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SUBSPACE BASED DAMAGE DETECTION TECHNIQUE: INVESTIGATION ON THE EFFECT OF NUMBER OF SAMPLES

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ABSTRACT: Damage detection techniques are the main tool in health monitoring to assess the functionality of structures. Among these techniques, statistical subspace-based damage detection technique is a robust method to evaluate the conditions of a structure without the need of evaluating its modal parameters. This can circumvent all the errors and difficulties in evaluating the modal properties of the structure while the changes in the eigen-structure of the data is identified indirectly. In our previous studies, the effects of damage location, damage ratio, and the noise in the data were investigated. In this study, the sensitivity of this technique to the number of samples of the data is addressed. The interaction of the number of measurements with damage and noise ratio is also investigated. For this purpose a bridge structure located in Reibersdorf, Austria, is considered. This structure is modelled and calibrated to the real test data; subsequently the damage is modelled in one of the elements for different damage ratios and excitation duration. It was demonstrated that this technique can operate robustly even with high noise present in the data by acquiring typical number of measurements especially for reference state data. Moreover, it was concluded that if the length of the reference data would not be large enough, increasing the length of the test data cannot much help in identifying the damage in the structure.

1. Introduction

Structural health monitoring is regarded as the main tool in assessing the functionality of existing structures. The importance of these techniques and researches becomes obvious by considering that failure of a structure can result in catastrophic lost.

Existing civil structures deteriorate by aging and under different loading conditions imposed from natural phenomena such as earthquakes, typhoons, flood and etc. Therefore, it is imperative to investigate the safety of continuing using these structures, especially after occurring major demands on the structure from these phenomena.

Numerous researches can be found in the literature and different approaches are proposed to deal with this problem by detecting possible damages in a structure. Some of these tests include sampling of the structure, which may affect the functionality of structure. These tests are named destructive tests. However the other type of the tests, namely non-destructive tests, do not involve with any action that can damage the structure or affect its functionality. Due to the need of continuation of the serviceability of the structure, more researchers have been focusing on the latter approach.

Nondestructive damage detection techniques can be categorized into two groups based on their requirements (Fan and Qiao, 2011): (I) local techniques, which need access to all parts of the structure or the location of damage if known, and (II) global damage techniques which use vibration data to evaluate global dynamic characteristics of the structure. Employing the local techniques may lead to interference in the operation of the structure and is not suitable for major structures. However, in the latter method there is no need to know or have access to the location of damage in priori.

The global techniques can be also categorized into two groups based on their approach to the problem. In the first category, the structural properties are identified and employed to assess the condition of the structure. The structural properties identified from these approaches include stiffness, damping, mass, load paths and boundary conditions (supports, connections, etc.). In the second category, the eigen-structure of the problem is employed to evaluate the safety condition of the structure. In these methods, modal properties such as natural frequencies, modal damping values and mode shapes are used to identify any changes in the structure. Any change in the structural properties leads to a change in the modal parameters of the structure. However generally, identifying the modal parameters in a structure is more practical and accurate than the structural properties.

The process of evaluating the modal parameters of a structure is also time consuming and it usually cannot be employed in real-time monitoring. Evaluation of these dynamic characteristics can be avoided by using statistical approaches, e.g. statistical subspace-based damage detection technique (SSDD) (Basseville et al, 2011, Basseville et al, 2004, Döhler et al, 2014, and Döhler and Mevel, 2013). This technique evaluates the global condition of structure by identifying changes in the eigen-structure of the problem. The damage can be detected by comparing a statistical model from the possibly damaged structure to the one obtained from a reference state. In other words, a subspace based residual function between these states is defined and compared using a χ^2 test. In this way, there is no need to estimate the natural frequencies and mode shapes, making this approach capable of being used in real-time monitoring of structures. It is demonstrated that this approach can also perform robustly under ambient excitations with changing statistics (Döhler et al, 2014, Döhler and Mevel, 2013, and Döhler and Hille, 2014).

In (Allahdadian et al, 2015), the performance of the SSDD technique was assessed for different damage types and ratios. It was shown that this technique can efficiently identify the damage occurred in different elements of the structure for various ratios. Moreover, the effect of the noise in the data was also investigated in our previous study (Allahdadian et al, 2015), in which it was demonstrated that this technique can detect damage from data with high noise ratios due to the fact that it monitors the change in the eigen-structure of the data.

In this study, we will investigate the effect of the number of samples taken for the test setups on the outcome of this technique. The effects of number of samples in reference state and in the test data from possibly damaged structure are considered. Moreover, the noise and damage ratio are the parameters involved, in order to study their interaction with the number of samples.

A bridge structure, i.e. S101, located at Reibersdorf, Austria, is investigated and simulated for this purpose. This structure was damaged artificially in a progressive manner and it was continuously measured during each damage level (Döhler et al, 2014). A finite element model of this structure was created and calibrated using the available measured data. The simulated measurements are generated by measuring the acceleration time histories of the nodes typically measured in a bridge structure. The noise ratio is then applied to the measured data and it is consequently processed by ARTeMIS software ®.

2. Stochastic subspace-based damage detection

Statistical subspace-based damage detection (SSDD) technique detects the damage in a structure by using a χ^2 test on a residual function (Basseville et al, 2011, Basseville et al, 2004, Döhler et al, 2014 and Döhler et al, 2013). Therefore, in this method, there is no need to compute and compare modal parameters of the reference and possibly damaged states of the system. In other words, this residual function represents the changes occurred to the model which can be caused by a damage in structure. These changes are identified in the eigen-structure of the problem.

2.1. Models and parameters

The dynamic system of the model can be considered as a discrete time state space model of

$$\begin{cases} X_{k+1} = FX_k + \varepsilon_k \\ Y_k = HX_k + \vartheta_k \end{cases} \tag{1}$$

where, the state is represented by $X \in \mathbb{R}^n$ and the measured output is $Y \in \mathbb{R}^r$. F also represents the state transition matrix and H shows the observation matrix with dimensions $n \times n$ and $r \times n$, respectively. The state noise, ε_k and measurement noise ϑ_k are assumed to be Gaussian unmeasured white noise with zero mean. The covariance of output measurements Y_k can be computed from the state space model (1) by

$$R_i = \mathbf{E}\left(Y_{k+i} Y_k^T\right) \tag{2}$$

in which operator $\mathbf{E}$ is the expectation function. With choosing parameters *q* and *p* such as $q \geq p+1$, the Hankel matrix $\mathbf{H}$ can be written as

$$\mathbf{H} = \begin{pmatrix} R_1 & R_2 & \dots & R_q \\ R_2 & R_3 & & R_{q+1} \\ \vdots & & \ddots & \vdots \\ R_{p+1} & R_{p+2} & \dots & R_{p+q} \\ & & & \end{pmatrix} \tag{3}.$$

As mentioned earlier the measurements are performed in a reference state and a possibly damaged state. The Hankel matrix of the measurements in reference state, $\mathbf{H}_0$, can then be computed from (2) and (3). This matrix is then decomposed using singular value decomposition in order to compute the left null space $\mathbf{S}$. Defining $\mathbf{H}$ for the possibly damaged state of the system, the left null space matrix **S** in the reference state is characterized by $\mathbf{S}^T\mathbf{H} = \mathbf{0}$ (Basseville et al, 2000, and Basseville et al, 2004). Therefore, the residual vector $\boldsymbol{\zeta}_n$ can be written as

$$\boldsymbol{\zeta}_n = \sqrt{n}\,\mathrm{vec}(\mathbf{S}^T\mathbf{H}) \tag{4}$$

in which, n represents the number of samples measured for computing $\mathbf{H}$. This residual can now be used in order to check if any change is made in the model due to damage. The residual vector $\boldsymbol{\zeta}_n$ is asymptotically Gaussian with zero mean in reference state; significant changes in its mean value indicates the structure is moved from its reference state. In order to check this change from the residual vector mean, the χ^2 test can be performed as following (Basseville et al, 2011, Basseville et al, 2004, Döhler et al, 2014 and Döhler et al, 2013).

$$\chi^2 = \zeta_n^T \Sigma^{-1} \zeta_n \tag{5}$$

Herein, Σ represents the covariance matrix of the residual in the reference state, and can be shown as

$$\Sigma = \mathbf{E}\left[\zeta_n \zeta_n^T\right] \tag{6}$$.

It is worth mentioning that the covariance of the input noise ε_k is assumed to not change between the reference state and the possibly damaged state. When using the residual defined in (4). In (Döhler et al, 2014, and Döhler and Mevel, 2013) it was shown that the modified residual

$$\tilde{\zeta}_n = \sqrt{n}\,\mathrm{vec}(\mathbf{S}^T \mathbf{U}_1) \tag{7}$$

is robust to changes in the covariance of the input noise ε_k in the same framework, where $\mathbf{U}_1$ is the matrix of the principal left singular vectors of $\mathbf{H}$.

By monitoring the value of χ^2 and comparing it to a threshold value, the state of the damage of the system can be estimated. This threshold can be simply evaluated using several data sets measured from the structure in its reference state. Subsequently, some other data sets measured from the reference state are used to check the threshold. Then the χ^2 value is computed for the possibly damaged structure. If the χ^2 value is computed to be higher than the threshold it can be inferred that the structure may be damaged. In other words, the amount of effect of damage on statistics of the measured data has a straight relation with the amount of the χ^2 value.

3. Data simulation

Simulating the damage in a structure and subsequently generating the ambient vibration test data can be a straightforward approach to evaluate damage detection techniques. This data can be an acceptable benchmark to evaluate the functionality of these techniques by providing control on the test conditions, e.g. structural properties and damage effects. In order to investigate the effect of noise on these techniques, a predefined amount of white noise is superposed to the simulated data.

Several points of the structure are excited using white noise excitation in all three directions. Different excitations are imposed on the structure in order to excite the structure as randomly as possible. This excitation can be done by acceleration or load forces in different points of the structure. Subsequently, the simulated data can be obtained by measuring acceleration time histories of the nodes typically measured and instrumented in a bridge.

The simulated data can then be analyzed in order to compute the natural frequencies and their corresponding mode shapes. These can be used to check which mode shapes can be captured by the simulated white noise excitation. Based on the positioning of the sensors and or insufficient excitation of the structure, some mode shapes may not be captured. For the latter, the excitation must be modified to impose an excitation to the structure close to the white noise in different points of the structure.

3.1. Damage simulation

In order to evaluate the functionality of the subspace-based damage detection technique, the ambient vibration test data can be simulated for different damage ratios. To simulate this data, a finite element model of the structure is created and then calibrated to the real structure. It should be mentioned that calibration of the structure does not have a straight effect on the damage detection technique. In other words, the damage detection technique should be able to detect the damage in any structural model including the uncalibrated one as long as the base of comparison is identical. However, in this study, calibration to a real structure is performed to obtain a realistic model.

The damage can be modeled in different locations by reducing the dimensions of one or some short elements corresponding to it. The amount of the damage can be presented by the ratio of this reduction. For each damage, ratio separate analysis model is created.

3.2. Noise superposition

The imposed noise on the data is created using a random generation algorithm. The simulated test data in each point and each direction is defined as a measurement channel. The probability distribution of the random generator is evenly distributed and its magnitude is chosen as a ratio, i.e. noise ratio, of the maximum value of each channel. Therefore, the maximum value, m_i, in each measurement channel, i, is evaluated and then by multiplying it to the noise ratio (N_r) the interval of the random numbers is defined. The random vector $\mathbf{R}_i$ can be evaluated from

$$\mathbf{R}_i = \text{random}(N_r m_i, \text{even}) \quad (8).$$

In the next step, the random vector $\mathbf{R}_i$ is added to the measured data $\mathbf{D}_i$ for the corresponding channel. Therefore, the modified measured data $\mathbf{ND}_i$ can be evaluated as

$$\mathbf{ND}_i = \mathbf{D}_i + \mathbf{R}_i \quad (9).$$

It should be noted that the mean of the generated noise vector, $\mathbf{R}_i$, is zero. Different noise ratios are investigated in this study, which will be described in section 4.2.

3.3. Number of samples

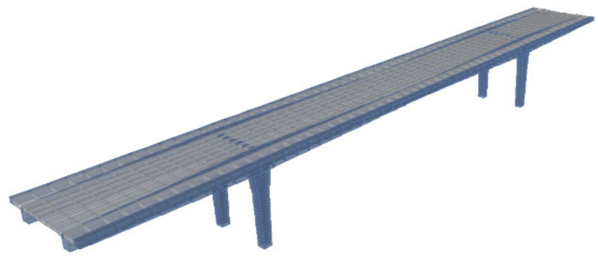
Number of samples of a discretized data depends on the frequency and duration of sampling. Therefore, by changing these values the number of the samples can be controlled. However, the frequency of sampling affects the precision of the data acquired especially in frequency domain. Depending on the general dynamic behaviour of a structure, the frequency of the sampling must be chosen. Among dynamic behaviours of a structure, the first natural frequency is the most popular benchmark to estimate the minimum threshold for the sampling frequency. Moreover, the maximum frequency that can be captured from a discretized data is the Nyquist frequency which equals to half of the sampling frequency. Therefore, the sampling frequency must be chosen as two times the maximum frequency, e.g. maximum natural frequency, which we are interested to capture.

Herein, we are interested to capture the effect of the data length. Therefore, the sampling frequency for all cases is constant; by changing the duration of the sampling, data with different length and number of samples can be generated.

4. Case study and discussion

The case study is the model of a bridge structure, namely S101, located in Reibersdorf, Austria. In (Döhler et al, 2014), this structure was progressively damaged and the ambient vibration data was recorded continuously to evaluate the SSDD method. In this study, the finite element model of this structure is used to simulate the damage in a specific location of the bridge, i.e. center of one of the main girders, with various extents. The finite element model is calibrated using the measured data from the bridge to have an accurate estimation of the behaviour of structure. The bridge structure and its finite element model are shown in Fig. 1. The natural frequencies of the analytical model and the bridge structure are also compared at Table 1.

(a)



(b)

Fig. 1 – (a) S101 bridge structure, Austria, and (b) its calibrated finite element model

Table 1 - Natural frequencies of the bridge structure in undamaged condition obtained from the measured data and finite element model

	Measured data (Hz)	Finite element model (Hz)
First bending mode	4.05	4.04
First torsional mode	6.30	6.08
Second bending mode	9.69	10.72
Second torsional mode	13.29	12.85
Third bending mode	15.93	19.58

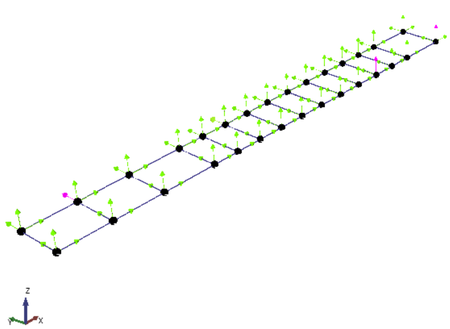
It can be inferred from Table 1 that the finite element model of the structure can be a good representative of the dynamic properties of the bridge. As mentioned in previous section, the purpose of this calibration is to have a realistic model of a bridge and it does not affect the assessment of the functionality of the damage detection technique. The effect of bearings in simulating the damage in other elements of the bridge is neglected. This effect was previously studied in (Allahdadian et al, 2015).

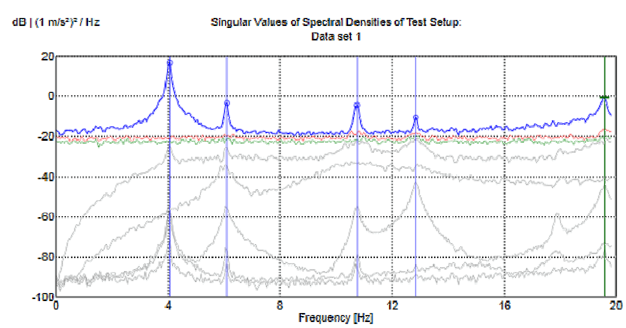
4.1. Damage and data simulation

As a demonstrative example, the damage is modeled only in the center of one of the main girders of the bridge. The reason of choosing the main girder for the damage location is because of its significant effect on the functionality of the bridge and investigating the sensitivity of the damage detection technique to such a damage. The effect of damage location in different element types was investigated in (Allahdadian et al, 2015), in which the data was assumed as pure and without noise. It is assumed that the noise ratio and damage location should not have any interaction that affects the functionality of the damage detection technique. However, this interaction is intended to be studied in future research papers.

In the girder, the damage is simulated by reducing a ratio, namely damage ratio, of its section dimension around the strong axis. The damage ratio varies among 20% (mild damage), 40% (medium damage) and 80% (severe damage).

The finite element model of the structure is excited with a white noise excitation as an acceleration time history in three directions. Moreover, the structure is vibrated by different white noise loads in various locations. The measured points to record acceleration time histories are illustrated in Fig. 2a. Spectral densities of the simulated data obtained from undamaged reference case are shown in Fig. 2b.





(a) **(b)**

Fig. 2 – (a) measuring-points corresponding to sensor locations; (b) Frequency domain decomposition of the simulated measurement data in undamaged structure

It is shown in Fig. 2b that the natural frequencies of the analytical model can be obtained from processing the simulated data accurately. Although, the structure is properly excited by white excitation, but some mode shapes cannot be captured. This stems from the configuration of the sensors and their resolution. As an example, the mode shapes associated to the longitudinal edges of the bridge cannot be captured by the sensors due to their small accelerations occurring in sensor locations.

4.2. Data length

In this study, the sampling frequency employed in all the cases is 500 Hz. This frequency is chosen constant and high (comparing to the first natural frequency of the structure) to eliminate its effect on the results. Therefore, by changing the duration of the measurements the number of samples is controlled. The durations of the simulated test data chosen in this study varies among 5, 7.5, 10, 12.5, 15, 17.5 and 20 minute. In reference state, the duration of each batch of data is chosen between 5, 10 and 15 minute, which correspond to, respectively, 20, 40 and 60 minute in total.

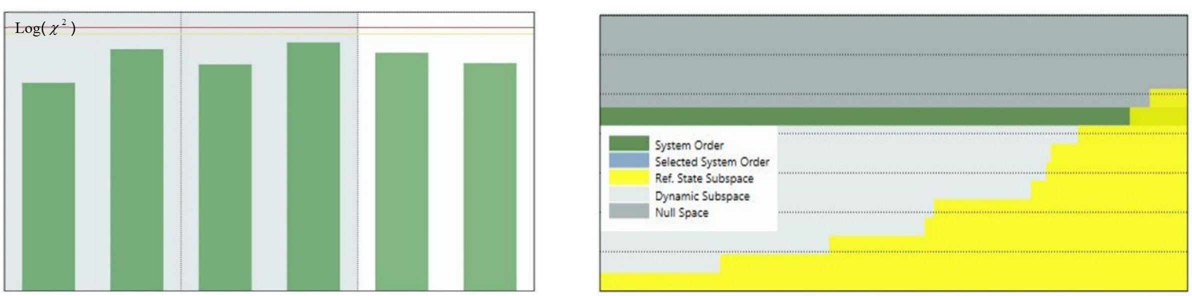
4.3. Noise addition

In order to investigate the effect of noise on the SSDD technique, a white noise vector is created using a random number generator and is applied to the data as mentioned in section 4. There are 90 sensors (channels) modeled for this bridge and the noise for each channel is applied based on the maximum response of that channel. Therefore, for each noise ratio, 90 vectors of time history of noise is created and applied to the data. It should be noted that the noise ratio in the reference state is different from the noise in measured data; for different noise ratios in reference state, different noise ratios in the measured data is investigated. The noise ratios chosen for this study are 10%, 30% and 80% in the measured data.

4.4. Damage detection and the results

The undamaged FE model of the structure is excited for six cases from which four (reference state) are used to create a threshold for the χ^2 value. The two remaining cases are then used to check the threshold (checking data). For each damage and noise ratio, the simulated data is recreated and the χ^2 test is performed. Subsequently, this value is compared to the computed thresholds from the reference state.

In each individual case the noise in reference state, checking and test data is equal and constant. Each case is corresponding to a specific duration of reference data. However, the damage and duration of the simulated test and checking data are changed. As an example for the reference state data, the reference state for the data without noise is shown in Fig. 3a. In order to validate the reference state, the null space of the Hankel matrix is illustrated in Fig. 3b, which shows that only a small portion of the singular values are more than the system order. This suggests that the reference state in both cases are reliable.

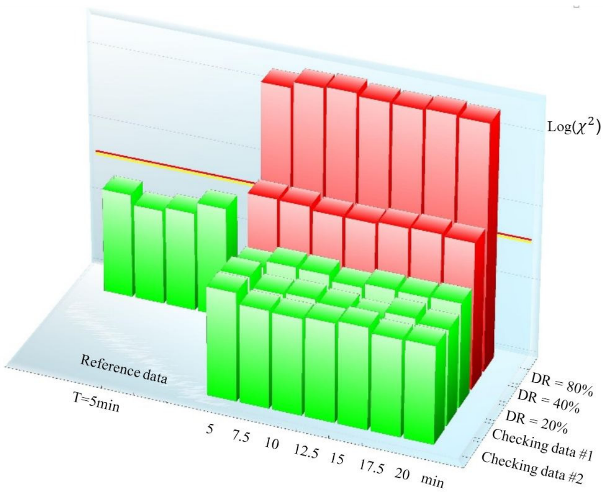


(a) **(b)**

Fig. 3 – (a) χ^2 values and thresholds of the reference state, and (b) validation of the reference state for data without noise

The threshold is computed for two significance values, namely critical zone for significance level 95% (shown with yellow line) and unsafe zone for significance level 99% (shown with red line). If the χ^2 test value computed from the structure becomes more than the yellow line, it suggests that the structure is in critical state. Similarly, if this value passes the red line, then the structure is estimated to be in unsafe conditions.

The χ^2 test values of the simulated data for different cases are illustrated in Fig. 4-6. The noise ratio in all the cases is 10% for Fig. 4, 30% for Fig. 5 and 80% for Fig. 6. Each figure includes two 3D graphs which are corresponding to a specific reference data duration. For each case, the χ^2 value is computed and compared to the threshold acquired from the reference state of the structure.



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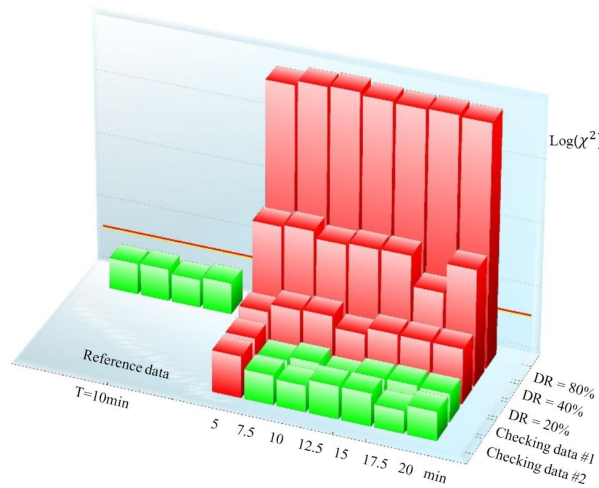

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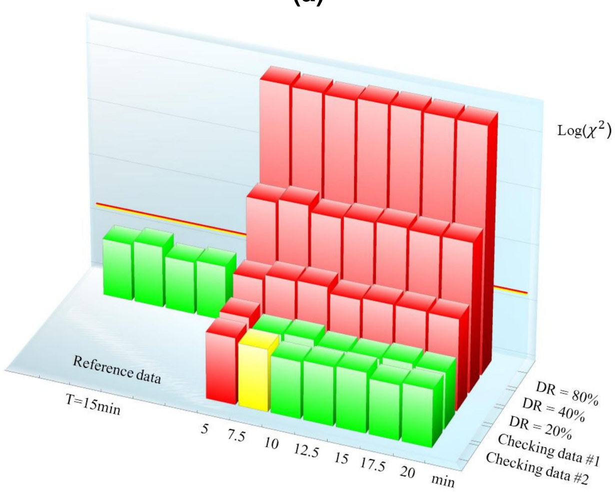
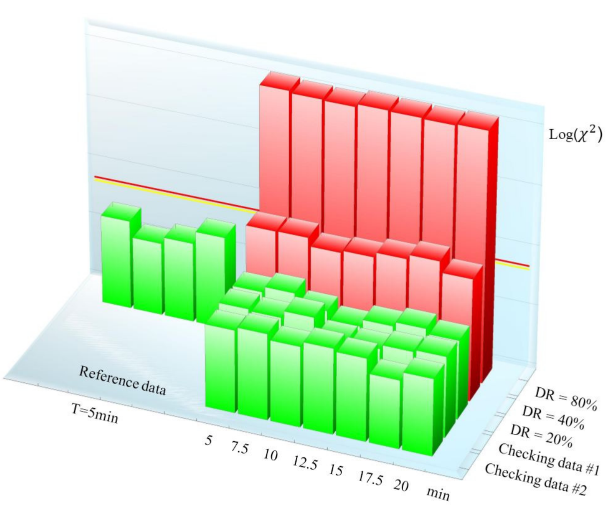
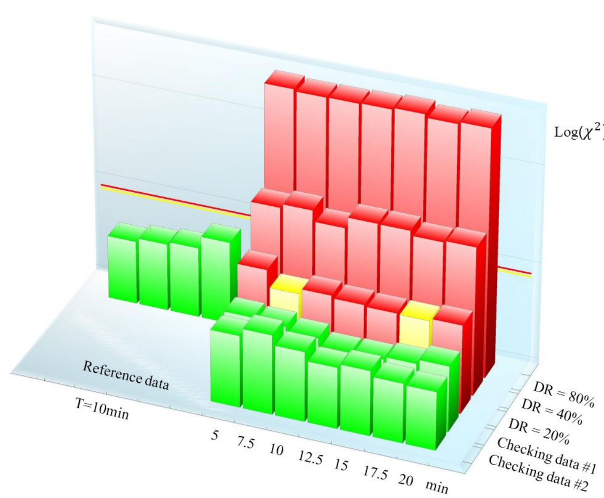


(c)

Fig. 4 - χ^2 test from SSDD technique for different damage ratio (DR) and with 10% noise in the data: (a) length of reference data is 5 minute, (b) 10 minute and (c) 15 minute.



(a)



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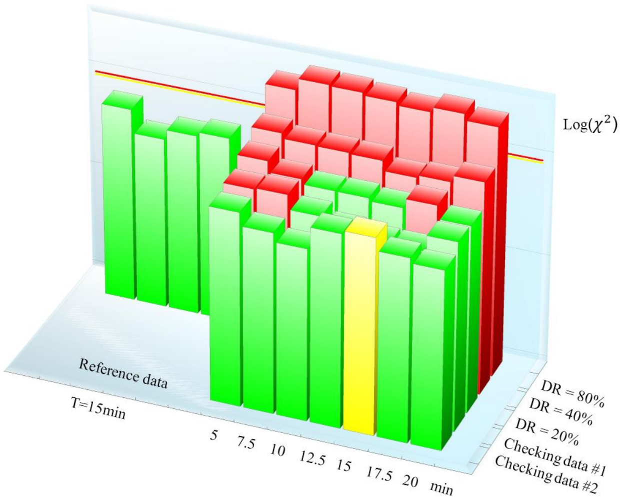
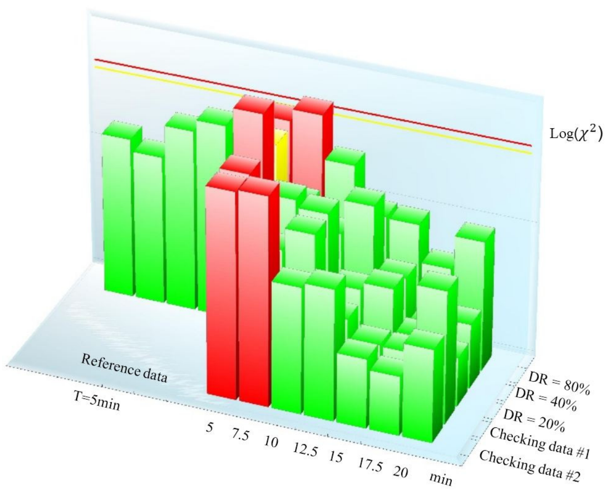
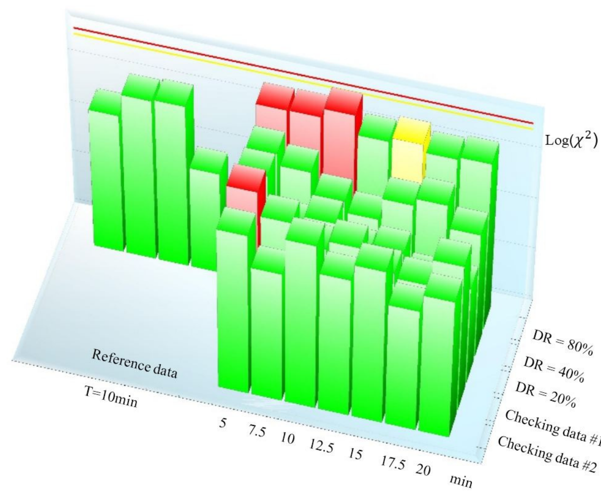


(c)

Fig. 5 - χ^2 test from SSDD technique for different damage ratio (DR) and with 30% noise in the data: (a) length of reference data is 5 minute, (b) 10 minute and (c) 15 minute.



(a)



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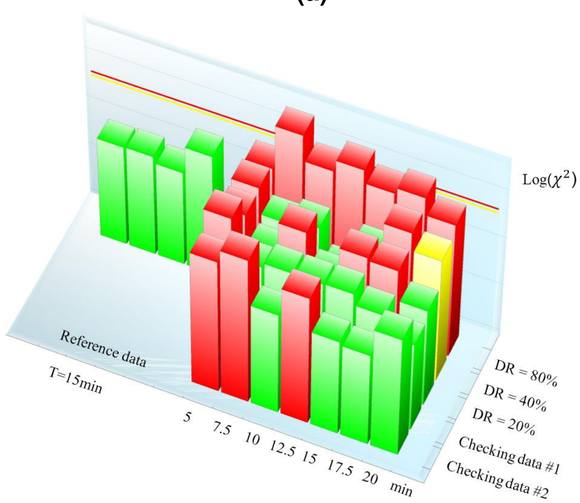

(c)

Fig. 6. χ^2 test from SSDD technique for different damage ratio (DR) and with 80% noise in the data: (a) length of reference data is 5 minute, (b) 10 minute and (c) 15 minute.

5. Discussion and conclusion

The following conclusions can be made by considering the results presented in this paper:

- Longer data length in the reference state helps to identify the damage in the structure. Especially when there is higher noise present in the data or the ratio of damage is not high, longer reference data can help to the damage detection. As an example, it can be seen in Fig. 6 in which the noise ratio is high, the severe damage can be detected only when the reference data length is high enough, i.e. 15 minute. Another example is that the mild damage in the structure cannot be detected for short reference data length, e.g. Fig. 4a and 5a, while it is detected using longer reference data, e.g. Fig. 4b and 5b. Similarly in Fig. 6c the damage can be detected with higher chance than in Fig. 6b.
- If the length of the reference data would not be large enough, increasing the length of the test data cannot much help in identifying the damage in the structure. As it is shown in Fig. 4a and 5a, by keeping the reference data constant and increasing the length of the test data, the mild damage in the structure cannot be detected.
- Enough length of test data is necessary to identify the damage ratio properly compared to the checking data. As an example, it can be seen in Fig. 4 that only by increasing the test data length from 5 minute the residual of the mild damage becomes efficiently larger than the one of the checking data.

For future study, investigating the data length effect in more depth is intended. Further study will be also focused on developing a formulation of this effect to evaluate and propose a threshold on the number of samples for both the reference and test data.

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