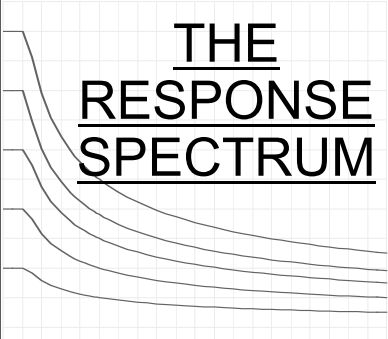
**The Canadian Society for Civil Engineering, Vancouver Section**





THE RESPONSE SPECTRUM

*A Technical Seminar on the Development
and Application of the Response Spectrum
Method for Seismic Design of Structures*

Response Spectrum Analysis for Structures and the NBCC 2005

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Incorporating Discussions and
Ideas From:
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Prof. Don Anderson, Ph.D. – UBC
Reza Anjam MA.Sc, P.Eng, RJC



1-2 June 2007 Vancouver, BC

Introduction

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This presentation looks at the Response Spectrum Dynamic Approach to structures and at some issues with respect to:

- NBCC 2005 requirements.
- Use of computer programs.
- Cautionary note – this is a complex topic (both static and dynamic) and computers may give a precise but not accurate solution.
- Always engage your own "on board" processor - thinking is encouraged.

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Outline	3
<ol style="list-style-type: none">1. Static Method (very brief)2. Why Do a Dynamic Analysis?3. Computer Modeling Issues (very brief – headings only)4. Basic Response Spectrum Dynamic Analysis Issues5. Basic NBCC 2005 Dynamic Analysis Issues	
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1 Static Method	4
<ul style="list-style-type: none">• The static method is based on dynamic analysis of “regular” structures.• It defines forces to apply to the structure which reproduce (more or less) the “correct” shear envelope up the building.• The F_t force at the top is period dependant and is intended to model higher mode effects.• The forces and their distribution, while roughly giving the correct dynamic shear, will overestimate the dynamic moments in the building.• The “J” Factor is used to “correct” the moment and bring it closer to the dynamic moment.	
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2 Why Do A Dynamic Analysis?

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Earlier Versions of NBCC (even in 1970) urged dynamic analysis be used for irregular buildings and for large eccentricities it was required unless the static torque effects were doubled.

Advantages are:

- Dynamic amplification of torque effects is captured.
- Changes in mass and stiffness are better modeled in a dynamic analysis. Podium building are a good example of this.
- Reductions in base shear in some torsionally eccentric buildings is captured.
- Reductions in overturning moments and displacements for tall, long period buildings when compared to the static approach.

Current computer programs make a dynamic spectrum analysis relatively simple to do once the model is built. However using the results requires a bit of understanding beyond a static analysis and this will be discussed in the next sections.

3 Some Computer Modeling Issues

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While live loads, snow loads, and wind loads tend to be independent of the structure (not quite true for wind loads on tall, long period structures), earthquake loads and displacements vary greatly with:

- *The building period* – basically the earthquake forces decrease and the displacements increase as the period increases.
- *The structural type* – for longer period buildings, uncoupled shear walls behave differently than coupled walls, moment frames, braced frames, and coupled walls – even for the same fundamental period

3 Some Computer Modeling Issues

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For this reason it is important to get the model to be as good an approximation as you can. Some of the items to think about in the model are:

- shear displacements
- finite joint sizes
- cracked “I” values in concrete, including the area of coupled walls.
- behaviour of tall walls through deep, below grade structures.
- diaphragm displacements at discontinuities of the lateral system.
- effect of footing rotations.
- modeling the diaphragm – rigid? membrane? plate?
- how to apply mass and in what units.
- how to model complex walls – as a “whole” element or as separate pieces.

3 Some Computer Modeling Issues

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Modeling frames (beams and columns), braced steel frames, and rectangular walls (coupled or uncoupled), is relatively straight forward.

However, irregular shapes (channels etc..) can lead to problems with figuring out where in the section the forces are acting and this makes design difficult. (both static and dynamic analysis)

Consider doing the following, using a channel shaped wall with unequal flanges as an example:

- Run once with the “wall” called up as one element. This will give overall shear, moments, torques and axial forces. Useful for checking and design of footings. But – what is going on in the flanges?
- Run again with the wall built up out of three separate named elements - two flanges and a web. The output will then be for the shear, moment, torque and axial force in each rectangular element. This makes design a bit more straight forward.

4 Some Computer Dynamic Spectrum Analysis Issues ⁹

- Understand the program!
- The “dynamic” result is a combination of mode shapes, so make sure there are enough to capture the building’s behaviour. Pick 3 times the number of floors up to 15 or so.
- Mode shapes are combined to get the “design” values. Pick “CQC” instead of “SRSS” if possible as it is better when eigenvalues (periods) are close together.
- Check the mode shapes, and the mass participation factors to make sure at least 90% of the mass has been captured into the analysis.
- If not, increase the number of mode shapes used until 90% or more has been captured. Make sure that enough mode shapes (higher modes) are included to pick up the response of podiums at the base.

4 Some Computer Dynamic Spectrum Analysis Issues ¹⁰

- Review the mode shapes, the periods and the participation factors to get a feel for what the building is doing and how it behaves.
- “Animate” the lower mode shapes individually and the combined result to see if they look right and get insight into the building behaviour. Often a flaw in the model can be spotted this way.
- Do a static run and compare to simple calculations as a check.
- Review the total weight and mass printed out as a check.
- Powerful graphical generators can be tricky and unexpectedly foul up the model. Check! Check! Check!

4 Some Computer Dynamic Spectrum Analysis Issues ¹¹

- The final results for moment, shear, displacement and drift are the result of an “SRSS” type combination of mode shapes that vibrate at different periods and so are not concurrent.

The result of this “SRSS” type combination is that:

- All the values are positive.
- The design forces for M, V and P for a member are not in equilibrium – and probably not concurrent.
- The lateral floor loads calculated are not in equilibrium with the base shear and moment.
- Drifts are an “SRSS” type summation of modal drifts and as such do not relate directly to the “SRSS” type displacements.
- **Avoid** back calculating any type of quantity from different quantities. It may be OK but can be dramatically different.

4 Some Computer Dynamic Spectrum Analysis Issues ¹²

- Values for **individual** mode shapes **are** consistent
- Design of odd shaped members resisting axial forces and moments can be puzzling when all values are positive. Run a static analysis to sort out which direction the forces are acting.
- The design of coupled walls for uplift using capacity design of the headers can capture a “dynamic” reduction of the uplift forces in the wall – see CSA A23.3 clauses and commentary.

5 NBCC 2005 Response Spectrum Dynamic Analysis 13
<p>5.1 Spectrum</p> <p>5.2 Scaling?</p> <p>5.3 Minimum Force Level (and deflections!)</p> <p>5.4 Minimum Force Level – Eccentric Building Issues</p> <p>5.5 Accidental Eccentricity</p> <p>5.6 P-Delta Effects</p>
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5.1 Response Spectrum – NBCC 2005 14												
<p>Site Properties Clause 4.1.8.4</p> <p>The Response Spectrum is given on a city by city basis similar to the way snow and wind loads are presented.</p> <p>The values are in Volume 2 – APP. C. and are given for natural periods of 0.2s, 0.5s, 1.0s and 2.0s.</p> <p>For Vancouver the values are:</p> <table style="margin-left: auto; margin-right: auto;"><tr><td>0.0s to 0.25</td><td>-</td><td>0.94g</td></tr><tr><td>0.5s</td><td>-</td><td>0.64g</td></tr><tr><td>1.0s</td><td>-</td><td>0.33g</td></tr><tr><td>2.0s</td><td>-</td><td>0.17g</td></tr></table>	0.0s to 0.25	-	0.94g	0.5s	-	0.64g	1.0s	-	0.33g	2.0s	-	0.17g
0.0s to 0.25	-	0.94g										
0.5s	-	0.64g										
1.0s	-	0.33g										
2.0s	-	0.17g										
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5.1 Response Spectrum – NBCC 2005

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The value at 4.0s and beyond is taken as half that at 2.0s. This produces a flat spectrum past 4.0s, which has some interesting consequences for long period buildings.

Interpolation is used between these values.

These values are multiplied by the soil factors which can increase or reduce them.

This is easily entered into an analysis program. See Fig 1

These values are in "gravity" units and factors must be applied in the program to make them consistent with the "mass units" used in the program.

5.1 Response Spectrum – NBCC 2005

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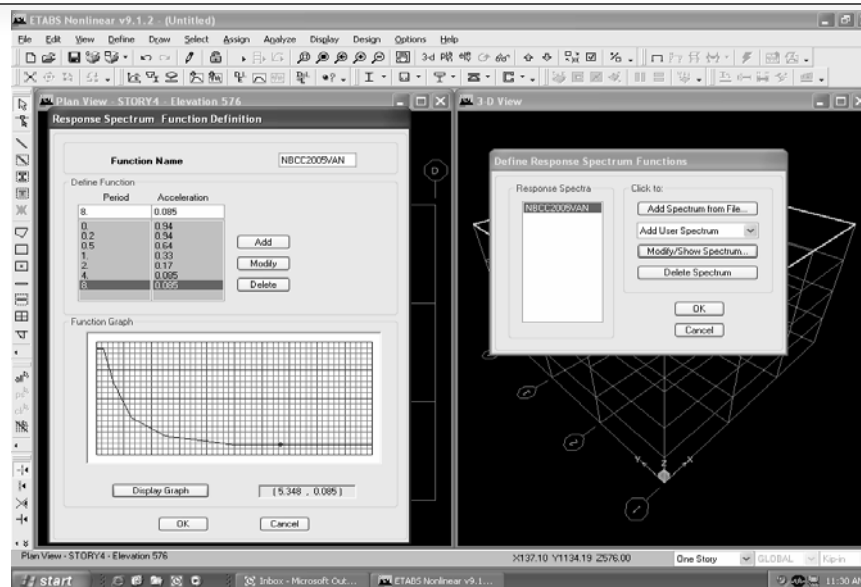


Fig 1

5.2 Scaling? – NBCC 2005

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In NBCC 1995 (and previous versions) Dynamic Response Spectrum Analysis is an alternate to the static analysis but the results are scaled such that the dynamic base shear is made equal to the static base shear.

If the calculated dynamic analysis first period was larger than the static formulae calculated period, the dynamic period could be used to calculate a reduced static base shear – but not less than 80% of the code formulae approach

5.2 Scaling? – NBCC 2005

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This was done to try and keep the base shear “analysis independent”, maintain a similar level of protection, and prevent designers from analysing very soft, flexible, long period, low force buildings based on “inappropriate” stiffness assumptions.

A few problems with this approach are:

- If the building was actually very stiff, the higher dynamic forces could be ignored.
- It can lead to very unconservative results for structures with large podiums or soft upper portions and stiff lower portions.

5.2 Scaling? – NBCC 2005

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2005 NBCC Approach (4.1.8.11, 4.1.8.12)

The NBCC 2005 Approach is different.

Essentially, the elastic response using the code spectrum is calculated and the results are divided by RdRo

- The values cannot be less than 80% of the static base shear for “regular” buildings and 100% of the static base shear for irregular buildings. If this shear is less, it must be scaled up to those minimum values. If the dynamic is higher than the static, then the dynamic results govern the design.
- Stiff buildings or podium buildings typically will have higher shears and must be designed for them.

Essentially – the only scaling allowed would be upwards in value.

5.3 NBCC 2005 – Minimum Force Level

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Minimum force level (and deflections) (4.1.8.11, 4.1.8.12)

Previous versions of NBCC raised the static base shear at long periods to account for higher modes and to provide a small degree of conservatism for long period (i.e. tall) buildings. This was done by allowing the design forces to fall off at $1/\sqrt{T}$.

However, NBCC 2005 uses a spectrum that falls off at $1/T$ in the long period range. The seismologists stated they do not have a lot of data for long periods and their confidence in the numbers decreases as the periods increase.

5.3 NBCC 2005 – Minimum Force Level

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Three Steps were taken to address this:

- The spectrum at 4 seconds was taken as 50% of the 2 second value.
- The spectrum runs flat after 4 seconds.
- The minimum static force was taken as that at 2 seconds. This becomes the lower bound force (or 80% of it does) for dynamic analysis.

We ran some parametric studies of buildings with heights of 200' to 800' and fundamental periods of 2s to 8s to examine the implications of the above. We also ran the results for a spectrum that kept falling off at 1/T past 4s for interest. The results are given in Table 1 and Table 2.

5.3 NBCC 2005 – Minimum Force Level

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Vancouver Shear Walls RdRo=5.6	W, kips	36,000	72,000	144,000
	H, ft	200	400	800
	Period, s	2	4	8
Dynamic (2005 code) Spectrum flat 4 sec	V	1,300	1,820	2,510
	M	115,000	230,000	830,000
	Disp	0.84	1.70	6.83
	Drift %	0.63	0.64	1.28
Static (2s) (2005 code) i.e., evaluated at 2 second values	V	1,350	2,700	5,400
	M	137,500	533,900	2,277,000
	Disp	1.48	6.87	28.16
	Drift %	1.11	2.58	5.28
Dynamic (2s) (2005 code) Dynamic scaled to .8xStatic(2s)	V	1,300	2,160	4,320
	M	115,000	272,967	1,428,526
	Disp	0.84	2.02	11.76
	Drift %	0.63	0.76	2.20
Dynamic2 (2005 code) Spectrum 1/T beyond 4 sec	V	1,300	1,820	2,280
	M	115,000	230,000	475,000
	Disp	0.84	1.68	3.57
	Drift %	0.63	0.63	0.67

Table 1

5.3 NBCC 2005 – Minimum Force Level

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Vancouver Moment Frames	W, kips	36,000	72,000	144,000
RdRo=6.8	H, ft	200	400	800
(coupled walls)	Period, s	2	4	8
Dynamic (2005 code)	V	870	870	1,520
Spectrum flat 4 sec	M	95,700	188,000	750,000
	Disp	0.79	1.55	5.85
	Drift %	0.5925	0.58	1.10
Static (2s) (2005 code)	V	900	1,800	3,600
i.e evaluated at 2 second values	M	131,000	544,000	2,191,000
	Disp	1.1	4.53	17.2
	Drift %	0.83	1.70	3.23
Dynamic (2s) (2005 code)	V	870	1,440	2,880
Dynamic scaled to .8xStatic(2s)	M	95,700	311,172	1,421,053
	Disp	0.79	2.57	11.08
	Drift %	0.5925	0.96	2.08
Dynamic2 (2005 code)	V	870	885	857
Spectrum 1/T beyond 4 sec	M	97,000	188,000	378,000
	Disp	0.79	1.55	2.9
	Drift %	0.5925	0.58	0.54

Table 2

5.3 NBCC 2005 – Minimum Force Level

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A few comments about the result:

- There is some thought being given to changing the lower bound to 4 second static forces.
- Wind and wind drift will be a major factor for the taller long period buildings and will govern in many cases.
- For the 8s frame building with a 1/T spectrum the base shear is 0.006 (0.6%) of the weight. (Hmmm!)
- Note the flat spectrum past 4s has some effect on base shears but a large effect on moments and deflections.

5.4 Minimum Force Levels – Eccentric Building Issues 25

Certain types of eccentric buildings (as well as applying the spectrum along a non-principal axis) will generate out of plane dynamic forces which do not appear in a static analysis. This raises the question as to what to compare to the static base shear:

- The component in the direction?
- The resultant?
- What???

There has been some discussion about this in CANCEE. The first thought was that using the component is too conservative, and to use the resultant instead. However this has some problems as well.

5.4 Minimum Force Levels – Eccentric Building Issues 26

At the moment the suggestion is to restrain the structure to vibrate in one direction only, determine the dynamic shear, and compare that to the static shear.

This is Professor Don Anderson's suggestion and is supported by Professor Jag Humar; it is based on the following argument (I think):

We compare to the static base shear to make sure the model is not strangely soft compared to "experience". Therefore check the model by doing this

This may not seem intuitive but the following will show that other methods can give very conservative answers that do not seem appropriate.

5.4 Minimum Force Levels – Eccentric Building Issues

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We will look at a 5 storey building that is 60m by 30m and is analyzed in 3 configurations.

- i. Doubly symmetric and concentric
- ii. Symmetric about the Y axis but very eccentric about the X axis.
- iii. Very eccentric about the X axis and slightly eccentric about the Y axis.

Note that since the height and weight are the same for all buildings, the static base shear is the same for all of them in both the X and Y directions.

Also note that for the static results, loads in the X direction only produce a base shear in the X direction, and Y loads only produce a Y direction base shear for all 3 cases.

The data presented is raw dynamic data for the elastic response ($RdRo=1.0$) as it is the comparisons that are of interest

The building with eccentricities in two directions is then analyzed Restrained in the Y direction and the Z rotation direction and it is seen that (not surprisingly) it gives the same result as the symmetric building for the X direction.

See the following figures and selected results.

5.4 Minimum Force Levels – Eccentric Building Issues

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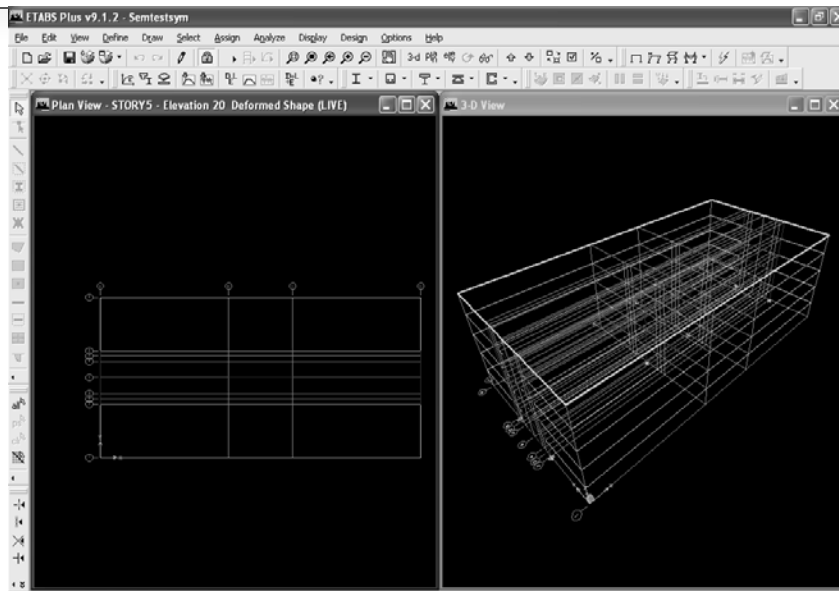


Fig 2

5.4 Minimum Force Levels – Eccentric Building Issues

29

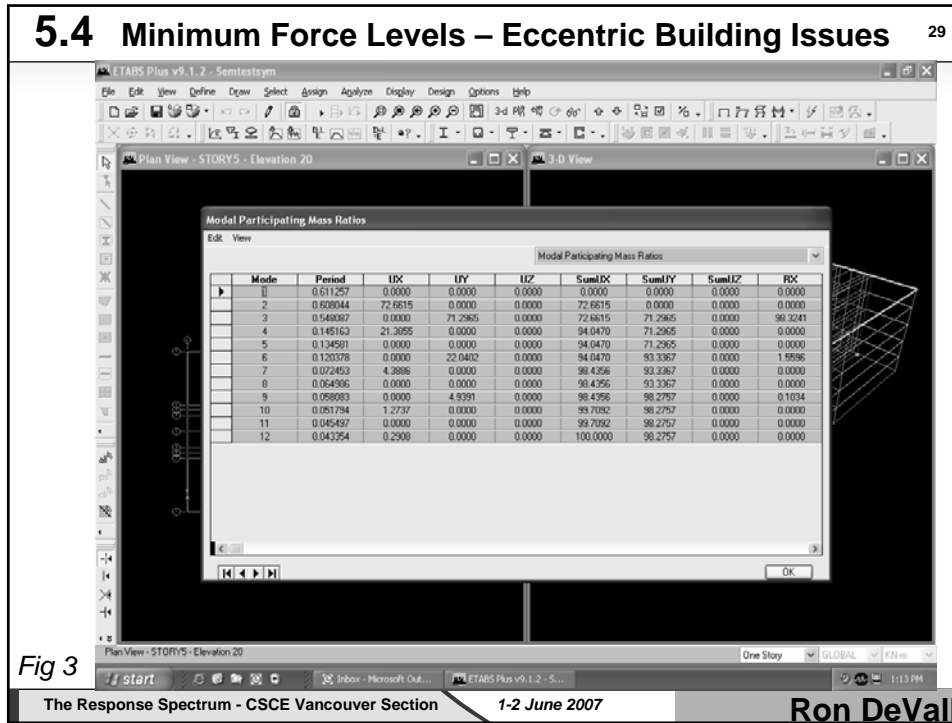


Fig 3

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5.4 Minimum Force Levels – Eccentric Building Issues

30

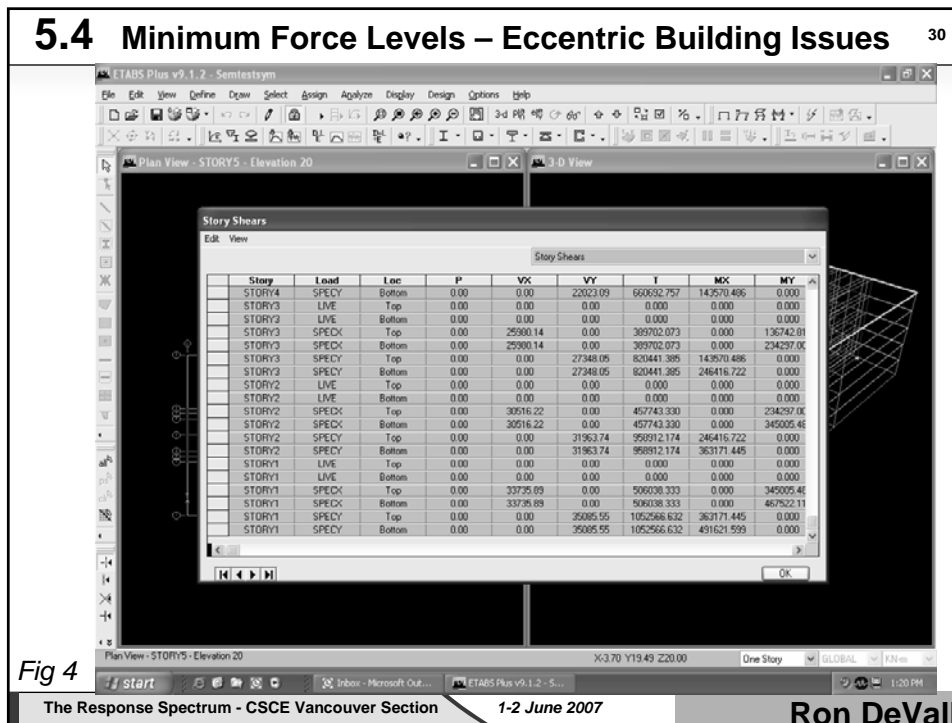


Fig 4

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5.4 Minimum Force Levels – Eccentric Building Issues 31

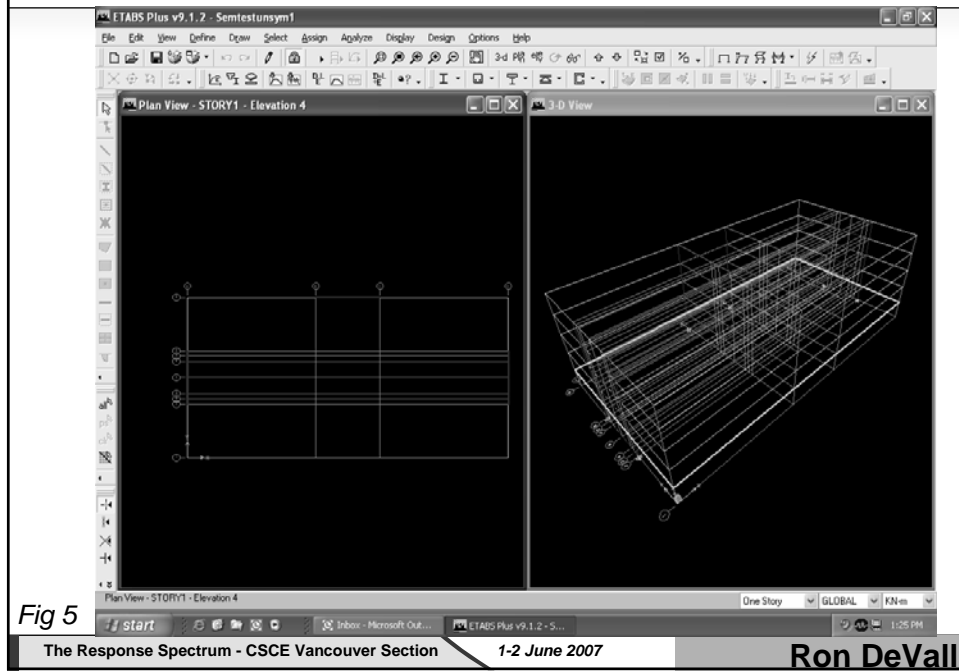


Fig 5

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5.4 Minimum Force Levels – Eccentric Building Issues 32

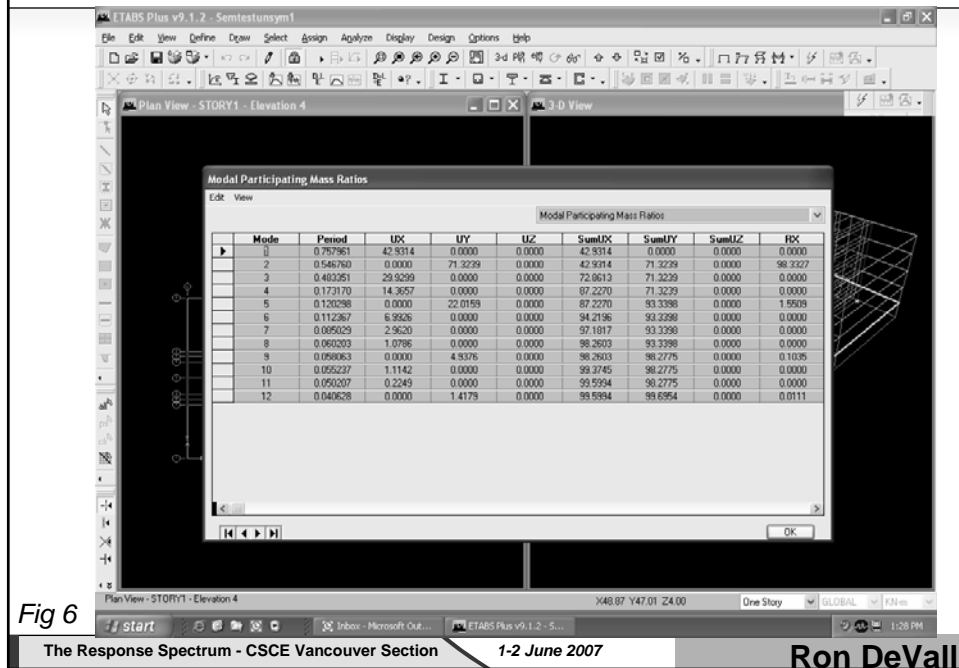


Fig 6

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5.4 Minimum Force Levels – Eccentric Building Issues

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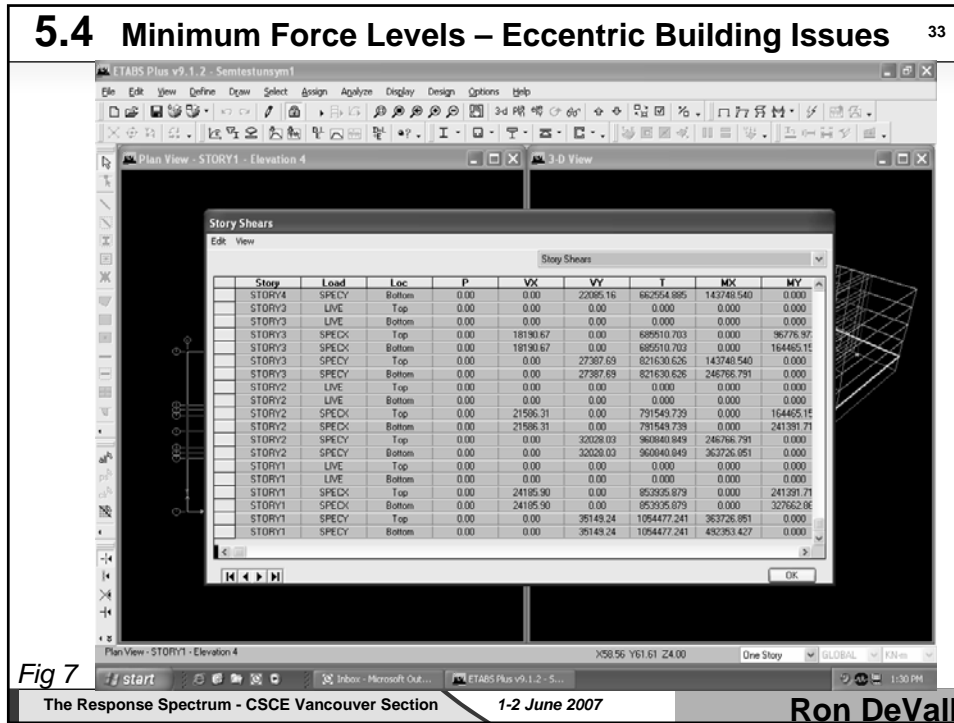


Fig 7

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5.4 Minimum Force Levels – Eccentric Building Issues

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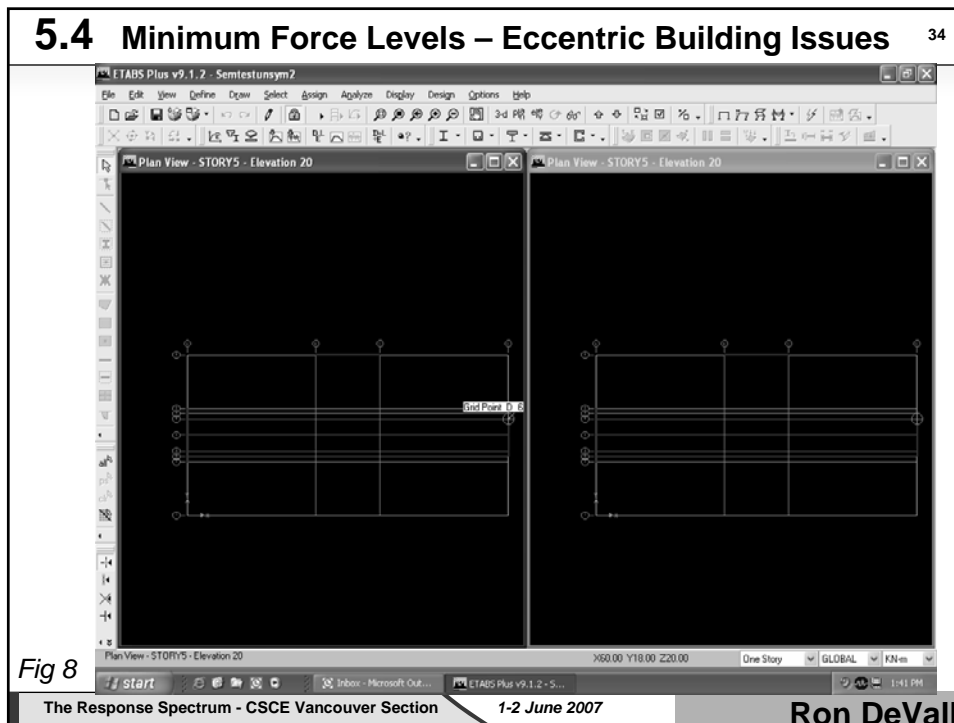


Fig 8

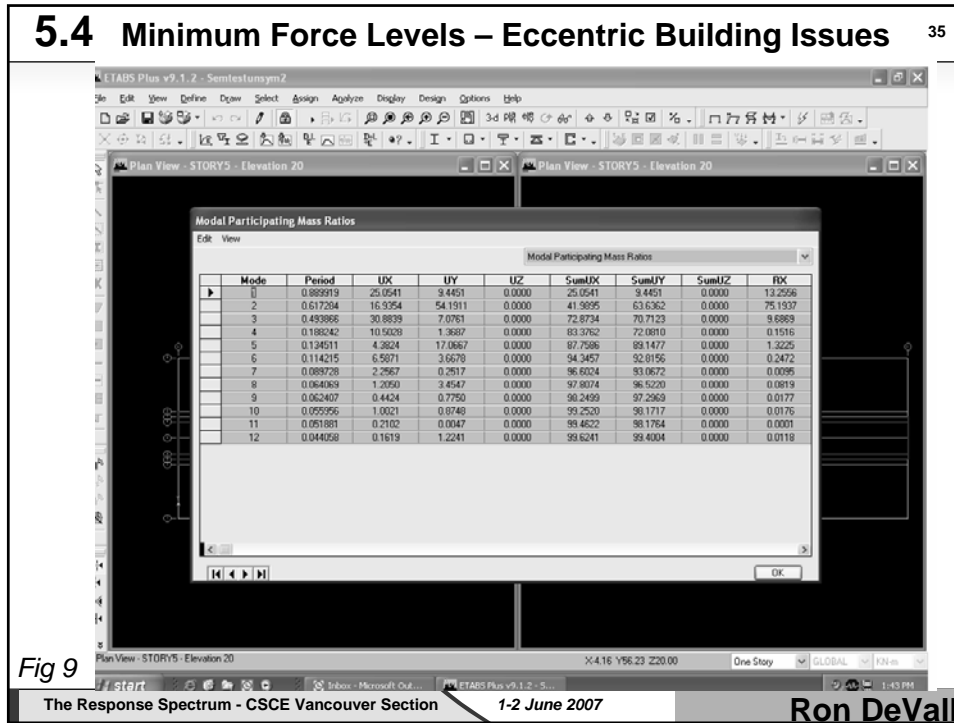
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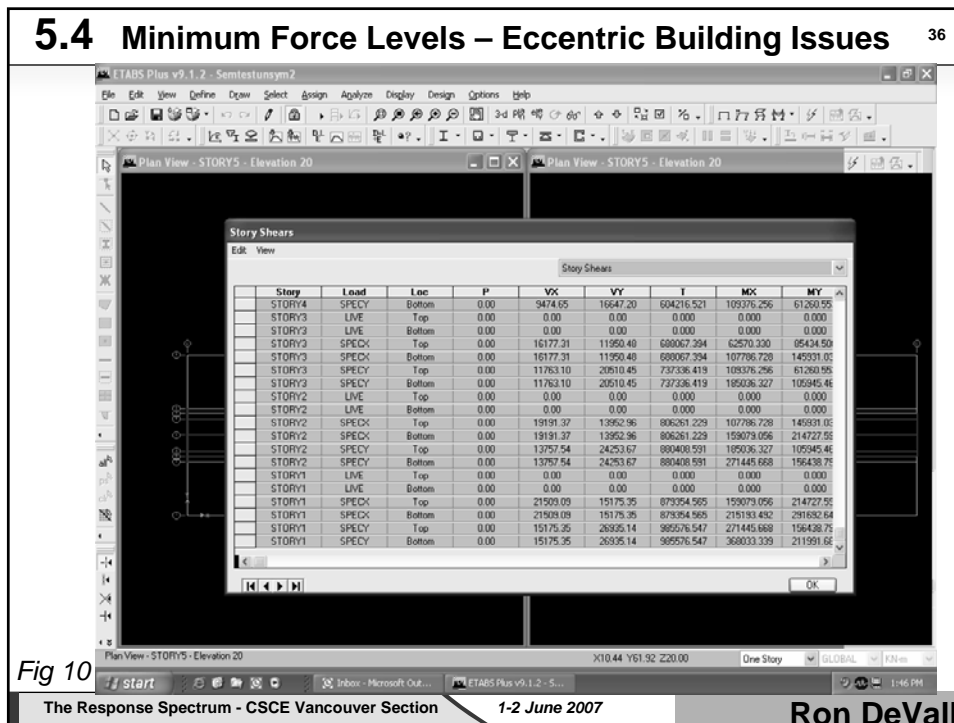
5.4 Minimum Force Levels – Eccentric Building Issues

35



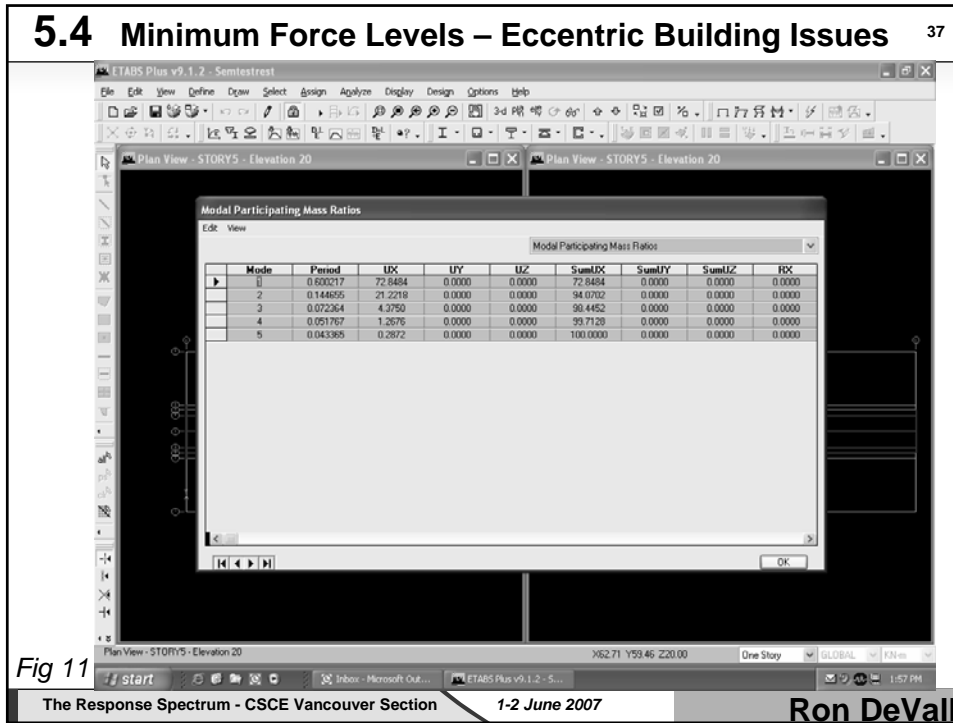
5.4 Minimum Force Levels – Eccentric Building Issues

36



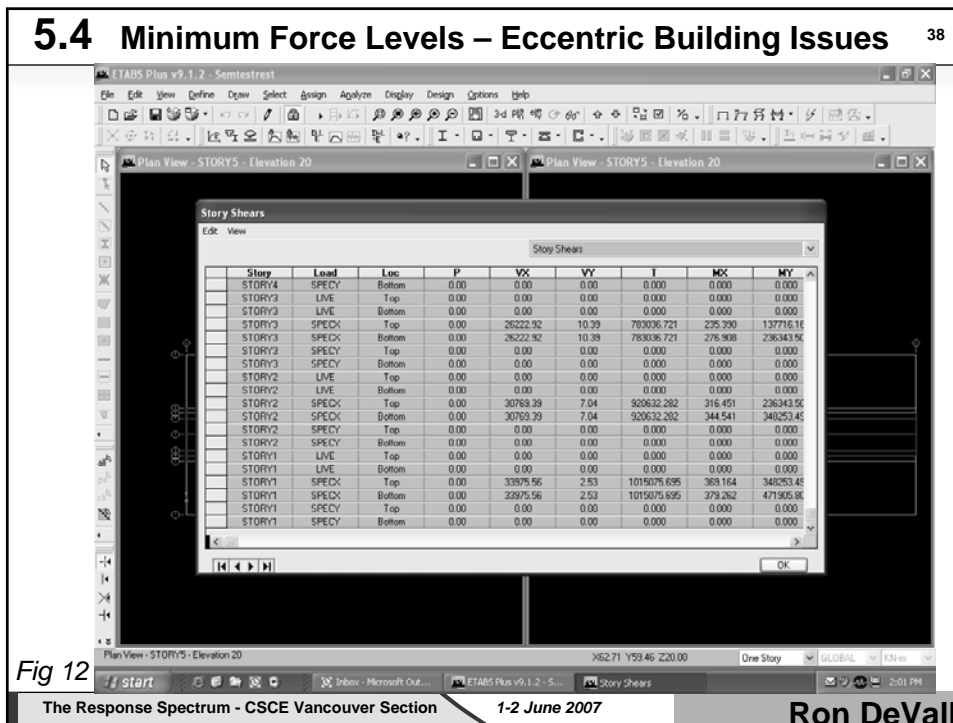
5.4 Minimum Force Levels – Eccentric Building Issues

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5.4 Minimum Force Levels – Eccentric Building Issues

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5.4 Minimum Force Levels – Eccentric Building Issues 39

The results are summarized in Table 3.

Periods in Second	Mode Shape	Symmetric Building	Y Symmetry Only	Unsymmetric 2 Axes	Restrained
	1	(TOR)0.611	0.758	0.89	0.60
	2	0.608	0.547	0.617	--
	3	0.548	0.483	0.494	--
Base Shear	Dynamic X				
	X Axis (kn)	33,735	24,185	21,509	33,975
	Y Axis (kn)	0.0	0.0	15,175	--
	Dynamic Y				--
	X Axis (kn)	0.0	0.0	15,175	--
	Y Axis (kn)	35,085	35,149	26,935	--

Table 3

5.4 Minimum Force Levels – Eccentric Building Issues 40

Review of *Table 3* raises several questions:

- What period to use for the static calculations? The fundamental period of the symmetric building is torsion. The unsymmetric buildings are clearly softened by the eccentricities and the lowest periods have large torsions in their mode shapes.
- The double unsymmetry produces out of plane forces. This may not be intuitive. However, for the first two buildings, deflection along X, even with rotation about Z, does not produce a C.M. deflection along Y. However, for the doubly unsymmetric building a deflection along X produces a Y displacement due to rotation about Z. For the dynamic case, this generates dynamic forces in the Y direction. This is **not** true for the static case.

What to compare to the X static force – the X component or the resultant?

- Looking at the “dynamic X” results it seems clear that scaling to the X component of the resultant is conservative and penalizes what is a “real” effect of base shear reduction for eccentric buildings. (This effect is discussed in “*Fundamentals of Earthquake Engineering*” by Newmark and Rosenblueth, 1971)

5.4 Minimum Force Levels – Eccentric Building Issues 41

Review of the static calculation also raises some questions.
The values are approximately:

- For the short period cut off of 0.667 times the 0.2 second spectral value.
 $V_b \cong 46,000\text{kn (+/-)}$
(NBCC does not address this cut-off if a dynamic is used for a short period building – this is a problem.)
- Static code value = $0.05(20)^{0.75}$
= 0.47 seconds X 2 = 0.94 seconds
- For T = 0.61 seconds
 $V_b = 41,300\text{ kn (+/-)}$
- For T = 0.758 seconds
 $V_b = 34,920\text{ kn}$
- For T = 0.89 seconds (not really a pure X direction period)
 $V_b = 29,200\text{ kn}$

For comparison to the static, a lower bound of 80% of the static can be used for regular buildings and 100% for irregular buildings.

Table 4 shows a few lower bound checks for the X direction.

5.4 Minimum Force Levels – Eccentric Building Issues 42

Scale Factor For Lower Bounds Calculated For Various Assumptions - X direction.				
	T	V Static	V Dynamic	Scale Factor
Symmetric Building	0.61	41,300 (0.8)	33,000	1.0
Scale to Restrained Case (Jag Humar, Don Anderson)	0.61	41,300 (1.0) (Unsymmetric buildings)	33,900	1.22
Singly Symmetric Use Actual Period (dubious)	0.758	34,900 (1.0)	24,200	1.44
Doubly Symmetric Use Actual Period (dubious)	0.89	29,300 (1.0)	21,500 (component)	1.36
	0.89	29,300 (1.0)	26,300 (resultant)	1.11

Table 4

5.4 Minimum Force Levels – Eccentric Building Issues

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Recommend:

- Use Professors Humar and Anderson's suggestion
- Restrain the structure to determine the period for determination of the static shear.
- This captures "suitability" of the stiffness of the model, which is the main reason for any kind of "comparing to static" approach.
- Determine any lower bounds and appropriate "upward" scale factors from this approach.
- It removes the uncertainty of how much of the fundamental period is softened by including torsional components.
- **It allows full capture of the "real" torsional behaviour of the model.**
- For the example buildings, it gives lower scaling values except for the dubious example where the period contains lots of "torsion" – and even here it is only about 10% high.

5.5 Accidental Eccentricity

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Accidental eccentricities are used by codes to account for torsional amplification and uncertainties in stiffness, mass distribution, and torsional effects of the ground motion.

Earlier pre 1995 NBCC codes and current U.S. codes allow shifting the mass to account for accidental torsional effects.

However, for the buildings that have a torsional fundamental period much greater than the lateral periods, shifting the mass often has little effect on the results. The dynamic effect is torsional **de-amplification** (see Newmark and Rosenblueth). Design on this basis produces soft and weak buildings in torsion. CANCEE judges this to be undesirable and requires that for this type of building the accidental torques should be applied statically which is difficult for torsionally soft buildings to deal with

The NBCC 2005 proposals allow shifting the mass a reduced amount of 0.05D for "not soft" torsional buildings but applying the accidental eccentricity (0.1D) statically for "torsionally soft" buildings. Since the static approach is quite demanding it allows loads from either the static method or a static load case developed from dynamic analysis. This last approach was not in NBCC 1995.

5.5 Accidental Eccentricity

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5.5.1 Static Approach Based on “Static Analysis”

This approach entails moving the point of application of the “Static Method” loads at each level $\times 0.1D$

This approach is reasonable for buildings dominated by the first mode. **However**, it is quite conservative compared to dynamic approaches for resultant torques at any level and for edge displacements for buildings dominated by higher modes (tall, long period buildings). This is because the static envelope loads produce an appropriate shear in the building, whereas the higher mode forces are often acting in different directions and maximums are also not concurrent.

You may wish to start with this approach, and if it creates difficulties go to the next section.

5.5 Accidental Eccentricity

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5.5.2 Static Approach Based on “Dynamic Analysis”

- i. Use the “floor loads” given by the dynamic analysis (very poor)
These are the “maximums” produced at each floor (SRSS of the modal floor loads) and are not concurrent nor always in the same direction. When added up the sum exceeds the base shear

This approach is even more conservative than the static approach from the static analysis in 5.5.1

- ii. “Pseudo Dynamic” static approach.
Generate a force at each floor by taking the differences between the dynamic shear at each floor. Loads developed this way will regenerate the shear envelope.
Use this force distribution multiplied by $0.1D$ to calculate a floor torque load to apply to the structure.
This may be an improvement on the static approach in 5.5.1, but it may not be much of an improvement as the static force of the static method are **based** on generating a shear envelope that reflects dynamic analysis.

5.5 Accidental Eccentricity

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5.5.2 Static Approach Based on “Dynamic Analysis”

- iii. “Real dynamic” static approach
- The following assumes that the analysis program (ETABS is one) does not do this automatically.
 - Export the mode shapes, floor masses, periods, participation factors and spectrum to a spread sheet.
 - Use basic principals and use the spread sheet to combine the above and generate the **modal** loads for the appropriate direction of spectral excitation.
 - Check the modal base shears against the program. (Check!, Check!)
 - Apply the “scale factors” if required.
 - Calculated the accidental modal torques applied to each level for each mode shape, accounting for sign.
 - Apply to the structure and sum results for the mode shape “accidental” torques using CQC or SRSS

This is quite an accurate approach but is a lot of work!!

5.5 Accidental Eccentricity

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5.5.2 Static Approach Based on “Dynamic Analysis”

- iv. Pseudo “real dynamic” approach

Repeat the steps in (iii.) up to calculation of the modal accidental torques applied to each floor. Then use the spread sheet to continue on as follows:

- Sum each of the modal accidental floor torques down the building to get the torsional resultant for each mode at each floor.
- Do an SRSS of the modal resultant torsions at each floor to get one value of the accidental torsional resultant at each floor.
- Use this envelope of torsion to back figure a single floor torque load at each floor by subtracting the different torsional resultant values at adjacent floors.
- These “back figured” floor torques (one per floor) will generate the SRSS torsional resultant at any level when summed over the floors above.
- Use these “backfigured” floor torques as a single load case in the analysis program. This is much easier to deal with.

5.6 P – Delta Effect

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5.6.1 General Background

While this seminar is for Dynamic Response Spectrum Analysis, many of the topics discussed here apply to a regular “static” earthquake analysis.

The “P” force to be considered is the total gravity effect – not just the vertical load in any element being considered.

We think of P – Delta effects as “non linear”, and they are with P. However for a given value of P, the equations (both differential and matrix) are linear and super position holds for other load cases. This is quite helpful.

It is important to note that all the methods that will be presented are approximations to a very difficult problem – dynamic stability

5.6 P – Delta Effect

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There is a fundamental difference in behaviour between wind loads and earth quake loads.

- *Wind loads* – Increasing P increases displacements and forces, and the usual “P – Delta” analysis works fine.
- *Earthquake Loads* – increasing P does not increase the maximum displacements until a difficult to determine critical value is reached – and then the displacement blows up. This is illustrated by figures from Prof. Bernal at Northwestern University. See Fig 13. It may be these curves that give us the idea that the displacements do not need to be increased for P – Delta effects.

The Canadian code approach is based on work like this and others, such as Jim Montgomery, P.h.D, P.Eng, and work by Tom Paulay. See Fig 14 and 15.

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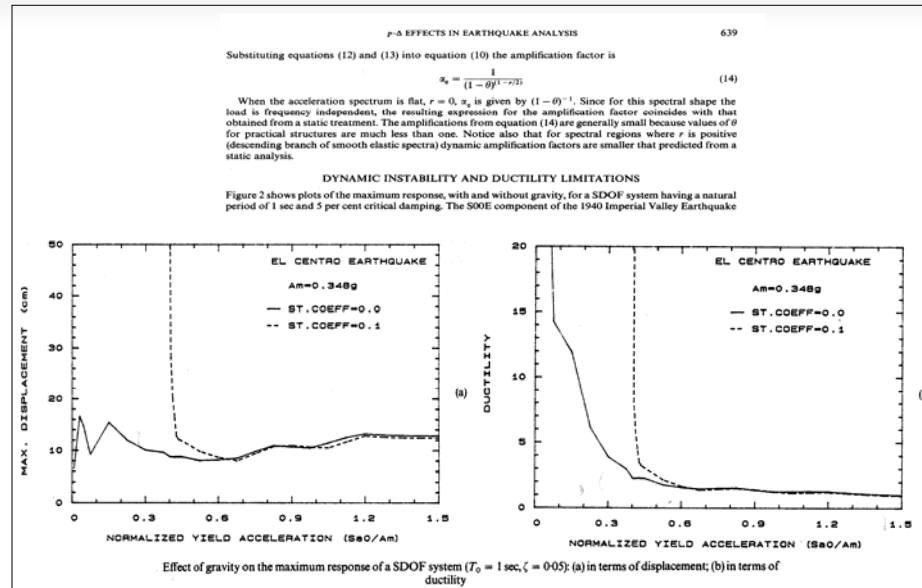


Fig 13

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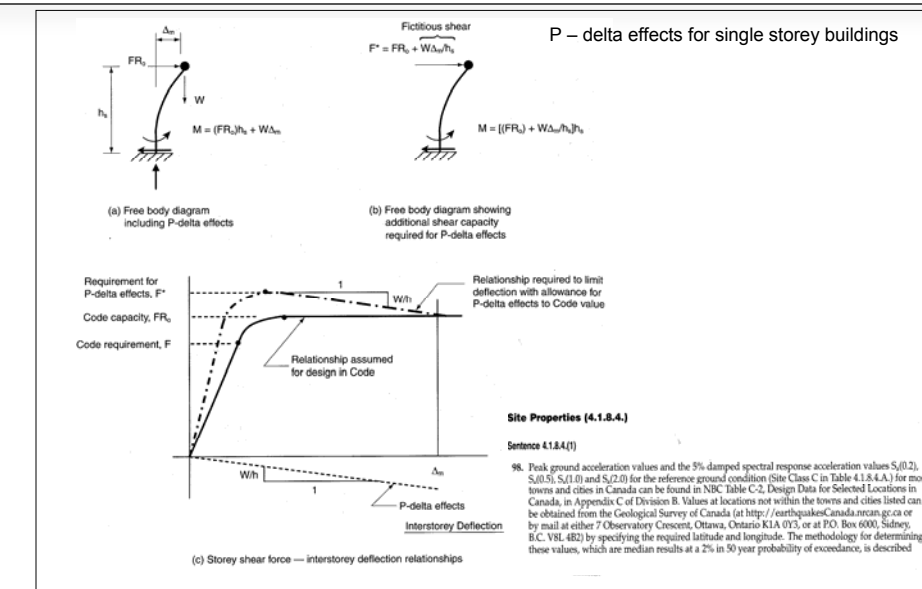


Fig 14

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Earthquake-induced forces, shears, overturning moments and torsional moments calculated at each storey level are to be multiplied by an amplification factor of $(1 + \theta_x)$ to allow for P-delta effects, where θ_x is a stability factor. The stability factor at level x is equal to

$$\theta_x = \frac{\sum_{i=x}^n W_i}{R_o \sum_{i=x}^n F_i h_i}$$

In the above expression

$\sum_{i=x}^n F_i$ is the seismic design shear force at the level under consideration, which is equal to the sum of the design lateral forces acting at and above the storey under consideration as determined in Sentence 4.1.8.11.(6).

$\sum_{i=x}^n W_i$ is that portion of the factored dead plus live load above the storey under consideration,

Δ_{max} is the maximum inelastic interstorey deflection as defined in Sentence 4.1.8.13.(2),

h_i is the interstorey height,

R_o is the overstrength-related force modification factor, and

$R_o \sum_{i=x}^n F_i$ is a measure of the capacity at the level under consideration.

The amplification factor of $(1 + \theta_x)$ need not be applied to displacements.

The procedure recommended to allow for P-delta effects is equivalent to proportioning the structure at each level x to resist an increased seismic shear force $\sum_{i=x}^n F_i'$ calculated from

$$\begin{aligned} \sum_{i=x}^n F_i' &= R_o \sum_{i=x}^n F_i + \sum_{i=x}^n W_i \frac{\Delta_{max}}{h_i} \\ &= R_o \sum_{i=x}^n F_i (1 + \theta_x) \end{aligned}$$

In calculating $\sum_{i=x}^n W_i$, the dead load factor and companion-load factors given in Load Case 5 of Table 4.1.3.2, should apply. The live load may be reduced for large tributary areas in accordance with Article 4.1.5.9. $\sum_{i=x}^n W_i$ is an estimate of the actual gravity load acting at the storey under consideration at the time of an earthquake.

With the seismic shear capacities at each storey increased to allow for P-delta effects, the ability of the strengthened structure to absorb inelastic energy during an earthquake is also increased. The interstorey deflections of the strengthened structure should be about the same as the deflections of the original structure with the P-delta effects taken to be zero.

If the stability factor, θ_x , calculated as described above is less than about 0.10, then P-delta effects can often be ignored. When the stability factor is more than 0.40, the structure should be redesigned to guard against potentially unstable buildings during extreme earthquakes.

Although the method described above is conservative in most cases, it cannot guard against the risk of dynamic instability when large inelastic deformations are expected, particularly when ductility demand is concentrated in a few storeys.

Modelling must take into account any other effects that might influence the lateral stiffness of the building, e.g. panel zone deformation in steel moment frames (Krawinkler, Bertero and Popov²⁷). Lateral stiffness is a particularly important parameter for two reasons:

- (i) the earthquake-induced load on the building is a direct function of the natural period, which itself is a direct function of lateral stiffness, and
- (ii) lateral stiffness is a major determinant of lateral displacement, which governs structural performance.

Fig 15

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This approach calculates a factor to apply to the design force values. It is based on the “expected, elastic” displacements and when the factor exceeds 1.4 the structure should be stiffened.

Note that while stiffness could be added, the approaches tend to “fix what is there” by adding strength instead. (note that often this results in an increase in stiffness as well.) The displacement is the same for both the P case and the P = 0.0 case.

Note also that the derivation of the equations for this approach tends to be frame based. It also addresses X and Y but no torsional effects. The strength increase required is based on the resistance including the “overstrength” effects. This philosophy holds true for the following discussions as well, i.e., the increase is added to the “actual” strength, not the factored strengths.

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5.6.2 Complications Introduced by Ductility

The code requirement is that the P – Delta effects are based on:

$$(\text{elastic forces}/R_d) + P \times \Delta_{\text{elastic}}$$

This means that it is not correct to do an elastic run with P-Delta effects and divide the results by $R_d R_o$ as this also divides the P-Delta component. The appropriate approach is more complicated.

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5.6.3 “Rough” Approach For Walls - No P-Delta in Program

- Check the periods, and make sure the torsional period is about the same or less than the X and Y periods.
- Use the principals of the NBCC commentary approach by:
 - i. Take the elastic displacement of the centroid of the weight and calculate the P-Delta base moment about X and Y ($M_{P\text{-Delta}}$)
 - ii. Calculate the base moment M_{R_d} using R_d with $R_o = 1.0$
 - iii. Calculate a factor “F” for X and Y
$$F = (M_{R_d} + M_{P\text{-Delta}})/M_{R_d}$$
If “F” is greater than 1.4, stiffen the structure.
 - iv. Apply to the program forces calculated using $R_d R_o$ for the respective direction of load.
 - v. Note this assumes any torsional effects will have multipliers less than or equal to the X and Y factors. The limit on the torsional period may make this so but it is an iffy assumption.

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5.5.4 Approach for walls – P-Delta in program -force multiplier approach, force increase approach

The following approach is based on values determined at the elastic response values. Because super position holds, spectrum values reduced for RdRo can also be used, with differences for P-Delta increased by RdRo

- i. Analyze the structure for RdRo = 1 and P = 0
- ii. Analyze the structure for RdRo = 1 and P = P
- iii. The difference in M, V, and Axial values is due to P-Delta effects at “elastic” displacements.

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- iv. These can:
 - Be reduced by dividing by Ro and adding directly to the design values to be resisted by factored resistances.
 - Or (an approximate shortcut) calculate the difference in moment/axial at the base for each element and determine a multiplier for each element

$$\text{Factor} = (M_{Rd} + M_{P\text{-Delta}})/M_{Rd}$$

This should capture torsion effects – but it also assumes the maximum factor is at the base. This may not be too bad an assumption for walls, but it should be confirmed.

- Or (an approximate shorter cut)
This is more-or-less the approach in the commentary
Perform the above calculation for the total base overturning moments and torque. Use the largest value of the calculated factor for all design values.

These “factors calculated at the base” may be a reasonable approximation for walls but may be poor for frames with a soft storey.

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5.6.5 P-Delta included in the program which also has a displacement multiplier in it.

This approach is handy because the effects at design force levels captures the P-Delta effect using increased displacements. This means all the design forces come out directly, including torsional effects if done properly.

However, a common way of doing this is to modify the stiffness matrix (for $R_d=4.0$ say) by terms such as $P \times 4\Delta$. Unfortunately this seems to be the same as $4P \times \Delta$. Etabs seems to do this, and this really softens up the building stiffness and increases the periods and displacements even as it may reduce the forces.

The question is – what building is being analyzed? Is this effect “correct”?

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Comment – suggest this is **not** correct and the structure should be brought back to the original ($P=P$) periods by tweaking the stiffness by various means such as stiffening elements or modifying E value.

If the change in periods associated with X, Y and Theta are about the same, then it is easy to change the E value to bring them all back to the original value.

However, if this cannot easily be done, then previous methods using factors can be used.

QUESTIONS ?????