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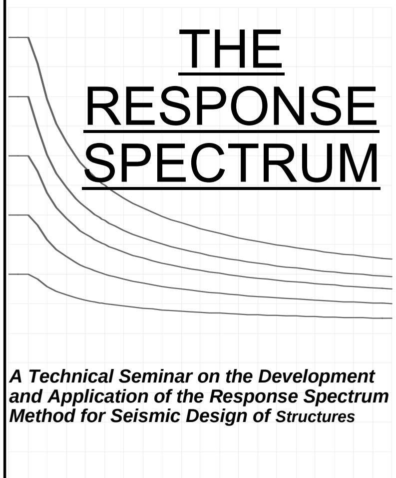

Application of Response Spectrum in Structural Engineering

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Topics to be covered:

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- Background
- Response Spectra Method of analysis
  - Mathematical background
  - Physical meaning of components
  - Examples
  - Damping
  - 3D analysis
- Summary

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Purpose of Analysis

- Predict, for a design earthquake, the force and deformation demands on the various components that compose the structure
- Permit evaluation of the acceptability of structural behavior (performance) through a series of **CAPACITY** vs **DEMAND** checks

NBCC 2005 Requirements:

4.1.8.7. Methods of Analysis

1) Analysis for design earthquake actions shall be carried out in accordance with the **Dynamic Analysis Procedure** as per **Article 4.1.8.12**. (see Appendix A), <u>except</u> that the Equivalent Static Force Procedure as per Article 4.1.8.11. may be used for structures that meet any of the following criteria:

a) for cases where $I_E F_a S_a(0.2)$ is less than 0.35,

b) regular structures that are less than 60 m in height and have a fundamental lateral period, T_a, less than 2 seconds in each of two orthogonal directions as defined in Article 4.1.8.8., or

c) structures with structural irregularity, Types 1, 2, 3, 4, 5, 6 or 8 as defined in Table 4.1.8.6. that are less than 20 m in height and have a fundamental lateral period, T_a, less than 0.5 seconds in each of two orthogonal directions as defined in Article 4.1.8.8.

4.1.8.12. Dynamic Analysis Procedures

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1) The Dynamic Analysis Procedure shall be in accordance with one of the following methods:
   a) Linear Dynamic Analysis by either the **Modal Response Spectrum Method** or the **Numerical Integration Linear Time History Method** using a structural model that complies with the requirements of **Sentence 4.1.8.3.(8)** (see Appendix A); or
   b) **Nonlinear Dynamic Analysis Method**, in which case a special study shall be performed (see Appendix A).
2) The **spectral acceleration values** used in the Modal Response Spectrum Method shall be the design spectral acceleration values S(T) defined in Sentence 4.1.8.4.(6)
3) The **ground motion histories** used in the Numerical Integration Linear Time History Method **shall be compatible with a response spectrum** constructed from the design spectral acceleration values S(T) defined in Sentence 4.1.8.4.(6) (see Appendix A).
4) The effects of **accidental torsional moments** ……

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What is the Response Spectrum Method (RSM)?

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The **Response Spectrum** is an estimation of maximum responses (i.e., acceleration, velocity and displacement) of a family of SDOF systems subjected to a prescribed ground motion.

The **RSM** utilizes the response spectrum to give the structural designer a set of *possible forces and deformations* a real structure would experience under earthquake loads.

For SDF systems, RSM gives quick and accurate peak response without the need for a time-history analysis.

For MDF systems, a true structural system, RSM gives a *reasonably* accurate peak response, again without the need for a full time-history analysis.

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The Modal Response Spectrum Method

What are the "ingredients"?

SEISMIC HAZARD

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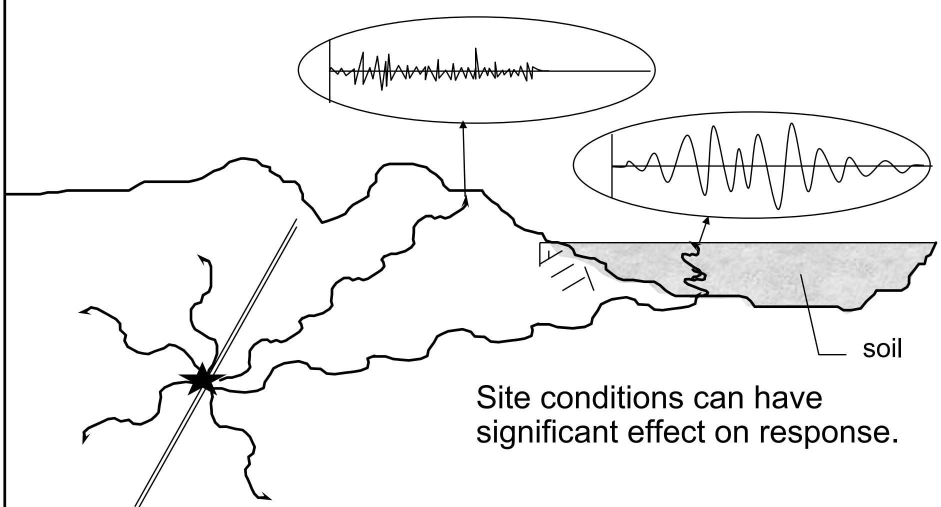
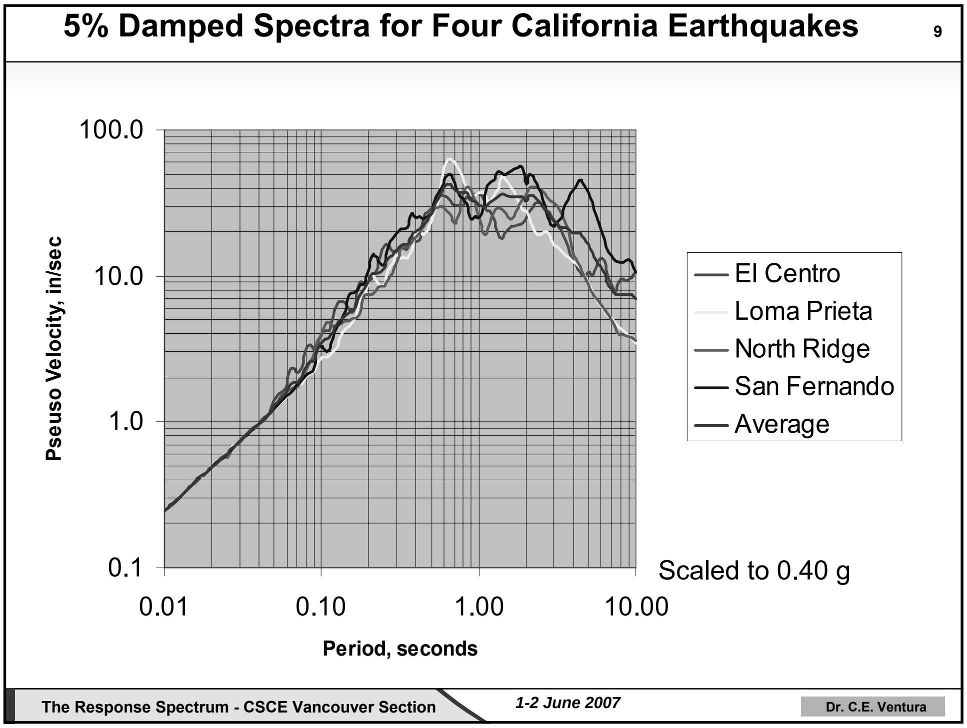


Site conditions can have significant effect on response.


5% Damped Spectra for Four California Earthquakes
9
100.0
10.0
1.0
0.1
0.01
0.10
1.00
10.00
Pseudo Velocity, in/sec
Period, seconds
El Centro
Loma Prieta
North Ridge
San Fernando
Average
Scaled to 0.40 g
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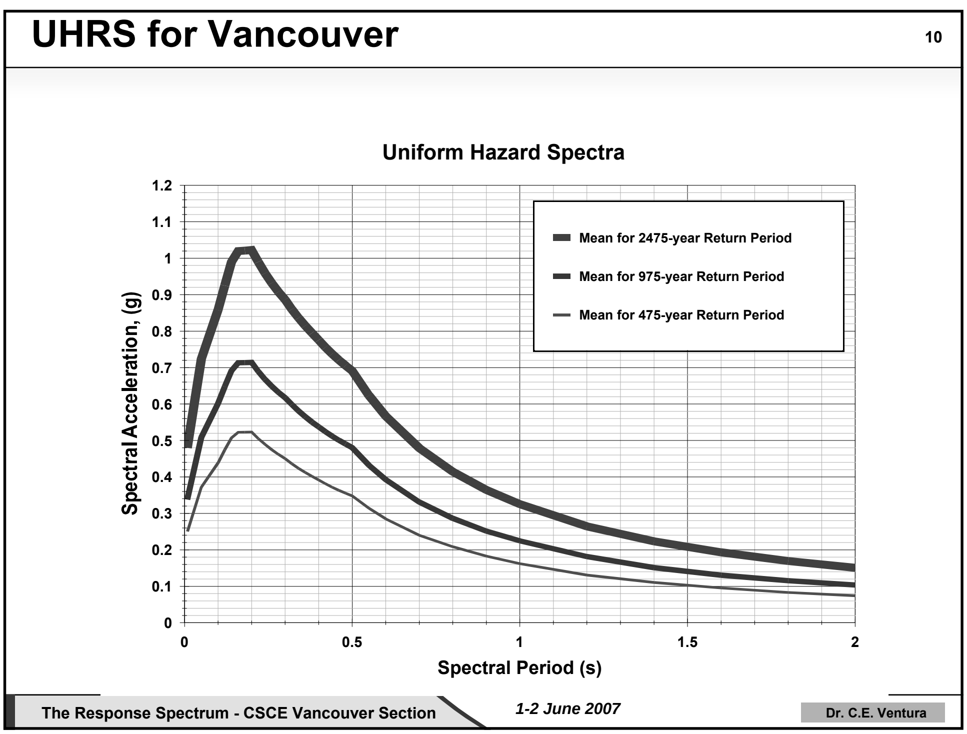
UHRS for Vancouver
10
Uniform Hazard Spectra
Spectral Acceleration, (g)
Spectral Period (s)
Mean for 2475-year Return Period
Mean for 975-year Return Period
Mean for 475-year Return Period
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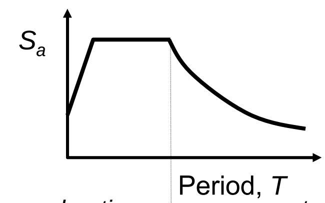
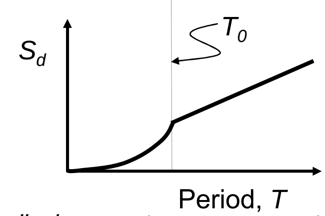
Design Spectra

- Acceleration response spectrum is shown to the right.
- Displacement response spectrum can be calculated assuming simple harmonic motion using expression below:

$$S_d = \frac{T^2}{4\pi^2} S_a g$$

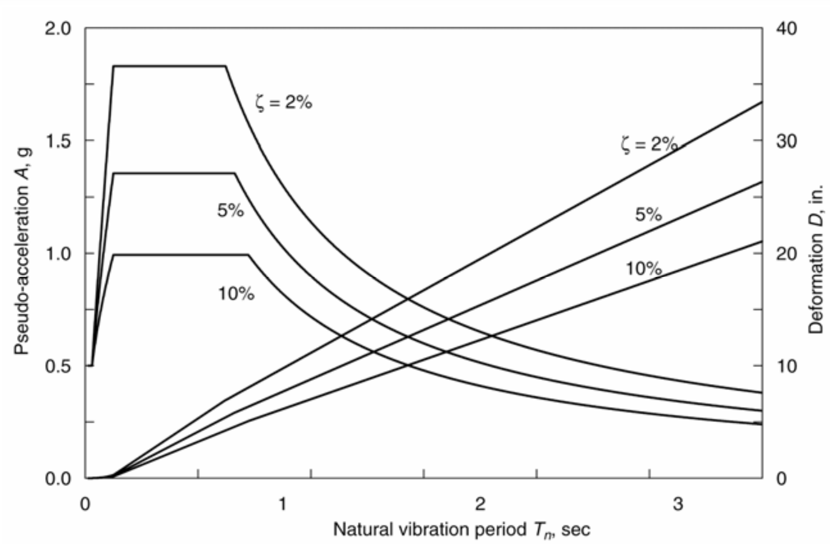


acceleration response spectrum



displacement response spectrum

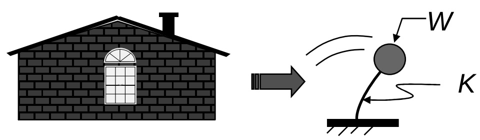
Elastic Design Spectrum



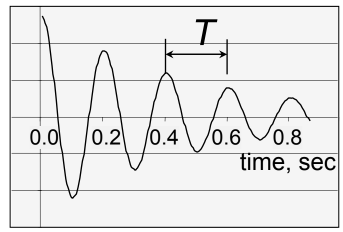
Linear Response of Structures

13

- Single-degree-of-freedom (**SDOF**) oscillators



Vibration Period $T = 2\pi\sqrt{\frac{W/g}{K}}$



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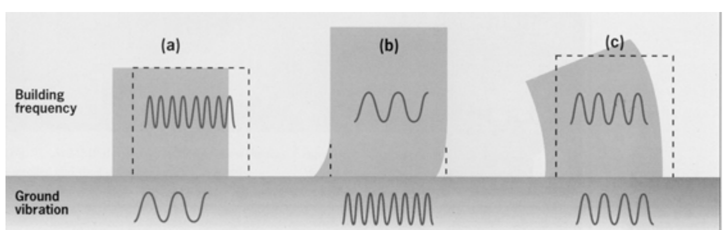
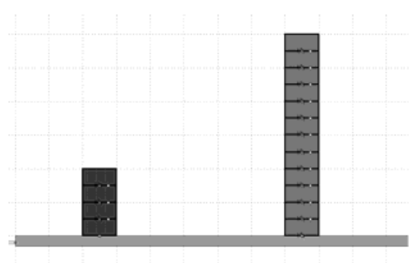
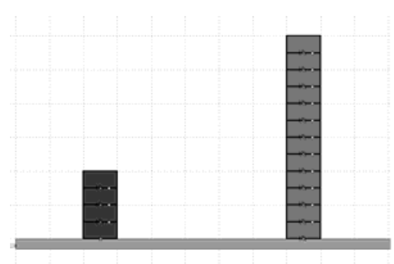
Ground shaking has different effects on buildings







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Multi-Story Structures

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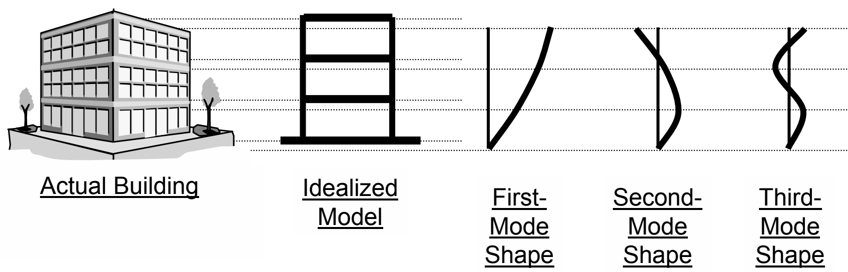
- Multi-story buildings can be idealized and analyzed as multi-degree-of-freedom (MDOF) systems.
- Linear response can be viewed in terms of individual modal responses.



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4.1.8.3.8 Structural Modelling

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Structural modelling shall be representative of the magnitude and spatial distribution of the **mass of the *building*** and **stiffness of all elements** of the SFRS, which includes stiff elements that are not separated in accordance with Sentence 4.1.8.3.(6), and shall account for:

a) the effect of the finite size of members and joints.

b) sway effects arising from the interaction of gravity loads with the displaced configuration of the structure, and

c) the effect of cracked sections in reinforced concrete and reinforced masonry elements.

d) other effects which influence the *buildings* lateral stiffness.

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Dynamic Equilibrium Equations – discrete systems 17

$$M\,a + Cv + Ku = F(t)$$

a = Node accelerations
v = Node velocities
u = Node displacements
M = Mass matrix
C = Damping matrix
K = Stiffness matrix
F(t) = Time-dependent forces

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A way to solve the equations of motion. 18

- This will be done by a transformation of coordinates from Normal Coordinates (displacements at the nodes) to Modal Coordinates (amplitudes of the natural Mode shapes).
- Because of the Orthogonality Property of the natural mode shapes, the equations of motion become uncoupled, allowing them to be solved as **SDOF** equations.
- After solving, we can transform back to the normal coordinates.

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Uncoupled Equations of Motion

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MDOF Equation of Motion: $M\ddot{u} + C\dot{u} + Ku = V(t)$

Transformation of Coordinates: $u = \Phi y$

Substitution: $M\Phi\ddot{y} + C\Phi\dot{y} + K\Phi y = V(t)$

Premultiply by Φ^T: $\Phi^T M\Phi\ddot{y} + \Phi^T C\Phi\dot{y} + \Phi^T K\Phi y = \Phi^T V(t)$

Using Orthogonality Conditions: Uncoupled Equations of Motion are:

$$\begin{bmatrix} m_1^* & & \\ & m_2^* & \\ & & m_3^* \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{Bmatrix} + \begin{bmatrix} c_1^* & & \\ & c_2^* & \\ & & c_3^* \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{Bmatrix} + \begin{bmatrix} k_1^* & & \\ & k_2^* & \\ & & k_3^* \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} V_1^*(t) \\ V_2^*(t) \\ V_3^*(t) \end{Bmatrix}$$

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Concept of Linear Combination of Mode Shapes

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(Transformation of Coordinates)

$$U = \Phi Y$$

$$U = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \begin{Bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{Bmatrix} \qquad U = \begin{Bmatrix} \phi_{11} \\ \phi_{21} \\ \phi_{31} \end{Bmatrix} Y_1 + \begin{Bmatrix} \phi_{12} \\ \phi_{22} \\ \phi_{33} \end{Bmatrix} Y_2 + \begin{Bmatrix} \phi_{13} \\ \phi_{23} \\ \phi_{33} \end{Bmatrix} Y_3$$

Mode Shape

Modal Coordinate = amplitude of mode shape

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Concept of Linear Combination of Mode Shapes

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$$\{u(t)\} = \sum_{n=1}^{N} \{\phi_n\} Y_n(t)$$

Mode shape

Response of SDOF

Modal Responses using Response Spectrum

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Maximum modal displacement $X_{n.Max} := \frac{L_n}{M_n} S_d(T_n, \xi_n) \cdot \phi_n$

Modal forces $F_{i.n.Max} := \frac{L_n}{M_n} S_a(T_n, \xi_n) \cdot \phi_n$

Modal base shears $V_{b.n.Max} := \frac{L_n^2}{M_n} S_a(T_n, \xi_n)\ \phi_n$

L_n, M_n, and φ_n are system parameters determined from the Modal Analysis Method.

$$\mathbf{L_n := \phi_n^T m \cdot 1} \qquad \mathbf{M_n := \phi_n^T m \cdot \phi_n}$$

$\mathbf{S_a(T_n, \xi_n)}$ System response from spectrum graph.

Modal Participation Factor, Γ_n

$$\Gamma_n = L_n/M_n$$
$$= \{\Phi_n\}^T[m]\{1\} / \{\Phi_n\}^T [m]\{\Phi_n\}$$

It is a measure of the contribution of each mode to the total Response of the system to the given type of excitation

It depends on how the mode shapes are scaled

Concept of Effective Modal Mass, m^*_n

$$m^*_n = L_n^2 / M_n$$
$$= \Gamma_n^2 \{\Phi_n\}^T [m]\{\Phi_n\}$$

- The sum of the effective modal mass for all modes is equal to the total structural mass.
- The value of effective modal mass is *independent* of mode shape scaling.

Practical value of Effective Modal Mass:

Use enough modes in the analysis to provide a total effective mass not less than 90% of the total structural mass (valid for base shear calculation only)

Effective modal masses and modal heights

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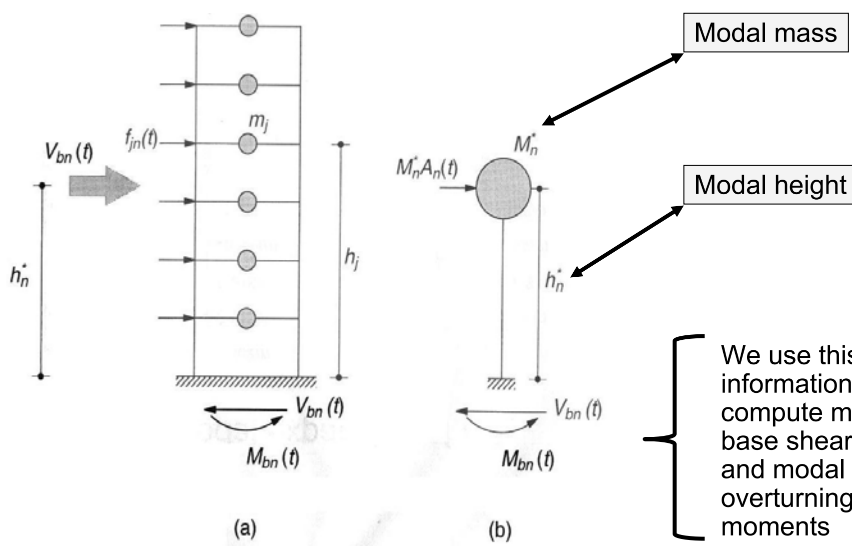

We use this information to compute modal base shears and modal base overturning moments

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Effective modal masses and modal heights

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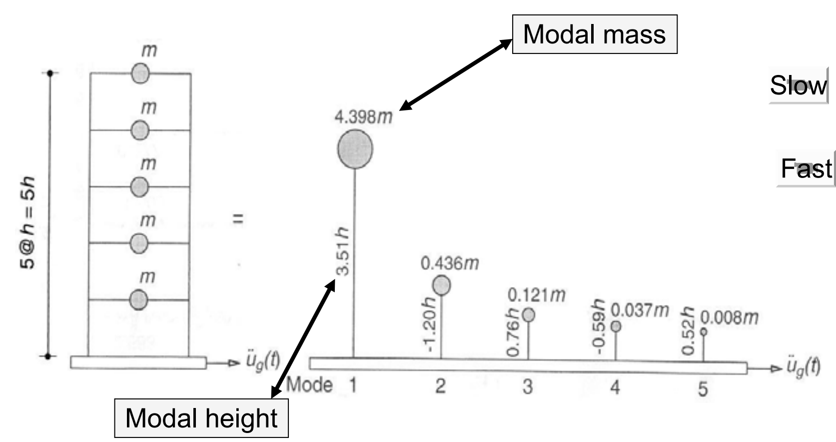

We can use this information to compute modal base shears and modal base overturning moments

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Response Spectrum Method steps

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Solution steps:

- Determine mass matrix, m
- Determine stiffness matrix, k
- Find the natural periods, $\mathbf{T_n}$ (or frequencies $\boldsymbol{\omega_n = 2\pi / T_n}$) and mode shapes $\boldsymbol{\varphi_n}$ of the system
- Compute peak response for the n^{th} mode, and repeat for all modes.
- Combine individual modal responses for quantities of interest (displacements, shears, moments, stresses, etc).

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Physical meaning of mode shapes

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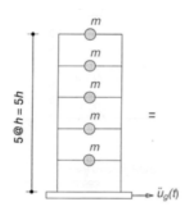
Regular building

Building with Podium

Base-Isolated building

Examine the behaviour of a simple 5 story building that represents these cases



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5 DOF with uniform mass and stiffness

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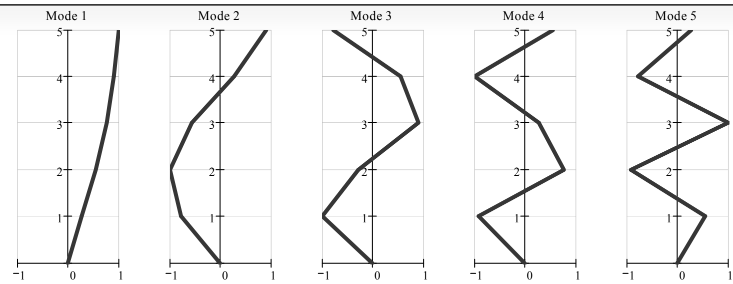

Modal Periods (in sec):

$T_1 = 2.001$ $T_2 = 0.685$ $T_3 = 0.435$ $T_4 = 0.338$ $T_5 = 0.297$

Modal Frequencies (in Hz):

$f_1 = 0.500$ $f_2 = 1.459$ $f_3 = 2.300$ $f_4 = 2.955$ $f_5 = 3.370$

Modal Masses (as a function of "m"):

$MM_1 = 4.398$ $MM_2 = 0.436$ $MM_3 = 0.121$ $MM_4 = 0.038$ $MM_5 = 0.008$

Modal Heights (as a function of "H"):

$MH_1 = 3.513$ $MH_2 = -1.204$ $MH_3 = 0.764$ $MH_4 = -0.594$ $MH_5 = 0.521$

Modal Participation Factors:

$\Gamma_1 = 1.252$ $\Gamma_2 = -0.394$ $\Gamma_3 = -0.208$ $\Gamma_4 = -0.116$ $\Gamma_5 = 0.053$

Total mass:

$\sum MM \cdot m = 1295.337$

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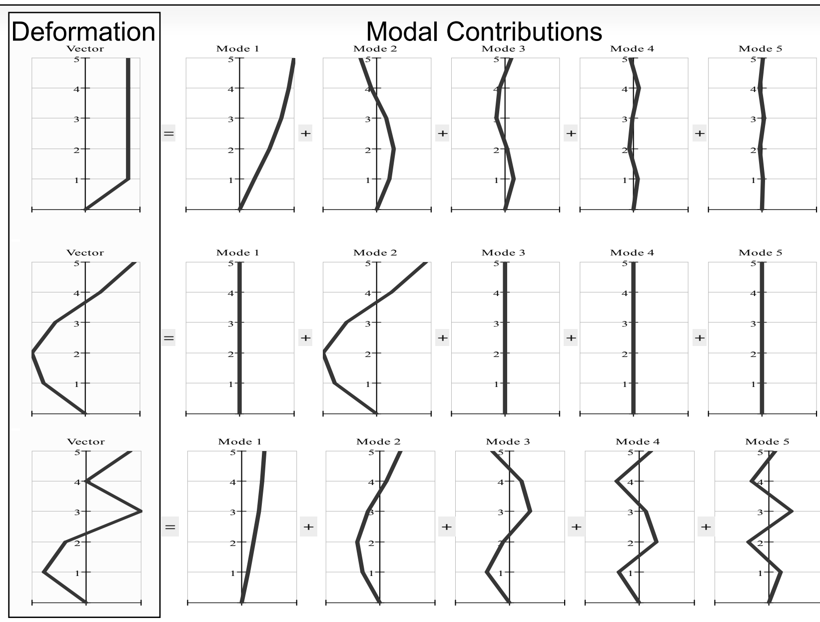
Modal Expansion (5 DOF with uniform mass and stiffness)

30



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Relative movement mode shapes

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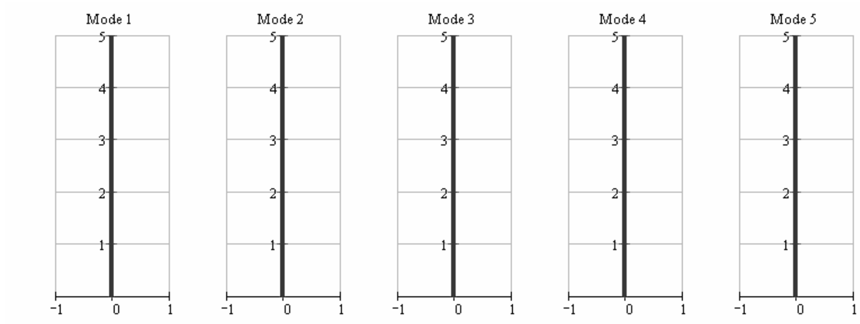
5 DOF with uniform mass and stiffness



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5 DOF – Base Isolated

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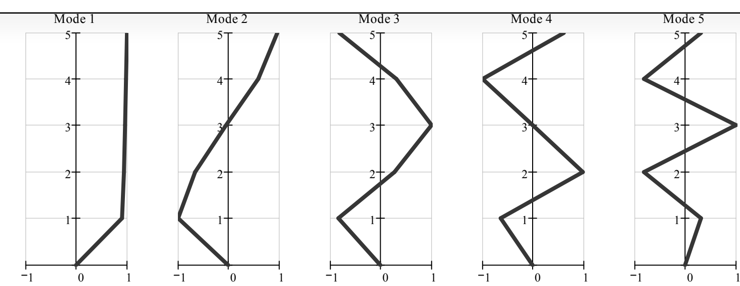

Modal Periods (in sec):

$T_1 = 5.866$ | $T_2 = 0.900$ | $T_3 = 0.482$ | $T_4 = 0.351$ | $T_5 = 0.299$

Modal Frequencies (in Hz):

$f_1 = 0.170$ | $f_2 = 1.111$ | $f_3 = 2.074$ | $f_4 = 2.845$ | $f_5 = 3.341$

Modal Masses (as a function of "m"):

$MM_1 = 4.994$ | $MM_2 = 0.006$ | $MM_3 = 0.000$ | $MM_4 = 0.000$ | $MM_5 = 0.000$

Modal Heights (as a function of "H"):

$MH_1 = 3.048$ | $MH_2 = -38.558$ | $MH_3 = 0.512$ | $MH_4 = -38.503$ | $MH_5 = 0.501$

Modal Participation Factors:

$\Gamma_1 = 1.037$ | $\Gamma_2 = -0.046$ | $\Gamma_3 = -0.012$ | $\Gamma_4 = -0.004$ | $\Gamma_5 = 0.002$

Total mass:

$\sum MM \cdot m = 1295.337$

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Relative movement mode shapes

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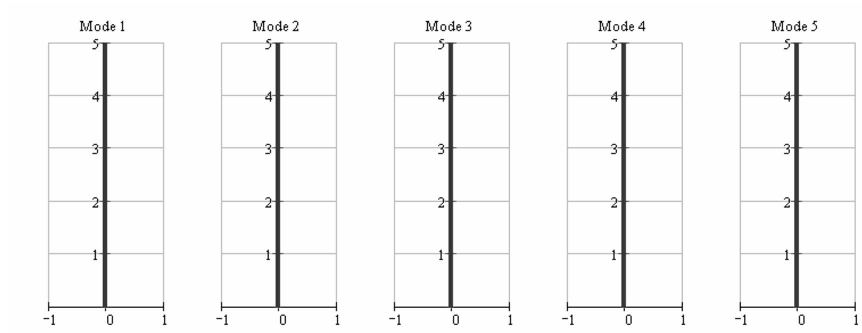
5 DOF – Base Isolated



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5 DOF – with Podium

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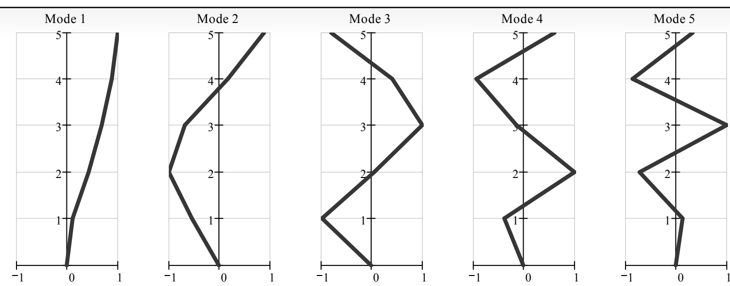

Modal Periods (in sec):

$T_1 = 2.001$ $T_2 = 0.724$ $T_3 = 0.523$ $T_4 = 0.407$ $T_5 = 0.341$

Modal Frequencies (in Hz):

$f_1 = 0.500$ $f_2 = 1.382$ $f_3 = 1.913$ $f_4 = 2.457$ $f_5 = 2.929$

Modal Masses (as a function of "m"):

$MM_1 = 3.316$ $MM_2 = 0.990$ $MM_3 = 0.634$ $MM_4 = 0.056$ $MM_5 = 0.004$

Modal Heights (as a function of "H"):

$MH_1 = 3.555$ $MH_2 = 0.102$ $MH_3 = 0.941$ $MH_4 = 0.114$ $MH_5 = 1.509$

Modal Participation Factors:

$\Gamma_1 = 1.327$ $\Gamma_2 = -0.658$ $\Gamma_3 = -0.441$ $\Gamma_4 = -0.169$ $\Gamma_5 = 0.046$

Total mass:

$\sum MM \cdot m = 1295.337$

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Relative movement mode shapes

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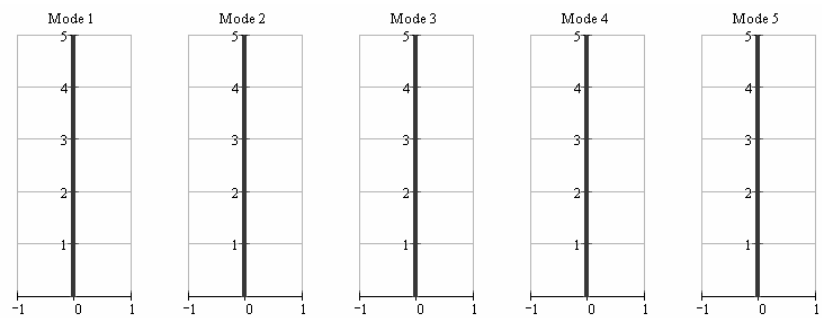
5 DOF – with Podium



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RSM – a sample calculations of a 5-storey shear-beam type building.

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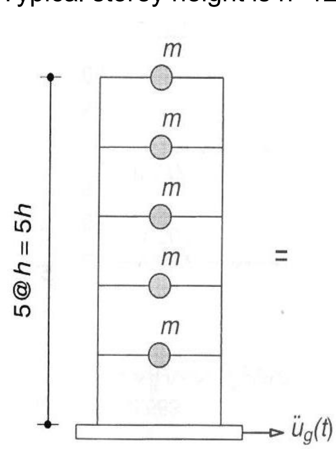
Typical storey height is h=12 ft.



(Example from Chopra's book)

$$\mathbf{m} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \textbf{100 kips/g}$$

$$\mathbf{k} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \times \textbf{31.54kip/in.}$$

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Natural vibration modes of a 5-storey shear building.

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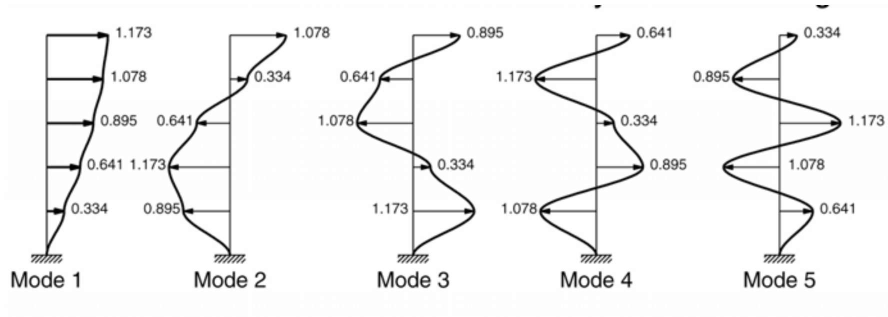
Mode shapes φ_n of the system:



T_1 = 2.01s T_2 = 0.68s T_3 = 0.42s T_4 = 0.34s T_5 = 0.29s

<u>Assume a damping ratio of 5% for all modes</u>

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Obtain values from Response Spectrum:

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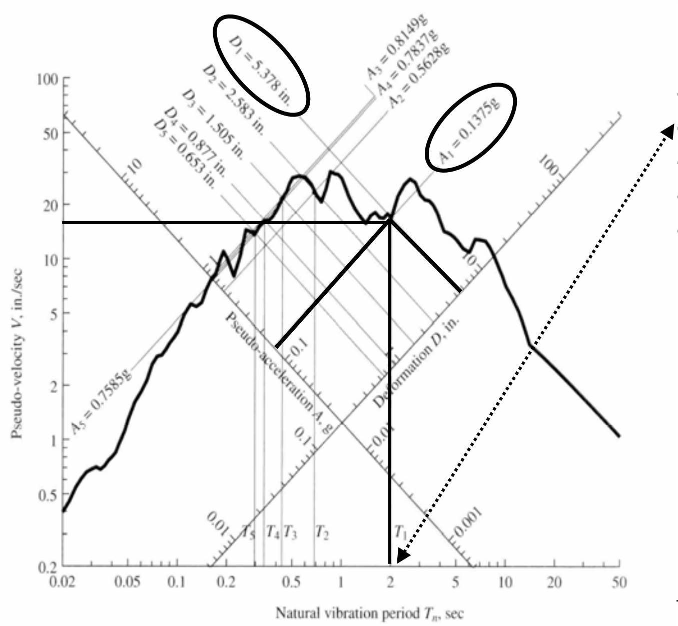

T_1 = 2.01s
T_2 = 0.68s
T_3 = 0.42s
T_4 = 0.34s
T_5 = 0.29s

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. . . Results:

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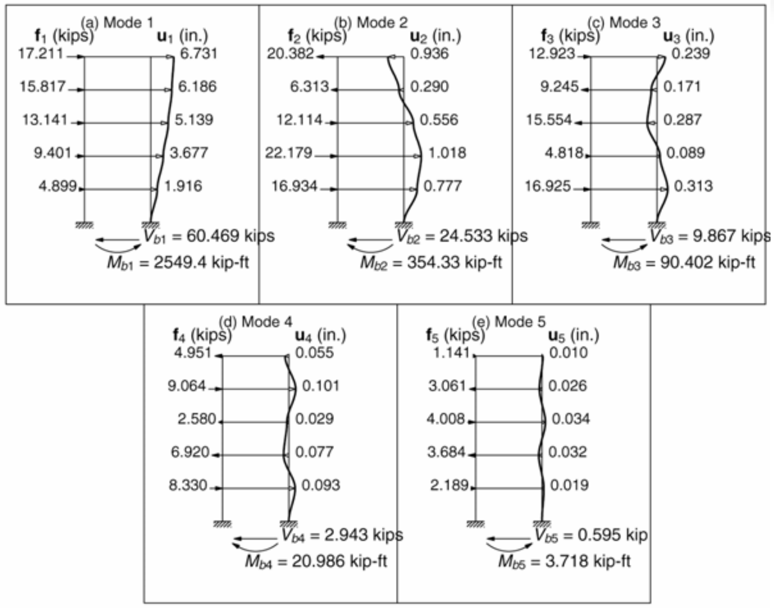

Modal Combinations to estimate peak response:

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- Modal maxima do not occur at the same time, in general.
- Any combination of modal maxima may lead to results that may be either conservative or unconservative.
- Accuracy of results depends on what modal combination technique is being used and on the dynamic properties of the system being analysed.
- Three of the most commonly used modal combination methods are:

Modal Combinations….

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a) Sum of the absolute values:

- leads to very conservative results
- assumes that maximum modal values occur at the same time
- response of any given degree of freedom of the system is estimated as

$$x_{i_{max}} \approx \sum_{n=1}^{L} \left| x_{i,n_{max}} \right|$$

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Modal Combinations…..

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b) Square root of the sum of the squares (SRSS or RMS):

- Assumes that the individual modal maxima are statistically independent.
- SRSS method generally leads to values that are closer to the "exact" ones than those obtained using the sum of the absolute values.
- Results can be conservative or unconservative.
- Results from an SRSS analysis can be significantly unconservative if modal periods are closely spaced.
- The response is estimated as:

$$x_{i_{max}} \approx \sqrt{\sum_{n=1}^{L} (x_{i,n_{max}})^2}$$

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Modal Combinations…. 43

c) Complete quadratic combination (CQC):

- The method is based on random vibration theory
- It has been incorporated in several commercial analysis programs
- A double summation is used to estimate maximum responses,

$$x_{i_{max}} \approx \sqrt{\sum_{n=1}^{L}\sum_{m=1}^{L} x_{i,n_{max}} \rho_{n,m} x_{i,m_{max}}}$$

ρ is a cross-modal coefficient (always positive), which for constant damping, z, is evaluated by

$$\rho_{n,m} = \frac{8z^2(1+r)r^{1.5}}{(1-r^2)+4z^2 r(1+r)^2}$$

Where $r = \rho_n / \rho_m$ and must be equal to or less than 1.0.

Similar equations can be applied for the computation of member forces, interstorey deformations, base shears and overturning moments.

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Modal combinations – for 5 DOF example: 44

Correct base shear is 73.278 kips (from time-history analysis)

ABSSUM: Summation of absolute values of individual modal responses

$V_b \leq \Sigma |V_{bn}|$ = 98.41 kips → grossly over-estimated

SRSS: Square root of sum of squares

$V_b = (\Sigma V_{bn}^2)^{1/2}$ = 66.07 kips
→good estimate if frequencies are spread out

CQC: $V_b = (\Sigma\Sigma V_{bi}\, \rho_{in} V_{bn})^{1/2}$ = 66.51 kips
→ good estimate if frequencies are closely spaced

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Damping

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$$M\,a + Cv + Ku = F(t)$$

Damping => Capacity of the system to dissipate energy

For mathematical simplicity we assume that damping is proportional to the relative velocity of the system (viscous damping), but….

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Damping – the big uncertainty!

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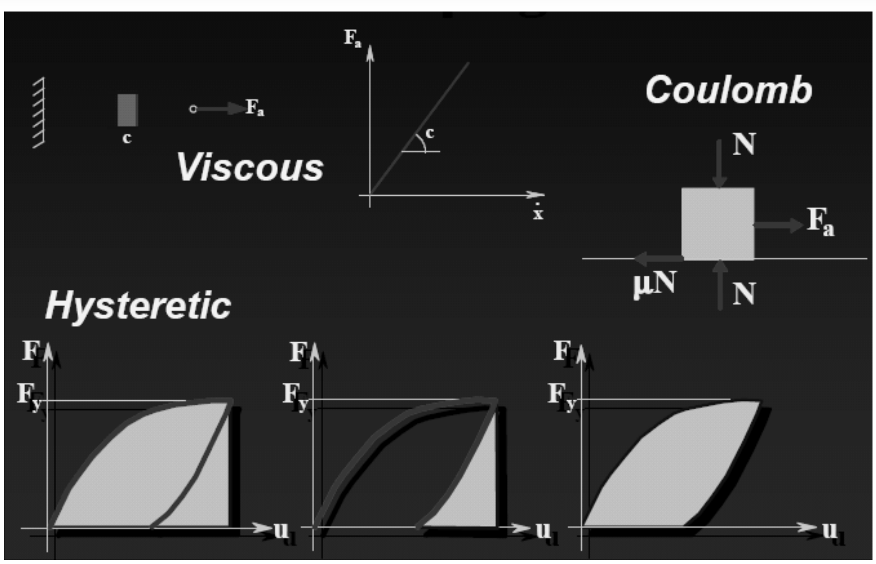

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Damping – the big uncertainty!

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We are oversimplifying a very complex problem, and the selection of appropriate viscous damping values carries a lot of uncertainty!

So, how do we deal with damping?

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Development of a Modal Damping Matrix

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In previous development, we have assumed:

$$\Phi^T C \Phi = \begin{bmatrix} c_1^* & & \\ & c_2^* & \\ & & c_3^* \end{bmatrix}$$

Two Methods Described Herein:

- Rayleigh "Proportional Damping"
- Wilson "Discrete Modal Damping"

That is, the mode shapes can uncouple the damping matrix

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Rayleigh Proportional Damping

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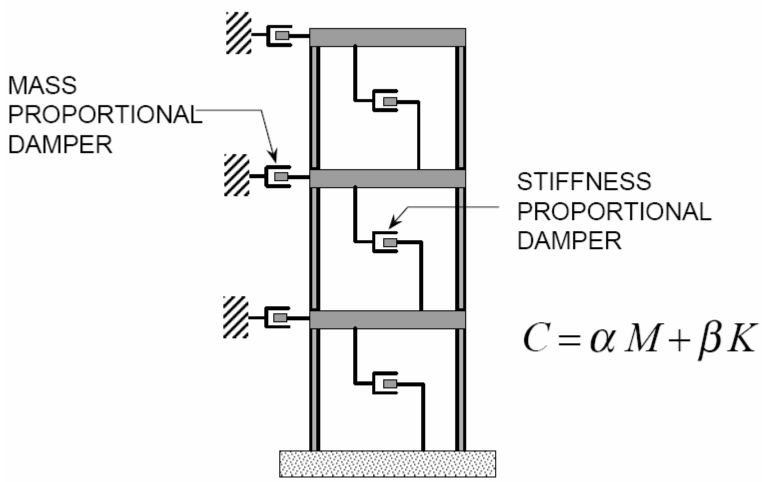

$$C = \alpha M + \beta K$$

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Rayleigh Proportional Damping

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Select Damping value in two modes, ξ_m and ξ_n

Compute Coefficients α and β:

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = 2\frac{\omega_m \omega_n}{\omega_n^2 - \omega_m^2} \begin{bmatrix} \omega_n & -\omega_m \\ -1/\omega_n & 1/\omega_m \end{bmatrix} \begin{Bmatrix} \xi_m \\ \xi_n \end{Bmatrix}$$

Form Damping Matrix $C = \alpha M + \beta K$

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Example of Rayleigh Proportional Damping

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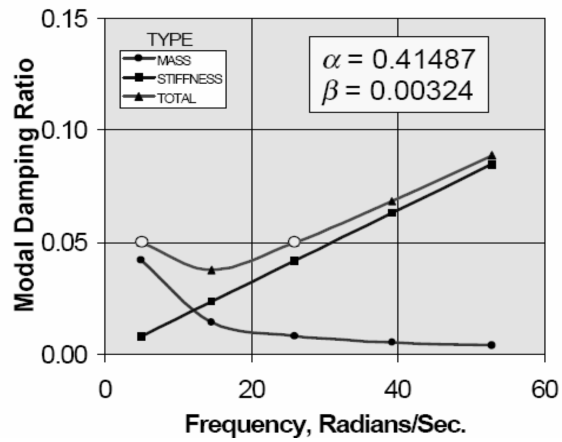
5% Critical in Modes 1 and 3

Structural Frequencies

Mode	ω
1	**4.94**
2	14.6
3	**25.9**
4	39.2
5	52.8



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Wilson Damping

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Directly Specify Modal Damping Values ξ_i^*

$$\Phi^T C \Phi = \begin{bmatrix} c_1^* & & \\ & c_2^* & \\ & & c_3^* \end{bmatrix} = \begin{bmatrix} 2m_1^*\omega_1\xi_1^* & & \\ & 2m_2^*\omega_2\xi_2^* & \\ & & 2m_3^*\omega_3\xi_3^* \end{bmatrix}$$

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Illustrative Example

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Analysis of 6-storey building

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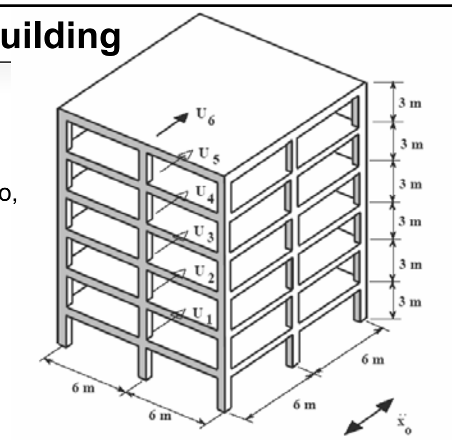
Study the response of the building to the N-S component of the recorded accelerations at El Centro, California, in May 18, 1940.



- All girders: witdh **b = 0.40 m,** depth **h = 0.50 m**.
- All columns have square section with a cross section dimension **h = 0.50 m**.
- Material of the structure has **E = 25 GPa**.
- The self weight of structure plus additional dead load is **780 kg/m²** and the industrial machinery, which is firmly connected to the building slabs, increases the mass per unit area by **1000 kg/m²**, for a total mass per unit area of **1780 kg/m²**.

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Dynamic properties:

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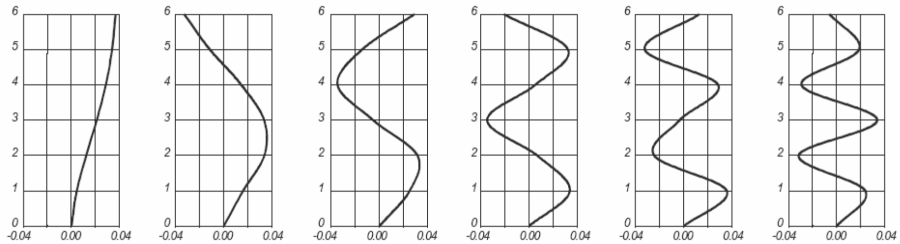

Mode 1 (T_1 = 1.16 s) Mode 2 (T_2 = 0.36 s) Mode 3 (T_3 = 0.20 s) Mode 4 (T_4 = 0.13 s) Mode 5 (T_5 = 0.092 s) Mode 6 (T_6 = 0.075 s)

The modal participation factors are:

$$\{\alpha\} = [\Phi]^T [M][\gamma] = \begin{Bmatrix} 34.970 \\ 13.540 \\ 8.2331 \\ 6.0279 \\ 4.4695 \\ 2.3861 \end{Bmatrix}$$

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The total effective mass is computed as:

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Mode	α_i	α_i^2	$\%M_{tot}$	$\%M_{tot}$ accumulated
1	34.970	1222.901	79.62%	79.62%
2	13.540	183.332	11.93%	91.55%
3	8.2331	67.784	4.41%	95.96%
4	6.0279	36.336	2.37%	98.33%
5	4.4695	19.976	1.30%	99.63%
6	2.3861	5.693	0.37%	100.00%

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Displacement Response Spectrum for El Centro Earthquake

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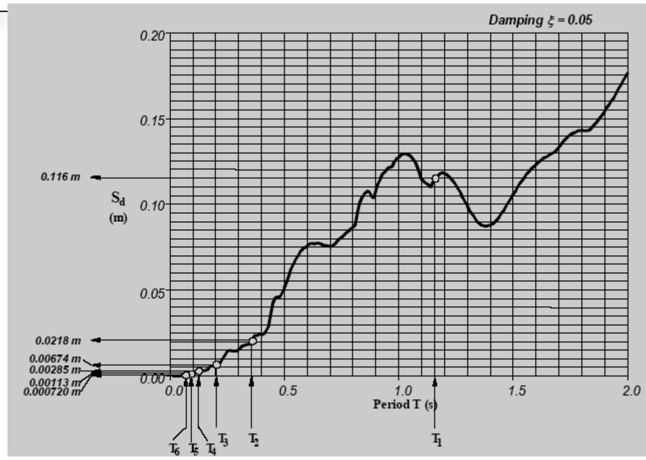

Mode	T_i (s)	$S_d(T_i,\xi_i)$ (m)
1	1.16	0.116
2	0.36	0.0218
3	0.20	0.00674
4	0.13	0.00285
5	0.092	0.00113
6	0.075	0.000720

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Maximum displacement values

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for the uncoupled degrees of freedom

Mode	α_i	$S_d(T_i,\xi_i)$ (m)	$(\eta_i)_{max} = \alpha_i \times S_d(T_i,\xi_i)$ (m)
1	34.970	0.116	4.0495
2	13.540	0.0218	0.29571
3	8.233	0.00674	0.055458
4	6.028	0.00285	0.017155
5	4.469	0.00113	0.0050639
6	2.386	0.000710	0.0017170

The maximum displacements for each mode are obtained from:

$$\left\{U^{(i)}_{mod}\right\} = \left\{\phi^{(i)}\right\}\left(\eta_i\right)_{max}$$

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Maximum lateral displacements for each mode

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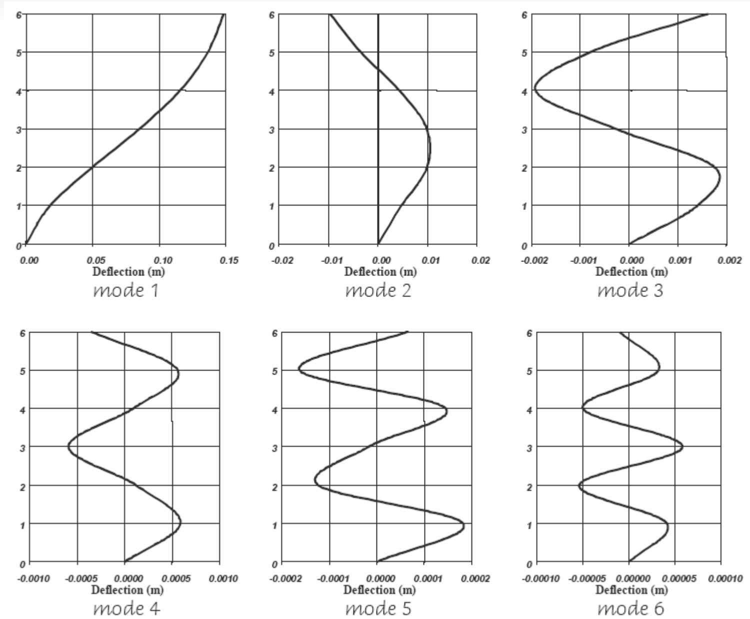

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Maximum story drift (%h) for each mode

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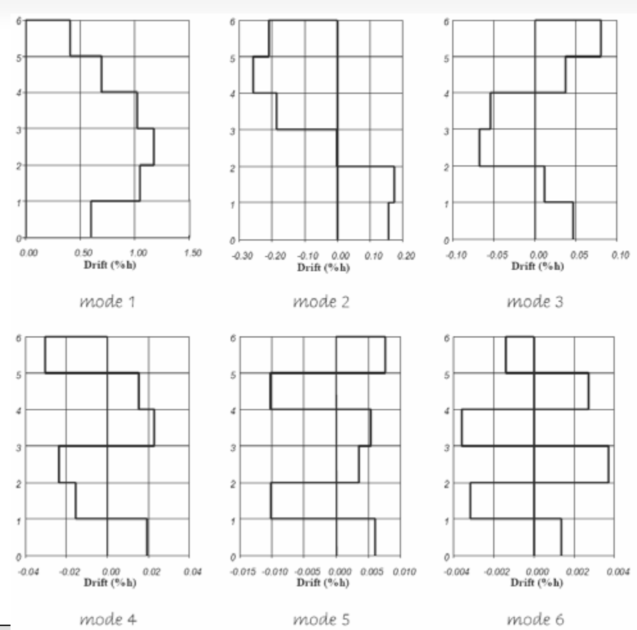

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Maximum modal forces for each mode

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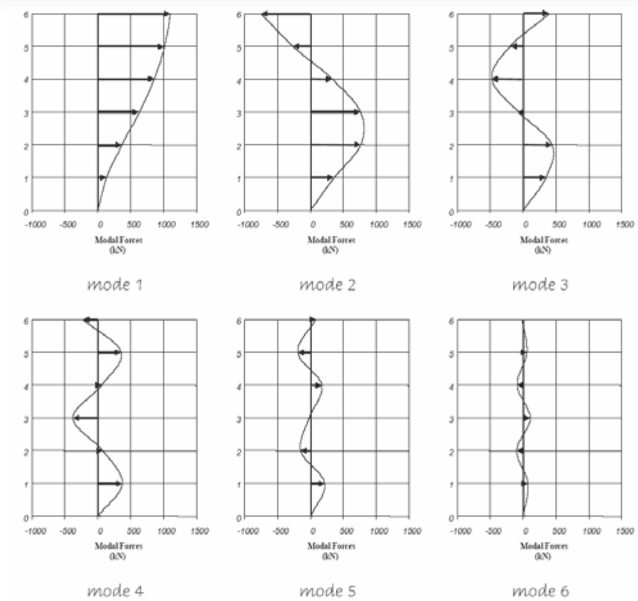
$$[F_{mod}] = [K_E][U_{mod}]$$



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Maximum story shear for each mode

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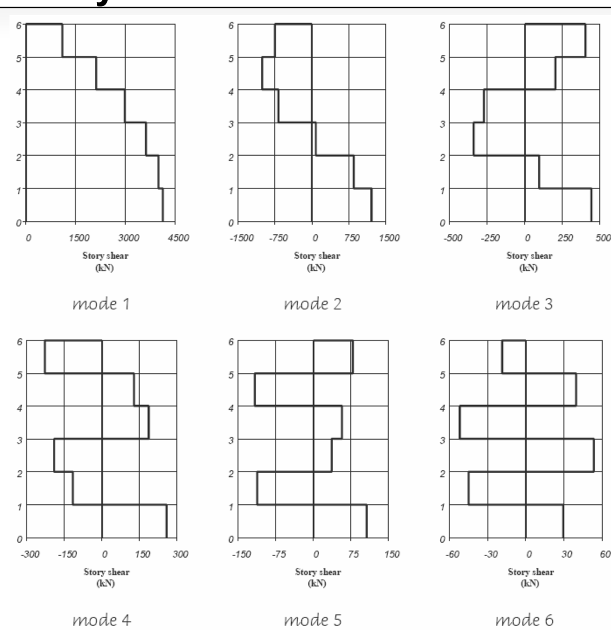

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Base shear (kN)

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$$\{V_{mod}\} = \{1\}^T [F_{mod}] = \{4122.1 \mid 1208.5 \mid 444.6 \mid 257.9 \mid 106.1 \mid 29.1\}$$

$V^{(1)}_{mod}$ $V^{(2)}_{mod}$ $V^{(3)}_{mod}$ $V^{(4)}_{mod}$ $V^{(5)}_{mod}$ $V^{(6)}_{mod}$

Overturning moment (kN · m)

$$M^{(i)}_j = \sum_{k=j+1}^{n} (h_k - h_j) \cdot F^{(i)}_j$$

Table 5 - Example 6 - Maximum story modal overturning moment

story	$M^{(1)}_{mod}$ (kN · m)	$M^{(2)}_{mod}$ (kN · m)	$M^{(3)}_{mod}$ (kN · m)	$M^{(4)}_{mod}$ (kN · m)	$M^{(5)}_{mod}$ (kN · m)	$M^{(6)}_{mod}$ (kN · m)
6	0.0	0.0	0.0	0.0	0.0	0.0
5	3324.9	-2246.7	1209.8	-679.2	237.8	-55.9
4	9698.6	-5287.8	1828.7	-290.9	-111.0	61.9
3	18652.9	-7333.6	1015.6	272.2	60.2	-93.3
2	29505.8	-7094.7	-6.9	-298.7	170.9	66.8
1	41466.8	-4558.2	282.4	-643.1	-162.9	-68.2
0	53833.1	-932.7	1616.3	130.7	155.3	19.2

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Overturning moment for each mode

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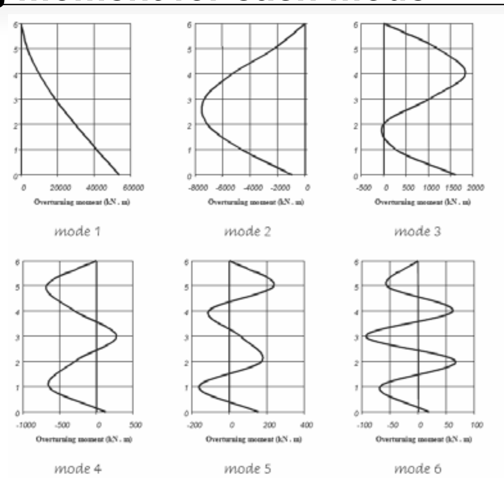

Maximum modal overturning moment at the base, in kN · m,

$$\{M_{mod}\} = \{h\}^T [F_{mod}] = \{53833 \mid -933 \mid 1616 \mid 131 \mid 155 \mid 19\}$$

$M^{(1)}_{mod}$ $M^{(2)}_{mod}$ $M^{(3)}_{mod}$ $M^{(4)}_{mod}$ $M^{(5)}_{mod}$ $M^{(6)}_{mod}$

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Maximum credible lateral displacements (m)

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$\{U^{(1)}_{mod}\}$	$\{U^{(2)}_{mod}\}$	$\{U^{(3)}_{mod}\}$	$\{U^{(4)}_{mod}\}$	$\{U^{(5)}_{mod}\}$	$\{U^{(6)}_{mod}\}$	↓ dof
0.148703	−0.009692	0.001618	−0.000355	0.000066	−0.000010	U_6
0.136429	−0.003428	−0.000790	0.000557	−0.000163	0.000032	U_5
0.115519	0.004295	−0.001915	0.000091	0.000144	−0.000050	U_4
0.084882	0.009854	−0.000280	−0.000592	−0.000017	0.000058	U_3
0.049588	0.009914	0.001754	0.000118	−0.000124	−0.000054	U_2
0.018061	0.004698	0.001397	0.000584	0.000181	0.000041	U_1

We now apply the SRSS procedure to each of the row of previous matrix. For example for the roof (6th story):

$$U_6^{max} = \sqrt{(0.148703)^2 + (-0.009692)^2 + (0.001618)^2 + (-0.000355)^2 + (0.000066)^2 + (-0.000010)^2} = 0.14903 \text{ m}$$

$$\{U^{srss}\} = \pm \begin{Bmatrix} 0.14903 \\ 0.13648 \\ 0.11560 \\ 0.08545 \\ 0.05059 \\ 0.01872 \end{Bmatrix} \begin{matrix} U_6 \\ U_5 \\ U_4 \\ U_3 \\ U_2 \\ U_1 \end{matrix} \quad \downarrow \text{dof}$$

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SRSS procedure for drift, storey shears

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$$\{\Delta^{srss}\} = \begin{Bmatrix} 0.0140 \\ 0.0223 \\ 0.0312 \\ 0.0354 \\ 0.0320 \\ 0.0188 \end{Bmatrix} m = \begin{Bmatrix} 0.47\%h \\ 0.74\%h \\ 1.04\%h \\ 1.18\%h \\ 1.07\%h \\ 0.62\%h \end{Bmatrix} \begin{matrix} 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{matrix} \quad \downarrow \text{story}$$

$$\{V^{srss}\} = \pm \begin{Bmatrix} 1417.6 \\ 2369.8 \\ 3080.3 \\ 3640.1 \\ 4080.2 \\ 4327.6 \end{Bmatrix} \begin{matrix} 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{matrix} \quad \downarrow \text{story}$$

Maximum credible base shear

$$V^{srss} = \sqrt{(4122.1)^2 + (1208.5)^2 + (444.6)^2 + (257.9)^2 + (106.1)^2 + (29.1)^2} = 4327.6 \text{ kN}$$

Maximum credible base overturning moment

$$M^{srss} = \sqrt{(53833.1)^2 + (-932.7)^2 + (1616.3)^2 + (130.7)^2 + (155.3)^2 + (19.2)^2} = 53865.8 \text{ kN} \cdot \text{m}$$

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Comparison of results

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Parameter	Example 5 Step-by-step Analysis	Example 6 Modal spectral Absolute value	Example 7 Modal spectral SRSS
Roof lateral displacement	0.149 m	0.160 m	0.149 m
Base shear	4 360 kN	6 170 kN	4 330 kN
Overturning moment	54 400 kN · m	56 700 kN · m	53 900 kN · m

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CAUTION!

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- Do **NOT** compute story shears from the story drifts derived from the SRSS of the story displacements.
- Calculate the story Shears in each mode (using modal drifts) and then SRSS the results.

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Weakness Of The Response Spectrum Methods

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{(1 - \frac{f_a}{F_{ex}}) F_{bx}} + \frac{C_{my} f_{by}}{(1 - \frac{f_a}{F_{ey}}) F_{by}} \leq 1.0$$

The Use Of The Maximum Peak Values Of f_a, f_{bx} and f_{by} Produces An Inconsistent Design

Axial Members Are Under Designed Compared To Bi-Axial Bending Members

SOLUTION ?

Use Design Checks As A Function Of Time

Dynamic Equilibrium Equations – 3D analysis

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$$M a + Cv + Ku = F(t)$$

F(t) = Time-dependent forces

$$= - M_x a_x - M_y a_y - M_z a_z$$

For 3D Earthquake Loading

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2-D versus 3-D Models

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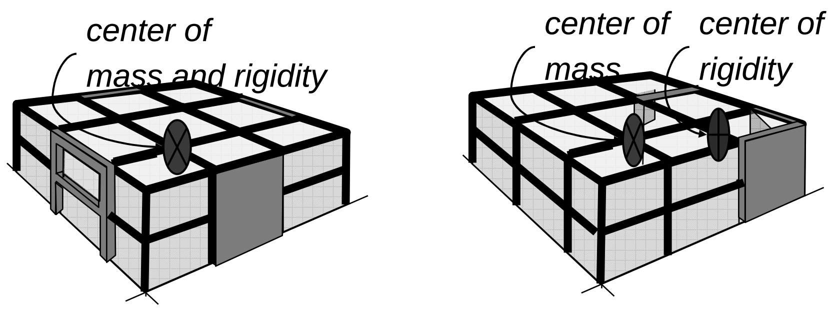
- 2D models adequate for structures with reasonably balanced mass and stiffness distributions.
- If center of mass and center of rigidity do not match, torsional response results, so 3D models are needed.



2-D Models OK   3-D Models Required

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Example of 3D behaviour of building

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- Constructed in 1993
- 32 – storey residential tower in Vancouver
- 3 levels of underground parking
- Concrete with central core shear walls

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AV testing of the City Tower building:

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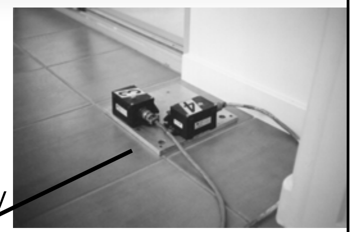
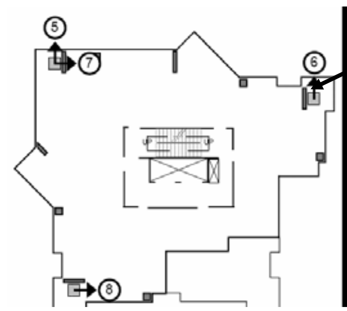
- Used Kinemetrics FBA-11 accelerometers
- Each record has 64K samples at 200 sps
- On-site data analysis
- All field work for each test conducted in one day





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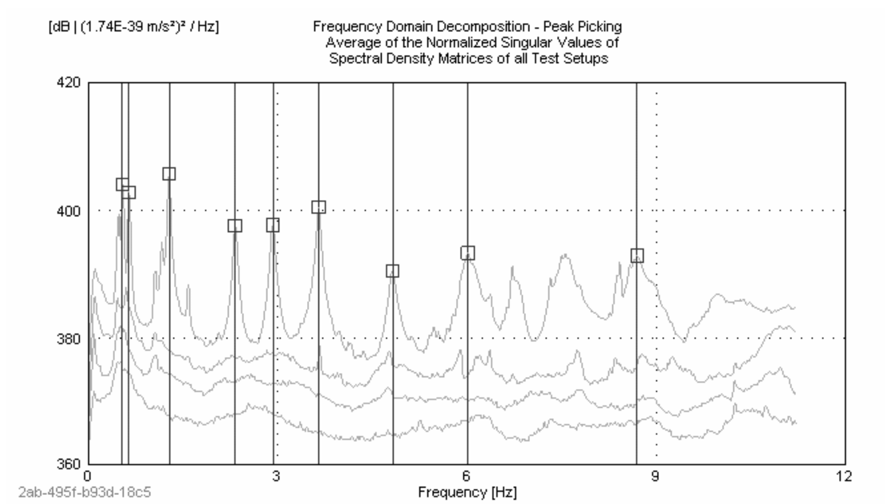
Results for CTC 32 – Spectral densities



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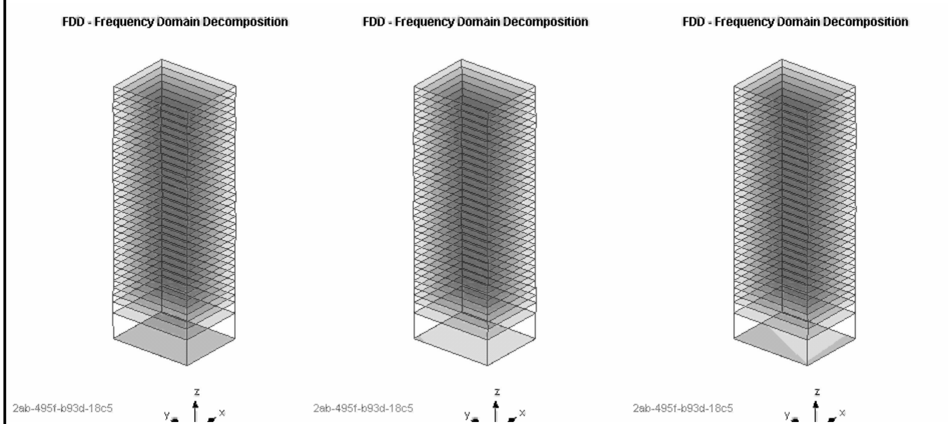
Mode Shapes for CTC 32



Mode 1	**Mode 2**	**Mode 3**
T = 1.83s	**T = 1.55s**	**T = 0.78s**

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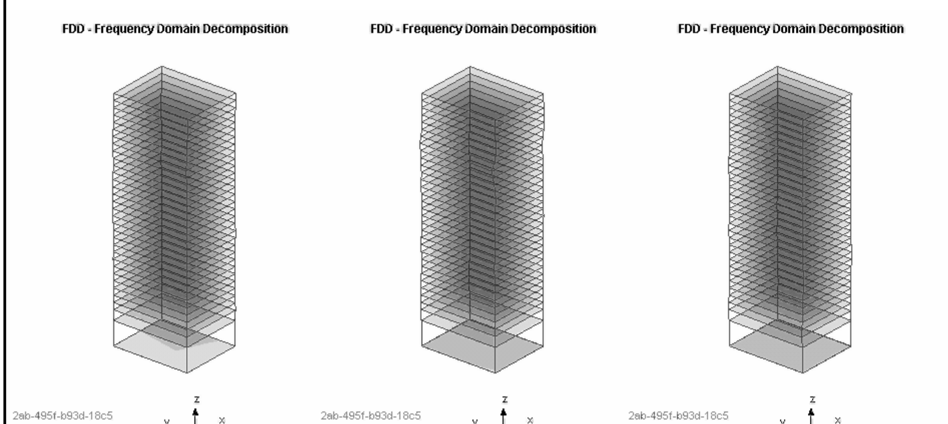
Mode Shapes for CTC 32



Mode 4	**Mode 5**	**Mode 6**
T = 0.43s	**T = 0.34s**	**T = 0.27s**

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THREE-DIMENSIONAL COMPUTER MODEL

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- Real and accidental torsional effects must be considered for all structures. Therefore, all structures must be treated as three-dimensional systems.
- Structures with irregular plans, vertical setbacks or soft stories will cause no additional problems if a realistic three-dimensional computer model is created.
- This model should be developed in the very early stages of design because it can be used for static wind and vertical loads, as well as dynamic seismic loads.
- Only structural elements with significant stiffness and ductility should be modeled. Non-structural brittle components can be neglected. However, shearing, axial deformations and non-center line dimensions can be considered in all members without a significant increase in computational effort by most modern computer programs.
- The rigid, in-plane approximation of floor systems has been shown to be acceptable for most buildings. For the purpose of elastic dynamic analysis, gross concrete sections are normally used, neglecting the stiffness of the steel. A cracked section mode should be used to check the final design.

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THREE-DIMENSIONAL COMPUTER MODEL

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The P-Delta effects should be included in all structural models. The effect of including P-Delta displacements in a dynamic analysis results in a small increase in the period of all modes. In addition to being more accurate, an additional advantage of automatically including P-Delta effects is that the moment magnification factor for all members can be taken as unity in all subsequent stress checks.

The mass of the structure can be estimated with a high degree of accuracy. The major assumption required is to estimate the amount of live load to be included as added mass. The lumped mass approximation has proven to be accurate. In the case of the rigid diaphragm approximation, the rotational mass moment of inertia must be calculated.

The stiffness of the foundation region of most structures can be modeled using massless structural elements. It is particularly important to model the stiffness of piles and the rotational stiffness at the base of shear walls.

The computer model for static loads only should be executed before conducting a dynamic analysis. Equilibrium can be checked and various modeling approximations can be verified using simple static load patterns. The results of a dynamic analysis are generally very complex and the forces obtained from a response spectra analysis are always positive. Therefore, dynamic equilibrium is almost impossible to check. However, it is relatively simple to check energy balances in both linear and nonlinear analysis.

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3D models:

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The current code defines an “irregular structure” as one that has a certain geometric shape or in which stiffness and mass discontinuities exist. A far more rational definition is that a “regular structure” is one in which there is a minimum coupling between the lateral displacements and the torsional rotations for the mode shapes associated with the lower frequencies of the system. Therefore, if the model is modified and “tuned” by studying the three-dimensional mode shapes during the preliminary design phase, it may be possible to convert a “geometrically irregular” structure to a “dynamically regular” structure from an earthquake-resistant design standpoint.

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Examples of 3D analysis

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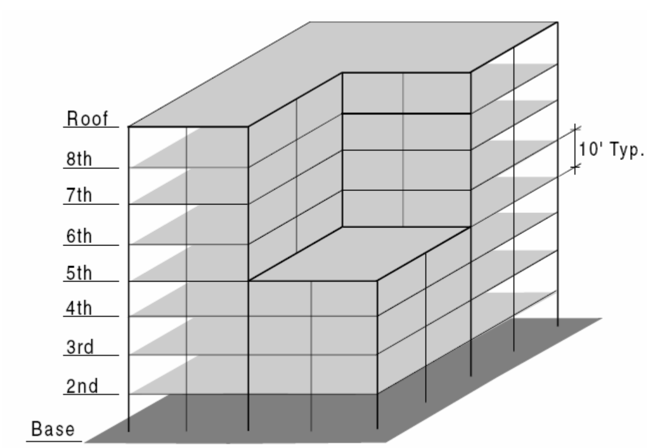

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Three Dimensional Base Forces and Moments

MODE	PERIOD	MODAL BASE SHEAR REACTIONS			MODAL OVERTURNING MOMENTS		
	Seconds	X-DIR	Y-DIR	Angle Deg.	X-AXIS	Y-AXIS	Z-AXIS
1	.6315	.781	.624	38.64	-37.3	46.6	-18.9
2	.6034	-.624	.781	-51.37	-46.3	-37.0	38.3
3	**.3501**	**.785**	**.620**	**38.30**	**-31.9**	**40.2**	**85.6**
4	.1144	-.753	-.658	41.12	12.0	-13.7	7.2
5	.1135	.657	-.754	-48.89	13.6	11.9	-38.7
6	**.0706**	**.989**	**.147**	**8.43**	**-33.5**	**51.9**	**2438.3**
7	.0394	-.191	.982	-79.01	-10.4	-2.0	29.4
8	.0394	-.983	-.185	10.67	1.9	-10.4	26.9
9	**.0242**	**.848**	**.530**	**32.01**	**-5.6**	**8.5**	**277.9**
10	.0210	.739	.673	42.32	-5.3	5.8	-3.8
11	.0209	.672	-.740	-47.76	5.8	5.2	-39.0
12	**.0130**	**-.579**	**.815**	**-54.63**	**-.8**	**-8.8**	**-1391.9**
13	.0122	.683	.730	46.89	-4.4	4.1	-6.1
14	.0122	.730	-.683	-43.10	4.1	4.4	-40.2
15	.0087	-.132	-.991	82.40	5.2	-.7	-22.8
16	.0087	-.991	.135	-7.76	-.7	-5.2	30.8
17	**.0074**	**-.724**	**-.690**	**43.64**	**4.0**	**-4.2**	**-252.4**
18	.0063	-.745	-.667	41.86	3.1	-3.5	7.8
19	.0062	-.667	.745	-48.14	-3.5	-3.1	38.5
20	.0056	-.776	-.630	39.09	2.8	-3.4	54.1
21	.0055	-.630	.777	-50.96	-3.4	-2.8	38.6
22	.0052	.776	.631	39.15	-2.9	3.5	66.9
23	**.0038**	**-.766**	**-.643**	**40.02**	**3.0**	**-3.6**	**-323.4**
24	**.0034**	**-.771**	**-.637**	**39.58**	**2.9**	**-3.5**	**-436.7**

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Three Dimensional Participating Mass - (percent)

MODE	X-DIR	Y-DIR	Z-DIR	X-SUM	Y-SUM	Z-SUM
1	34.224	21.875	.000	34.224	21.875	.000
2	23.126	36.212	.000	57.350	58.087	.000
3	2.003	1.249	.000	59.354	59.336	.000
4	13.106	9.987	.000	72.460	69.323	.000
5	9.974	13.102	.000	82.434	82.425	.000
6	.002	.000	.000	82.436	82.425	.000
7	.293	17.770	.000	82.729	90.194	.000
8	7.726	.274	.000	90.455	90.469	.000
9	.039	.015	.000	90.494	90.484	.000
10	2.382	1.974	.000	92.876	92.458	.000
11	1.955	2.370	.000	94.831	94.828	.000
12	.000	.001	.000	94.831	94.829	.000
13	1.113	1.271	.000	95.945	96.100	.000
14	1.276	1.117	.000	97.220	97.217	.000
15	.028	1.556	.000	97.248	98.773	.000
16	1.555	.029	.000	98.803	98.802	.000
17	.011	.010	.000	98.814	98.812	.000
18	.503	.403	.000	99.316	99.215	.000
19	.405	.505	.000	99.722	99.720	.000
20	.102	.067	.000	99.824	99.787	.000
21	.111	.169	.000	99.935	99.957	.000
22	.062	.041	.000	99.997	99.998	.000
23	.003	.002	.000	100.000	100.000	.000
24	.001	.000	.000	100.000	100.000	.000

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Simple 3D building

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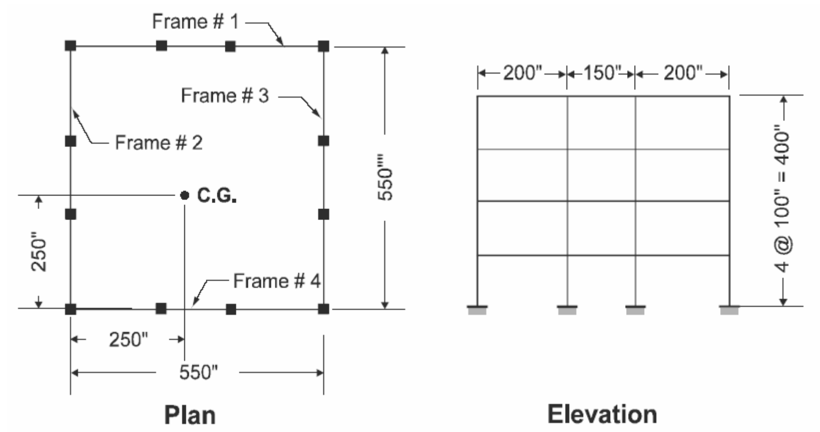

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Base Shears in Each Frame for First Five Modes

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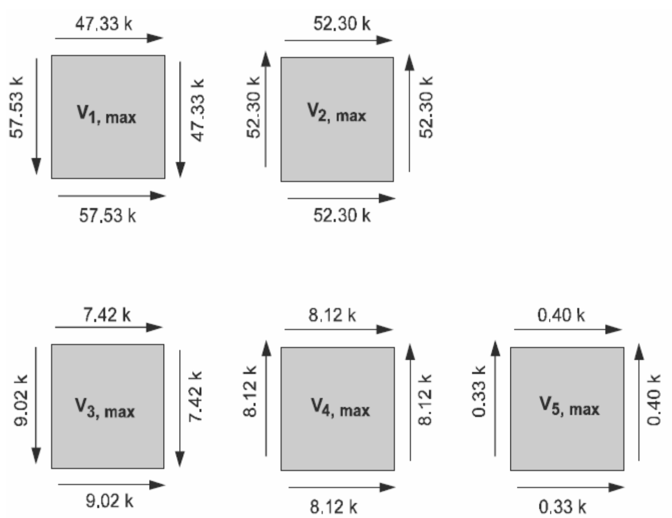

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Comparison of Modal Combination Methods

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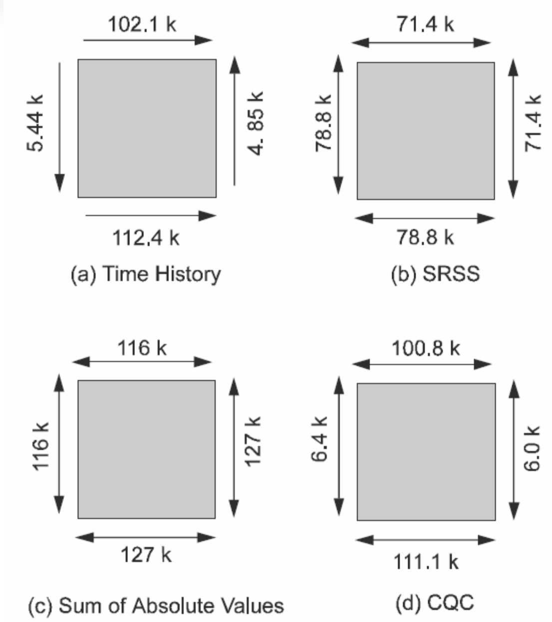

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RSM – Summary

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Response spectrum method offers a standardized solution to evaluate structures according to the National Building Code of Canada. The method is simple, straightforward, yet powerful that the designer can assess his design in a timely and efficient manner – ie. back of envelope calculations.

With the use of powerful computer hardware and computer modeling software available today, RSM offers a way for a designer to quickly verify and understand the sometimes non-intuitive results obtained from those sophisticated tools.

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References & Notes

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Some of the slides included here were kindly provided by Dr. Mete Zosen and Dr. Luis Garcia of Purdue University

Notice

While the instructors have tried to be as accurate as possible, they cannot be held responsible for the designs of others that might be based on the material presented in this course and these notes. The material taught at this course is intended for the use of professional personnel competent to evaluate the significance and limitations of its contents and recommendations, and who will accept the responsibility for its application. The instructors and the sponsoring organizations disclaim any and all responsibility for the applications of the stated principles and for the accuracy of any of the material taught at the course and contained in these notes.

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