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A NEW FLEXIBILITY-BASED INDEX FOR DAMAGE IDENTIFICATION IN LINEAR-SHAPED STRUCTURES

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ABSTRACT: Early damage detection in engineering structures has an important role in Structural Health Monitoring (SHM) to prevent some calamitous events. In this paper, a new method for damage identification in linear-shaped structures is presented. Linear-shaped structure is defined as a structure that its elements can be arranged only on a straight line. By using Grey System Theory (GST) and flexibility curvature, a new index is introduced for damage prognosis. GST has been developed for inspecting the correlation between two regular sequences by a geometrical-based comparison with a reference sequence that is weak with limited measured data. The applicability of the presented method is demonstrated by studying several damage patterns on two numerical examples of linear-shaped structures, named shear frame and beam. In addition, to generalize the applicability of the suggested method for real SHM programs, the modal data are contaminated by some random noises and the presented method is employed for damage identification. Moreover, the impact of the number of available modal data on calculating flexibility matrix is investigated. The obtained results emphasize the robustness of the presented method for damage identification in linear-shaped structures.

1. Introduction

Structural damage prognosis is the most important part of Structural Health Monitoring (SHM) programs, which is devoted for identifying damage in structural systems. Early damage detection not only can prevent some calamitous events, but also it can decrease additional costs for repairing structures. Researches try to identify structural damage by practical approaches and in this regard, different methods were proposed in the literature. Despite major differences among different damage identification methods, all of them are based on analyzing structural feedbacks. Analyzing vibrational characteristics of the monitored structure is one of the well-known strategies for damage identification. Generally, damage is defined as some deterioration in physical properties of a structure. On the other hand, vibrational characteristics, such as natural frequencies and related mode shape vectors, depend on the physical properties of the monitored structure. Therefore, by studying vibrational properties, we can find valuable information about structural damage. A complete review of vibrational-based methods can be found in (Fan and Qiao, 2011). Some methods use signal processing approaches for damage localization (Kim and Melhem, 2004; Bagheri, Ghodrati Amiri and Seyed Razzaghi, 2009; Ghodrati Amiri, Jalalinia, Zare Hosseinzadeh and Nasrollahi, 2015). These methods employ mathematical decomposition approaches for extracting the main sensitive features of the vibrational feedbacks to structural defects. Goyal and Pabla (2015) presented a complete review of these approaches.

Other type of damage prognosis methods try to localize structural damage by direct mathematical-based strategies (Bernal, 2006; Catbas, Gul and Burkett, 2008; Yan, Ren and Huang, 2012; Yan, Dyke and Irfanoglu, 2012; Ghodrati Amiri, Zare Hosseinzadeh, Bagheri and Koo, 2013). Ndambi, Vantomme and Harri (2002) studied different damage indices, which utilized eigen-frequencies and/or mode shape derivatives and compared their ability for damage localization in reinforced concrete beams. They made a conclusion about good performance of the strain energy method for damage assessment. Xia and Hao (2003) proposed a statistical-based index for damage identification via natural frequency changes. Limongelli (2011) localized damages in frames by interpolating the operational mode shapes via spline function. Yan, Ren and Huang (2012) introduced modal strain energy as a damage-sensitive parameter and used it by considering statistical concepts for damage identification in beams.

Despite the good performance of the above-mentioned methods, researchers try to present simple and practical methods, which can localize structural damage using as few as possible input data. This paper presents a new method for damage localization by introducing a novel damage index based on flexibility curvature and Grey System Theory (GST). By utilizing GST, we can judge about geometrical correlation between two vectors. Finally, the presented method is verified by studying different damage patterns on a fifteen-story shear frame. Moreover, we generalize this method for damage identification in beams by considering only the translational degrees of freedom (DOFs) which can be interpreted as a condition in which an incomplete set of modal data are used for SHM. Results show the good and acceptable performance of the proposed method for damage localization in linear-shaped structures.

2. Grey System Theory

Grey System Theory employs a geometrical-based comparison for measuring amount of correlation between two regular sequences, especially, when weak and limited measured data are available (Deng, 1989). GST reflects the degree of approaching two geometrical curves by calculating Grey Relation Coefficients (GRCs). It is clear that the bigger coefficients show a big approaching between the baseline and the test sequences.

Consider the reference sequence ($\mathbf{A}_0$) and the test sequence ($\mathbf{A}_t$) as below:

$$\mathbf{A}_0 = \{a_0(1), a_0(2), ..., a_0(n)\}^T \tag{1}$$

$$\mathbf{A}_t = \{a_t(1), a_t(2), ..., a_t(n)\}^T \tag{2}$$

where $a_0(n)$ and $a_t(n)$ are n-th points on the geometrical curves. GRCs can be calculated as follows:

$$\eta_t(n) = \frac{\min_t \min_n \beta + \alpha \max_t \max_n \beta}{\beta + \alpha \max_t \max_n \beta} \tag{3}$$

in which $\eta_t(n)$ is GRC in the n-th point. α is defined as distinguishable coefficient which is a number between 0 and 1, and in this study, we select α=0.5. In addition, β is the grey variant as follows:

$$\beta = |a_0(n) - a_t(n)| \tag{4}$$

Generally, $\eta_t(n)$>0.9 shows a complete relation between the reference point and the test point. More details about GST can be found in (Fu, Zheng, Zhao and Xu, 2001; Zare Hosseinzadeh, Bagheri and Ghodrati Amiri, 2013). It is worth noting that GST is used for damage identification in few static-based methods (Chen, Zhu and Chen, 2005; Abdo, 2012). Recently, Zare Hosseinzadeh, Bagheri and Ghodrati Amiri (2013) used it for damage localization and quantification by analyzing the first mode shape slope as a vibrational characteristic, which is sensitive to structural damage.

3. Proposed Method

This section presents details of the proposed method for damage localization in linear-shaped structures.

Generally, a linear-shaped structure is defined as a structure in which all elements can be arranged only on a straight line. In other words, each member or each element of a linear-shaped structure is introduced by two nodes and each node has only one DOF. Shear frame is the best example of linear-shaped structures in which each story can be defined as an element. By considering these conditions, the free vibration problem for a structure with N DOFs and N_e elements can be presented as below:

$$\mathbf{K}\boldsymbol{\varphi}_i = \lambda_i \mathbf{M}\boldsymbol{\varphi}_i \tag{5}$$

where **K** and **M** are global stiffness and mass matrices, respectively. In addition, λ_i and $\boldsymbol{\varphi}_i$ are the i-th eigenvalue and related eigenvector, respectively. It should be noted that the vector $\boldsymbol{\varphi}_i$ is normalized with respect to the global mass matrix.

By utilizing the first m modes' data (i.e. the first m eigenvalues and related eigenvectors), the flexibility matrix ($\mathbf{F}_m$) can be estimated as follows:

$$\mathbf{F}_m = \boldsymbol{\Phi}\mathbf{O}^{-1}\boldsymbol{\Phi}^T \tag{6}$$

in which $\boldsymbol{\Phi}$ is a matrix of the first m modes' eigenvectors and **O** is a diagonal matrix that is defined as below:

$$\mathbf{O} = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_N \end{bmatrix} \ , \ \sqrt{\lambda_1} < \sqrt{\lambda_1} < \dots < \sqrt{\lambda_N} \tag{7}$$

This paper employs diagonal members of the flexibility matrix before and after damage for introducing a new damage localization index. We can calculate the flexibility matrix using the first m modes' data for intact structure ($\mathbf{F}_m^u$) and damaged structure ($\mathbf{F}_m^d$), using Eq. (6). Then, we define vectors $\mathbf{v}^u$ and $\mathbf{v}^d$ by considering the diagonal members of the $\mathbf{F}_m^u$ and $\mathbf{F}_m^d$ as follows:

$$\mathbf{v}^u = \left\{ \mathbf{F}_m^u(1,1) \quad \mathbf{F}_m^u(2,2) \quad \dots \quad \mathbf{F}_m^u(i,i) \quad \dots \quad \mathbf{F}_m^u(N,N) \right\}^T \tag{8}$$

$$\mathbf{v}^d = \left\{ \mathbf{F}_m^d(1,1) \quad \mathbf{F}_m^d(2,2) \quad \dots \quad \mathbf{F}_m^d(i,i) \quad \dots \quad \mathbf{F}_m^d(N,N) \right\}^T \tag{9}$$

These two vectors can represent intact and damaged states. As mentioned before, this paper is aimed at proposing a damage localization index based on the numerical calculation of the vector curvature. In this study, the Finite Central Differentiation Procedure is employed for numerical calculation of curvature. This procedure can be defined as:

$$v_i'' = \frac{v_{(i+1)} - 2v_i + v_{(i-1)}}{L_{(i+1)}L_{(i-1)}} \tag{10}$$

where L_i is the distance between i-th and (i+1)-th nodes. By applying the GST, we can measure amount of correlation between the calculated curvature vectors. For this purpose, the curvature vectors of the intact structure ($\mathbf{v}^u$) and damaged structure ($\mathbf{v}^d$) are considered as reference and test sequences, respectively. Then, the GRCs are calculated based on the presented approach in the previous section:

$$\mathbf{GRC} = \left\{ \eta_t(1) \quad \eta_t(2) \quad \dots \quad \eta_t(i) \quad \dots \quad \eta_t(N) \right\}^T \tag{11}$$

Finally, for each point, the proposed damage index (DI) is defined as:

$$DI_i = \frac{\Omega_i}{\max\left(\Omega_1, \Omega_2, \dots, \Omega_i, \dots, \Omega_N\right)} \ , \ i = 1, 2, \dots, N \tag{12}$$

where Ω_i is the i-th member of the $\boldsymbol{\Omega}$ which is presented as below:

$$\boldsymbol{\Omega} = \mathbf{1.0} - \mathbf{GRC} \tag{13}$$

As mentioned before, this method is applicable to linear-shaped structures. The performance of the proposed damage index can be summarized as below:

Each member or element of a linear-shaped structure has two nodes and for each node, we can calculate the suggested damage index (*DI*) based on the presented approach. The *i*-th element will be introduced as a damaged element if both calculated *DI* are so close to 1.0. It is worth noting that if one of the element's nodes were fixed by support, in this case the calculated *DI* in the free node will be used for judging about its health.

In the following, the efficiency of the proposed method is demonstrated by studying several damage patterns on two numerical examples of linear-shaped structures.

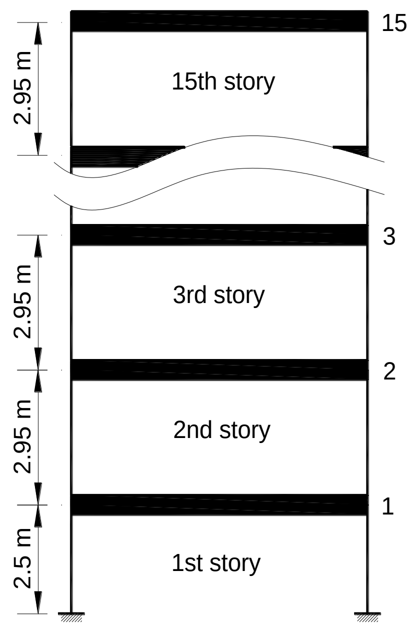

Fig. 1 – Fifteen-story shear frame

Table 1 – Physical properties of fifteen-story shear frame.

Story No.	Mass (ton)	Stiffness (MN/m)
1~5	60	6.0
6~10	45	6.0
11~15	25	4.5

Table 2 – Damage patterns in the fifteen-story shear frame.

Damage pattern	Damage location	Damage Severity (%)
(1)	Story 7	15
(2)	Stories 9, and 15	5, and 25
(3)	Stories 4, 8, and 12	20, 20, and 10

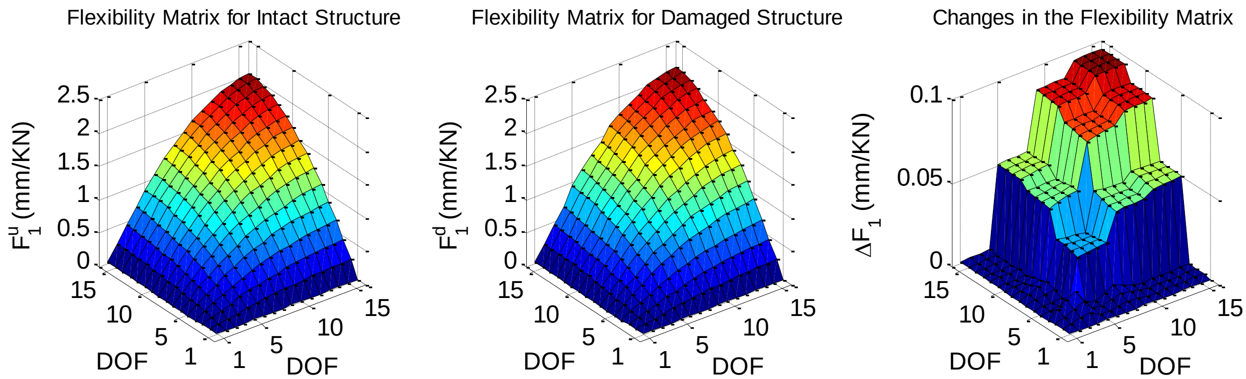

Fig. 2 – Flexibility matrix and its change for intact and damaged shear frames (third pattern)

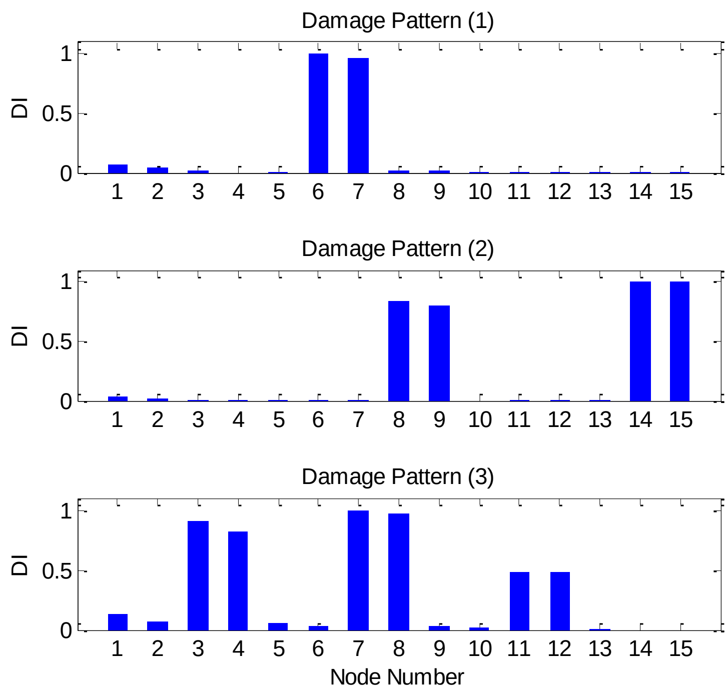

Fig. 3 – Damage detection results of shear frame using the first mode's data

4. Numerical Examples

4.1. A Fifteen-Story Shear Frame

In the first example, a fifteen-story shear frame is considered (Fig. 1). The physical properties and the studied three damage patterns are listed in Tables 1 and 2, respectively. It should be noted that these damage patterns are simulated by reducing the stiffness of the damaged stories, based on the defined damage levels. The calculated flexibility matrix using only the first mode's data, for intact and damaged structures (based on the third damage pattern), and the changes between them ($\Delta \mathbf{F}_1$) are shown in Fig. 2. From this figure, it is obvious that although there is not any distinguishable difference between the flexibility matrices in the damaged and undamaged states, we can find some irregularity in the $\Delta \mathbf{F}_1$. This irregularity is sensible in the diagonal members. Therefore, it can be concluded that inspecting amount of correlation between diagonal members of the flexibility matrix in damaged and undamaged states can be a damage-sensitive parameter.

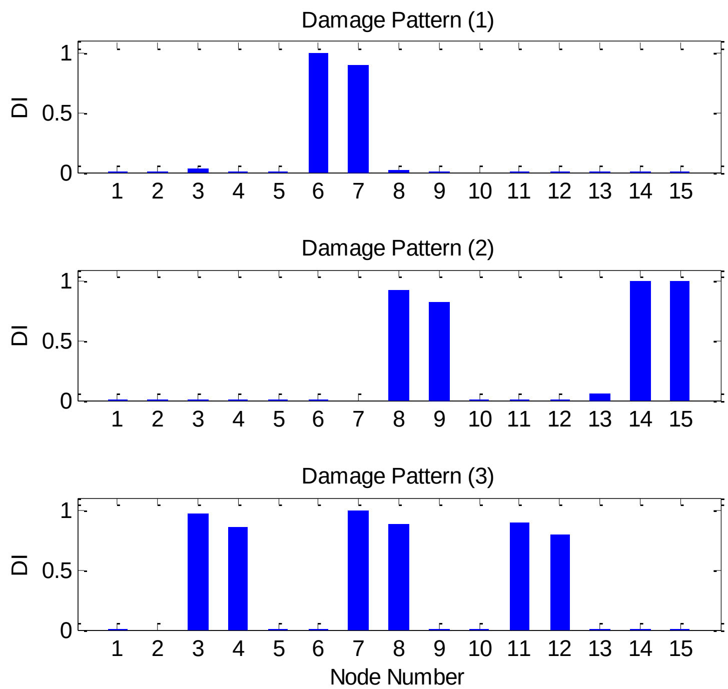

Fig. 4 – Damage detection results of shear frame using the first three mode's data

In the following, we apply the presented method for damage identification in the mentioned three damage cases using only the first and the first three modes' data. The obtained results for an ideal case (noise free state) are shown in Figs. 3 and 4 for three damage patterns. By inspecting these results, it can be concluded that the presented method is able to localize simulated damages with high level of accuracy and by increasing the number of utilized modes' data for calculating flexibility matrix; the index can perform precisely, especially in the healthy members.

Despite the good performance of the presented method in ideal case, for generalizing its applicability in real SHM programs, we should examine it when the input data are polluted with some random noises. This issue is studied by contaminating the natural frequencies with 5% uniformly distributed random noises. Fig. 5 shows the obtained results for such a condition in which the problem is solved by considering only the first mode's data. Similar to the noise free state, the results indicate the good performance of the presented method for defect localization in the presence of random noises.

4.2. Simply Supported Beam

This example is devoted to generalize the presented method for damage identification in another type of linear-shaped structures, named beams. For this purpose, a simply supported beam is considered. Based on Fig. 6, the finite element model of this structure consists of ten elements and nine free nodes, with two DOFs in each node. The physical properties of this structure are as below: Modulus of elasticity E=25 GPa, and mass density ρ=2500 kg/m^3. In addition, cross sectional area and the moment of inertia of elements are A=0.35 m^2, I=0.01429 m^4, respectively.

To generalize the presented method for damage localization in beams, we should reduce the structural model in a way that only one DOF is allotted to each node. By this modification, not only can we use the presented method for damage prognosis in beams, but also we can apply method to such SHM programs in which only a limited number of sensors were installed on the beam. We use Guyan static reduction method (Guyan, 1965) for applying above addressed modification. Guyan's method is based on separating mass and stiffness matrices to sub matrices related to the slave and master DOFs. Readers can find more details about this method in (Guyan, 1965). Therefore, we can use presented *DI* for health monitoring of beams when structural model is reduced by considering only the translational DOFs.

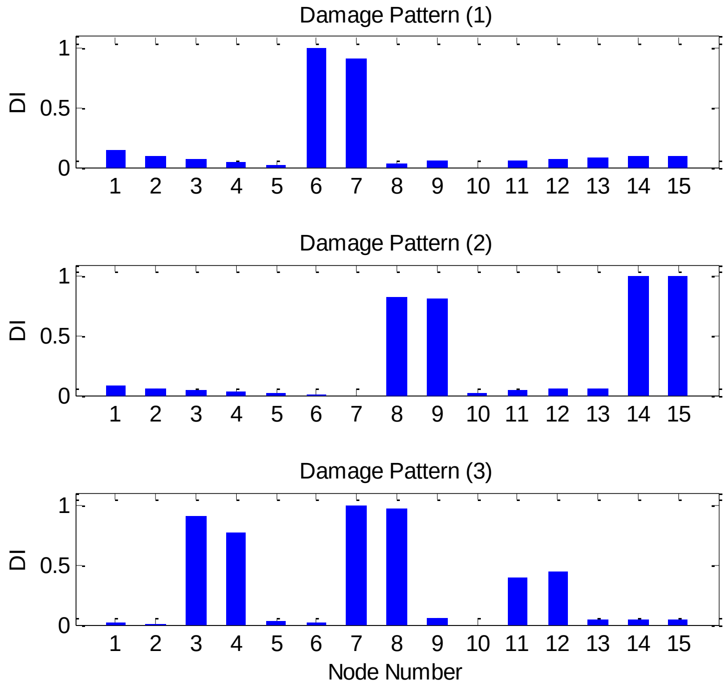

Fig. 5 – Damage detection results of shear frame using the first mode's data with 5% noise

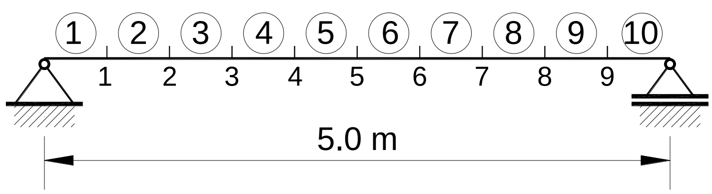

Fig. 6 – Finite element model of simply supported beam

Table 3 – Damage patterns in the simply supported beam.

Damage pattern	Damage location	Damage Severity (%)
(1)	Element 5	25
(2)	Elements 1, and 6	15, and 10

In this section, the applicability of the presented modification is investigated by simulating two damage patterns on the introduced simply supported beam. The details of these patterns are summarized in Table 3. First pattern consists of a single damage scenario and the second one is devoted for simulating multiple damage case. The obtained results for noise free state are shown in Fig. 7. In this study, the flexibility matrix is calculated by employing only the first mode's data. As mentioned in Section 3, the proposed *DI* will be so close to 1.0 in the free nodes of the damaged elements. Here, we peruse obtained results for the damage pattern (2), for instance. Based on the obtained results, *DI* for the first, fifth and sixth nodes are so close to 1.0. The first element has only one free node and therefore, this element is considered as a damaged element. In addition, both calculated *DI* for the free nodes of the sixth element are close to 1.0 and as a result, this element is reported as a damaged element. Therefore, the good performance of the presented approach for damage identification in beams can be concluded.

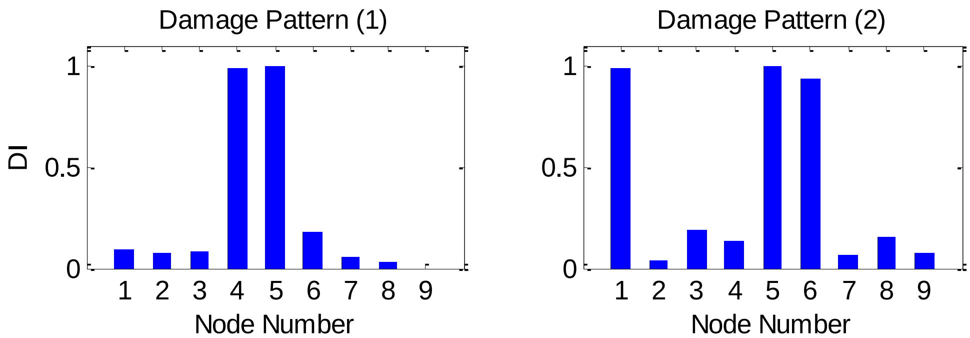

Fig. 7 – The obtained results for two damage patterns of simply supported beam

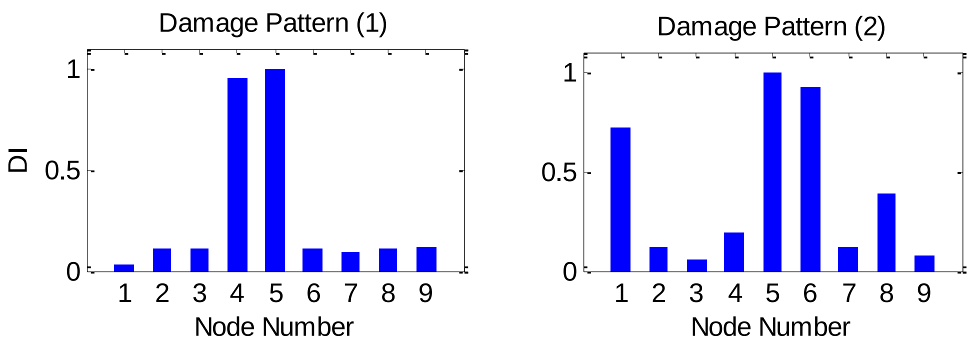

Fig. 8 – The obtained results for two damage patterns of simply supported beam with 5% noise

In addition, Fig. 8 shows the obtained results for a condition in which 5% random noises are added to input data. Results emphasize the viable applicability of the presented method for damage identification. Moreover, by considering the presented modification, we can introduce the proposed method as a powerful strategy for damage identification, when a limited number of sensors were installed on the beam.

5. Conclusions

In this paper, a new method for damage localization in linear-shaped structures was presented. Linear-shaped structures is defined as a structure that its elements can be arranged only on a straight line. By applying Grey System Theory on the flexibility curvature, a new damage index was introduced. The applicability of the proposed method for damage localization was investigated by studying three different damage patterns on a fifteen-story shear frame. Moreover, the presented method was generalized for damage identification of beams when only the translational DOFs were considered. Obtained results emphasized the good and stable performance of the proposed method for defect localization and introduced it as a powerful method for damage identification in linear-shaped structures.

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