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INFLUENCE OF FRICTION MODELS ON OPTIMAL PERFORMANCE OF FRICTION DEVICES FOR ASEISMIC DESIGN OF BUILDINGS

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ABSTRACT

Hysteretic behavior of friction devices utilizing dry sliding has been typically modeled with Coulomb friction having a constant coefficient of friction. However, the basic laws for typical sliding materials and experimental investigations show non-linear relationship between friction and sliding velocity, which includes stiction and Stribeck effect. The friction models considered in this investigation are (1) Coulomb friction model (Model FM1) – frictional resistance at stick and sliding stage remain constant, and (2) Comprehensive friction model (Model FM2) – frictional model considers the effect of stiction and Stribeck effect. In this paper the importance and influence of stiction and Stribeck effect in determining the optimal performance of friction devices in reducing response under seismic excitations has been described.

Introduction

The design parameters that control the influence of friction devices used for aseismic design of structures are the damper locations in the building frame, slip loads at device level, and the stiffness of the braces in which the devices are installed. Filiatrault and Cherry (1990) have developed specialized algorithm to obtain the optimum slip-load distribution for the friction devices modeled as Coulomb's friction by minimizing a relative performance index (RPI) derived from energy concepts. They also developed slip-load spectrum for quick evaluation of optimum slip load. The spectrum takes into account the properties of the structure and of the ground motion anticipated at the site. From this study an important conclusion was drawn that the optimum slip load depends on the frequency and amplitude of the ground motions and is not strictly the structural property. It was also found to be linearly proportional to the peak ground acceleration (PGA). Moreschi (2000) and Asahina et al. (2004) have followed a genetic algorithm approach to obtain optimum slip-load at device level and optimal configuration within the frames. The available procedure to get optimum slip-load provides acceptable response reduction to the frame. But, the effect of different performance indices (from different response parameters) has not been addressed in these studies. The optimum slip-load may be different for different performance indices. The past studies have prescribed procedures for determining optimal slip load and distribution of friction devices within frame structures based on Coulomb friction model. Comprehensive friction model, which considers the effects of stiction and Stribeck, may alter the effectiveness of present practice. Earthquakes are also

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random phenomena with both uncertain intensities and frequency contents. So ground motion characterization also needs to be taken into consideration for robust configuration.

In this paper, the response of four-storey example frame building with friction devices has been investigated. The paper discusses the following aspects: (i) Evaluation of various dimensionless performance indices to characterize the seismic efficiency of friction devices, and (ii) Influence of stiction and Stribeck effect in simulating the optimal seismic performance of friction devices.

Dry Friction Models

The following two models have been considered in the investigation of response behavior of frame structure with friction device (Fig. 1), where *F* is the friction force and $\dot{u}$ is the relative velocity.

Coulomb Friction Model (Model FM1)

This is the most frequently used model, proposed over 200 years ago and is represented in Fig. 1(a). In this model, the coefficient of friction remains constant and the friction force is expressed as

$$F = \mu F_N \operatorname{sgn}(\dot{u}) \tag{1}$$

where F_N is the normal load (controllable prestress force) on the sliding surface, F is the frictional resistance same for both stick and sliding stage, μ is the coefficient of sliding friction, $\dot{u}$ is relative sliding velocity, and sgn($\dot{u}$) is the signum function that assumes a value of +1 for positive sliding velocity and –1 for negative sliding velocity.

Comprehensive Friction Model (Model FM2)

It has been observed through various experimental studies that friction force does not remain constant. The frictional resistance of dry friction modeled based on experimental and theoretical investigations by Wang and Shieh (1991) shows that the friction force during sliding, *F*, obeys the following exponential law:

$$\begin{aligned} F &= \left(F_d + (F_s - F_d)\, exp\left(-\left| \frac{\dot{u}}{\dot{u}_s} \right| \right) \right) sgn(\dot{u}) \\ F_d &= \mu_d F_N \\ F_s &= \mu_s F_N \end{aligned} \tag{2}$$

where F_d is the lower bound limit of the sliding frictional resistance and F_s is its upper bound limit, μ_d is the coefficient of sliding friction at relatively large velocity and μ_s is the coefficient of sliding friction at the point of zero velocity. *F* is the frictional resistance at sliding stage and varies from the upper bound limit (F_s) to the lower bound limit (F_d). The variation of friction resistance is a function of sliding velocity ($\dot{u}$) and Stribeck velocity ($\dot{u}_s$). The Stribeck velocity ($\dot{u}_s$) can be regarded as the decay rate of the sliding friction coefficient. The typical friction force variation for realistic friction model is shown in Fig. 1(b).

Mathematical Formulation of Equations of Motion

The mathematical formulation of multi-degree-of-freedom (MDOF) frame structure with friction slider mounted on Chevron brace (Fig. 2) has been presented herein. The structure is considered as a two-dimensional (2-D) shear building. Two degrees-of-freedom are present on each floor, corresponding to the horizontal displacement of the storey and the brace, respectively, relative to the ground, as shown in

Fig. 2(a). Simple friction energy-dissipation devices with slotted bolted connection (SBC) has been considered, where the sliding plate within the vertical plane is connected to the centerline of beam soffit as shown in Fig. 2(b). So it may be noted that the structure weight does not have any effect to the normal load. The sliding plate having slotted holes is sandwiched between two clamping plates. The clamping plates are rigidly mounted on the Chevron brace and connected to the sliding plate through prestressed bolts. The slotted holes facilitate the sliding of the sliding plate over the frictional interface at a constant controllable prestress force. The placement of sliders in vertical plane of the beam ensures that only the prestress force controls the normal load on the sliding surface. The presence of two friction interfaces for each bolt doubles the frictional resistance. In the formulation of the MDOF structure, the structure degrees-of-freedom is denoted with subscript f and the brace with device degrees-of-freedom with subscript d. Two lumped mass models, one for the free frame structure and another for the brace with device, are required to idealize the dynamic behavior of the structure.

Through the entire solution process, the equations of motion are split into two subsets with sub-indices *st* representing the stick phase (non-sliding phase) and *sl* representing the sliding phase respectively. The motion of any storey of the structure consists of either of two phases: (1) non-sliding or stick phase wherein the frictional resistance ($\mathbf{F_{st}}$) between the floor and the device has not been overcome, and (2) sliding or slip phase in which sliding frictional resistance ($\mathbf{F_{sl}}$) exceeds and the friction force, and acts opposite to the direction of the relative velocity between the floor and friction device. Linear behavior of the structure with friction devices is assumed at both stick and sliding stage of response. The overall response for each storey consists of series of non-sliding and sliding phases. The number of active degree of freedom ranges between N (all the devices in non-sliding phase) and $2N$ (all devices are in sliding phase). If the total number of non-sliding floors are denoted by n_{st} and total number of sliding floors n_{sl}, then the total number of degrees of freedom at any instant of time is equal to $n_{st}+2\times n_{sl}$.

The generalized governing equations of motion in matrix form can be given as:

$$\mathbf{M\ddot{u}}_{st+sl} + \mathbf{C\dot{u}}_{st+sl} + \mathbf{Ku}_{st+sl} = -\mathbf{Mr}\ddot{u}_g - \mathbf{F}_{f+sl} \tag{3}$$

where **M**, **C**, and **K** are the mass, damping and stiffness matrices, respectively, **r** is the force-influence vector, **u** represents the displacement degrees of freedom relative to the base of the structure and u_g is the ground displacement. The over dot represent derivatives with respect to time. The friction force vector is represented as **F**, and the matrices are given as:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_f & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_d \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \mathbf{C}_f + \mathbf{C}_{d2} & \mathbf{C}_{d3} \\ (\mathbf{C}_{d3})^T & \mathbf{C}_{d1} \end{bmatrix}, \mathbf{K} = \begin{bmatrix} \mathbf{K}_f + \mathbf{K}_{d2} & \mathbf{K}_{d3} \\ (\mathbf{K}_{d3})^T & \mathbf{K}_{d1} \end{bmatrix},$$
$$\mathbf{u}_{st+sl} = \begin{Bmatrix} \mathbf{u}_{f,st+sl} \\ \mathbf{u}_{d,st+sl} \end{Bmatrix}, \mathbf{r} = \begin{Bmatrix} \mathbf{r}_f \\ \mathbf{r}_d \end{Bmatrix}, \mathbf{F} = \begin{Bmatrix} +\mathbf{F}_{f+sl} \\ -\mathbf{F}_{f+sl} \end{Bmatrix}, \mathbf{r}_f = \mathbf{1}, \mathbf{r}_d = \mathbf{1} \tag{4}$$

where

$$\mathbf{M}_d = \begin{bmatrix} m_{d1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & m_{dN} \end{bmatrix},$$
$$\mathbf{C}_{d1} = \begin{bmatrix} c_{d1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & c_{dN} \end{bmatrix}, \mathbf{C}_{d2} = \begin{bmatrix} c_{d2} & 0 & 0 \\ 0 & \ddots c_{dN} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{C}_{d3} = \begin{bmatrix} 0 & -c_{d2} & 0 \\ 0 & 0 & \ddots -c_{dN} \\ 0 & 0 & 0 \end{bmatrix}, \tag{5}$$

$$\mathbf{K}_{d1}=\begin{bmatrix}k_{d1} & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & k_{dN}\end{bmatrix},\ \mathbf{K}_{d2}=\begin{bmatrix}k_{d2} & 0 & 0\\ 0 & \ddots k_{dN} & 0\\ 0 & 0 & 0\end{bmatrix},\mathbf{K}_{d3}=\begin{bmatrix}0 & -k_{d2} & 0\\ 0 & 0 & \ddots -k_{dN}\\ 0 & 0 & 0\end{bmatrix}$$

In the above equations, $\mathbf{M_f}$, $\mathbf{C_f}$, and $\mathbf{K_f}$ are the $N \times N$ mass, damping and stiffness matrices of the structure excluding the bracing members, $\mathbf{M_d}$, $\mathbf{C_{d1}}$, $\mathbf{C_{d2}}$, $\mathbf{C_{d3}}$, $\mathbf{K_{d1}}$, $\mathbf{K_{d2}}$ and $\mathbf{K_{d3}}$ are $N \times N$ mass, damping and stiffness matrices of the brace with friction device, respectively. The damping property of the free frame (excluding the brace with device) structure may be different from the same of the brace with device. So the complete structure is non-classical damped system. The non-classical damping matrix [**C**] for the structure is obtained by first evaluating the classical damping matrix for the free frame, [$\mathbf{C_f}$], based on the damping ratios appropriate for the structure (Chopra 2001).

The structure and the brace degree of freedoms (DOFs) at any storey satisfy the following conditions during the stick phase:

$$\begin{aligned}\ddot{\mathbf{u}}_{f,st} &= \ddot{\mathbf{u}}_{d,st}\\ \dot{\mathbf{u}}_{f,st} &= \dot{\mathbf{u}}_{d,st}\\ \mathbf{u}_{f,st}-\mathbf{u}_{d,st} &= \text{constant}\end{aligned} \tag{6}$$

In Eq. 3, stick or non-sliding phase of a particular floor requires that the corresponding friction force satisfy the equation,

$$\left|F_{f,st}\right| < F_{st} \tag{7}$$

The friction force vector consisting of the friction force in all the devices is given by:

$$\begin{aligned}\mathbf{F}_{f,st} = {} & \mathbf{M}_{f,st}\ddot{\mathbf{u}}_{f,st+sl}+\mathbf{M}_{f,st}\mathbf{r}_f\ddot{u}_g+(\mathbf{C}_{f,st}+\mathbf{C}_{d2,st})\dot{\mathbf{u}}_{f,st+sl}+\\ & \mathbf{C}_{d3,st}\dot{\mathbf{u}}_{d,st+sl}+(\mathbf{K}_{f,st}+\mathbf{K}_{d2,st})\mathbf{u}_{f,st+sl}+\mathbf{K}_{d3,st}\mathbf{u}_{d,st+sl}\end{aligned} \tag{8}$$

In Eq. 8, $\mathbf{F_{f,st}}$ is the vector of frictional resistance of all friction devices at stick stage. When the condition in Eq. 7 is not satisfied for any floor, that floor enters into the sliding phase. Then the corresponding brace degree-of-freedom at the floor level also becomes active in the equations of motion.

The maximum frictional resistance in stick stage (F_{st}) and frictional resistance in sliding stage (F_{sl}) for a friction device for different friction models are given by Eq. 1 and Eq. 2, respectively. The direction of sliding of a brace degree of freedom can be found from the following relationship:

$$\text{sgn}(\dot{u}_f - \dot{u}_d) = -\frac{F_{f,st\,max}}{\left|F_{f,st\,max}\right|} \tag{9}$$

The response of the structure always starts in the stick phase. This phase of response continues until the unbalanced frictional resistance of any floor exceeds the maximum frictional resistance of the brace with device at that floor. It is important to note that the number of stories experiencing stick and sliding conditions varies continuously through the entire response phase. When the relative sliding velocity ($\dot{u}_f$-$\dot{u}_d$) at any floor becomes zero or changes its sign during motion, then the brace with device at that storey may or may not enter the stick phase. It may reverse its direction of sliding or have a momentary halt and continue in the same direction. The status of motion during transition phase can be evaluated from Eq. 7. The equations of motion corresponding to the floor is changed to the appropriate stick or sliding equations before evaluating response during the next time-step.

Performance Indices

To characterize the effectiveness of the friction devices, six dimensionless performance indices have been considered. All these indices are defined as the ratios between the peak values of a certain response quantity (displacements, acceleration, base shear, strain energy, input energy, and dissipated energy) of the frame with friction devices, and the peak value of same responses of the bare or braced frame structure. The response quantities in the indices are peak quantities for full time-history of response and among all the floors. Consequently, these indices are dimensionless and always positive with their value range usually between 0 and 1. Values close to zero indicate excellent performance of the friction devices in reducing the maximum response while values close to 1 or higher indicate ineffectiveness of the friction devices. In the present formulation, by considering stick or sliding frictional resistance (slip load) equal to zero, the response for free frame structure can be obtained. Similarly the response of brace frame structure can be obtained by considering stick or sliding frictional resistance (slip load) as infinitely large. The following different indices have been considered for the performance evaluation:

(i) Inter-storey drift ratio (IDR): This ratio is expressed as

$$IDRF = \frac{\text{Peak inter-storey drift of the structure with friction devices}}{\text{Peak inter-storey drift of the free frame structure}},$$
$$IDRB = \frac{\text{Peak inter-storey drift of the structure with friction devices}}{\text{Peak inter-storey drift of the braced frame structure}} \tag{10}$$

This ratio accounts for the reduction of inter-storey drift response that mostly characterizes the level of damage in structural members and in the non-structural members including the occupational and functional components.

(ii) Absolute acceleration ratio (AAR): This ratio is defined as

$$AARF = \frac{\text{Peak absolute acceleration of the structure with friction devices}}{\text{Peak absolute acceleration of the free frame structure}},$$
$$AARB = \frac{\text{Peak absolute acceleration of the structure with friction devices}}{\text{Peak absolute acceleration of the braced frame structure}} \tag{11}$$

This ratio accounts for the reduction of acceleration response that mostly characterizes the level of damage in non-structural members and human comfort conditions.

(iii) Drift and acceleration ratio (DAR): This ratio is defined as (Moreschi 2000)

$$DARF = \frac{1}{2}\left(\frac{\text{Peak inter-storey drift of the structure with friction devices}}{\text{Peak inter-storey drift of the free frame structure}} + \frac{\text{Peak absolute acceleration of the structure with friction devices}}{\text{Peak absolute acceleration of the free frame structure}}\right),$$
$$DARB = \frac{1}{2}\left(\frac{\text{Peak inter-storey drift of the structure with friction devices}}{\text{Peak inter-storey drift of the braced frame structure}} + \frac{\text{Peak absolute acceleration of the structure with friction devices}}{\text{Peak absolute acceleration of the braced frame structure}}\right) \tag{12}$$

It is noted that this parameter gives equal weight to the deformation and acceleration related responses and represents a combined contribution of the two factors.

(iv) Base shear ratio (BSR): This ratio is defined as

$$BSRF = \frac{\text{Peak base shear of the structure with friction devices}}{\text{Peak base shear of the free frame structure}},$$
$$BSRB = \frac{\text{Peak base shear of the structure with friction devices}}{\text{Peak base shear of the braced frame structure}} \tag{13}$$

The base shear is used as a basic design parameter and low values of BSR indicate corresponding reduction in design earthquake forces.

(v) Relative performance index (RPI): This ratio is defined as (Filiatrault and Cherry 1990)

$$RPIF = \frac{1}{2}\left(\frac{SEA}{ASEF} + \frac{SEM}{SEMF}\right),$$
$$RPIB = \frac{1}{2}\left(\frac{SEA}{ASEB} + \frac{SEM}{SEMB}\right) \tag{14}$$

where *SEA* = strain energy area, is the area under the strain-energy time history for the system with friction devices, *ASEF* = strain energy area for free frame structure, *ASEB* = strain energy area for braced frame structure, *SEM* = peak strain energy for the system with friction devices, *SEMF* = peak strain energy for the free frame structure, and *SEMB* = peak strain energy for the braced frame structure. If *RPIF* or *RPIB* is equal to 1, then the response corresponds to the behavior of a free or braced frame structure. This index behaves similarly to the IDR as it is derived from strain energy concept. The information obtained from RPI is similar to the information provided by IDR and is often used in published literature.

(vi) Index for Work Done by devices (IWD): This index is defined as

$$IWD = 1 - \frac{W_D}{W_I} \tag{15}$$

where W_D = area of cumulative frictional work done, and W_I = area of cumulative input energy. For both the bare frame and braced frame structures this index is equal to unity. This index does not directly represent the reduction of structural response, but quantifies the effectiveness of the friction devices in dissipating energy.

All the performance indices have been normalized with respect to both the free frame and the braced frame structural responses. This enables assessment of response reduction and performance enhancement of the structures with friction devices while also including the stiffening influence of the bracing system.

Example System

A four-storey steel frame building with friction devices (Fig. 2) has been considered for evaluating the seismic performance of friction devices. The braces with damping devices exhibit highly non-linear behavior. The effect of energy dissipation due to viscous damping in the brace members is normally very small compared to the work done by friction sliding. So the viscous damping in the brace has been neglected. The structural damping ratios of the free-frame have taken as 2% of its critical damping. The example building has the following member properties (Dimova et al. 1995):

Floor masses: $m_{f1} = m_{f2} = m_{f3} = 41610.0\,\text{kg}$, and $m_{f4} = 40820.0\,\text{kg}$
Moment of Inertia of ground and first floor columns: 8740.8 cm^4
Moment of Inertia of second and third floor columns: 7117.5 cm^4
Moment of Inertia of roof girders: 12486.0 cm^4
Moment of Inertia of floor girders: 15567.0 cm^4
Moment of Inertia of bracing members: 77.8 cm^4
Cross sectional area of bracing members: 7.57 cm^2
Mass of bracings and friction dampers: $m_{d1} = m_{d2} = m_{d3} = m_{d3} = 23.0\,\text{kg}$
Stiffness of bracing members: $k_{d1} = k_{d2} = k_{d3} = k_{d4} = 28.6\,\text{MN/m}$

The fundamental time period of the free frame and braced frame buildings are 1.00 s and 0.56 s, respectively. The total normal load (F_N) on the sliding surface is equal to $2n_bF_{Ni}$, where n_b is the number of prestress bolts, and F_{Ni} is the prestress force in a single bolt. All the bolts in a particular friction device are assumed to have the same prestress force. The value of coefficient of friction has been considered for steel on steel slider as reported by Bilkay and Anlagan (2004). Based on their investigations the following friction parameters have been used for friction models: (1) Minimum coefficient of sliding friction (sliding stage, μ_d) = 0.16, (2) Maximum coefficient of sliding friction (stick stage, μ_s) = 0.29, and (3) Stribeck velocity $\dot{u}_s$ = 0.025 m/s.

In this investigation, the time-history responses of the example system have been evaluated for an ensemble of nine earthquake records (Patro 2006). The ground motions chosen in this study cover wide variety of earthquakes having different peak ground accelerations (PGA), frequency content and duration. Three time histories have been selected for each of soft soil (FSR1-3), alluvium soil (FMR1-3), and hard soil (FHR1-3).

Evaluation of Performance Indices

The performance of the friction devices has been evaluated for a range of prestress forces. The prestress force has been varied from 1% (F_N = 16 kN) to 51% (F_N = 825 kN) of total floor weight (1625 kN) of structure. The same prestress force has been applied to all the friction devices. The performance indices used to evaluate the performance of friction devices must represent the overall response of the building. The various performance indices, which are discussed earlier, have been evaluated for comprehensive friction model (Model FM2) for the example building subjected to all nine ground motions, and their results are shown in Fig. 3(a-i). In Fig. 3(a-i) performance indices are normalized with fixed braced frame response. It has been observed that the optimum prestress force varies between indices. It is found that optimum prestress force at indices using absolute floor acceleration differs significantly from those using floor displacements or inter-storey drift. The widely used Relative Performance Index (RPI) closely matches the pattern of drift response index (IDR) and may not be a suitable representative of the overall response. It should also be noted that the uses of friction devices reduce the structural responses through a combination of energy dissipation and increase in lateral stiffness. The proper selection of prestress force or slip load in the friction devices can simultaneously reduce both the inter-storey drift and the acceleration responses. Therefore, the performance index of Eq. 12 (DAR), that considers both the acceleration and the drift, is most useful. By minimizing the drift acceleration ratio, there is reduction in the peak inter-storey drifts, coupled with reduction in the peak structural accelerations. The other performance indices such as Base Shear Ratio (BSR) and Index for Work Done (IWD) also represent the overall structural response and are useful in assessing the effectiveness of the friction devices.

The Drift Acceleration Ratio (DAR) has been used to evaluate the difference between the comprehensive friction model (Model FM2) and Coulomb friction model (Model FM1) for prediction of optimal response of the example system. The results are shown in Fig. 4(a-i) after normalization with free frame structure response. It can be seen in Fig. 4 that the optimal structural performance is obtained for prestress force in the range of 300 kN – 500 kN for all ground motions except FMR1 and FHR1 when considering Coulomb friction model (Model FM1). In contrast it can be seen that for all ground motions the prestress force of 100 kN – 300 kN provides the maximum reduction in responses when considering Model FM2. It is to be noted that the optimum prestress force for model FM1 is approximately fifty percent more that of the optimum prestress force for model FM2. The results in Fig. 4 also show that the use of Coulomb friction model consistently underestimates the response, thereby overestimating the effectiveness of the friction devices. This clearly demonstrates that the use of Coulomb friction model is not very appropriate for the design of structures with friction devices.

Discussions and Conclusions

The paper presents the aseismic response behavior of structures with friction devices. The friction force has been modeled using Coulomb friction model (Model FM1), and the comprehensive friction model (Model FM2) that includes both stiction and Stribeck effects. An ensemble of nine ground motions recorded on different soil conditions has been considered to evaluate the effectiveness of friction devices for vibration control. It is found that the prestress force (normal load on sliding surface) is the most important parameter for the design of the friction devices. It is also observed that comprehensive friction model influences on the evaluation of optimal prestress force to reduce the seismic response of the system.

Based on the investigations presented in this paper, the following main conclusions can be drawn:

- Most structures with friction-based energy dissipation systems are designed based on Coulomb friction; however behavior of actual friction is more complex and includes the stiction and Stribeck effects. The friction model should represent the actual behavior.
- The optimal performance is influenced by the friction model. The use of comprehensive model (FM2) results in lower optimal prestress force as compare to Model FM1.
- Among various indices, the drift acceleration ratio is suitable for evaluating the optimal prestress force for friction-based supplemental system.
- The optimum prestress force may be very different for different response quantities of interest such as relative floor displacement, storey drift or absolute floor acceleration.

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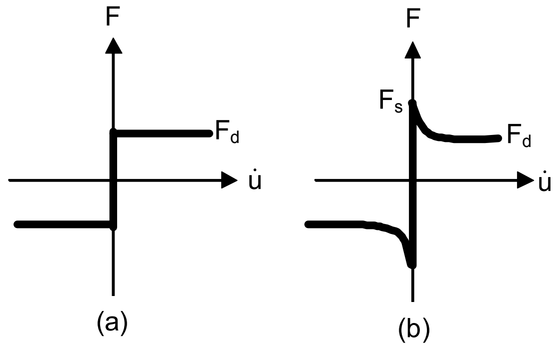

Figure 1. Dry friction model.

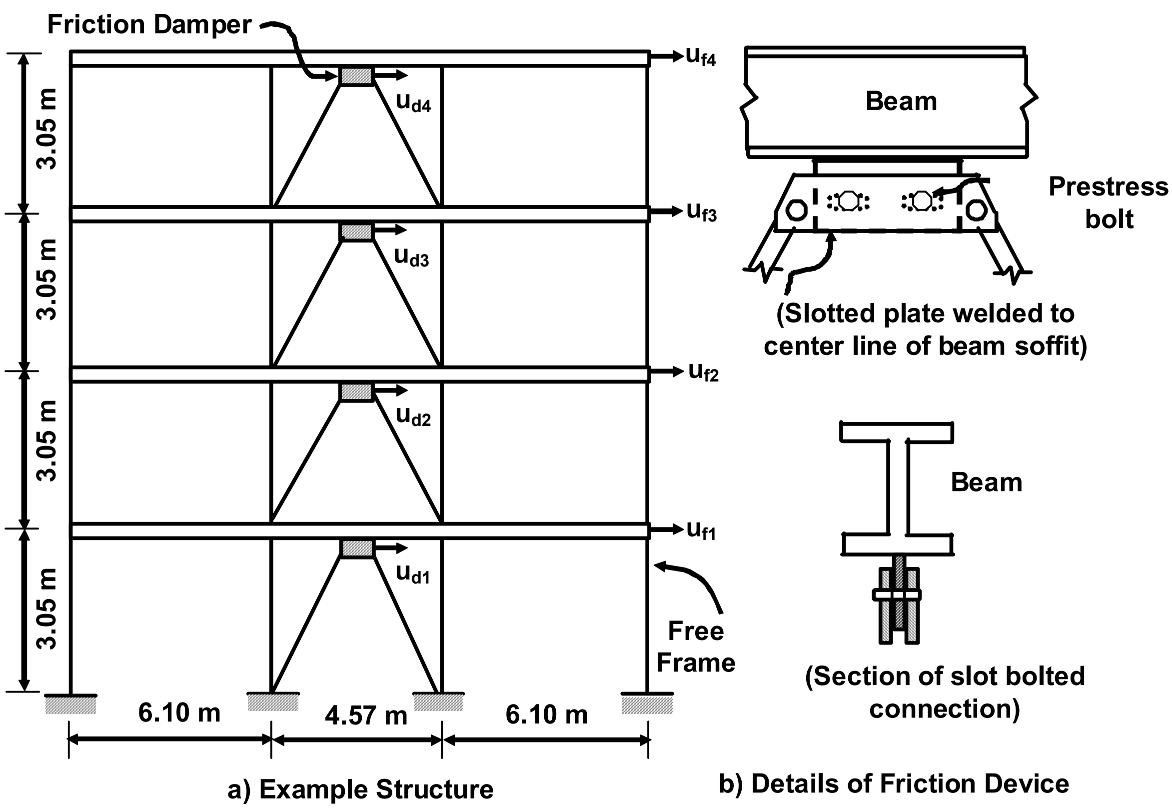

Figure 2. Schematic diagram of four-storey building with friction devices (Dimova et al. 1995).

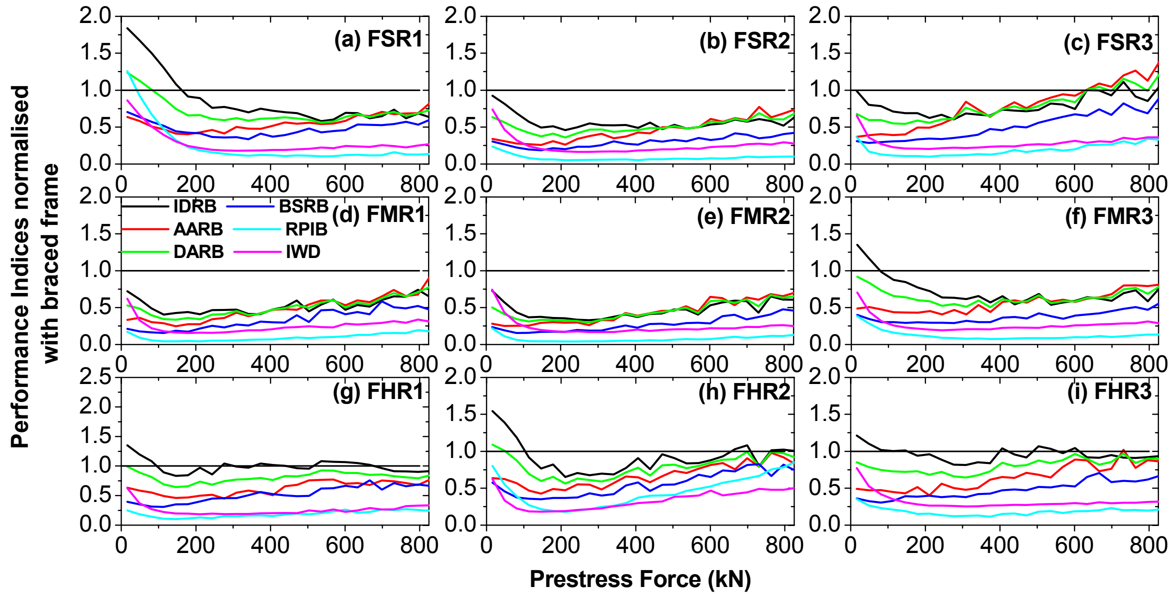

Figure 3. Performance indices, normalized with braced frame structure response, for different ground motions (Friction Model FM2).

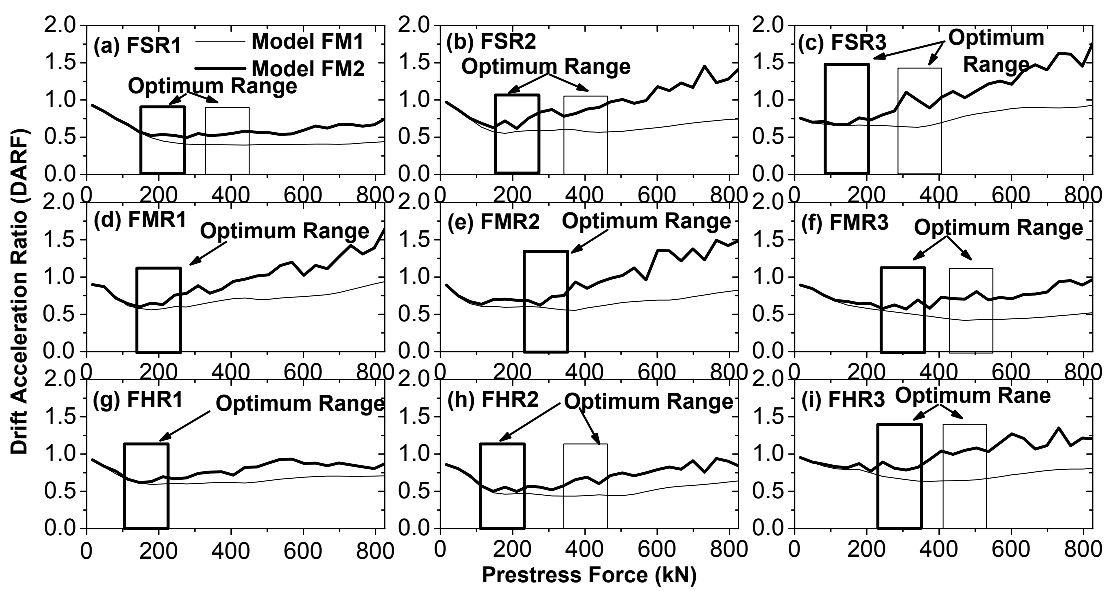

Figure 4. Drift acceleration ratio normalized with free frame structure response (DARF) subjected to different ground motions.