Ninth Canadian Conference on Earthquake Engineering
Ottawa, Ontario, Canada
26-29 June 2007

SIMPLIFIED SEISMIC ANALYSIS OF ASYMMETRIC ELASTIC SYSTEMS WITH SUPPLEMENTAL DAMPING

J.-L. Lin[1] and K.-C. Tsai[2]

ABSTRACT

A building in which the center of stiffness (CR) is not coincident with center of mass (CM) along only one horizontal plan axis is defined as a one-way asymmetric building. Further, if such a building is constructed with supplemental damping, e.g. viscous dampers in the braces, it becomes a non-proportionally or non-classically damped system. Its damping matrix can not be diagonalized from the mode shapes of the undamped system. Not counting the time-consuming direct integration of the equation of motion for the original multiple-degree-of-freedom (MDOF) system, the conventional seismic analysis of the noted structures can be categorized into two types. One is based on the use of complex mode shapes and the other one on neglecting the off-diagonal elements of the transformed damping matrix. The shortcomings of the two approaches are that they are either too complicated for practicing engineers or result in unacceptable errors. An approximate method is proposed for the seismic analysis of this kind of structure. The proposed method is a modal analysis method that uses two-degree-of-freedom (2DOF) modal equations. Since the proposed 2DOF modal equations inherit the non-proportional damping property of the original MDOF system, the modal translation and rotation are not proportional even in an elastic state. Three numerical examples, which include two one-story and one three-story prototype buildings, are worked out in this research. The results are compared with those obtained by the direct integration of the equations of motion for the original MDOF system and the typical simplified modal analysis by neglecting the off-diagonal elements in the transformed damping matrix. These examples illustrate that the use of the proposed method can effectively improve the accuracy of the analytical results, without significantly increasing the computational efforts.

Introduction

Although the research on the dynamic responses of non-proportionally damped symmetric structural systems was conducted at a much earlier time (Moh *et al*. 1965; Itoh 1973), the study of non-proportionally damped asymmetric structures was performed much later (Goel 1998). Based on the literature review conducted by Goel (2001), the methods for analysis of non-proportionally damped systems are categorized and their corresponding shortcomings are stated briefly as follows. The first approach is to integrate directly the coupled equations of motion, which is numerically inefficient for

---

[1]Doctoral student, Dept. of Civil Engineering, National Taiwan University, Taipei, Taiwan
[2]Professor, Dept. of Civil Engineering, National Taiwan University, Taipei, Taiwan
Director, National Center for Research on Earthquake Engineering, Taipei, Taiwan

systems with a large number of degrees of freedom. Thus, Clough and Mojtahedi (1976) proposed to directly integrate a truncated set of coupled modal equations, which is more efficient than dealing with the complete set of coupled equations of motion in discrete coordinates. The other approach is the mode superposition method using complex mode shapes (Igusa *et al*. 1984; Veletsos and Ventura 1986; Goel 2000), which results in doubling of the size of the eigenvalue problems and difficulties associated with the use of complex numbers in the dynamic response analysis. Another approach is the hybrid time-domain, frequency-domain procedure (Ibrahimbegovic *et al*. 1990; Claret and Venancio-Filho 1991), which solves the coupled modal equations iteratively in time domain. However, this method cannot be implemented on most commercially available structural analysis programs. Finally, the last approach, which is the most common and simplest approach, is to simply neglect the off-diagonal elements in the transformed damping matrix. It is appealing to the design professionals because it enables the use of the traditional modal analysis methods.

Warburton and Soni (1977) have studied the accuracy of the last approach, and have proposed a condition involving the natural frequencies and the elements of the transformed damping matrix which, if fulfilled, limits response calculation errors to a specific range. Goel (2001) investigated the effects of neglecting off-diagonal terms of the transformed damping matrix on the seismic response of non-proportionally damped one-way asymmetric systems. The specific aim of that study was to identify the range of system parameters for which this simplification could be used without introducing significant errors in the response. Goel (2001) concluded that the aforementioned approximate method was suitable for use over a wide range of parameters. The error parameter, defined by Warburton and Soni (1977), becomes excessive when the value of the normalized supplemental damping eccentricity $\bar{e}_{sd}$ is close to -0.5. This indicates that the approximate method should not be used for asymmetric-plan systems with a large normalized supplemental damping eccentricity.

The main objective of this study is to investigate the effectiveness of modal analysis for elastic one-way asymmetric buildings with supplemental damping by using the 2DOF modal equations, instead of single-degree-of-freedom (SDOF) modal equations. The SDOF modal equation is obtained by neglecting the off-diagonal terms of transformed damping matrix. The 2DOF modal equation is obtained by resolving each diagonal element of mass, damping and stiffness matrices into a 2×2 matrix. Each 2DOF modal equation is represented by a corresponding 2DOF modal stick (Lin and Tsai 2006a). Moreover, the inelastic properties of each 2DOF stick can be determined by the corresponding push-over curves (Lin and Tsai 2006a). It is found that the proposed method can be applied for a much wider range of parameters without introducing significant errors; therefore, it improves the accuracy of modal analysis and is more appealing to design professionals for practical use.

Theoretical Background

SDOF Modal Equations

The equation of motion for a typical *N*-story building where each floor is represented by a rigid diaphragm with two DOFs (one is translational DOF and the other one is rotational DOF) is:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\boldsymbol{\iota}\ddot{u}_g \tag{1}$$

where the **M**, **C**, **K** represent the mass, damping and stiffness matrices related to the deformation **u**(t), $\boldsymbol{\iota}$ is the influence vector, and $\ddot{u}_g(t)$ is the ground acceleration. The damping matrix can be expressed as

$$\mathbf{C} = \mathbf{C}_0 + \mathbf{C}_{sd} \tag{2}$$

where $\mathbf{C}_0$ is the inherent damping matrix and $\mathbf{C}_{sd}$ is the damping matrix due to supplemental dampers. The matrix $\mathbf{C}_0$ is defined as

$$\mathbf{C}_0 = \alpha\mathbf{M} + \beta\mathbf{K} \tag{3}$$

in which α and β are determined from the damping ratios of two specific modes. The transformed damping matrix is equal to

$$\mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi} = \begin{bmatrix}\boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_{2N}\end{bmatrix}^T \mathbf{C}\begin{bmatrix}\boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_{2N}\end{bmatrix} \tag{4}$$

in which $\boldsymbol{\varphi}_i$ is the ith mode shape of the undamped system. In general, the transformed damping matrix is not a diagonal matrix for structures with supplemental dampers. The approximate procedure is based on neglecting the off-diagonal terms of the transformed damping matrix. Therefore, Eq. 1 can be decomposed into 2N SDOF modal equations of motion:

$$\ddot{D}_n + 2\omega_n\xi_n\dot{D}_n + \omega_n^2 D_n = -\ddot{u}_g(t) \qquad n = 1 \sim 2N \tag{5}$$

in which D_n is the nth modal coordinate. The corresponding damping ratio, ξ_n, and the square of the circular frequency, ω_n^2, are

$$\xi_n = \frac{\boldsymbol{\varphi}_n^T\mathbf{C}\boldsymbol{\varphi}_n}{2\omega_n\boldsymbol{\varphi}_n^T\mathbf{M}\boldsymbol{\varphi}_n} \qquad \omega_n^2 = \frac{\boldsymbol{\varphi}_n^T\mathbf{K}\boldsymbol{\varphi}_n}{\boldsymbol{\varphi}_n^T\mathbf{M}\boldsymbol{\varphi}_n} \tag{6}$$

By solving Eq. 5, the displacement history of the non-proportionally damped system is approximated as:

$$\mathbf{u}(t) = \sum_{n=1}^{2N}\mathbf{u}_n(t) = \sum_{n=1}^{2N}\Gamma_n\boldsymbol{\varphi}_n D_n(t) \tag{7}$$

in which Γ_n is the nth modal participating factor defined as:

$$\Gamma_n = \frac{\boldsymbol{\varphi}_n^T\mathbf{M}\boldsymbol{\iota}}{\boldsymbol{\varphi}_n^T\mathbf{M}\boldsymbol{\varphi}_n} \tag{8}$$

2DOF Modal Equations

The construction and verification of 2DOF modal sticks corresponding to 2DOF modal equations for seismic analysis of inelastic asymmetric structures can be found in Lin and Tsai (2006a). Here, the application of 2DOF modal equations on one-way asymmetric elastic structures with supplemental damping is studied. The right-hand side of Eq. 1 is the seismic load and can be written as:

$$-\mathbf{M}\boldsymbol{\iota}\ddot{u}_g(t) = -\mathbf{s}\ddot{u}_g(t) = -\sum_{n=1}^{2N}\mathbf{s}_n\ddot{u}_g(t) = -\sum_{n=1}^{2N}\Gamma_n\mathbf{M}\boldsymbol{\varphi}_n\ddot{u}_g(t) \tag{9}$$

It is assumed that only the nth undamped modal displacement, $\mathbf{u}_n$, of the non-proportionally damped system will be excited under the load, $-\mathbf{s}_n\ddot{u}_g(t)$, thus,

$$\mathbf{M}\ddot{\mathbf{u}}_n + \mathbf{C}\dot{\mathbf{u}}_n + \mathbf{K}\mathbf{u}_n = -\mathbf{s}_n\ddot{u}_g(t) \quad n = 1 \sim 2N \tag{10}$$

The mass, damping and stiffness matrices shown in Eq. 10 are partitioned as:

$$\mathbf{M} = \begin{bmatrix}\mathbf{m} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_0\end{bmatrix}_{2N\times 2N} \qquad \mathbf{C} = \begin{bmatrix}\mathbf{c}_{zz} & \mathbf{c}_{z\theta}\\ \mathbf{c}_{\theta z} & \mathbf{c}_{\theta\theta}\end{bmatrix}_{2N\times 2N} \qquad \mathbf{K} = \begin{bmatrix}\mathbf{k}_{zz} & \mathbf{k}_{z\theta}\\ \mathbf{k}_{\theta z} & \mathbf{k}_{\theta\theta}\end{bmatrix}_{2N\times 2N} \tag{11}$$

in which $\mathbf{m}$ and $\mathbf{I}_0$ are the mass and moment of inertia of the building system, respectively. The subscript

z and θ, which are shown in Eq. 11, denote the sub-matrices relating to translational and rotational degree of freedoms, respectively. The nth undamped modal displacement is also partitioned as:

$$\mathbf{u}_n = \begin{bmatrix} \mathbf{u}_{zn} \\ \mathbf{u}_{\theta n} \end{bmatrix}_{2N\times 1} = \Gamma_n \begin{bmatrix} \boldsymbol{\varphi}_{zn} D_{zn} \\ \boldsymbol{\varphi}_{\theta n} D_{\theta n} \end{bmatrix}_{2N\times 1} = \Gamma_n \begin{bmatrix} \boldsymbol{\varphi}_{zn} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\varphi}_{\theta n} \end{bmatrix}_{2N\times 2} \begin{bmatrix} D_{zn} \\ D_{\theta n} \end{bmatrix}_{2\times 1} \tag{12}$$

in which φ_{zn} and $\varphi_{\theta n}$ are the components of the nth mode shape associated with translational and rotational DOFs, respectively. When D_{zn} is equal to $D_{\theta n}$, i.e. $D_{zn}=D_{\theta n}=D_n$, Eq. 12 is the same as the conventional definition of $\mathbf{u}_n$. By pre-multiplying both sides of Eq. 10 with $\begin{bmatrix} \boldsymbol{\varphi}_{zn} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\varphi}_{\theta n} \end{bmatrix}_{2N\times 2}^T$ and substituting Eq. 12 into it, Eq. 10 becomes

$$\mathbf{M}_n \ddot{\mathbf{D}}_n + \mathbf{C}_n \dot{\mathbf{D}}_n + \mathbf{K}_n \mathbf{D}_n = -\mathbf{M}_n \mathbf{1} \ddot{u}_g(t) \quad n = 1 \sim 2N \tag{13}$$

in which

$$\mathbf{D}_n = \begin{bmatrix} D_{zn} \\ D_{\theta n} \end{bmatrix}_{2\times 1} \quad \mathbf{M}_n = \begin{bmatrix} \boldsymbol{\varphi}_{zn}^T \mathbf{m} \boldsymbol{\varphi}_{zn} & 0 \\ 0 & \boldsymbol{\varphi}_{\theta n}^T \mathbf{I}_0 \boldsymbol{\varphi}_{\theta n} \end{bmatrix}_{2\times 2} \quad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times 1}$$
$$\mathbf{C}_n = \begin{bmatrix} \boldsymbol{\varphi}_{zn}^T \mathbf{c}_{zz} \boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{zn}^T \mathbf{c}_{z\theta} \boldsymbol{\varphi}_{\theta n} \\ \boldsymbol{\varphi}_{\theta n}^T \mathbf{c}_{\theta z} \boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{\theta n}^T \mathbf{c}_{\theta\theta} \boldsymbol{\varphi}_{\theta n} \end{bmatrix}_{2\times 2} \quad \mathbf{K}_n = \begin{bmatrix} \boldsymbol{\varphi}_{zn}^T \mathbf{k}_{zz} \boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{zn}^T \mathbf{k}_{z\theta} \boldsymbol{\varphi}_{\theta n} \\ \boldsymbol{\varphi}_{\theta n}^T \mathbf{k}_{\theta z} \boldsymbol{\varphi}_{zn} & \boldsymbol{\varphi}_{\theta n}^T \mathbf{k}_{\theta\theta} \boldsymbol{\varphi}_{\theta n} \end{bmatrix}_{2\times 2} \tag{14}$$

Equation 13 is the nth modal equation of motion with two degrees of freedom. It has been mathematically proved that Eq. 13 is eventually equivalent to Eq. 5 for proportionally damped elastic systems (Lin and Tsai 2006a). Each 2DOF modal equation of motion, Eq. 13, can be modeled by a 2DOF stick with translational DOF, D_{zn}, and rotational DOF, $D_{\theta n}$ (Lin and Tsai 2006a). D_{zn} and $D_{\theta n}$ are denoted as modal translation and modal rotation of the nth mode, respectively. Furthermore, the modal damping matrix $\mathbf{C}_n$ given in Eq. 14 can be represented as:

$$\mathbf{C}_n = \begin{bmatrix} \boldsymbol{\varphi}_{zn} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\varphi}_{\theta n} \end{bmatrix}_{2N\times 2}^T \mathbf{C} \begin{bmatrix} \boldsymbol{\varphi}_{zn} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\varphi}_{\theta n} \end{bmatrix}_{2N\times 2} \tag{15}$$

If $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$, then

$$\mathbf{C}_n = \begin{bmatrix} \boldsymbol{\varphi}_{zn} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\varphi}_{\theta n} \end{bmatrix}_{2N\times 2}^T (\alpha\mathbf{M} + \beta\mathbf{K}) \begin{bmatrix} \boldsymbol{\varphi}_{zn} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\varphi}_{\theta n} \end{bmatrix}_{2N\times 2} = \alpha\mathbf{M}_n + \beta\mathbf{K}_n \tag{16}$$

Therefore, if $\mathbf{C} \neq \alpha\mathbf{M} + \beta\mathbf{K}$, then $\mathbf{C}_n \neq \alpha\mathbf{M}_n + \beta\mathbf{K}_n$.

It follows that a non-proportionally damped MDOF system will result in 2N non-proportionally damped 2DOF modal equations with modal translation not equal to modal rotation even in an elastic state. This is closer to the actual condition. From the modal displacement history, $\mathbf{D}_n(t)$, which is solved by direct integration of Eq. 13, the total displacement history of the non-proportionally damped one-way asymmetric system is obtained as:

$$\mathbf{u}(t) = \sum_{n=1}^{2N} \mathbf{u}_n(t) = \sum_{n=1}^{2N} \Gamma_n \begin{bmatrix} \boldsymbol{\varphi}_{zn} D_{zn}(t) \\ \boldsymbol{\varphi}_{\theta n} D_{\theta n}(t) \end{bmatrix}_{2N\times 1} \tag{17}$$

The 2DOF modal stick has two sub-modes–one is active and the other one is spurious. The active one is the same as the conventional SDOF modal stick, and the spurious one has no contribution to the elastic responses of a proportionally damped 2DOF modal stick (Lin and Tsai 2006a). In addition, as shown in the above discussion, the 2DOF modal stick will be non-proportionally damped when the original MDOF structure is a non-proportionally damped system. Thus, the 2DOF modal stick can take the out-of-phase motions, which are between the modal translation and the modal rotation, into account.

The 2DOF modal equations possess the property of non-proportional damping at the expense of increasing the DOF by one in the modal coordinate. However, this can be easily handled by commercially available structural analysis programs. Therefore, the proposed simplified method still keeps the advantage found in the conventional approximate method which has none of the difficulties associated with the use of complex numbers.

Analytical Example

Selected Structural System, Ground Motion and Basic Assumptions

A one-story and a three-story asymmetric building with viscous dampers (Fig. 1) are analyzed by three methods. They include the direct integration of the equation of motion, modal analysis by SDOF and 2DOF modal equations. The analysis of the same one-story building, but without dampers, is also carried out. The objective is to see whether or not the analytical results of the proportionally damped system obtained by modal analysis using 2DOF and SDOF modal equations are the same. All of the beams and columns are symmetric making the CR on the geometric center of each floor. It is assumed that CM is on the right side of the CR (see Fig. 1). Therefore, the left side of CR is the stiff side. It is found in this research that the analytical error resulting from the use of SDOF approach is greater when the center of supplemental damping (CSD) is on the stiff side of each floor. Thus, the CSD is purposely placed on the stiff side in the following examples in order to illustrate the accuracy of the proposed 2DOF approach. According to the investigation of error in response of the one-story asymmetric building (Goel 2001), the error introduced by conventional approximate method is over 20% when the normalized supplemental damping eccentricity, $\bar{e}_{sd} = e_{sd}/a$, is equal to -0.5. The values of e_{sd} and a represent the distance from CM to CSD and the plan dimension of the system perpendicular to the ground motion, respectively. In this study, the CM is eccentrically such that the value of $\bar{e}_{sd}$ is equal to -0.75, which is appropriate for illustrating the effectiveness of the proposed simplified method. The damping coefficient of dampers, C_z, along the Z-axis is determined by the supplemental damping ratio, ξ_{sd}, (Goel 1998):

$$C_z = 2m\omega_z\xi_{sd} \tag{18}$$

in which ω_z is the circular frequency of transverse vibration. The supplemental damping ratios, ξ_{sd}, used in these two prototype buildings are both equal to 30%. The properties of the three buildings are shown in Table 1 to Table 4 in which the units are *kN*, *m* and *sec.* The *n*th column vector of matrix Φ shown in Table 2 and 4 is the *n*th mode shape, φ_n, of the MDOF building. Therefore, the sub-column vectors of the upper three elements and the lower three elements of φ_n, shown in Table 4, are equal to φ_{zn} and $\varphi_{\theta n}$, respectively. The matrix Φ shown in Table 2 and 4 has been normalized, which makes $\Phi^T\mathbf{M}\Phi$ equal to an identity matrix. The diagonal elements of matrix $\Phi^T\mathbf{C}\Phi$ and Λ, shown in Table 2 and 4, are the values of $2\omega_n\xi_n$ and ω_n^2, respectively, which are used in each SDOF modal equation of motion given in Eq. 5.

Table 1. Properties of one-story buildings.

	M		C				K	
			C_0		C_{sd}			
w/o damper	9.45	0	8.10	3.09	0	0	4594	5176
	0	23.03	3.09	42.02	0	0	5176	48547
with damper	9.45	0	8.10	3.09	115.30	389.1	4594	5176
	0	23.03	3.09	42.02	389.1	1313.3	5176	48547

Table 2. The undamped eigenvectors, Φ, eigenvalues, Λ, and transformed damping matrix of one-story buildings.

	$\Phi=\begin{bmatrix}\phi_{z1} & \phi_{z2}\\ \phi_{\theta 1} & \phi_{\theta 2}\end{bmatrix}$		$\Lambda=\begin{bmatrix}\omega_1^2 & 0\\ 0 & \omega_2^2\end{bmatrix}$		$\Phi^T C \Phi$	
w/o damper	-0.319	-0.066	413.49	0	0.813	0
	0.042	-0.204	0	2180	0	1.868
with damper	-0.319	-0.066	413.49	0	4.383	15.309
	0.042	-0.204	0	2180	15.309	67.525

Table 3. Properties of three-story building.

M		9.45					
		0	9.45			symm.	
		0	0	9.45			
		0	0	0	23.03		
		0	0	0	0	23.03	
		0	0	0	0	0	23.03
C	C_0	12.214					
		-13.785	32.24			symm.	
		4.6137	-21.724	40.873			
		10.784	-14.988	4.8543	78.188		
		-14.988	32.664	-23.734	-99.789	221.99	
		4.8543	-23.748	42.726	32.239	-156.33	283.92
	C_{sd}	157.04					
		-157.04	314.09			symm.	
		0	-157.04	314.09			
		-176.67	176.67	0	198.76		
		176.67	-353.35	176.67	-198.76	397.52	
		0	176.67	-353.35	0	-198.76	397.52
K		6970					
		-9740	21120			symm.	
		3260	-15350	27220			
		7620	-10590	3430	51200		
		-10590	23080	-16770	-70510	1.53E+05	
		3430	-16780	30190	22780	-1.10E+05	1.97E+05

Table 4. The undamped eigenvectors, Φ, eigenvalues, Λ, and transformed damping matrix of three-story building.

$\Phi = \begin{bmatrix} \varphi_{z1} & \cdots & \varphi_{z6} \\ \varphi_{\theta 1} & \cdots & \varphi_{\theta 6} \end{bmatrix}$	0.2466					
	0.1748	0.0528			symm.	
	0.0719	0.0203	-0.2028			
	-0.0488	0.1574	-0.0367	-0.0133		
	-0.0349	0.1124	0.0313	0.0391	-0.1033	
	-0.0145	0.0469	0.0412	-0.0488	-0.1280	0.1434
$\Lambda = \begin{bmatrix} \omega_1^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \omega_6^2 \end{bmatrix}$	85.237					
	0	362.18			symm.	
	0	0	1000.1			
	0	0	0	3443		
	0	0	0	0	4188.2	
	0	0	0	0	0	14168
$\Phi^T C \Phi$	5.280					
	-1.612	1.294			symm.	
	3.220	-0.706	37.966			
	-1.821	0.617	6.444	77.950		
	-1.011	0.217	-12.003	-3.860	10.186	
	0.585	-0.199	-2.159	-23.619	1.274	27.960

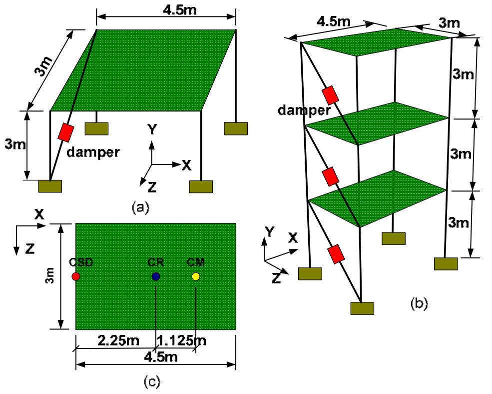

Figure 1. (a) one-story building (b) three-story building (c) floor plan.

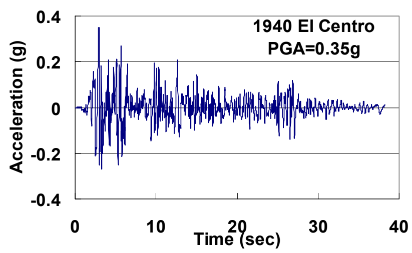

Figure 2. 1940 El Centro earthquake NS component.

The ground acceleration record used in this study is the NS component of 1940 El Centro earthquake shown in Fig. 2; it is scaled to peak ground acceleration (PGA) equal to 0.14g and 0.1g for one-story and three-story building, respectively. The ground acceleration record is applied along the Z-axis and all the buildings remain elastic. The floors are simulated as rigid diaphragms and, therefore, only the Z-axial translation and Y-axial rotation are considered in these analytical examples. Although real modes of the building are coupled in translation and rotation, by comparing the components of mode shapes, one can identify whether the mode is translational or rotational dominant. The inherent damping ratios of the two specific modes are assumed to be 2%. The noted two modes are the first Z-axial translation-dominant mode and the first Y-axial rotation-dominant mode. The inherent damping of the MDOF building is modeled by Rayleigh damping.

Seismic responses of one-story and three-story buildings

In the following the results of response history obtained by direct integration of equation of motion, and modal analyses using SDOF and 2DOF modal equations are denoted as RHA, SMA and 2MA, respectively. The modal responses of the one-story building without viscous dampers obtained by 2MA are shown in Figs. 3(a) and 3(b). It is observed that the modal translation, D_{zn}, is equal to the modal rotation, $D_{\theta n}$, for proportionally damped elastic systems. Hence, the total responses of such a system, obtained by RHA, 2MA and SMA, are all the same, as shown in Figs. 3(c) and 3(d). The 2×2 $\mathbf{M}_n$, $\mathbf{C}_n$ and $\mathbf{K}_n$ matrices, n=1~2, of the one-story building with viscous damper, defined in Eqs. (13) and (14), could be found in Lin and Tsai (2006b). The sum of the four elements of matrix $\mathbf{M}_n$ is equal to one. Additionally, the sums of the four elements of matrix $\mathbf{C}_n$ and $\mathbf{K}_n$ are equal to the value of the nth diagonal element of matrix $\Phi^T\mathbf{C}\Phi$ and Λ, respectively (Lin and Tsai 2006b). The modal translation and rotation of the elastic one-story building with viscous damper calculated by 2MA are no longer equal as shown in Figs. 4(a) and 4(b). This just reflects the non-proportional damping effect. The total response of this non-proportionally damped system, obtained by RHA, 2MA and SMA, are shown in Figs. 4(c) and 4(d). It is found that the analytical results obtained by 2MA are almost the same as those obtained by RHA. However, the errors in the peak translational and rotational responses obtained by SMA are up to 37.6% and 59.6%, respectively.

The $\mathbf{M}_n$, $\mathbf{C}_n$ and $\mathbf{K}_n$ matrices, n=1~6, of the three-story building, defined in Eqs. (13) and (14), could be found in Lin and Tsai (2006b). The sum of the four elements of matrix $\mathbf{M}_n$ is equal to one. Again, the sums of the four elements of matrix $\mathbf{C}_n$ and $\mathbf{K}_n$ are equal to the value of the nth diagonal element of matrix $\Phi^T\mathbf{C}\Phi$ and Λ, respectively (Lin and Tsai 2006b). The first two modal responses of the three-story building with viscous dampers calculated by 2MA are shown in Figs. 5(a) and 5(b). The modal translation, D_{zn}, is not equal to modal rotation, $D_{\theta n}$, which reflects the non-proportional damping effect. The total responses of this non-proportionally damped system, obtained by 2MA and SMA, which are both compared with RHA, are shown in Figs. 5(c) and 5(d), respectively. It is found that the error in rotational response is larger than that in translational response obtained by SMA. The errors in the peak translation and rotation at roof are 1% and 16.4%, respectively. However, the analytical results obtained by 2MA are almost the same as those obtained by RHA.

From the results obtained in these examples, it is seen that the 2DOF modal equations of motion possessing the non-proportional damping characteristic are more appropriate for the modal analysis of non-proportionally damped one-way asymmetric buildings. Furthermore, the incorrect prediction of translation and rotation at CM by SMA may amplify the error in the prediction of corner translation, which would diminish the applicability of the conventional approximate method to asymmetric structures. The proposed 2MA procedure provides a better alternative to deal with this kind of problem.

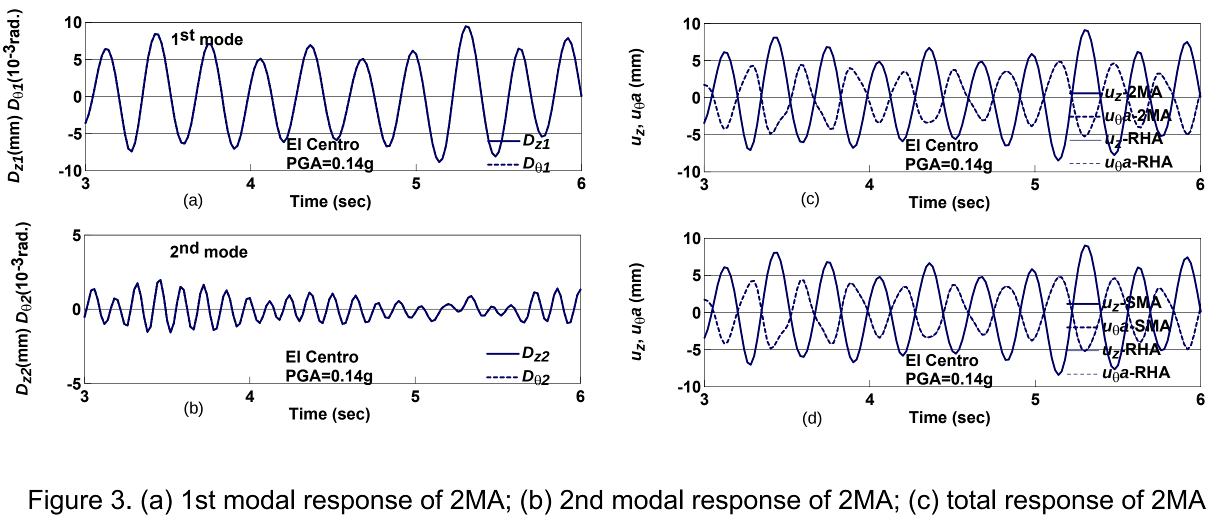

Figure 3. (a) 1st modal response of 2MA; (b) 2nd modal response of 2MA; (c) total response of 2MA and RHA; (d) total response of SMA and RHA for one-story building without damper.

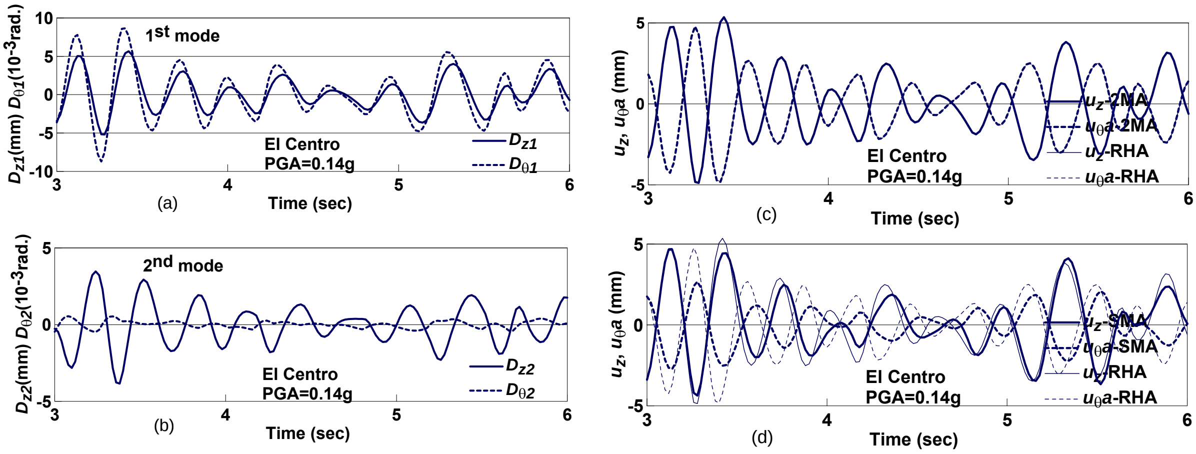

Figure 4. (a) 1st modal response of 2MA; (b) 2nd modal response of 2MA; (c) total response of 2MA and RHA; (d) total response of SMA and RHA for one-story building with damper.

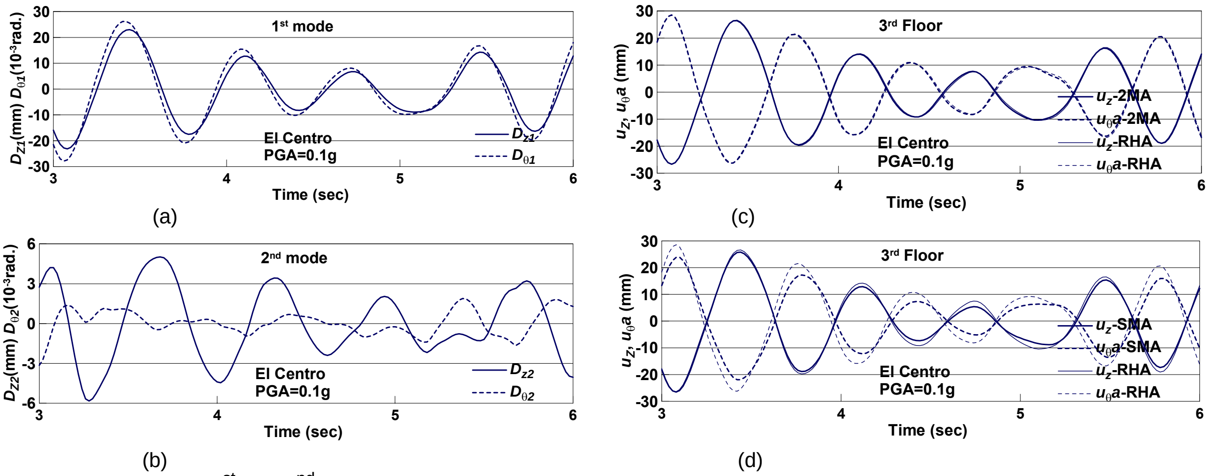

Figure 5. The (a) 1st (b) 2nd modal response of three-story building obtained by 2MA; Total translational and rotational response at the 3rd floor obtained by (c) 2MA and RHA (d) SMA and RHA.

Conclusions

This study develops a method of analyzing the seismic response of one-way asymmetric buildings with supplemental damping. The proposed method is similar to conventional modal analysis except that it is based on the solution of the 2DOF modal equations instead of SDOF modal equations. The proportionalities of damping matrices in 2DOF modal equations depend on that of the damping matrix for the original MDOF system. Hence, the method allows the modal translation and modal rotation in a 2DOF modal equation to be different from each other. The predicted response is closer to the realistic structural behavior, a result that could not be achieved by the use of conventional SDOF modal equation. The accuracy of the analytical result obtained by the proposed method is illustrated by three numerical examples in this study. The proposed method inherits the advantages of conventional modal analysis without the complexity of other developed procedures. The seismic analysis of non-proportionally damped two-way asymmetric building merits further study.

Acknowledgments

The financial support sponsored by the National Center for Research on Earthquake Engineering (NCREE) in Taiwan is gratefully acknowledged.

References

Claret, A.M. and Venancio-Filho, F., 1991. A modal superposition pseudo-force method for dynamic analysis of structural systems with non-proportional damping, *Earthquake Engineering and Structural Dynamics*, 20(4), 303-315.

Clough, R.W. and Mojtahedi, S., 1976. Earthquake response analysis considering non-proportional damping, *Earthquake Engineering and Structural Dynamics*, 4(5), 489-496.

Goel, R.K., 1998. Effects of supplenmental viscous damping on seismic response of asymmetric-plane systems, *Earthquake Engineering and Structural Dynamics*, 27, 125-141.

Goel, R.K., 2000. Seismic behavior of asymmetric buildings with supplemental damping, *Earthquake Engineering and Structural Dynamics*, 29, 461-480.

Goel, R.K., 2001. Simplified analysis of asymmetric structures with supplemental damping, *Earthquake Engineering and Structural Dynamics*, 30, 1399-1416.

Ibrahimbegovic, A., Chen , H.C., Wilson, E.L. and Taylor, R.L., 1990. Ritz method for dynamic analysis of large discrete linear systems with non-proportional damping, *Earthquake Engineering and Structural Dynamics*, 19(6), 877-889.

Igusa, T, Der Kiureghian, A. and Sackman, J.L., 1984. Modal decomposition method for stationary response of non-classically damped systems, *Earthquake Engineering and Structural Dynamics*, 12(1), 121-136.

Itoh, T., 1973. Damped vibration mode superposition method for dynamic response analysis, *Earthquake Engineering and Structural Dynamics*, 2, 47-57.

Lin, J.L. and Tsai, K.C., 2006a. Simplified seismic analysis of asymmetric building systems, *Earthquake Engineering and Structural Dynamics*, (accepted for publication).

Lin, J.L. and Tsai, K.C., 2006b. Simplified seismic analysis of one-way asymmetric elastic systems with supplemental damping, *Earthquake Engineering and Structural Dynamics*, (accepted for publication).

Moh, Z.L., Goldberg, J.E. and Bogdanoff, J.L., 1965. On modal distribution of damping, *Engineering Experimental Station Technical Bulletin No 74*, West Virginia University Bulletin.

Veletsos, A.S. and Ventura, C.E., 1986. Modal analysis of non-classically damped linear systems, *Earthquake Engineering and Structural Dynamics*, 14(2), 217-243.

Warburton, G.B. and Soni, S.R., 1977. Error in response calculations for non-classically damped structures, *Earthquake Engineering and Structural Dynamics*, 5(4), 365-376.