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INELASTIC SEISMIC RESPONSE OF BUILDINGS BASED ON A MODAL PUSHOVER ANALYSIS

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ABSTRACT

A newer analysis technique called modal pushover analysis has been proposed recently for inelastic seismic analysis of multi-story buildings. In this method, the modal response of a structure is first obtained by subjecting the structure to pushover analyses using loads corresponding to different modal responses. The total response of the structure is then obtained by square-root-of-sum-of-square (SRSS) technique. This paper is concerned with the verification of the modal pushover analysis technique as applied to inelastic seismic response of a 40-story building. The first three modes of vibrations considered in this study. The story overturning moments obtained through the modal pushover analysis were compared to the corresponding moments obtained through the inelastic modal decomposition method, which indicated a good correlation between the two methods. The pushover analyses results were also compared to the corresponding results obtained through nonlinear direct integration time history analyses. The paper presents a detailed comparison between modal pushover analysis results and the inelastic modal decomposition results. Based on these comparisons the validity of the assumptions and approximations associated with the modal pushover analysis had been discussed.

Introduction

The nonlinear static seismic analysis and design procedure, known as the pushover analysis, uses a simplified nonlinear analysis to estimate the seismic demands of structures. This analysis method is based on the suggestion that the response of a multi degree of freedom structure can be related to the response of an equivalent single degree of freedom (SDOF) system. This implies that the response is controlled by a dominant single mode, and that the shape of this mode remains constant throughout the time history. Since the response of low-rise structures is generally dominated by the single mode of vibration, pushover analysis, in general, lead to good predictions of seismic demands of low-rise structures (Seneviratna and Krawinkler, 1997). Response of high-rise structures is, however, controlled by higher modes of vibrations. As the height of structures increases, the assumptions associated with the conventional pushover analysis become inappropriate. It has been found that for taller structures pushover analysis gives very different results compared to the time history analysis. Researchers have identified that the differences in results are due to the influence of higher modes of vibration (Sangarayakul and Warnitchai, 2004).

Various studies have been conducted to improve the performance of a conventional pushover analysis when applied to multi-story structures. Some researchers (Bracci et al., 1997; Gupta and Kunnath, 2000)

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have focused on an adaptive force distribution procedure. Chopra and Goel, (2002) presented a modal pushover analysis procedure to incorporate the higher mode effects on the seismic demands of high-rise structures. This procedure comprises of separate pushover analysis for each mode of vibration. These modal results are then combined by the square-root-of-sum-of-square (SRSS) combination rule. The modal pushover analysis procedure was evaluated by Chintanapakdee and Chopra, (2003) using three to eighteen-storyed framed buildings under various ground motions. The total responses (mainly story drift) were directly compared with the responses from nonlinear time history analysis, hereafter called as direct integration (DI) method. Since there were no other analysis tools available to evaluate the modal responses based on the modal pushover analysis procedure, the contribution of errors from separate modes was unclear. Recently, Sangarayakul and Warnitchai, (2004) have developed an approximate method for decomposing total inelastic dynamic responses of tall buildings into simple modal responses. Decomposed inelastic modal responses are advantageous to evaluate each mode demand estimated by the modal pushover analysis procedure. The primary objective of this study was to evaluate validity of the the modal pushover analysis using inelastic modal decomposition method.

Modal Pushover Analysis and Underlying Assumptions

The modal pushover analysis procedure consists of separate pushover analysis for each mode of vibration. It uses the force patterns similar to the corresponding elastic mode shapes. The modal pushover analysis procedure is based on several assumptions. The following assumptions were made during the development of modal pushover analysis procedure.

- The coupling between modes arising from yielding of the system can be neglected.
- The elastic mode-shapes-force-pattern can approximately produce inelastic modal displacements after the structure yields.
- SRSS combination rule can be applied to combine inelastic modal responses.

Most of these assumptions are necessary so as to be able to use the dynamic properties of elastic structures in an inelastic static analysis. But in reality, the structures may behave in a very different manner in the inelastic range. Therefore, errors may arise from the use of the above assumptions; however, these assumptions might be valid under certain circumstances. The modal pushover analysis procedure sounds intuitively plausible as long as the assumptions are valid. These assumptions are acceptable within the elastic limit. But beyond the elastic limit, these assumptions are not strictly valid. Note that in linear elastic system, response can be easily uncoupled by using modal matrix. Each modal response is independent to each other. For inelastic system, there might be strong contribution on modal response from other modes. These coupling between inelastic modal responses are neglected during the development of the modal pushover analysis procedure. But strong coupling has been observed, when the structure is subjected under some ground motions. The SRSS combination rule also misleads the results at the plastic hinges especially when the structure yields in more than one mode.

Response History for Inelastic MDOF System

The differential equations governing the response of a multistory inelastic building to horizontal earthquake ground motion are given by

$$\boldsymbol{m}\ddot{\boldsymbol{u}} + \boldsymbol{c}\dot{\boldsymbol{u}} + \boldsymbol{f}_s = -\boldsymbol{m}\boldsymbol{i}\ddot{u}_g(t) \quad (1)$$

and,

$$\boldsymbol{f}_s = \boldsymbol{f}_s(\boldsymbol{u}, sign\ \dot{\boldsymbol{u}}) \quad (2)$$

where, $\boldsymbol{u}$ is the vector of N lateral floor displacements relative to the ground; $\boldsymbol{m}$ and $\boldsymbol{c}$ are the mass and classical damping matrices of the system; $\boldsymbol{i}$ is influence vector whose each element is equal to unity. $\boldsymbol{f}_s$ represents the lateral resisting force vector induced by stiffness.

Expanding the displacements of the inelastic system in terms of the natural vibration modes of the corresponding linear system, we get

$$\boldsymbol{u}(t) = \sum_{n=1}^{N} \boldsymbol{\phi}_n q_n(t) \tag{3}$$

Initially mass, damping and stiffness of the structural element of the inelastic system can be defined by same as in elastic system. Hence mass- and classical damping-orthogonality property can be used for the inelastic system within the elastic range. Substituting Eq. 3 into 1, pre-multiplying by $\boldsymbol{\phi}_n^T$, and using orthogonality properties, we get

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \frac{F_{sn}}{M_n} = -\Gamma_n \ddot{u}_g(t) \tag{4}$$

where, $$F_{sn} = F_{sn}(\boldsymbol{q}, sign\ \dot{\boldsymbol{q}}) = \boldsymbol{\phi}_n^{\mathrm{T}} \boldsymbol{f}_s(\boldsymbol{u}, sign\ \dot{\boldsymbol{u}}) \tag{5}$$

and, $$\Gamma_n = \frac{L_n}{M_n}, \quad L_n = \boldsymbol{\phi}_n^{\mathrm{T}} \boldsymbol{m}\boldsymbol{i}, \quad M_n = \boldsymbol{\phi}_n^{\mathrm{T}} \boldsymbol{m}\boldsymbol{\phi}_n \tag{6}$$

The resisting force F_{sn} depends on the modal co-ordinates $\boldsymbol{q_n}(t)$, implying coupling of modal co-ordinates because of yielding of the structure.

Target Roof Displacements

The basis for developing a modal pushover analysis procedure is the uncoupled modal response history analysis procedure. It is assumed that coupling between modal co-ordinates $\boldsymbol{q_n}(t)$ arising from yielding of the system can be neglected. Note that resisting force in Eq. 5 depends upon modal co-ordinate vector $\boldsymbol{q}$ and sign of its derivative $\dot{\boldsymbol{q}}$. But the assumption implies that nth mode modal co-ordinate $\boldsymbol{q_n}(t)$ has dominant contribution to develop resisting force in corresponding mode. The effects of other modes can be neglected. Hence resisting force for nth F_{sn} can be only the function of q_n and sign of $\dot{q}_n$. Then the Eq. 5 can be written as,

$$F_{sn} = F_{sn}(q_n, sign\ \dot{q}_n) \tag{7}$$

With this simplification and replacing $q_n(t)$ by $\Gamma_n D_n(t)$, Eq. 4 can be rewritten in the form of nth mode inelastic SDOF system as follows.

$$\ddot{D}_n + 2\zeta_n \omega_n \dot{D}_n + \frac{F_{sn}}{L_n} = -\ddot{u}_g(t) \tag{8}$$

where,

$$F_{sn} = F_{sn}(D_n, sign\ \dot{D}_n) = \boldsymbol{\phi}_n^{\mathrm{T}} \boldsymbol{f}_s(D_n, sign\ \dot{D}_n)\} \tag{9}$$

Eq. 8 represents the equation of motion for an inelastic SDOF system. Here, ζ_n and ω_n may be interpreted as natural frequency and damping ratio of the nth-mode of corresponding linear MDOF system. Eq. 9 gives force deformation relation of nth-mode inelastic SDOF system. Solution of Eqs. 8 and 9 through out the time domain is termed as modal response history analysis procedure procedure. Then the target roof displacement for nth mode is determined by,

$$u_{rno} = \Gamma_n \phi_{rn} D_n \quad (10)$$

where,

$$D_n = \max |D_n(t)| \quad (11)$$

Now, the pushover analysis for each mode is performed where the structure is subjected to monotonically increasing lateral forces associated with nth mode with an invariant height-wise distribution until the target roof displacement of corresponding mode, u_{rno}, is reached. Note that in linear elastic system, contribution on nth mode from other modes becomes zero. But for inelastic system there may have significant contribution on modal response from other modes. Such possible contributions from other modes are neglected during the development of modal pushover analysis procedure. As a result, it may lead for a possible source of significant error in modal pushover analysis procedure.

Evaluation Techniques

Inelastic Modal Decomposition

The inelastic modal decomposition was used to decompose total inelastic dynamic responses of wall buildings into approximate modal responses. The method is based on the equivalent linear concept, where a nonlinear structure is represented by a set of equivalent linear models. One linear model is used for representing only one vibration mode of the nonlinear structure. Linear properties of models are determined with the help of total inelastic response from nonlinear time history analysis. The mode shape and modal properties (frequency, damping, etc) of each individual mode are directly identified from the inelastic response time histories. The inelastic modal decomposition method was used, here, for the evaluation of the MPA procedure.

Properties of Linear Models

A 40-storyed wall building was modeled by a simplified lumped mass system as shown in Figs. 1 (a) and (b). The structural modeling of this building is described later. Inelastic modal responses of this simplified system are represented by series of linear models shown in Fig. 1 (c). Linear model used here are Bernoulli-Euler cantilever beam having distributed physical properties. The beam span is divided into the plastic hinge region ($0 \le x \le l_p$), and the remainder, elastic region ($l_p < x \le h$). Within each region, the flexural stiffness *EI*, the mass per unit length μ , and the stiffness-proportional damping *G* are uniformly distributed. The governing equation for such system under the ground motion $\ddot{u}_g(t)$ is given by,

$$\mu \ddot{u} + a_0 \mu \dot{u} + G \dot{u}^{iv} + EIu^{iv} = -\mu \ddot{u}_g \quad (12)$$

where

$$EI = \begin{cases} EI_e & \text{for } l_p < x \le h \\ EI_p & \text{for } 0 \le x \le l_p \end{cases} \quad (13)$$

and
$$G = \begin{cases} a_1\, EI_e & \text{for } \; l_p < x \le h \\ a_1\, EI_e + c\, EI_p & \text{for } \; 0 \le x \le l_p \end{cases} \tag{14}$$

where $u(x,t)$ is the transverse displacement response relative to the base. Primes and Roman numeral denote an x derivative and over-dots denote a time derivative. When the responses are confined within the limit of linear elastic behavior, EI_p in the plastic hinge region is equal to EI_e, and the damping factor c is zero. Eq. 12 then represents a uniform cantilever beam with the Rayleigh damping applied proportionally to mass and stiffness via the factors a_0 and a_1 respectively. These factors are assigned such that a damping ratio of 5% is obtained in the first two modes. When the responses exceed the elastic limit, normally the flexural rigidity in the plastic hinge zone decreases ($EI_p \le EI_e$) and the additional damping factor c increases ($c \ge 0$). Eq. 12 can only be solved using complex mode analysis

Initially, the EI_p and c of linear models are unknown. To consider responses of n mode, we have to solve n sets of Eq. 12. First three modes were considered in this study. We have n equations to solve for 2n variables (n numbers of EI_p and n numbers of c). It is not possible to solve such equations by straight forward manner. One possible way is to iterate these 2n parameter values until the sum of all modal responses computed from these n linear models best match with the total inelastic response computed from the nonlinear time history analysis. The response quantity used here in this matching is bending moment. The index to quantify the accuracy of the matching is therefore given by:

$$e = \int_{t_a}^{t_b}\int_{0}^{h} \left[M_M(x,t) - M_E(x,t) \right]^2 dx\, dt \tag{15}$$

where M_M is the total bending moment computed from nonlinear time history analysis, M_E is the sum of modal bending moments computed from n equivalent linear models, t_a is the starting time of the first cycle of flexural yielding, and t_b is the ending time of the last cycle of flexural yielding.

The parameter identification begins by assuming the initial values of EI_{pi} and c_i of each equivalent linear model, where i = 1 to n. The corresponding equations of motion of the form shown by Eq. 12 are then solved, and the corresponding coordinates $u_i(x,t)$ are obtained. By this way, bending moment contributed by the i^{th} mode, M_{Ei}, can be determined. Once all modal bending moment from n equivalent linear models are determined, the total bending moment (M_E) can be computed by:

$$M_E(x,t) = \sum_{i=1}^{n} M_{Ei}(x,t) = \begin{cases} \displaystyle\sum_{i=1}^{n} EI_e\, u_i''(x,t) & \text{for } \; l_p < x \le h \\ \displaystyle\sum_{i=1}^{n} EI_{pi}\, u_i''(x,t) + c_i EI_{pi}\, \dot{u}_i''(x,t) & \text{for } \; 0 \le x \le l_p \end{cases} \tag{16}$$

At this stage, the index e can be computed by using Eq. 15. If e is higher than an acceptable value, then all 2n parameter values will be adjusted and the whole process will be repeated until the acceptable value of e is attained.

Modal Responses

Best parameters EI_p and c of each linear model that minimize the value of e in Eq. 15, are used to compute model responses. With these parameter Eq. 12 is solved using complex mode analysis to get modal displacement time history, $u(x,t)$. Other modal responses time history (moment $M(x,t)$ and shear $V(x,t)$) are computed by,

$$M_i(x,t) = EI\, u_i''(x,t) \quad \text{and} \quad V_i(x,t) = EI\, u_i'''(x,t) \tag{17}$$

Numerical Investigation

Structural Modeling

A forty-story building was selected for the case study. Height of each floor was set as 3.65 meter. The structural configuration was assumed in such a way that the lateral-load-resisting system was provided by slender reinforced concrete structural walls and the gravity-load-carrying system was provided by building frames. Modes of lateral deformation were generally controlled by flexural bending of walls because the lateral stiffness of walls was normally much higher than that of the frames. The building was modeled as a vertical cantilever beam with equal masses lumped at each story level as shown in Fig. 1 (b). Beam elements were used to model the section of wall between adjacent floors. To achieve the desirable modes of inelastic deformation, a ductile flexural plastic hinge zone was allowed to form only at the base of the structure when the yield moment, M_y, was exceeded. It is expected that the propagation of plastic hinge is limited up to 10% of the total height from the base. Well known bilinear material model with 5% strain hardening was used to represent the hysteretic inelastic moment–rotation relation for all plastic hinges. The structure considered in this study was designed in accordance with UBC, (1997). The fundamental period of structure assumed to be equal to the value given by Uniform Building Code, i.e. T = 0.0488 $h^{3/4}$, where h is the total wall height in meters. The viscous damping was represented by Rayleigh damping, with 5% damping ratio in the first two modes. SAP-2000, structure analysis software, was used for the modal pushover analysis. Nonlinear time history analysis was carried out using DRAIN-2DX.

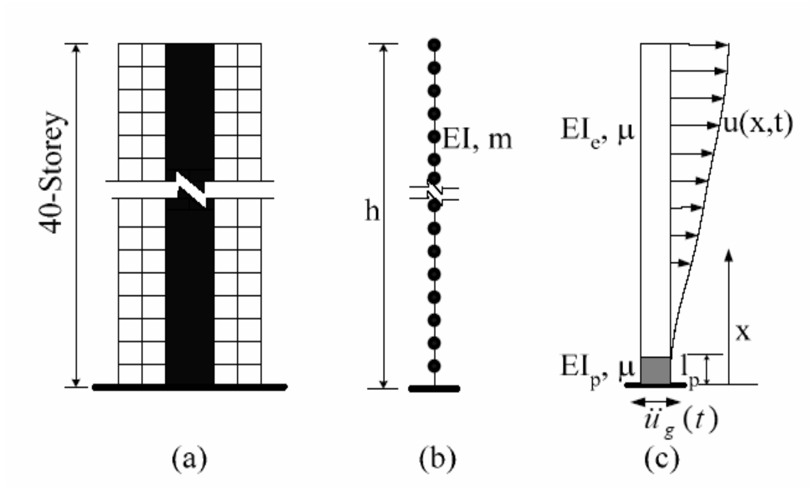

Figure 1. (a) wall building; (b) simplified model; (c) equivalent linear wall model.

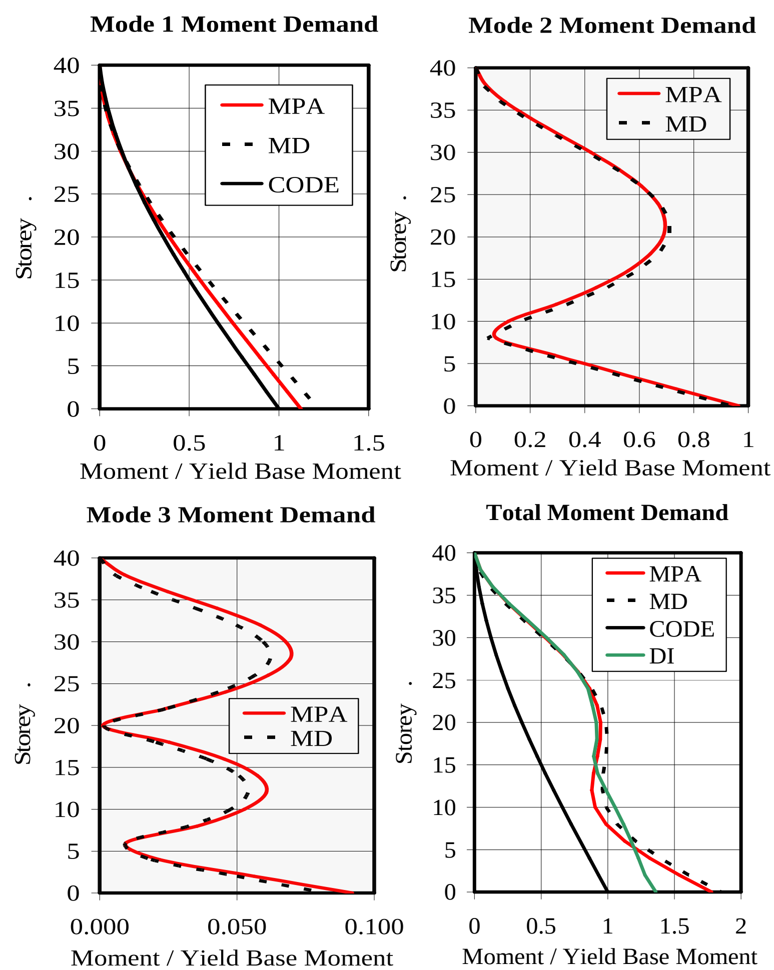

Figure 2. Moment demand under the ANFS x 1.4 ground motion.

Results and Discussions

Investigation on the modal pushover analysis procedure begins with the Northridge (Arleta-Nordhoff Fire Station), ANFS, ground motion. The ground motion was scaled by 1.4 to achieve sufficient inelastic deformation such that ductility of equivalent first mode single degree of freedom, μ (SDOF), becomes nearly 2.0. Fig. 2 depicts the story overturning moment demands of the structure along its height. Total responses are also presented with inelastic time history analysis results. Results from both modal pushover analysis and modal decomposition exhibit general matching and the total overturning moments are also well estimated except at the base. Even though base moment demand estimation by the MPA procedure and MD are similar, they are significantly higher than the results from nonlinear time history analysis. The SRSS combination rule was applied to get total demand from modal demand. The overestimation of base moment demand can be explained by the possible error on the SRSS combination rule. The total moment remains constant to yield moment (M_y) at the plastic hinges in inelastic range. But MPA uses same structure for all mode of analysis with out considering the effect of one mode response to another mode. Hence, the structure considered for second mode still may remains elastic and moment can still increase even though the first mode is already yielded. In this way, significant error on total moment appears immediately after the yielding of structure in first mode. The structure also can yield in first and second mode separately under the strong ground motions. Considering only two fundamental modes and assuming that the structure is yielded in both modes, the total base moment is given as $\sqrt{M_y^{\,2} + M_y^{\,2}} = 1.414 * M_y$. But it is not possible to increase total moment beyond the yield moment. However, it can be increased by small amount due to strain hardening. This confirms that the one source of error in moment demand at plastic hinge zone is SRSS combination rule which may overestimate the moment by 41% when two yielded modes are considered. Additions of the responses from higher modes further contribute on overestimation in responses at the plastic hinges.

Base moment envelop was computed and has been given in Fig. 3. It is observed that the results from the MPA procedure and MD are quite matching with nonlinear time history analysis within the elastic range. The MPA and MD results start to divert when the total moment reaches to yield moment. Structure starts to yield in first mode at scale factor equal to 0.8. Note that modal response seems still elastic when total moment reaches yield moment. Therefore, significant error on the MPA starts to appear immediately after the total moment equal to yield moment. The error continues to increase until the structure yields in second mode. After that, error remains almost constant (40-50%). Third mode moment is comparatively less. Therefore, there is no significant increment in total moment after the structure yields in second mode. On the other hand, it was observed that the SRSS combination rule seems reasonably acceptable for the estimation of shear at

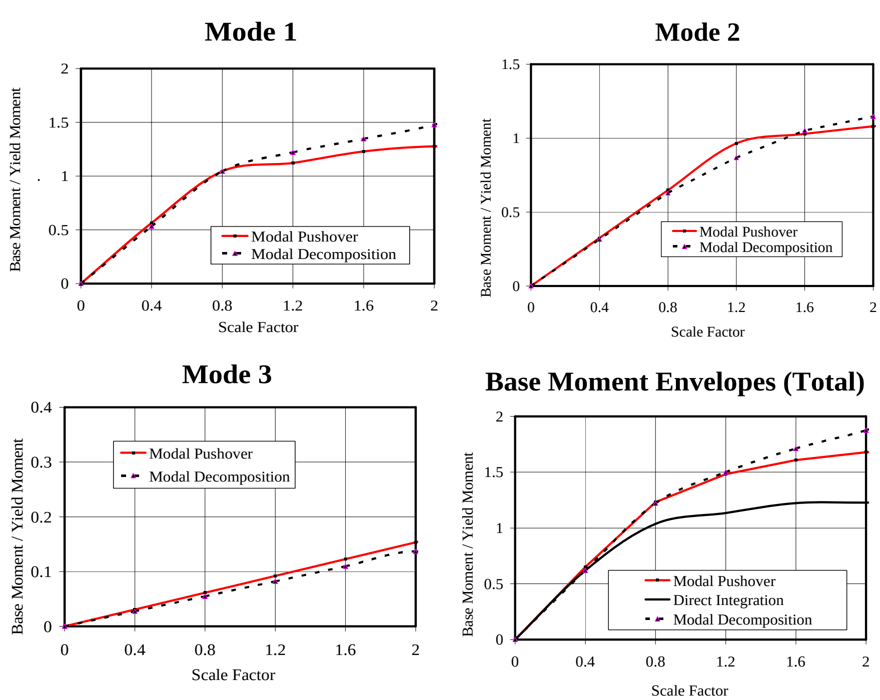

Figure 3. Base moment envelope under the ANFS x 1.4 ground motion.

plastic hinge zone. It is due to the fact that shear can still increase at plastic hinge when the moment reaches to yield value. The results for shear, however, are not shown here.
Contrary to the results under the ANFS ground motion, modal demands are significantly differing when structure is subjected to Lomaprieta (Gilroy Array #3 station), LP-GA3 ground motion. LP-GA3 ground motion was normalized by a scale factor 1.2. Responses based on these analyses are presented in Fig. 4 and 5. It can be observed that the MPA procedure shows significant error in upper stories for LP-GA3 ground motion. The error was even more for the storey shear.

From the results obtained from inelastic modal decomposition, it can be observed that the

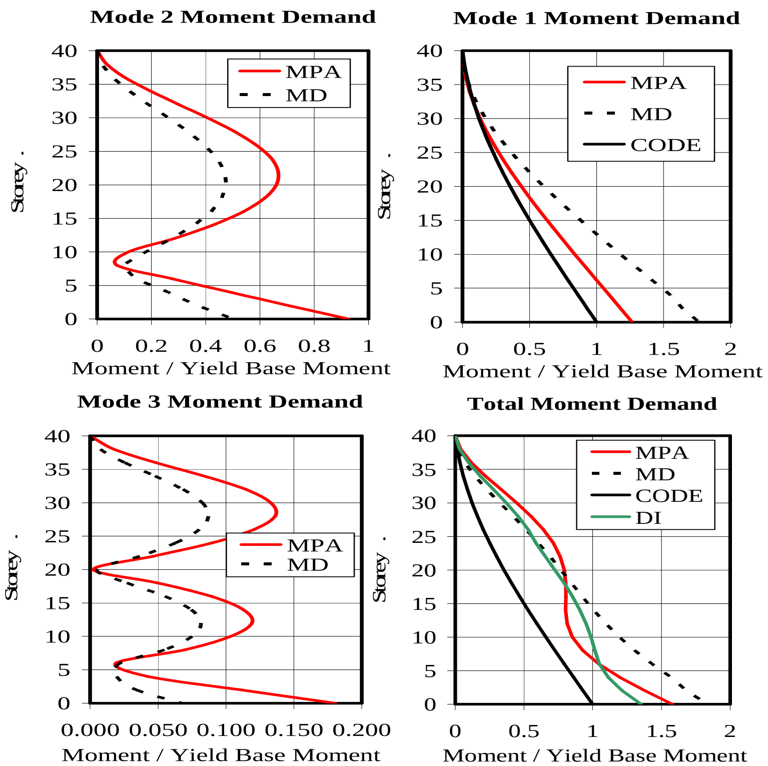

Figure 4. Moment Demand under the LP-GA3 x 1.2 ground motion.

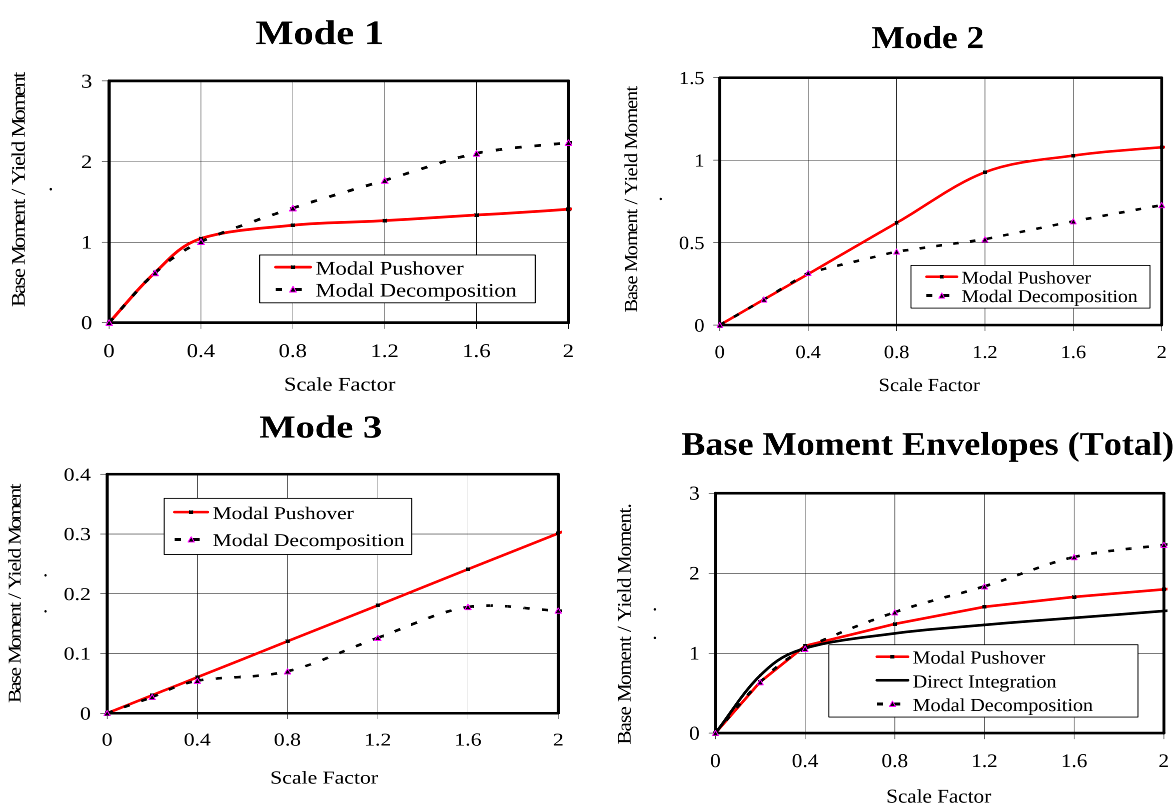

Figure 5. Base moment envelope under the LP-GA3 x 1.2 ground motion.

inelastic modal responses are uncoupled each other when the structure is subjected under ANFS ground motion. But, inelastic modal responses are strongly coupled each other when the structure is subjected under LP-GA3 ground motion. In other words, when the structure yields in the first mode, it gives yielding effect on second mode. Modal pushover analysis cannot identify the coupling effect between the modes, because it uses separate models for each mode of analysis.

Furthermore, a comparative study of modal time history response of two (ANFS and LP-GA3) ground motion was carried out. Fig. 6 (a) shows the first and second mode base moment time history for ANFS ground motion. It is observed from Fig. 6 (a) that the peak responses for the first and the second mode lie at two separate time instant. For the first mode, the peak response occurs after 7 sec. But the second mode peak response occurs before 7 sec, where the first mode response is very low. Hence, the time instant for two modal peak responses are well separated. Therefore, there is no strong coupling between two modal peak responses. This is the case in which modal pushover analysis seems to be working well.

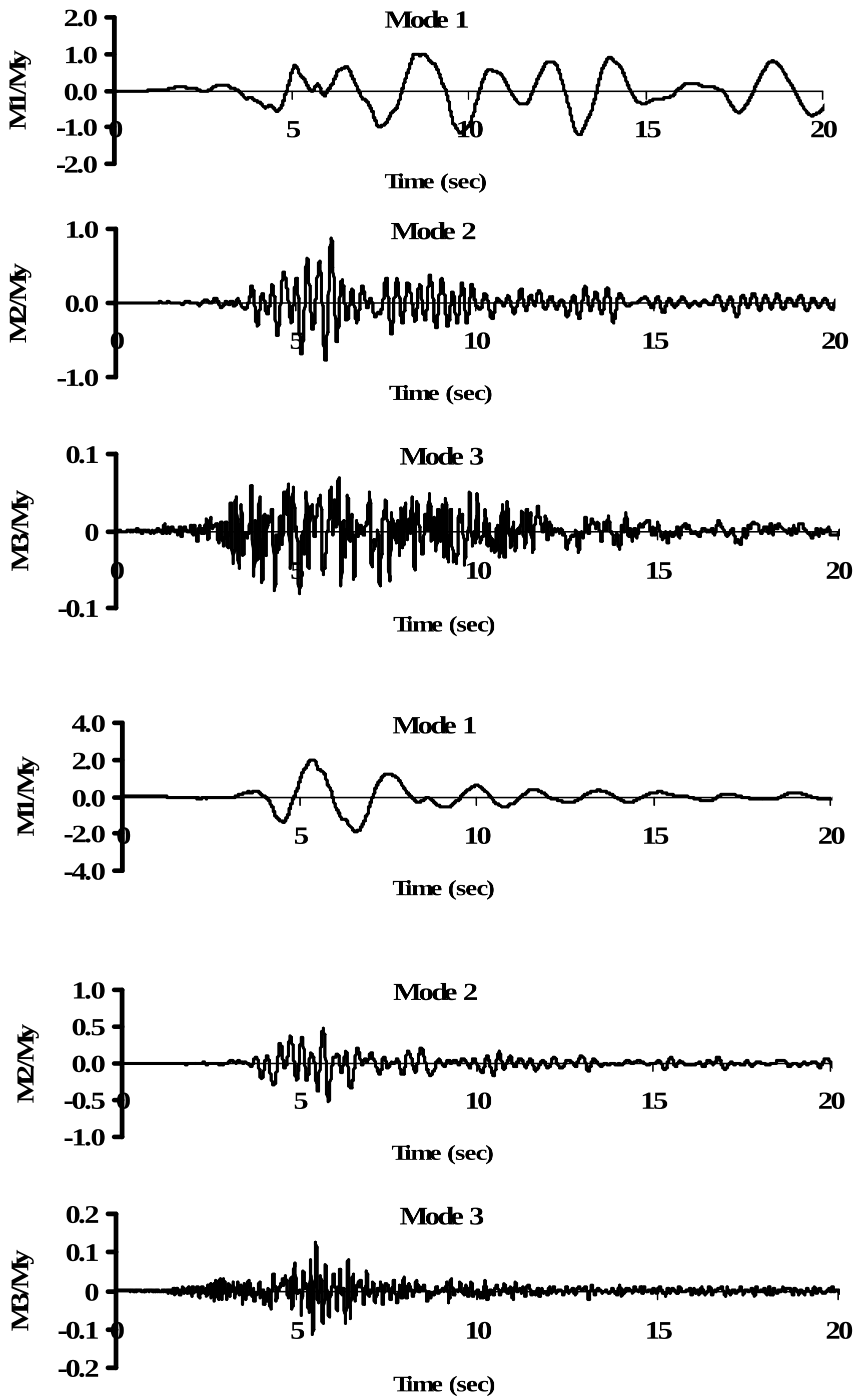

Figure 6. Decomposed base moment time history of 40 storey wall building subjected on
(a) ANFS (b) LP-GA3 ground motion

Fig. 6 (b) depicts the modal time history response of the structure under LP-GA3 ground motion. The modal time history response of the structure under LP-GA3 shows that the first and second mode responses reach to peak at the same time instant (5.5 sec). The second mode response gets yielding effect from the first mode. Therefore the second mode response diverts from the linear response at the scale factor of 0.4, even though the second mode base moment is very less than yield moment. Modal pushover analysis is unable to consider this coupling effect. For such ground motion the MPA procedure seems not to be working properly.

Conclusions

Recently developed modal pushover analysis (MPA) procedure seems to be capable of giving significantly improved results over the conventional pushover analysis. The MPA procedure is based on several assumptions and approximations. Most of these assumptions are made to use dynamic properties of elastic structures. But, the structures may behave in a very different manner in the inelastic range. Sometimes, the MPA procedure is known to show significant errors. These errors arise from assumptions and approximations. It is observed that accuracy on the MPA procedure and reliability of assumptions extensively depends upon the ground motion characteristics. The modal pushover analysis procedure assumes that the coupling between modes arising from yielding of the system can be neglected. This assumption seems valid when peak of modal responses occurs at different time instant. Modal pushover analysis seems very accurate for ground motions that have peak modal responses at different time instant.

Results from the modal pushover analysis procedure may be misleading for the ground motions that have peak modal responses at the same time instant. The errors are mainly due to the assumption of uncoupling of modal responses. For some ground motions, modal properties of higher modes may significantly change due to yielding of first mode. Effect of coupling becomes more important when the contribution on the response from the higher mode is significant (e.g., storey shear for high-rise structures). Use of the SRSS combination rule seems to be acceptable with some exceptions. The moment on the plastic hinge zone is significantly over estimated for strong ground motion due to the SRSS combination rule.

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