The 11th Canadian Conference on Earthquake Engineering
Canadian Association for Earthquake Engineering

APPLICATION OF SEMI-ACTIVE TUNED MASS DAMPER IN CONTROLLING THE SESIMIC RESPONSE OF A BULIDING

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ABSTRACT: Tuned mass dampers (TMD) are found to be quite effective in controlling the vibration in tall buildings. Integrating a TMD with a semi-active control system enables it to perform more efficiently. Magnetorheological (MR) fluid damper provides an attractive choice for developing semi-active control system for a TMD in a tall building. However, difficulties arise in modelling complexities for MR fluid dampers, especially when the seismic performance of a building integrated with these dampers need be evaluated through dynamic time history analysis. In this paper a simplified, yet accurate mathematical model has been developed to represent the MR dampers in order to integrate them in the structural model of a building. To demonstrate the effectiveness of semi-active TMD system in controlling the seismic response of a tall building, a forty-storey steel-frame building designed according to the Canadian standard, has been studied with and without semi-active and passive TMDs. It has been shown that the semi-active control system modifies structural response more effectively than the classic passive TMD in both mitigation of maximum displacement and reduction of the settling time of the building.

1. Introduction

Tuned mass dampers (TMD) are passive control devices that are usually installed at the roof of a building to control its dynamic response due to wind or an earthquake. Tuned mass damper consists of a mass, a spring, and a damper anchored or attached to the main structure. During wind or ground excitation, a part of the vibration energy of the main structure is transferred to the TMD system. The TMD parameters (e.g., mass, spring and damping) are typically optimized during the design to have maximum energy dissipated by the TMD system. Generally, TMD is tuned for the fundamental frequency of the main structure as the first mode typically has the largest participation in structural response. Thus, TMD which is tuned for the first mode is most effective when the first mode dominates the response. However, the first mode does not always dominate the response of the structure under wide-band excitation frequencies. That could be the case in seismic loading of the structure where multiple modes are significant or in a wind loading which mainly excites the structure in the second frequency and causes the resonance in the second mode of vibration. The traditional passive TMD systems operate in a narrow frequency band and they are basically effective for the vibration frequency for which they are tuned. Therefore, when the difference between the excitation frequency and the tuned frequency increases the TMD system becomes less effective. Several innovative methodologies are available to widen the narrow frequency band of the TMD systems such as designing multiple tuned mass dampers, active tuned mass dampers, hybrid mass dampers and semi-active tuned mass dampers.

The history of using TMD system goes back to 1928 (Ormondroyd and Hartog 1928), while the concept of semi-active tuned mass damper came much later in 1983 (Hrovat *et al.* 1983). Hrovat *et al.*(1983) introduced a semi-active TMD for wind-induced vibrations in high rise building. A semi-active tuned mass damper (SATMD) is a TMD which is equipped with a semi-active device and associated control system. Pinkaew and Fujino (2001) investigated the control effectiveness of a SATMD with variable damping device under harmonic excitation. Ji *et al.* (2005) compared four different control algorithms in SATMD and concluded that among the four control algorithms, DBG (displacement-based ground hook) and clipped optimal algorithm show the best performance under the earthquake loading. Yang *et al.* (2010) investigated vibration suppression of structures under random base excitation using MR-based SAMD system. It should be noted that application of SATMD to seismic vibration control are mainly based on laboratory test or model building which utilize small MR dampers under harmonic, random or only one seismic excitation. However, in the present research work, a large size MR damper has been implemented to design SATMD for controlling the seismic response of a high-rise building for which no reference work is available. The lateral load resisting system of the building considered here consists of typical steel moment-resisting frames designed based on the current versions of the relevant codes and standards in Canada. The response of the structure subjected to a set of seismic ground motions with low, medium and high frequency contents has been calculated to investigate the performance of the building with semi-active tuned mass damper in comparison to that of passive and active TMD systems.

2. Building Details

A typical 40-story steel building has been designed according to the provisions of the National Building Code of Canada (NBCC 2010) and the relevant standard (CAN/CSA-S16-09). This building will be used as a benchmark to investigate the effectiveness of the optimally designed semi-active tuned mass system under different seismic excitations. It should be mentioned that in these stages of design TMD is not considered. Once the semi active tuned mass damper and structural design is finalized, the member's sizes in steel moment frames can be refined according to new reduced forces and deformations. The building has nine bays of six meters in one direction and five bays of six meters in another direction and is 122 meters in height as shown in Fig.1.

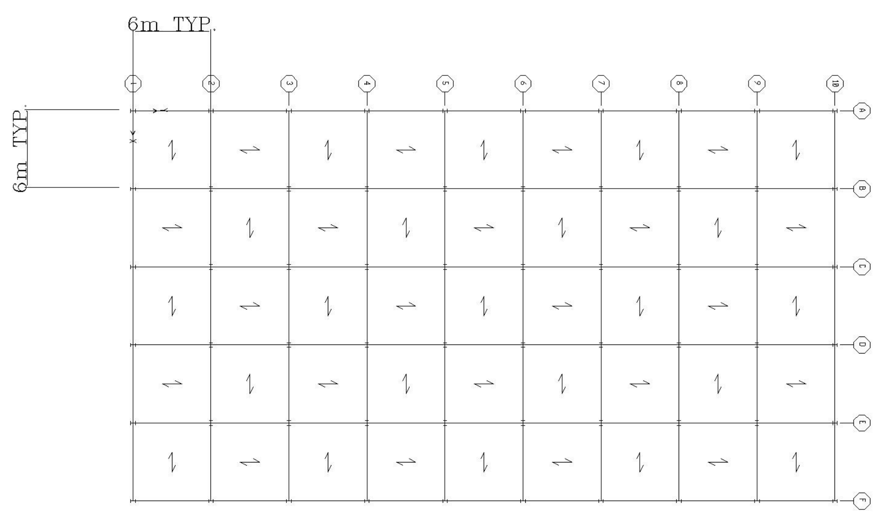

Fig. 1 Beam frame plan and columns layout of 40 story building

The lateral load resisting system of building consists of moment resistant steel frames. The dead and live loads are estimated to be 6 kPa and 2.4 kPa, respectively. The seismic and wind load provisions of NBCC (2010) have been applied to estimate the lateral loads. Since the earlier version of the building code NBCC (2005), the equivalent static load-based design procedure is not applicable for buildings higher than 60 m (Yousuf and Bagchi 2010, El Kafrawy *et al.* 2011). In the present case where the building height exceeds the limit of 60 m, the equivalent static load method has been used only for preliminary proportioning of the structural members, while the response spectrum analysis has been used for determining the member forces for the detailed design. The steel structural design has been done using CSA-S16 (2009) standard, and the CISC handbook (2010). The ETABS software has been used for the analysis and design of the Structure. The columns and beams section used in the building are listed in Table 1.

3. Finite Element Model

A three dimension finite element (FE) model of the forty-story building presented in Section 2 has been developed in the ETABS software for the purpose of analysis and design. The FE model has about 7000 degrees of freedom (DOFs) which makes it computationally very expensive for analysis and development of a control system. Thus, the 3D model has been simplified here to an equivalent 2D FE model (shear story building).

Table1 Summary of the column and beam sections

Floor number	Column size	Beam size
1	WWF600x551	W610x101
2-5	WWF500x456	W610x101
6-10	WWF450x409	W610x101
11-15	WWF450x342	W610x101
16-20	WWF400x303	W510x101
21-25	WWF400x243	W610x91
26-30	w610x195	W610x91
31-35	W460x144	W530x82
36-39	W410x114	W460x67
40	Depend on the position of mass(W410x114 mostly)	Depend on the position of mass(W460x67 mostly)

It should be noted that the simplification has been done in a way that the main DOFs (main floor displacements and velocities) of the system are retained and represented appropriately to the 2D model such that the modal and dynamics characteristics of the 2D and 3D FE models are comparable. Among three main dynamic characteristics of the structure, the mass and damping in the 2D model are kept the same as those in the 3D model. However, the stiffness of the 2D model has been altered to achieve the appropriate dynamic responses according to the 3D FE model. While the floor mass and damping properties are kept the same in both the models, structural stiffness matrix in the 2D model has been modified to provide the best agreement between the first mode of vibration of the 3D and 2D FE models. Then, the dynamic response of the 3D and 2D FE models due to random or seismic loading are compared to justify the modification of the stiffness matrix. The response of top roof floor of the 3D and 2D FE models under El Centro ground excitation has been shown in Figure 2.The 3D model has been developed using ETABS while 2D model has been implemented in Simulink. The first four vibration modes of the building are shown in Fig. 3.The periods of vibration corresponding to these modes are 5.76 s, 2.02 s, 1.22 s and 0.80 s, respectively; which account for a participation of 95%.As the building is symmetric and mass participation of the torsional modes is negligible, the torsional effect can be neglected. In that case, a two-dimensional analysis of the structure would produce acceptable response of the structure.

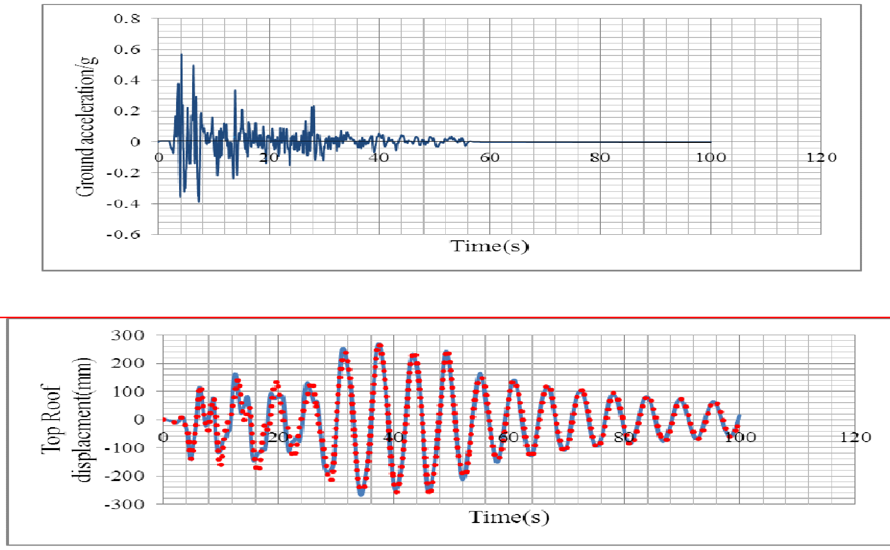

Fig. 2 Top: El Centro ground excitation Bot:3D model in Red,2D model in Blue

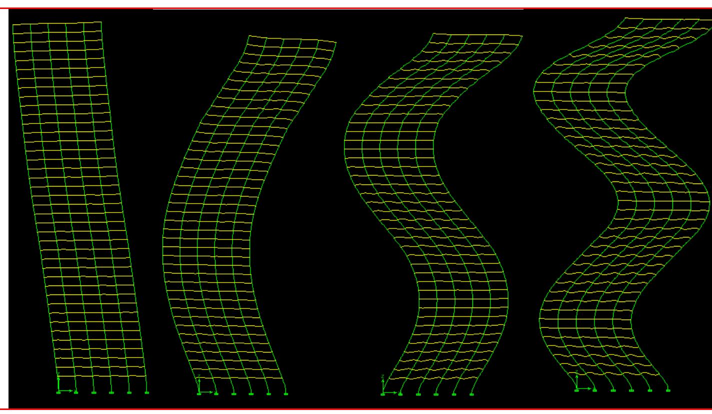

Fig. 3 Vibration mode shapes of the building

4. Tuned mass damper (TMD) design

TMD consists of a mass, a spring, and a damper. A simple arrangement of TMD is shown in Fig. 4. The mass is typically limited to the maximum magnitude that can be installed in a structure. TMD mass ratio (TMD mass divided by the main structural mass) has a positive direct effect on the structural response and it is recommended that for all practical purposes the mass ratio be mainly limited to about 10 present. For a tall building with height of forty-story such as the one considered in this study, the mass ratio can be between 1.5 to 2 percent (Watakabe *et al.* 2001). Watakabe *et al.* (2001) installed TMD with a mass ratio of 1.73% in a 39-story building. On the other hand, considering the assigned TMD mass ratio, the spring and damping coefficients of TMD system should be carefully designed to provide optimal structural vibration suppression. Here, the stiffness and damping of the TMD system have been altered from 20% to 200% of the values corresponding to the perfect tuning condition and the response of the structure during ground motion has been determined accordingly. Although this approach provides a pair values for the stiffness and damping of an optimal TMD system (based on minimum displacement), it is observed that different ground motion needs different pair of optimal stiffness and damping ratio for the TMD system. The pilot structure used in this research (i.e., the 40 story building) has been investigated using different ground motions. In case of perfect tuning, the natural frequency of a TMD should be equal to the fundamental frequency of the main structure. For the mass ratio of 1.5%, perfect tuning requires that:

$$m_{TMD} = 0.015 \times M_{structure}$$
$$\omega_{TMD} = \omega_{struture} \tag{1}$$

Thus;

$$\frac{k_{TMD}}{m_{TMD}} = \omega^2{}_{struture} \quad ; \quad c_{TMD} = 2 \times \xi_{structure} \times m_{TMD} \times \omega_{struture} \tag{2}$$

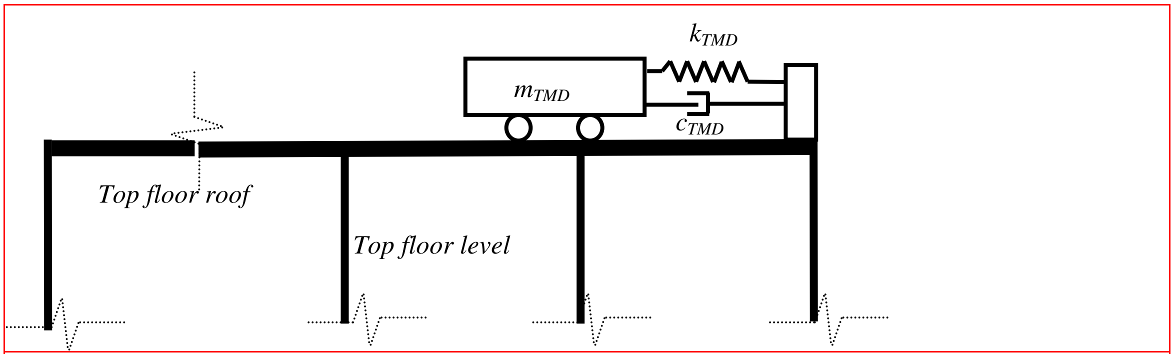

Fig. 4 Typical arrangement of TMD system

5. 5. Control algorithms

Equations of the motion of seismically excited structure in the finite element form can be expressed as (Ogata 2010):

$$M_S \ddot{Z} + C_S \dot{Z} + K_S Z = \Lambda U - M_S \Gamma \ddot{x}_g \tag{3}$$

Where, M_s, C_s and K_s are the mass, damping and stiffness matrices of the system. U is the control force vector, Λ is the control force location matrix, and Γ is the direction matrix related to the base acceleration, $\ddot{x}_g$.

The equations of the motion as given by Eq. (3) are transformed into a state space representation (also known as the "time-domain approach") to provide a convenient way to model and control a system with multiple inputs and outputs.

Fig. 5 shows a block diagram of the state space representation of equations of motion. The vector of states (*X*) is multiplied by matrix *A* to give dynamic forces of the structure. In addition vector of states (*X*) is used to compute control forces by multiplying the gain matrix (*K*) by it (U=-K*X). Computationally speaking, this procedure consider ground excitation as the input signal and tries to minimize the response of the system by introducing the control force into the system (structure).This control force (in this case, the force in TMD) is limited to the capacity of the actuators or that of the semi-active damper (e.g. MR damper).

The control force obtained from Eq. (14) should be augmented by a semi-active control strategy in order to be used with a semi-active control system. A semi-active control system with an MR damper can only apply passive force, F_{MR} which is opposite to direction of the relative velocity of the two ends of the MR damper, v_{rel}, an their relationship can be expressed as:

$$F_{MR} \propto -sign(v_{rel}) \tag{15}$$

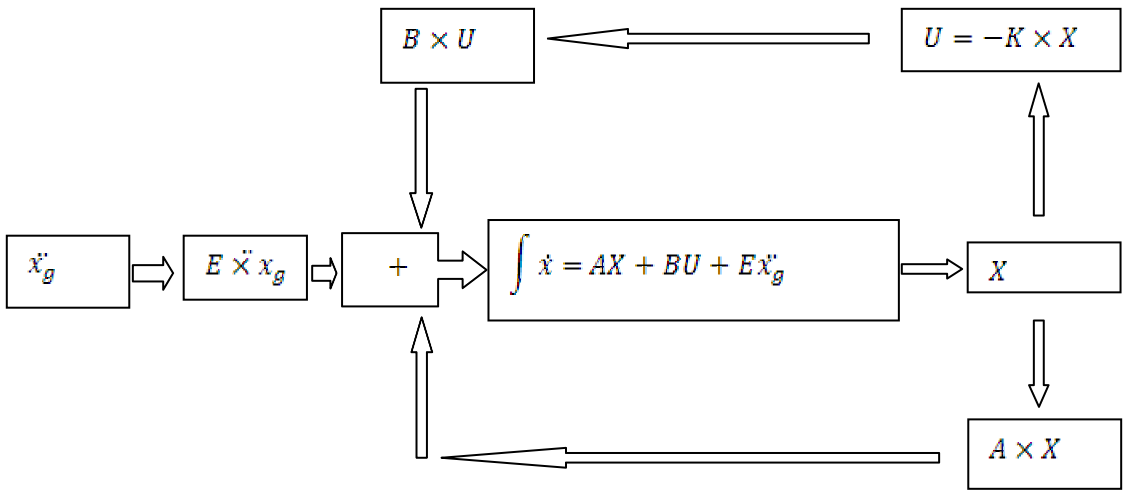

Fig. 5 State space flow chart for SATMD and TMD analysis

6. 6. The semi-active tuned mass damper

One type of semi-active control device is magneto-rheological (MR) fluid damper. The MR fluid is very sensitive to the applied magnetic field and changes from liquid to semi–solid material in millisecond. This unique feature enables to build a variable damping device with MR fluid (MR damper) with minimal power requirements. In this study, a 200 kN (20 ton) MR damper similar to that studied by Yang *et al.* (2002) has been utilized in the semi-active tuned mass damper (SATMD). The 200 KN MR damper has 8 cm stroke length, however practically the stroke of 40 cm is required (Watakabe *et al.* 2001). The MR damper can be modeled using the modified Bouc-Wen model as reported in Yang *et al.* (2002), while the input signal is low pass filtered. This filtering helps the numerical procedure to be stable but generates error when control algorithms use this input to provide commands to MR damper, especially when cutting frequency is as low as 5 Hz. In the current research, a simple phenomelogical model developed for 20 ton MR damper has been used (Esteki *et al.* 2011, 2014). In this model, the damping force in the MR device is related to the strut velocity and the current which governs the yield stress of the MR fluid. The model can very well approximate MR damper force in random vibration. The maximum useful current for MR damper which is used in this study and also used by Yang *et al.* (2002) is 1 Amp. This current is chosen from design chart provided by manufacturer.

7. Analyses of the building equipped with SATMD

In current research the performance of 40-story tall, steel structure equipped with semi-active tuned mass damper or SATMD has been studied. To better realize the characteristics and functionality of the SATMD system designed here, the analysis of the structure has been carried out for six different earthquake ground motion records selected from the database maintained by the Pacific Earthquake Engineering Research Center (PEER) at the University of California Berkeley. The selected ground motion records contain low, medium or high frequency contents. The results are shown in time domain as well as frequency domain. All earthquake records are scaled to 0.2g to represent in the seismicity of Vancouver, Canada where the building being studied is assumed to be located. Kobe ground acceleration record and its Fast Fourier Transform are shown in Fig. 6. Kobe ground acceleration record has duration of about 20 seconds and its FFT analysis shows frequency contents to be in the range of 1 to 5 Hz. The roof displacement of the structure due to the scaled Kobe ground acceleration is shown in Fig. 7. As it can be seen from the figure, while both TMD and SATMD reduce the response of the structure for up to 20 seconds, SATMD performs better than the passive TMD. After 20 seconds, SATMD shows its superior performance by suppressing the vibration completely, while passive TMD system does not damp out the vibration that effectively.

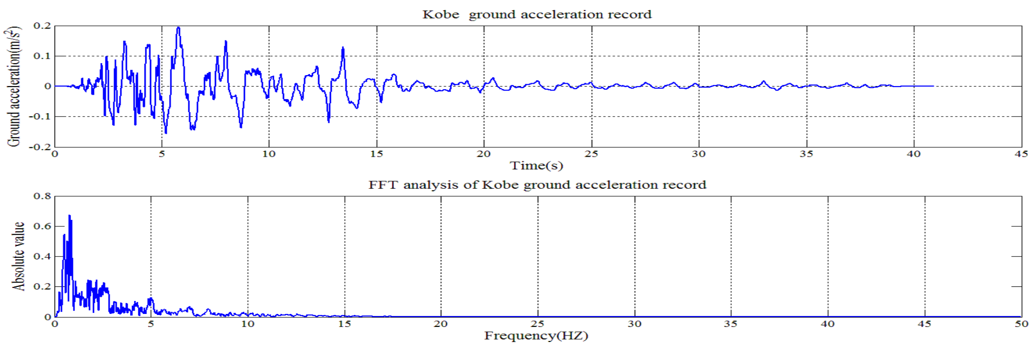

Fig. 6 Kobe ground acceleration record and its FFT diagram

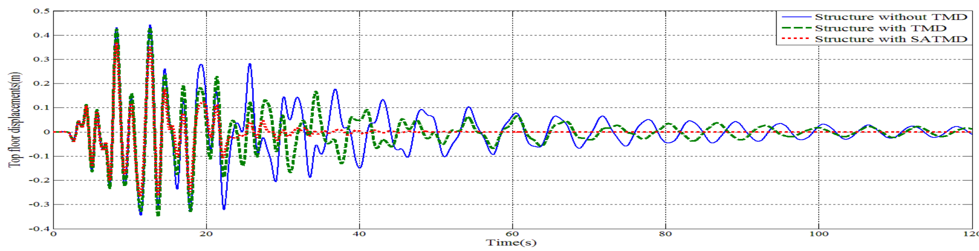

Fig. 7 Roof displacement of the structure due to Kobe ground motion in time domain

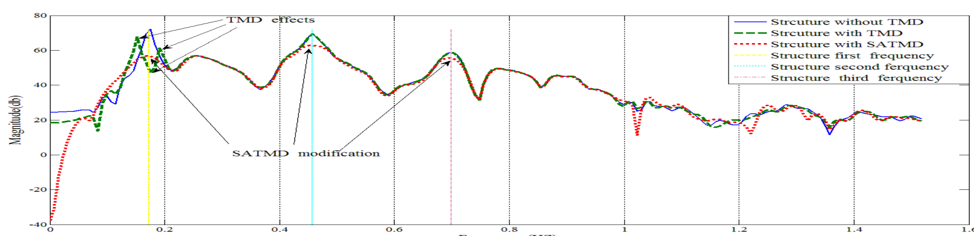

Fig. 8 Roof displacement of the structure due to Kobe ground motion in frequency domain

The performance of TMD and SATMD can be better studied in the frequency domain. Fig. 8 shows the comparison of the frequency response of the uncontrolled structure with those of controlled structures using passive TMD and SATMD systems. It can be observed that, TMD decreases the first vibration mode appreciably; however it generates other two peaks in right and left side of fundamental frequency. SATMD generally performs better in the given frequency range and clearly suppress the vibration in the above mentioned two peaks generated by passive TMD. In addition. it can be seen from Fig. 8 that SATMD not only reduces the response at the first mode frequency but also reduces the frequency responses at higher modes such as the second and third modes. This means that SATMD increases the effactivness of TMD for the second and third modes of vibration; or in the other words, it expands the frequency band of a TMD system.The same concoulsion can be deducted for Irpinia and Kocaeli earthquake records (not shown here to conserve space).The maximum diplacment reduction in roof displacement and settling time for three seismic records with low frequency contents is provided in Table 2. The seismic records with low frequency contents are choosen since they induce a large deformation in tall structure because of their low frequency contents would be closer to the natural frequency of the structure. Table 2 clearly shows the superior performance of SATMD system as compared to a conventional TMD system. As it can be seen from Table 2, the SATMD reduces maximum displacements more effectively than the conventional TMD system, and the main advantage of the SATMD is observed in the reduction of the setting time which is almost less than half of that of TMD.

Table 2 Reduction in the maximum displacements, and settling time for low frequency seismic records

Ground motion record	Maximum reduction in the peak displacements (percent)		Settling time (s)	
	TMD	SATMD	TMD	SATMD
Kobe	2.5	17	67	27
Irpinia	3.8	16.5	90	34
Kocaeli	2.9	6	159	64

The behavior of TMD and SATMD in the cases of earthquake records with medium and high frequency contents (including Tabas, Nahanni and Upland) is also investigated. The response of the building to these earthquakes is Summarized in Table 6. Since the distribution of frequency of these seismic records do not match the fundamental frequency of the structure, they cause very small deformation. The roof displacement is about 150 mm due to the Tabas ground motion record; and 60 mm and 10 mm due to Nahanni and Upland records, respectively. These displacements for a tall building which is 120 m high are quite negligible. Similar levels of improvements in response of structure is found when it is equipped with SATMD as compared to passive TMD, both in time domain and frequency domain. The maximum reduction in displacements and settling time for the seismic records with medium and high frequency ranges in the 40 story building with TMD and SATMD are shown in Table 3.

Table 3 Reduction the maximum displacements, and settling time for the ground motion records medium and high frequency ranges

	Maximum reduction in displacements (%)		Settling time (s)	
	TMD	SATMD	TMD	SATMD
Tabas	6	11	70	30
Nahanni	21	27	35	15

8. Tuning of the control system

The performance of control system directly affects the response of SATMD system. LQR control algorithm uses optimization to determine the gain matrix in order to optimize the cost function. Thus, defining various cost functions will results in different gain matrices and different control forces, and consequently different structural responses. The cost function, J, in Eq. (11) is a combination of the state vector (displacements and velocities) and the control force vector weighted by the coefficients defined by Q and R matrices, respectively. Therefore, Q and R matrices can change the cost function, gain matrix, control force, and the structural response, as explained in Section IV. R is a unit matrix multiplied by a constant coefficient which should be determined so that the gain matrix can yield applicable control forces. In the current research, the response of the structure with SATMD system using three different Q matrices has been investigated. Q_1 (Eq. (19)) considers both displacement and velocity, Q_2 (Eq.(20)) considers only velocity, and Q_3 (Eq.21) considers only displacement. In using Q3 in Eq.(11), the structural displacements will be included in the cost function and the object of the control system is to minimize the structural displacements (except TMD mass displacement). On the other hand, if matrix Q1 and Q2 are used, the control system will minimize both structural displacements and velocity. The response of the structure has been computed using Q1, Q2 and Q3, and illustrated in Fig.9.

$$Q_1 = \begin{bmatrix} 1_{41\times41} & 0_{41\times41} \\ 0_{41\times41} & 1_{41\times41} \end{bmatrix};\ Q_2 = \begin{bmatrix} 0_{41\times41} & 0_{41\times41} \\ 0_{41\times41} & 1_{41\times41} \end{bmatrix};\ Q_3 = \begin{bmatrix} 1_{41\times41} & 0_{41\times41} \\ 0_{41\times41} & 0_{41\times41} \end{bmatrix} \qquad (21)$$

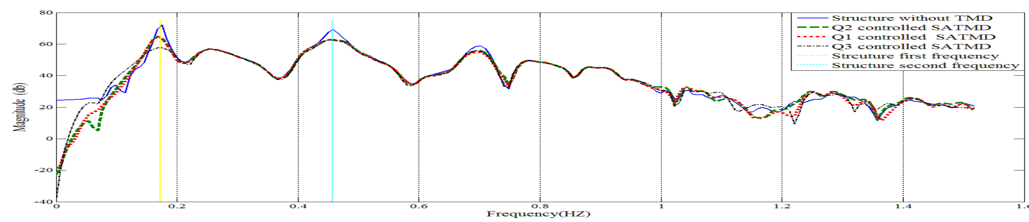

Fig. 9 Roof displacement of the structure with different *Q* weighting matrix due to Kobe ground motion in frequency domain

As it can be seen from Fig. 9, the variation of *Q* matrix greatly alters the results of the SATMD system. The best results for minimizing the top floor displacement is reached by *Q* matrix defined in Eq.(21). Eq.(21) results in a cost function which contains the structural displacements only.

9. Conclusions

In this paper a MR-based semi-active tuned mass damper system (SATMD) has been designed to control the vibration of a 40-story tall steel building structure designed according to the National Building Code of Canada (NBCC 2010) and the relevant standard for steel design (CAN/CSA-S16-09). The LQR control algorithm has been used to find the optimal damping forces. The response of the uncontrolled structure and controlled structure equipped with traditional passive TMD system and SATMD system under different earthquake records with low, medium and high frequency contents have been determined and compared. It has been shown that SATMD system has superior performance as compared to the traditional TMD system in reducing the displacement demand as well as the settling time of the structure. SATMD system can suppress the vibration in a wide range of frequencies in contrast to TMD system which is tuned at a particular frequency. To minimize the structural response due to a seismic excitation, an optimal *Q* matrix for LQR control system in TMD application has been proposed. Furthermore, it has been illustrated that in a practical range of the auxiliary mass in a tall building (1.5% to 2.5% of building mass), the response of structure does not change appreciably. Finally, it has been shown that in spite of the obvious advantages of a semi-active control system versus the passive control system in reducing the seismic response of tall structures, the active control system is the most effective one to be used with a TMD system provided the power requirements met for the actuator produce a large push and pull forces, in case of an earthquake. SATMD may remain operational even in the case of a power failure during an earthquake and provide effective vibration control to a structural system.

10. Acknowledgement

Support of the Natural Sciences and Engineering Research Council (NSERC) is gratefully acknowledged.

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