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PARAMETRIC STUDY OF DISPLACEMENTS IN SELF-CENTERING SINGLE-DEGREE-OF-FREEDOM SYSTEMS

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ABSTRACT: Modern seismic design has enabled engineers to design structures to avoid collapse and save lives, but repairs afterwards can be expensive and time-consuming. Therefore, the focus of seismic design is moving beyond collapse prevention to also control structural damage. With this additional objective, the concept of a self-centering system that returns to its original position after a major earthquake is becoming more appealing. A self-centering system can be described in terms of the initial period, the linear limit, the energy dissipation parameter, and the stiffness in the nonlinear range. Therefore, studying the influence of these parameters on the seismic response of a generalized single-degree-of-freedom (SDOF) system gives insight into the design and analysis of many different self-centering systems. Previous research on self-centering SDOF systems has been limited to systems with a relatively high strength and no significant effect. In this study, the influence of different parameters is explored by considering 2,688,000 nonlinear dynamic analyses of self-centering SDOF systems. Increasing the period or reducing the linear limit generally increases the peak displacements. However, changes to the linear limit are less significant when the linear limit is already low. Reducing the energy dissipation also increases the peak displacements. The stiffness in the nonlinear range is relatively unimportant when it is positive, but it becomes increasingly important when it becomes negative.

1. Introduction

The main objective of seismic design is to prevent collapse and allow safe evacuation after a rare large earthquake. To achieve this goal, structural systems have traditionally been designed to ensure that certain elements dissipate energy through inelastic deformation. This methodology has proved to be effective in recent major earthquakes [e.g. Kam et al. 2010, Clifton et al. 2011]. However, even if people survived, the buildings were often so damaged that reoccupancy was not possible without expensive and time-consuming repair, and sometimes even demolition and reconstruction were needed [e.g. The New Zealand Herald 2012]. The need for reconstruction is related to the structures' residual deformation [e.g. Iwata et al. 2005, McCormick et al. 2008].

Because of the ongoing effort to control residual deformations, self-centering structures are attracting increasing attention. A self-centering system usually depends on a gap opening mechanism to limit earthquake-induced forces, and it will return to its original position because of its self-weight and prestressing. This self-centering principle can be applied to precast concrete frame rocking joints [e.g. Priestley and MacRae, 1996], unbonded post-tensioned precast concrete frames [e.g. Roh and Reinhorn, 2010] and walls [e.g. Kurama et al. 1999], steel moment frame connections (Fig. 1(a)) [e.g. Christopoulos et al. 2002] and controlled rocking steel braced frames (Fig. 1(b)) [e.g. Wiebe et al. 2013]. As these systems can limit seismic forces without structural damage, there is not normally a limit on ductility capacity, but only deformation capacity.

While these systems have different mechanisms, the cyclic behaviour of each can be described as a flag-shaped hysteresis, as shown in Fig. 1(c). If a self-centering structure is to be designed, this flag-shaped hysteresis must be fully defined to achieve a certain performance target (e.g. lateral displacement). Thus, it is important to study the seismic response of generalized SDOF self-centering systems.

Previous studies [Christopoulos et al. 2002, Seo and Sause, 2005] have explored the influence of the parameters that define a flag-shaped hysteresis on ductility demands and showed that by adjusting the energy dissipation and nonlinear stiffness, self-centering systems can achieve similar ductility demands as traditional systems with elastoplastic hysteresis. In these studies, Christopoulos et al. (2002) implicitly considered force reduction factors R in the range of $4.5-8.5$ and Seo and Sause (2005) used values of $2-6$. However, recent studies [Wiebe and Christopoulos, 2014] have suggested that a larger value of R can be used while still achieving reasonable displacement limits. On the other hand, none of these studies have considered the effect of a negative nonlinear stiffness caused by significant $P-\Delta$ effects.

This paper extends previous studies to also consider large force reduction factors and negative nonlinear stiffnesses. After defining the relevant parameters and the ground motions used, the results are presented to show the influence of different parameters.

2. Definition of Parameters

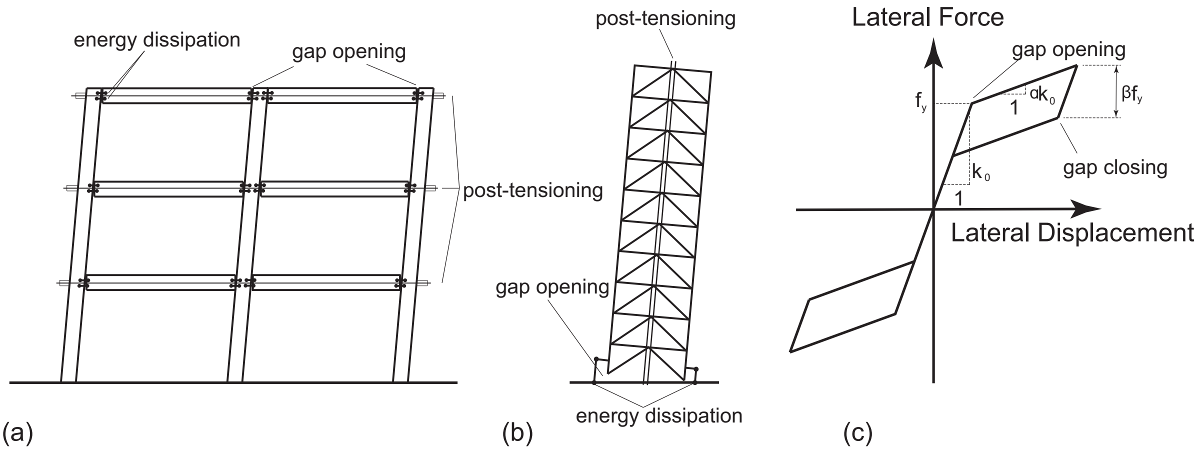

Fig. 1 – a) Self-centering moment resisting frame; (b) controlled rocking steel braced frame; (c) flag-shaped hysteresis

As shown in Fig. 1(c), the behaviour of any self-centering system can be idealized by four parameters: initial stiffness k_0, linear limit f_y, nonlinear stiffness αk_0, and energy dissipation parameter β. In this study, the initial stiffness will be defined by its relation to the natural period:

$$T_0 = 2\pi\sqrt{\frac{m}{k_0}} \quad (1)$$

The nonlinear stiffness will be defined in terms of a tangent period:

$$T_{\tan} = 2\pi\sqrt{\frac{m}{|\alpha|k_0}} \times \mathrm{sgn}(\alpha) \quad (2)$$

For traditional elastoplastic systems, both the initial stiffness and the nonlinear stiffness are determined by the structural stiffness distributions, so it is appropriate to define the nonlinear stiffness using α as a fraction of the initial stiffness [e.g. Miranda, 2000]. The first studies on self-centering systems adopted this convention [e.g. Christopoulos et al. 2002, Seo and Sause, 2005]. However, for self-centering systems, after the gap opens, the nonlinear stiffness is determined by the post-tensioning and is nearly independent of the initial stiffness [e.g. Wiebe and Christopoulos, 2014]. When designing a self-centering system, the post-tensioning (or αk_0) is normally selected before all structural members have been defined, so the initial stiffness is unknown. Therefore $T_{\tan}$ is easier to control during initial design than α. Another advantage of this definition is that it separates the nonlinear stiffness completely from the initial stiffness, decoupling these two parameters from a research perspective. Note that by this definition the tangent period is negative when $\alpha < 0$, and $T_{\tan} = \infty$ for systems with zero nonlinear stiffness.

The range of SDOF parameters considered in this study is summarized in Table 1. All analyses use a initial stiffness proportional damping ratio of 5%, a time step of $0.001\mathrm{s}$, and Newmark's scheme with constant average acceleration.

Table 1 – Range of SDOF parameters considered.

Parameter	Considered values
Initial Period T_0 (s)	0.05~1.0 (increments of 0.05) 1.0~3.0 (increments of 0.10)
Tangent Period $T_{\tan}$ (s)	-20, -10, -5, 1, 1.5, 2.0, 3.0, 5.0, 8.0, 10.0, 20.0, ∞
Force Reduction Factor R	2, 4, 6, 8, 10, 15, 20, 30, 50, 100
Energy Dissipation Parameter β	0, 0.1, 0.2, 0.4, 0.6, 0.8, 1.0
Damping Ratio ζ	5% (proportional to initial stiffness)

3. Ground Motion Suite

The input for the nonlinear dynamic analysis in this study comes from the record set #1A that was selected in a study for the PEER transportation system research program [Baker et al. 2011]. The records used here are the broad band ground motions that represent the dominant hazard in active seismic regions with large earthquakes at small distances. They are selected for soil sites where the structural vibration period and the distance to active faults are unknown, i.e. period- and site-independent. The individual and median elastic response spectra of these records are shown in Fig. 2.

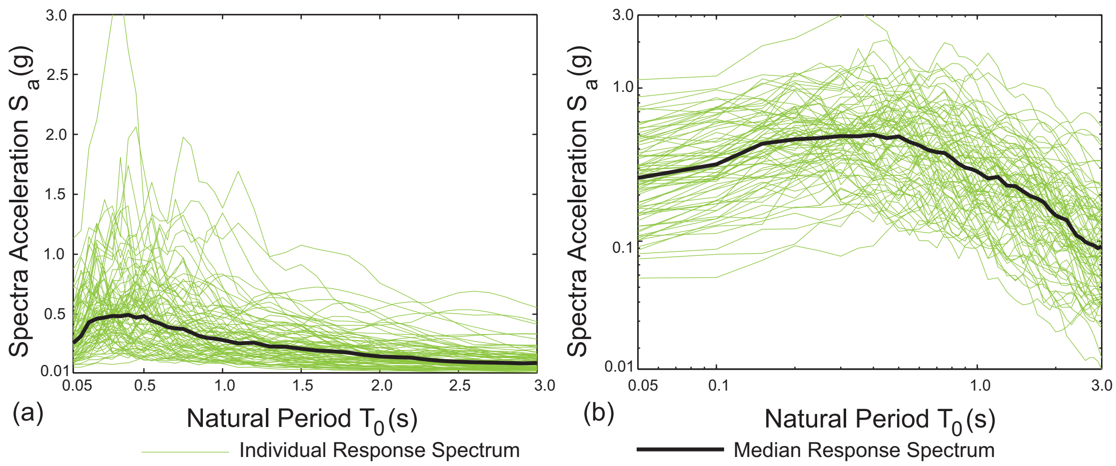

Fig. 2 – Elastic response spectra of ground motions considered:

(a) plot with linear scaling of the axes and (b) plot with log-log scale of the axes

4. Results and Discussion

The median responses of typical self-centering systems with zero nonlinear stiffness ($T_{\tan} = \infty$) and intermediate energy dissipation ($\beta = 40\%$) are shown in Fig. 3, as well as individual values. Like an elastic system, the median lateral displacement of a self-centering system generally increases as the initial period increases, as indicated in Fig 3(a). Fig. 3(b) shows that the peak displacement also increases as the linear limit reduces, but only up to a force reduction factor of about $R = 10$. These trends also hold for larger values of T_0 or β. Just as ground motions' elastic spectra are highly variable, the seismic responses of self-centering systems are also highly scattered. For example, in the case of $T_0 = 1.0\mathrm{s}$ in Fig. 3(a), the largest response to an individual record (about $500mm$), is more than ten times the smallest (less than $50mm$).

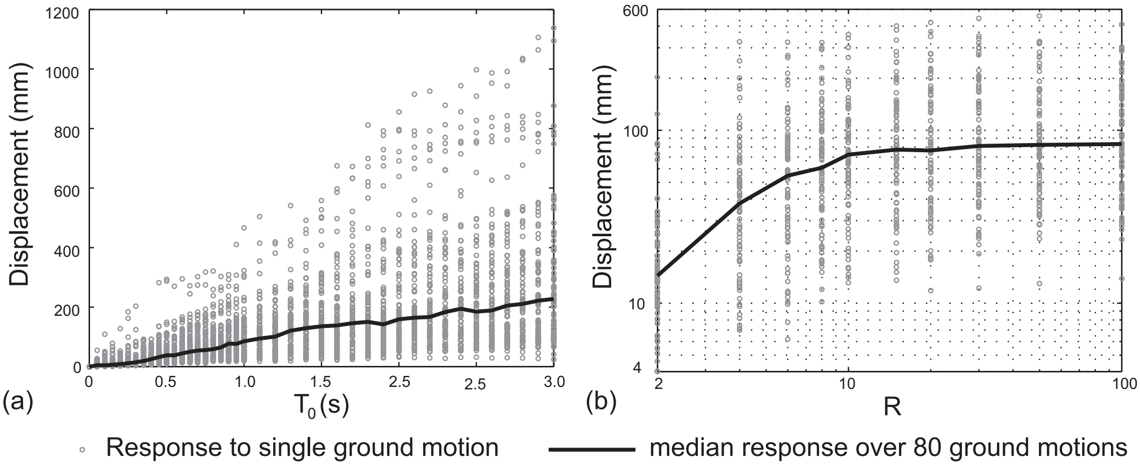

Fig. 3 – Displacement affected by (a) initial period and ground motion when $R = 2, T_{\tan} = \infty, \beta = 0.4$ and (b) linear limit and ground motion when $\mathrm{T}_0 = 0.3\mathrm{s}, T_{\tan} = \infty, \beta = 0.4$

In the following discussion, the results will be presented in terms of displacement coefficient C_R. For each ground motion, C_R is defined as the ratio of the peak inelastic displacement of a self-centering system to that of an elastic system with the same initial period:

$$C_R = \frac{\Delta_{inelastic}}{\Delta_{elastic}} \tag{3}$$

The displacement ratio is used rather than displacement ductility because it is a more useful parameter for a self-centering system. Unlike traditional systems, there is not necessarily any yielding associated with the nonlinear response in self-centering systems. Therefore, to express the displacement capacity of a self-centering system as a multiple of the displacement associated with the onset of nonlinearity is not as meaningful as it is for a traditional yielding system.

4.1. Influence of Linear Limit

To investigate the influence of the linear limit, the nonlinear stiffness is fixed as zero ($T_{\tan} = \infty$) for analyses in this discussion. Then R is changed for 3 different cases: no hysteretic energy dissipation ($\beta = 0$), intermediate hysteretic energy dissipation ($\beta = 40\%$) and full hysteretic energy dissipation ($\beta = 100\%$).

Fig. 4 shows that reducing the linear limit by changing R from 2 to 6 leads to a significant increase in displacement for all levels of hysteretic energy dissipation, and this increase is larger at shorter initial periods. For example, when $\beta = 40\%$ and R is increased from 2 to 6, the median C_R value increases by 97%, 51% and 17% for $T_0 = 0.5\,\mathrm{s}$, $1.0\,\mathrm{s}$ and $2.0\,\mathrm{s}$ respectively. However, further reduction in the linear limit (i.e. increasing R) has relatively little influence on the displacement. For the case of $\beta = 40\%$ and $T_0 = 1\,\mathrm{s}$, the median C_R value increases only about 30% when R is increased from 8 to 30. For some cases, the response of a system with a lower linear limit (larger R value) is slightly smaller than the response of a system with a higher linear limit (smaller R value). This phenomenon is the subject of ongoing research and is beyond the scope of this paper.

For traditional systems, a larger R value is not acceptable because it means significant nonlinear responses and large ductility demands on the structure. These large nonlinear responses usually result in large residual displacements, which make structural repairs expensive [e.g. Iwata et al. 2005, McCormick et al. 2008]. However, for a self-centering system, a larger displacement only means further deformation of the post-tensioning and energy dissipation devices, which may not even cause yielding, much less failure. Therefore, the results discussed above indicate that a larger R can be used, as long as the displacements are acceptable.

At very short periods, the effect of R is even more significant. For the case of $T_0 = 0.1\,\mathrm{s}$ and $\beta = 40\%$, the median C_R values for $R = 2$ and $R = 4$ are 6.8 and 30.5 respectively, the latter of which is approximately a 7% drift for a $3.5\,\mathrm{m}$ one-storey building. Even if this drift is less than the displacement capacity of the structure, it far exceeds the code-specified drift limit of 2.5% [NBCC 2010] and thus would not be acceptable. For the case of $T_0 = 0.05\,\mathrm{s}$ and $\beta = 40\%$, $C_R = 38.5$ even for $R = 2$. These observations suggest that self-centering systems may not be appropriate for extremely stiff buildings. Elastic design may be more advisable to avoid residual drifts.

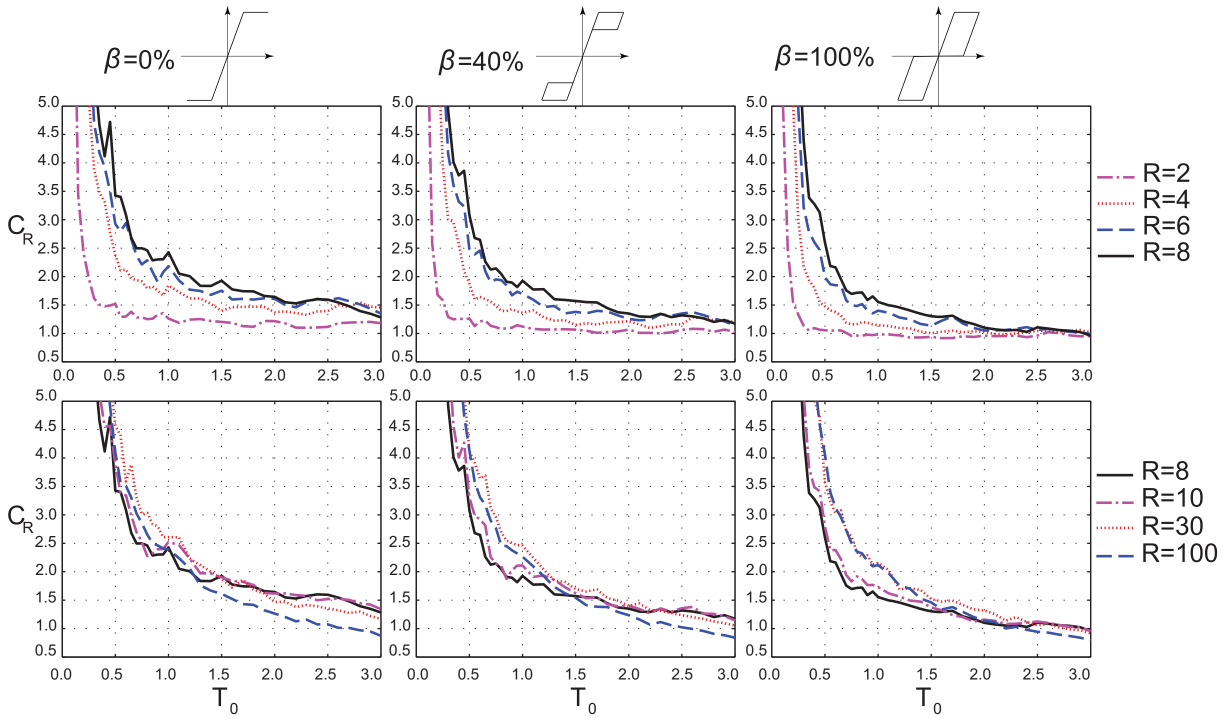

Fig. 4 – Influence of linear limit on median responses ($T_{\tan} = \infty$)

4.2. Influence of Hysteretic Energy Dissipation

The energy dissipation parameter β quantifies the energy dissipation apart from the assumed inherent viscous damping. Fig. 5 shows the effect of β for a system with zero nonlinear stiffness ($T_{\tan} = \infty$) and high initial stiffness ($T_{\tan} = 0.5\,\mathrm{s}$). For all values of R, the displacement generally decreases with increasing β. The effect of β on C_R is relatively larger when R is smaller. For example, increasing β from 0% to 100% reduces C_R by 30% when $R = 2$, while this decrease is only 15% when $R = 50$. For $R = 100$, the displacement remains approximately constant regardless of β.

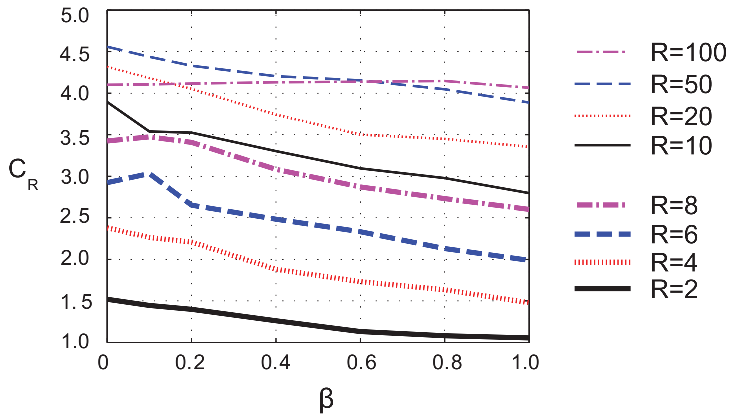

Fig. 5 – Effect of β on median C_R for $T_0 = 0.5\,\mathrm{s}, T_{\tan} = \infty$

Fig. 6 shows that the effect of hysteretic energy dissipation is more significant at very short periods in all cases, where a small difference in β causes a noticeable change in C_R. However, in the intermediate to long period range ($T_0 \geq 0.5\,\mathrm{s}$), the decrease in the median C_R is almost a constant over different initial periods when β is increased. For instance, when $R = 8$, $T_{\tan} = \infty$ and $1.0\,\mathrm{s} \leq T_0 \leq 3.0\,\mathrm{s}$, increasing β from 0% to 20% causes C_R to decrease by a consistent factor of about 8%. Also the influence of β diminishes as the value of β becomes larger. For example, in the case of $R = 8$, $T_{\tan} = \infty$ and $1.0\,\mathrm{s} \leq T_0 \leq 3.0\,\mathrm{s}$, increasing β from 0% to 20% reduces C_R by about 8% (C_R decreases by about 0.14) while increasing β from 80% to 100% reduces C_R by only about 4% (C_R decreases by about 0.05).

The observation from Fig. 5 that the value of β affects systems with higher linear limit (smaller R value) more than for those with lower linear limit is not valid for all cases. Actually, when R is increased from 2 to 8, the influence of β become more significant, but when R is further increased to 30 or 100, the effect of β becomes negligible, which can be observed by comparing different rows in Fig.6. Fig. 6 also shows an interaction between β and $T_{\tan}$. When $T_{\tan}$ is close to T_0, the system is nearly elastic, so β has almost no influence on the median response. However, this situation is rare in practice because the nonlinear period is normally much longer than the initial period.

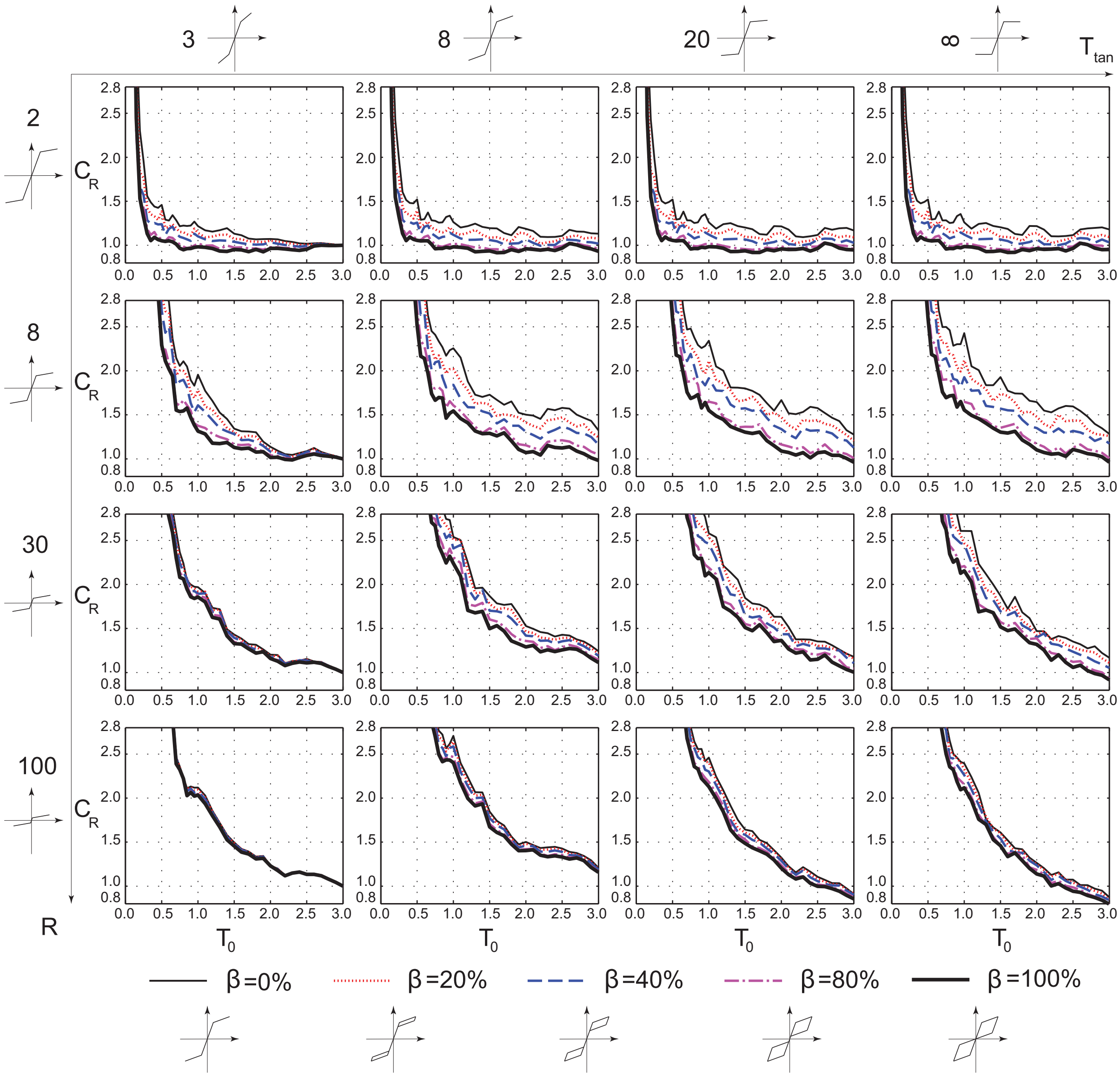

Fig. 6 – Influence of β on median C_R

4.3. Influence of Nonlinear Period

Fig. 7 shows the influence of nonlinear period for different values of R and T_0. When the system is relatively stiff in the nonlinear range ($1\text{s} < T_{\tan} < 5\text{s}$), C_R generally increases with $T_{\tan}$, especially for large R values. For example, in the case of $T_0 = 0.5\text{s}$ and $\beta = 40\%$, increasing $T_{\tan}$ from 1s to 5s increases C_R by 85% for $R = 10$, but only by 9% for $R = 2$. This is because the shorter $T_{\tan}$ is, the closer T_0 and $T_{\tan}$ become, which means that the system is nearly elastic with $C_R = 1.0$. However, this situation where $T_{\tan}$ is close to T_0 is rare in practice. When $T_{\tan}$ becomes longer ($8\text{s} < T_{\tan} < \infty$), the influence of $T_{\tan}$ on C_R becomes different. For large values of R ($R \geq 10$), C_R remains constant or slightly decreases with increasing $T_{\tan}$, while for small values of R ($R < 10$), C_R tends to increase slightly or remain constant. Thus the tangent period generally does not have much influence when it is a practical positive value.

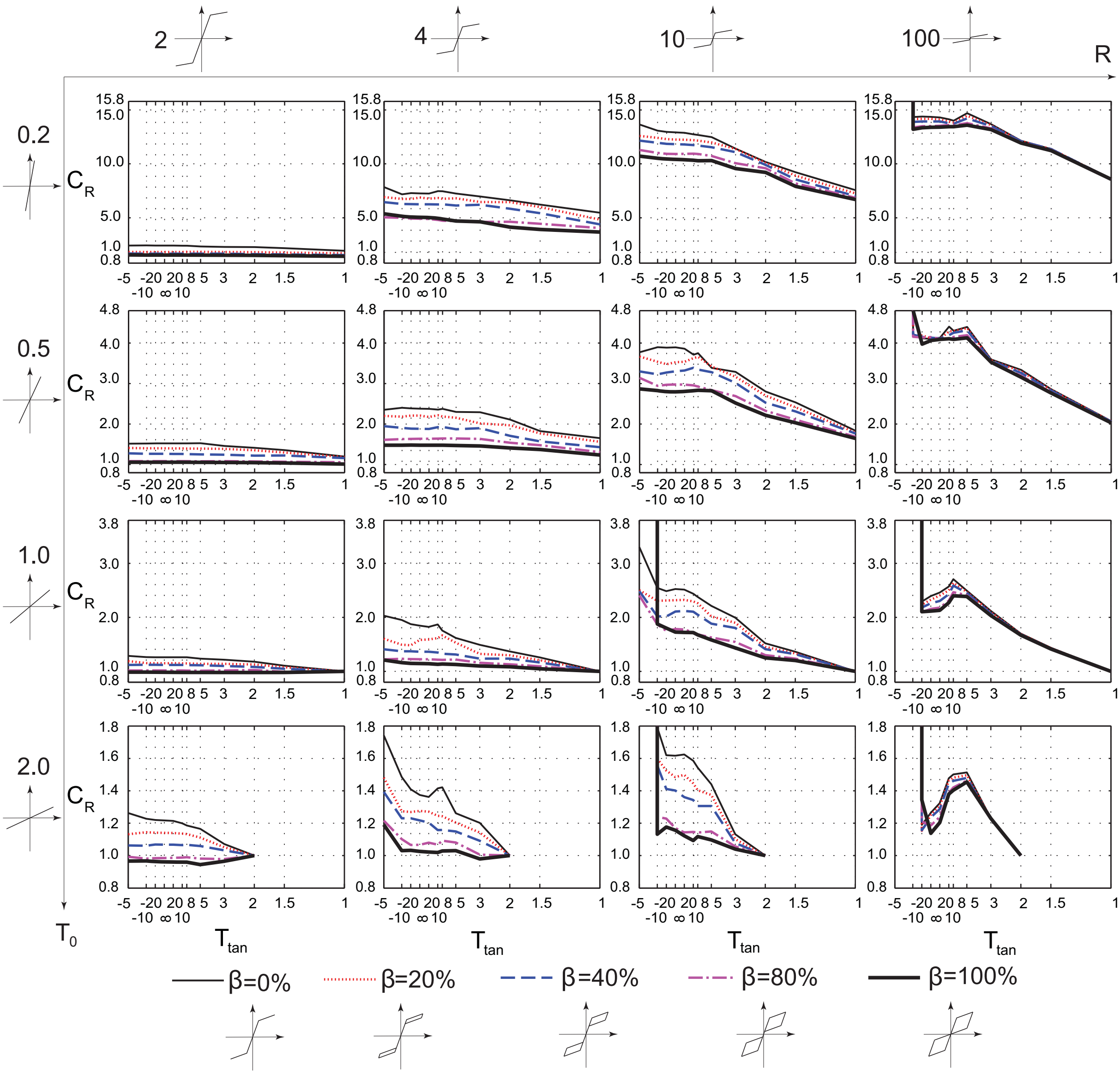

Fig. 7 – Influence of $T_{\tan}$ on median C_R

When the nonlinear period is negative and its absolute value is large ($-\infty < T_{\tan} \le -10\,\mathrm{s}$), systems with a high linear limit (e.g. $R = 2, 4$) and a short initial period (e.g. $0.2\,\mathrm{s} \le T_0 \le 1.0\,\mathrm{s}$) show similar responses as those with positive long nonlinear periods. However, when the nonlinear period becomes shorter ($-5 \le T_{\tan} \le -10\,\mathrm{s}$) or the linear limit is reduced further ($R \ge 10$), the system is prone to instability. This is seen as a vertical line in Fig. 7, where the median displacement is very large ($C_R > 10^3$). At longer initial periods (e.g. $T_0 = 2.0\,\mathrm{s}$), the system is more sensitive to the negative stiffness. The instability occurs because the negative stiffness is large enough to cause the system to enter the second or fourth quadrant in the force-displacement relationship while the displacement is still increasing, which means the system become physically unstable. A lower linear limit (larger R values), a longer initial period (lower peak force) or a shorter nonlinear period (steeper drop in nonlinear range) makes this happen more readily.

When instability occurs, increasing β can decrease the displacement before the critical nonlinear period is reached, but it cannot prevent the instability. In Fig.7, the appearance that increasing β is more effective for long period ($T_0 = 2.0\,\mathrm{s}$) is caused only by different scales of the y-axis in different rows. In fact, increasing β is more effective at short periods, as discussed in section 4.2.

5. Conclusions

This paper has presented a parametric study on the seismic response of self-centering systems. As occurs for an elastic system, increasing the initial period or reducing the linear limit generally increases the peak displacement, although not as a proportion of the displacements of an elastic system with the same initial period. Reducing the linear limit can double the peak displacement when the force reduction factor is small, but the peak displacements become less sensitive to the linear limit when the force reduction factor is already greater than about 10. The effect of linear limit is more pronounced at short initial periods. Increasing the hysteretic energy dissipation generally reduces the peak displacements up to 50% when comparing the case of maximum self-centering hysteretic energy dissipation to that of no hysteretic energy dissipation, but additional energy dissipation is less effective when it is added to a system that already has a high level of energy dissipation. Also, the energy dissipation parameter affects short period systems ($T_0 \le 0.5\,\mathrm{s}$) more than long period ones, and it influences systems with intermediate linear limits ($4 \le R \le 10$) more than those with lower and higher linear limits. Finally, as long as the nonlinear stiffness stays in practical positive levels, it is relatively unimportant. However, when it becomes negative, the response becomes unstable, especially if the linear limit is low, the initial stiffness is small, or the nonlinear stiffness is much less than zero. In these situations, collapse becomes likely. Increasing hysteretic energy dissipation can only suppress nonlinear response before a critical negative stiffness that can cause collapse is reached, but cannot prevent the collapse caused by negative stiffness.

Further research is ongoing to explain the mechanism behind these phenomena and to develop expressions to predict the peak displacements of self-centering SDOF systems.

6. Acknowledgement

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7. References

BAKER, Jack, LIN, Ting, SHAHI, Shrey, "New ground motion selection procedures and selected motions for the PEER transportation research program", *PEER Report,* Pacific Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley, CA, March 2011.

CHRISTOPOULOS, Constantin, FILIATRAULT, Andre, FOLZ, Bryan, "Seismic response of self-centering hysteretic SDOF systems", *Earthquake Engineering and Structural Dynamics,* Vol. 31, No. 5, May 2002, pp. 1131-1150.

CHRISTOPOULOS, Constantin, FILIATRAULT, Andre, UANG, Chia-Ming, FOLZ, Bryan, "Posttensioned energy dissipating connections for moment-resisting steel frames", *Journal of Structural Engineering,* Vol. 128, No. 9, September 2002, pp. 1111-1120.

CLIFTON, Charles, BRUNEAU, Michel, MACRAE, Gregory, LEON, Roberto, FUSSELL, Alistair, "Steel structures damage from the Christchurch Earthquake series of 2010 and 2011", *Bulletin of the New Zealand Society for Earthquake Engineering,* Vol. 44, No. 4, December 2011, pp. 298-318.

IWATA, Yoshihiro, SUGIMOTO, Hirokazu, KUWAMURA, Hitoshi, "Reparability limite of steel structural buildings based on the actual data of the Hyogoken-Nanbu Earthquake", *Journal of Structural and*

Construction Engineering: Transactions of AIJ, No. 588, February 2005, pp. 165-172.

KAM, Weng, PAMPANIN, Stefano, DHAKAL, Rajesh, GAVIN, Henri, ROEDER, Charles, “Seismic performance of reinforced concrete buildings in the september 2010 Darfield (Cantebury) Earthquake”, *Bulletin of the New Zealand Society for Earthquake Engineering,* Vol. 43, No. 4, December 2010, pp. 340-350.

KURAMA, Yahya, SAUSE, Richard, PESSIKI, Stephen, LU, Lewu, “Lateral load behaviour and seismic design of unbonded post-tensioned precast concrete walls”, *ACI Structural Journal,* Vol. 96, No. 4, July 1999, pp. 622-632.

LAM, Nelson, WILSON, John, HUTCHINSON, Graham, “The ductility reduction factor in the seismic design of buildings”, *Earthquake Engineering and Structural Dynamics,* Vol. 27, No. 7, July 1998, pp. 749-769.

MA, Xiang, “Seismic design and behaviour of self-centering braced frames with controlled rocking and energy-dissipating fuses”, *PhD Dissertation,* Stanford University, Stanford, CA, August 2010.

MCCORMICK, Jason, ABRANO, Hiroshi, IKENAGA, Masahiro, NAKASHIMA, Masayoshi, “Permissible residual deformation levels for building structures considering both saftety and human elements”, *The 14th World Conference on Earthquake Engineering, Beijing, China,* Paper ID 05-06-0071.

MIRANDA, Eduardo, “Inelastic displacement ratios for structures on firm sites”, *Journal of Structural Engineering,* Vol. 126, No. 10, October 2000, pp. 1150-1159.

NRCC (National Research Council Canada), “National Building Code of Canada 13th edition”, Ottawa, Ontario: National Research Council of Canada, 2010.

PRIESTLEY, Nigel, MACRAE, Gregory, “Seismic tests of precast beam-to-column joint subassemblages with unbonded tendons”, *PCI Journal,* Vol. 41, No. 1, January-February 1996, pp. 64-81.

ROH, Hwasung, REINHORN, Andrei, “Nonlinear static analysis of structures with rocking columns”, *Journal of Structural Engineering,* Vol. 136, No. 5, May 2010, pp. 532-542.

SEO, Choung-Yeol, SAUSE, Richard, “Ductility demands on self-centering systems under earthquake loading”, *ACI Structural Journal,* Vol. 102, No. 2, March 2011, pp. 275-285.

The New Zealand Herald, “Govt reveal post-quake rebuild costs for Greater Christchurch”, Internet reference, URL: http://www.nzherald.co.nz/nz/news/article.cfm?c_id=1&objectid=10852452, published December, 2012, accessed March, 2015.

WIEBE, Lydell, CHRISTOPOULOS, Constantin, “Mitigation of higher mode effects in base-rocking system by using multiple rocking joints”, *Journal of Earthquake Engineering,* Vol. 13, No. S1, April 2009, pp. 83-108.

WIEBE, Lydell, CHRISTOPOULOS, Constantin, TREMBLAY, Robert, LECLERC, Martin, “Mechanisms to limit higher mode effects in a controlled rocking steel frame. 1: concept, modelling, and low-amplitude shake table test”, *Earthquake Engineering and Structural Dynamics,* Vol. 42, No. 7, June 2013, pp. 1053-1068.

WIEBE, Lydell, CHRISTOPOULOS, Constantin, “Performance-based seismic design of controlled rocking steel braced frames. I: methodological framework and design of base rocking joint”, *Journal of Structural Engineering,* Available online ahead of print, DOI: 10.1061/(ASCE)ST. 1943-541X.0001202.