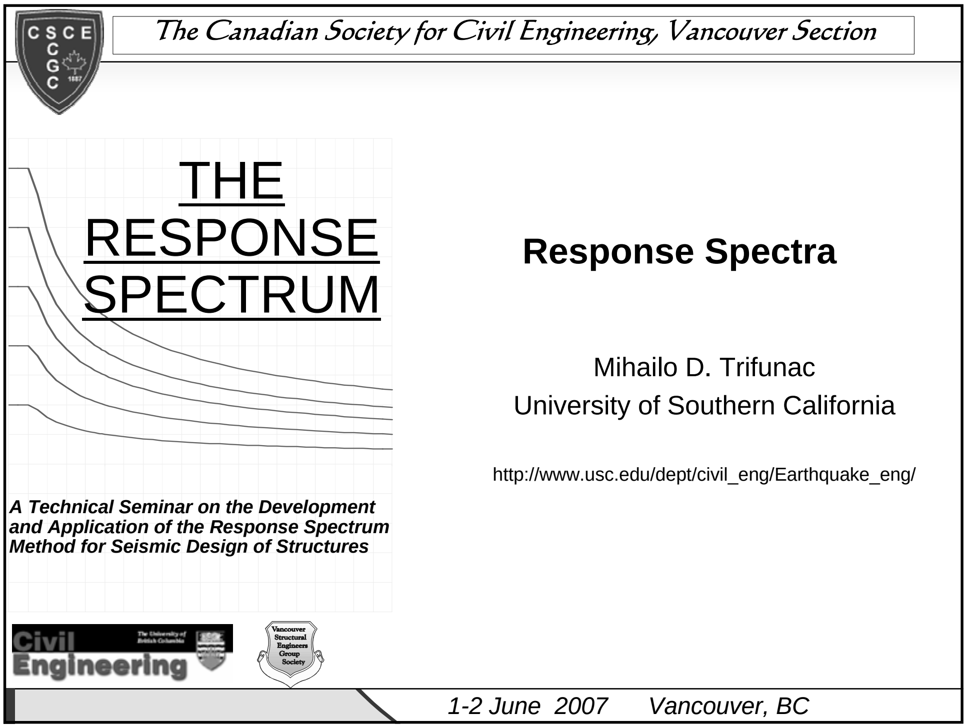

The Canadian Society for Civil Engineering, Vancouver Section

THE RESPONSE SPECTRUM

A Technical Seminar on the Development and Application of the Response Spectrum Method for Seismic Design of Structures

Response Spectra

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http://www.usc.edu/dept/civil_eng/Earthquake_eng/

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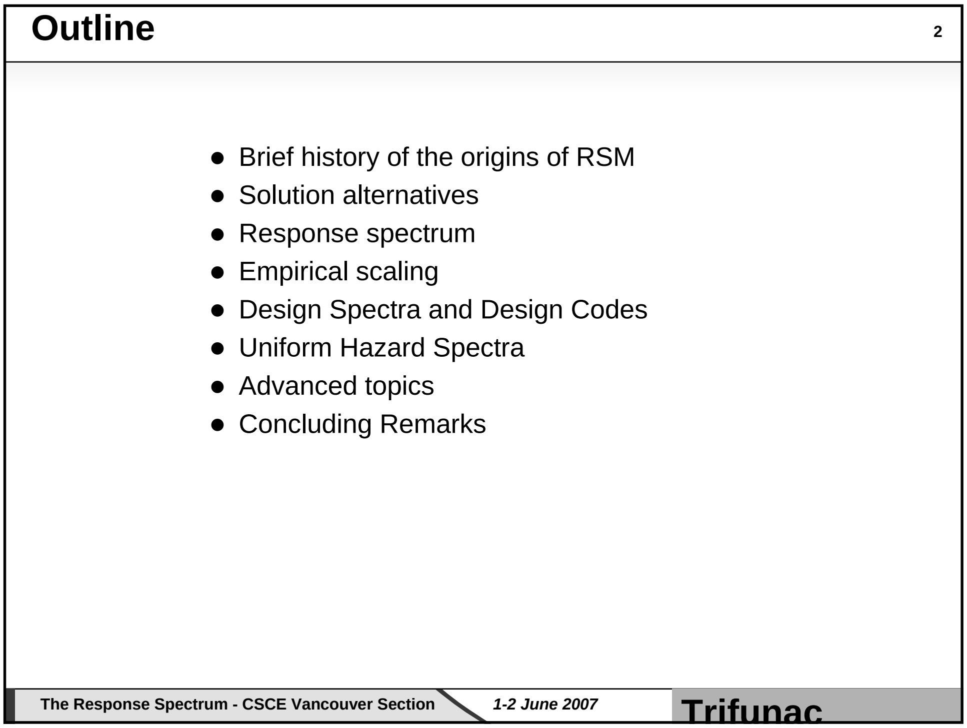


Outline

- Brief history of the origins of RSM
- Solution alternatives
- Response spectrum
- Empirical scaling
- Design Spectra and Design Codes
- Uniform Hazard Spectra
- Advanced topics
- Concluding Remarks

Brief History

The first practical steps, which initiated the engineering work on the design of earthquake resistant structures, accompanied the introduction of the *seismic coefficient* (*shindo* in Japan, and *rapporto sismico* in Italy, for example), and started to appear following the destructive earthquakes in San Francisco, California, in 1906, Messina-Reggio, Italy, in 1908 , and Tokyo, Japan, in 1923. The first seismic design code was introduced in Japan in 1924 following the 1923 earthquake. In California the work on the code development started in 1920s, but it was not after the Long Beach earthquake in 1933 that the Field Act was finally adopted in 1934 .

A static load, typically equal to 5 to 10 percent of the building weight, was applied horizontally, to simulate earthquake action. No dynamic analysis was required.

Brief History

In early 1900s, at most universities, engineering curricula did not include advanced mathematics and mechanics, both essential for teaching analysis of the dynamic response of structures. This lack in theoretical preparation is reflected in the view of C. Derleth , civil engineering professor and Dean of Engineering at U.C. Berkeley, who commented after the 1906 earthquake:

Many engineers with whom the writer has talked appear to have the idea that earthquake stresses in framed structures can be calculated, so that rational design to resist earthquake destruction can be made, just as one may allow for dead and live loads, or wind and impact stresses. Such calculations lead to no practical conclusions of value.

A comment by A. Ruge, the first professor of engineering seismology at Massachusetts Institute of Technology, that “the natural tendency of average design engineer is to throw up his hands at the thought of making any dynamical analysis at all..”, made three decades later, shows that the progress was slow.

Brief History

In 1929, at University of Michigan, in Ann Arbor, first lectures were organized in the Summer School of Mechanics, by S. Timoshenko. In southern California, studies of earthquakes, and the research in theoretical mechanics, were expanded significantly when R.Millikan, became the first president of Caltech, in 1921. Millikan completed his Ph.D. studies in Physics, at Columbia University, in 1895, and following recommendation of his advisor M. Pupin spent a year in Germany. This visit to Europe appears to have influenced many of Millikan's later decisions while recruiting the leading Caltech faculty two decades later. In 1921 H.O. Wood invited Millikan to serve on the Advisory Committee in Seismology. The work on that committee and Millikan's interest in earthquakes were also significant for several subsequent events. In 1926 C. Richter, and in 1930 B. Gutenberg joined the seismological laboratory. In the area of applied mechanics, Millikan invited Theodor von Karman, and in 1930 von Karman became the first director of the Guggenheim Aeronautical Laboratory. It was Millikan's vision and his ability to anticipate future developments, which brought so many leading minds to a common place of work, creating environment, which made the first theoretical formulation of the concept of the response spectrum method possible.

Brief History

This year we commemorate the 75-th anniversary of the formulation of the concept of the Response Spectrum Method (RSM) in 1932. Since 1932 the RSM evolved into the essential tool and the central theoretical framework, in short a conditio sine qua non, for Earthquake Engineering. The mathematical formulation of the RSM first appeared in the doctoral dissertation of M.A. Biot (1905-1985) in 1932, and in two of his papers (Biot 1933; 1934). Biot defended his Ph.D. thesis at Caltech, in June of 1932 , and presented a lecture on the method to the Seismological Society of America meeting, which was held at Caltech, in Pasadena, also in June of 1932. Theodore von Karman, Biot's advisor, played the key role in guiding his student, and in promoting his accomplishments. After the method of solution was formulated, Biot and von Karman searched for an optimal design strategy. A debate at the time was whether a building should be designed with a soft first floor, or it should be stiff throughout its height, to better resist earthquake forces.

Solution alternatives

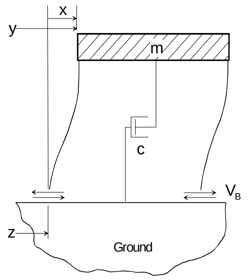
Solution of the differential equations which describe response of structures can be viewed in terms of (1) waves (D’Alembert in a memoir of the Berlin Academy in 1750), or (2) using vibrational approach in terms of characteristic functions (mode-shapes) (Bernoulli, in a memoir of the Berlin Academy in 1755). Mathematical principles and methods associated with the latter have been published by Rayleigh (Theory of sound 1877).

Response spectrum method is based on the vibrational representation of the solution, where with each mode shape and its natural frequency is associated one equivalent single-degree-of-freedom (SDOF) system . Then for linear systems the response is represented as a superposition of responses of those equivalent SDOF systems. Therefore the analysis of linear response of n-degree-of-freedom systems can be reduced to a study of individual SDOF systems, one at a time.

Response spectrum method employs the response of SDOF system to earthquake excitation as a basis, to construct the response of linear multi-degree-of-freedom systems. Therefore we begin by examining the response of SDOF system.

Response spectrum

SINGLE - DEGREE - OF - FREEDOM SYSTEM



m = mass

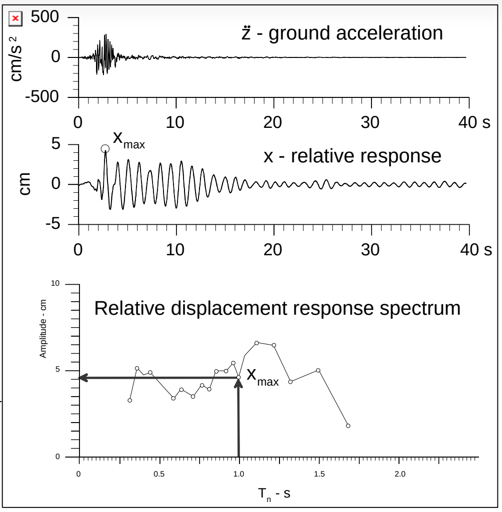
c = damping constant

V_B = base shear

x = relative motion of m

y = absolute motion of m

z = absolute motion of ground



$$m\ddot{x}+c\dot{x}+kx=-m\ddot{z} \qquad \omega_n^2=\frac{k}{m} \qquad \varsigma=\frac{c}{2\sqrt{km}}$$

$$\ddot{x}+2\omega_n\varsigma\dot{x}+\omega_n^2x=-\ddot{z}$$

$$x(t)=\frac{-1}{\omega_n\sqrt{1-\varsigma^2}}\int_0^t \ddot{z}(\tau)e^{-\varsigma\omega_n(t-\tau)}\sin\omega_n\sqrt{1-\varsigma^2}\,(t-\tau)d\tau.$$

For efficient computation of response spectra see NOTE B

Response Spectrum

The relative displacement x(t) is important for earthquake-resistant design because the strains in the structure are directly proportional to the relative displacements. The total shear force V_B , for example, exerted by the columns on the ground is V_B (t) = kx(t).

The exact relative velocity is

$$\dot{x}(t) = -\int_0^t \ddot{z}(\tau) e^{-\varsigma\omega_n(t-\tau)} \cos\omega_n\sqrt{1-\varsigma^2}(t-\tau)d\tau \ + \frac{\varsigma}{\sqrt{1-\varsigma^2}}\int_0^t \ddot{z}(\tau) e^{-\varsigma\omega_n(t-\tau)} \sin\omega_n\sqrt{1-\varsigma^2}(t-\tau)d\tau$$

The absolute acceleration of the mass m is obtained by further differentiation.

$$\ddot{y}(t) = \omega_n \frac{\left(1-2\varsigma^2\right)}{\sqrt{1-\varsigma^2}}\int_0^t \ddot{z}(\tau) e^{-\varsigma\omega_n(t-\tau)} \sin\omega_n\sqrt{1-\varsigma^2}(t-\tau)d\tau \ + \frac{\varsigma}{\sqrt{1-\varsigma^2}}\int_0^t \ddot{z}(\tau) e^{-\varsigma\omega_n(t-\tau)} \cos\omega_n\sqrt{1-\varsigma^2}(t-\tau)d\tau.$$

Of primary interest, for engineering applications, are the maximum absolute values of relative displacement, SD, velocity, SV, and acceleration, SA experienced during the earthquake response (see equations A.9 to A.11 in Note A).

Response Spectrum

Following approximate relationships exist between the spectral quantities SD, SV, and SA :

$$SD \approx \frac{T}{2\pi} SV \qquad SA \approx \frac{2\pi}{T} SV.$$

For engineering applications, it is convenient to use the following approximations

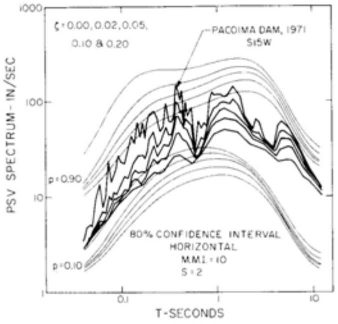
$$PSV = \frac{2\pi}{T} SD \qquad PSA = \left(\frac{2\pi}{T}\right)^2 SD$$

PSV and PSA are called “pseudo velocity” and “pseudo acceleration”.

FOURIER SPECTRA AND RESPONSE SPECTRA

It can be shown that Fourier Amplitude Spectra are same as the Relative Velocity Spectra for zero damping and evaluated at the end of excitation (see NOTE A). This provides valuable link between seismological and engineering spectral characterizations of strong earthquake ground motion.

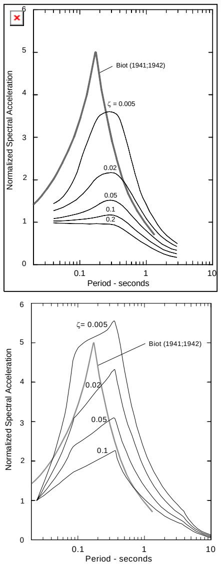
Empirical Scaling



When it is necessary to work with a spectrum of actual ground motions, one can search the database of recorded accelerograms and use a spectrum computed directly from the chosen accelerogram (as in this figure; Trifunac 1978). Such spectra will have irregular shape and will reflect the properties of the recording site and of the earthquake. To obtain the corresponding smooth amplitudes, empirical scaling can be used.

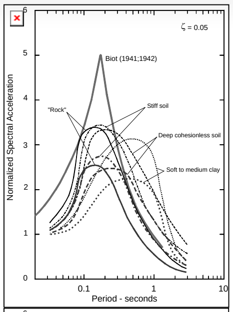
Empirical scaling of spectral amplitudes can be developed for regions with significant number (more than about 200) of recorded strong motion accelerograms, where the data exists on local soil and geological site conditions at the recording stations, and where magnitudes and intensities of the contributing earthquake evens are documented. There are many examples in literature on how this was done in many areas. A recent review dealing with empirical scaling of PSV amplitudes can be found in the work of Lee (2007) (see NOTE C).

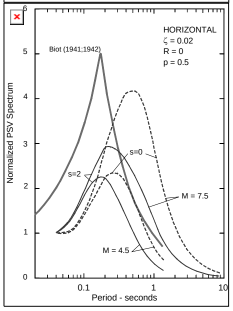
Design Spectra



Design spectra [of Biot, Housner (top left), Newmark and co-workers (bottom left), for example] have been developed by enveloping the spectra of recorded accelerograms available at the time (see NOTE D). All spectral shapes have been normalized to unit peak acceleration, so that for design work the spectral amplitudes could be scaled by the suitably chosen peak acceleration only. This approach required experience and considerable judgment, since complex characteristics of strong shaking and of the nature of structural response had to be included in the proper choice of the design peak acceleration. In this approach the spectral shape approximated the role of dynamic amplification and of the relative contribution of the higher modes to the overall response.

Design Spectra



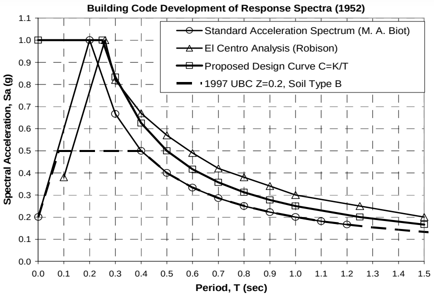


In early 1970s the shapes of the design spectra were refined to reflect the soil site conditions, and in one case also the geological site conditions and the earthquake magnitude or site intensity (see NOTE D). It was found that “soft” sites amplify the long period spectral amplitudes and diminish the short period amplitudes.

Size of the earthquake has profound effect on the shape of normalized spectral amplitudes. Larger magnitude events generate more long period energy and thus lead to spectral shapes which have larger long period amplitudes.

In the selection of site specific spectra for design, many different earthquake sources can contribute to the set of possible ground motions, each producing different amplitudes and spectral shapes. By combining all those contributions and considering their relative probabilities of occurrence we arrive at the concept of Uniform Hazard Spectrum (see NOTE E).

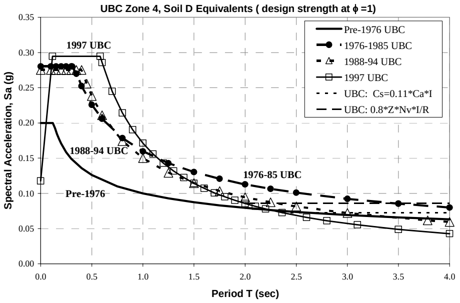
Design Codes



The basis for the development of current seismic building code provisions had their beginnings in the 1950s. A Joint Committee of the San Francisco Section of ASCE and the Structural Engineers Association of Northern California prepared a “model lateral force provision” based on a dynamic analysis approach and response spectra (see NOTE F).

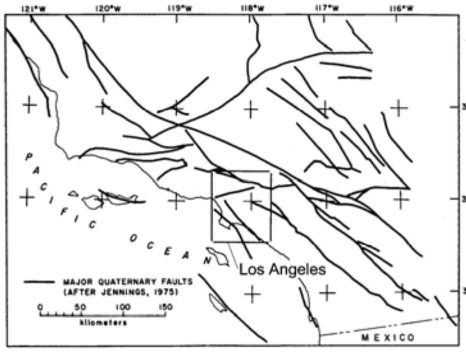
The Proposed Design Curve, was based on a compromise between a Standard Acceleration Spectrum by M. A. Biot and the analysis of El Centro accelerogram by E. C. Robison. It is interesting to note that the Biot curve with PGA of 0.2g has a peak spectral acceleration of 1.0g at a period of 0.2 s. The curve then descends in proportion to 1/T (i.e., constant velocity). If the peak spectral acceleration is limited to 2.5 times the PGA, the Biot spectrum is very close to the 1997 UBC design spectrum for a PGA of 0.2g (dashed line without symbols in the Figure). These values were considered consistent with the current practice and the weight of the building included a percentage of live load.

Design Codes



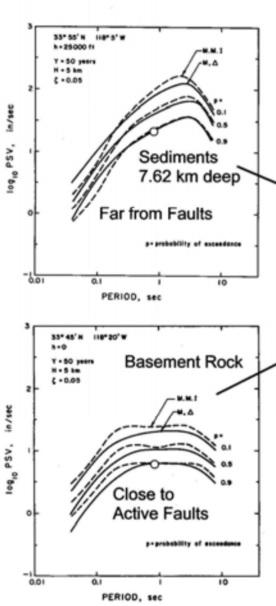
Over the years, the UBC went through many revisions, influenced by earthquake events such as the San Fernando, Loma Prieta, or Northridge earthquakes and by data relating to soil effects. The comparable curves shown in the Figure have been adjusted to represent strength design response spectra and include factors representing soil classification type D. At this level of design, the structures would be expected to remain linear-elastic with some reserve capacity before reaching yield. In order to survive major earthquake ground motion the structure is expected to experience nonlinear post yielding response.

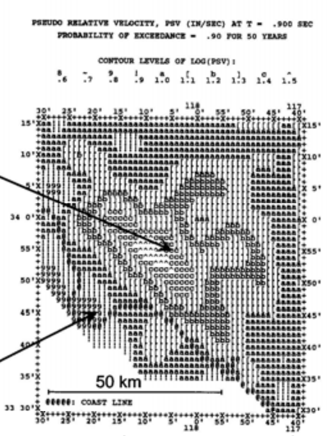
Uniform Hazard Spectra



The first important step in computation of Uniform Hazard is the formulation of the regional seismicity model. For each active fault different occurrence rates, occurrence models, and maximum earthquake magnitudes need to be specified. Next, suitable regional attenuation laws have to be developed, specific for the quantity being analyzed (peak amplitude, spectral amplitude, duration, energy, power, strain, curvature, etc.). By repeating the calculations at a suitable grid of points, Uniform Hazard Maps can be prepared. Such microzonation maps serve as a balanced design tool, and can be used directly, or as a starting point for rational preparation of regional design codes. Such a microzonation map is illustrated in the next slide, for PSV amplitudes, given exposure time, and for a selected probability of being exceeded.

Uniform Hazard Spectra

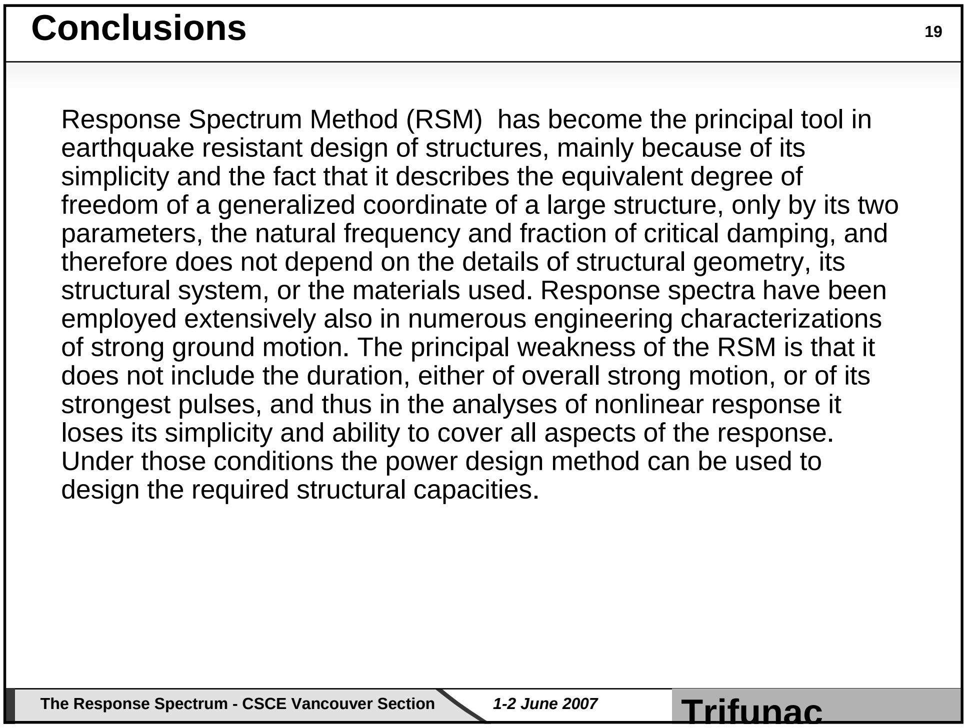




Los Angeles area hazard map, on the right, shows Log(PSV) at T=0.9 s, for 0.9 probability of exceeding, for exposure time of 50 years, and for a 1’x1’ (1.5x1.8 km) grid. The open circles in the figures on the left show the PSV amplitudes for the two sites shown by arrows. By reading from many such maps, for different oscillator periods, and for different probabilities of being exceeded the two UHS for PSV can be constructed on the left.This example shows how different UHS can be even in the same general area. Long periods are amplified (top left figure) by deep sediments. Short period UHS amplitudes (left bottom) are amplified by proximity to the Palos Verdes and Newport-Inglewood faults.

Advanced Topics

The basic model employed to describe the response of a simple structure to only horizontal earthquake ground motion, is the single-degree-of-freedom system (SDOF) experiencing rocking , relative to the normal to the ground surface, and assuming that the ground does not deform in the vicinity of the foundation. In more advanced vibrational representations of the response, additional components of earthquake excitation (three translations and three rotations), dynamic instability, soil-structure interaction, spatial and temporal variations of excitation, differential motions at different support points, and nonlinear behavior of the stiffness (of soil and of structure) can be considered, but the structure usually continues to be modeled by mass-less columns, springs, dashpots, and with rigid mass (see NOTE G). For analyses of response in the vicinity of faults, where strong ground motion can contain powerful and large pulses, solutions in terms of nonlinear wave propagation represent ideal methods capable of providing directly the information needed in the design (inter-story drifts and the zones of potential strain localization).



Conclusions

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Response Spectrum Method (RSM) has become the principal tool in earthquake resistant design of structures, mainly because of its simplicity and the fact that it describes the equivalent degree of freedom of a generalized coordinate of a large structure, only by its two parameters, the natural frequency and fraction of critical damping, and therefore does not depend on the details of structural geometry, its structural system, or the materials used. Response spectra have been employed extensively also in numerous engineering characterizations of strong ground motion. The principal weakness of the RSM is that it does not include the duration, either of overall strong motion, or of its strongest pulses, and thus in the analyses of nonlinear response it loses its simplicity and ability to cover all aspects of the response. Under those conditions the power design method can be used to design the required structural capacities.

The Response Spectrum - CSCE Vancouver Section *1-2 June 2007* **Trifunac**