the ends. Using Equation 5 or Equation 12, scale up the quarter span shear force and use the obtained values in the design.

9. Summary, Conclusion and Recommendations

The seismic response of one storey buildings with flexible diaphragms is significantly influenced by the flexibility of the diaphragm. Diaphragm flexibility leads to the magnification of bending moment and shear force along the length of the diaphragm at mid span and quarter span, respectively. An analytical study is carried out, using a large set of buildings and spectrum compatible ground motions, to examine the level of increase in the internal forces and to provide methods to account for such magnification in the design. The following conclusions are reached from the present study.

1. The flexibility of the diaphragm causes a non-uniform distribution of seismic forces along the length of the diaphragm. Higher modes participation further increases the inertia forces near mid-span. The concentration of inertia forces near mid-span leads to significant increase in bending moment at mid span and shear force at the quarter span.
2. The contribution of higher modes and consequently the magnification of bending moments and shear forces is enlarged when the excursion of the system into the nonlinear range is increased by the use of larger force reduction factors in the design of the LLRS. On the contrary, higher post yield stiffness values decrease the contribution of higher modes and the magnification of internal forces.
3. The FEMA parabolic distribution of inertia forces provides good estimates of the bending moment and shear force when the LLRS is elastic. Nonlinearity in the LLRS increases the mid span bending moments and the in span shears beyond those obtained from parabolic distribution. Therefore, the internal forces obtained from the parabolic distribution should be scaled up based on the equations suggested in the study.
4. Magnification of internal forces is slightly greater in the eastern region than in the western region because the spectral shape for the east is steeper leading to greater contribution from the higher modes. However, the differences are not large, and the single set of equations proposed here may be used for all regions.
5. Further studies are recommended using buildings with different types of LLRS and buildings with different LLRS layout.

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