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APPLICATION OF PUSHOVER PROCEDURES FOR THE SEISMIC ANALYSIS OF PLAN-ASYMMETRIC BUILDINGS

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ABSTRACT

This paper illustrates an investigation on the effectiveness of pushover procedures for plan-asymmetric buildings. In particular, attention was focused on the application of the modal pushover analysis (MPA). An extensive numerical study was performed considering RC spatial frames with different values of eccentricity between center of mass and center of stiffness and of moment of inertia of floor mass. The results of pushover analyses were compared with those of a series of non-linear dynamic analyses which were carried out considering different earthquake records. The response parameters examined in the comparison were top displacement at the center of mass and roof rotation as well as story displacement and inter-story drift of the frames parallel to the direction of the seismic action. The results showed a good performance of the MPA procedure also for the structures characterized by large values of eccentricity and of moment of inertia of floor mass.

Introduction

The pushover analysis is now recognized as one of the standard methods for estimating the seismic demand not only for research purposes but also for practical applications, and it is included in guidelines and design codes (BSSC 1997, CEN 2003). In comparison with linear elastic methods it is able to give more information on the structural response to seismic action, for example the lateral strength or the distribution of inelastic deformations. In comparison with non-linear dynamic analysis it is simpler and it avoids the dependence on the input motion. The non-linear static procedure, however, is affected by several levels of approximation, and the reliability of results depends also on the type of structure. In particular one of the major difficulties lies in the definition of a lateral force distribution which is capable of reproducing the inertia forces during the seismic response. This problem is evident especially when high-rise or irregular buildings are studied. Recently various research studies proposed procedures for applying pushover analysis in such cases. These procedures involve in general more complexity than the standard pushover analysis based on a single time-invariant lateral force pattern. The procedure called modal pushover analysis (Chopra and Goel 2002, Chintanapakdee and Chopra 2003) was developed for including the contributions of more than one mode of vibration. Adaptive procedures (Gupta and Kunnath 2000, Pinho and Antoniou 2004) were proposed for considering the variation of lateral load distribution during the inelastic response.

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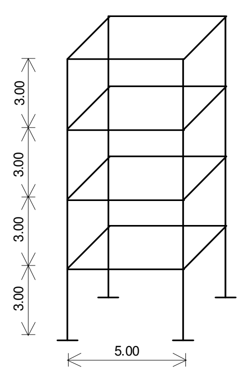
The major part of earlier research focused on the planar analysis of building, regular or irregular in elevation. In the past years some proposals were presented for applying the pushover analysis to the spatial analysis of plan-asymmetric buildings (Kilar and Fajfar 1998, Faella and Kilar 1998, Moghadam and Tso 2000, Penelis and Kappos 2002). One of them is the extension to the spatial analysis of the modal pushover procedure (Chopra and Goel 2004), initially developed and verified in the case of planar analysis. The purpose of this work was to evaluate the application of pushover procedures, and in particular of the modal pushover analysis, for plan-asymmetric buildings. For these buildings the coupling of translational and rotational modes of vibration becomes a fundamental aspect of the response. To this purpose a set of RC spatial frames was designed considering the same geometric configuration and different levels of irregularity in plan. The effectiveness of modal pushover analysis was studied through the comparison with non-linear dynamic analyses, which were performed considering various earthquake records. A fiber model was used for the non-linear analyses of RC structures.

Selected buildings and non-linear model

All the considered buildings have the same structural configuration, which is a spatial RC frame characterized by four square floors and one span in both lateral directions x and y (Fig. 1). The design was carried out according to the new Italian Seismic Code (OPCM 2003), similar to the last version of Eurocode 8 (CEN 2003). Square cross-sections with a width of 0.3 m were adopted for columns and rectangular cross-sections with a width of 0.3 m and a depth of 0.5 m were adopted for beams. The cylinder strength of concrete was set equal to 30 MPa and the yield strength of reinforcing steel was set equal to 430 MPa. The design response spectrum was defined considering a rigid soil type and a peak ground acceleration equal to 0.35 g. The buildings were designed following the provisions for the high ductility class. In particular the force reduction factor was set equal to 5.4 and the capacity design procedure was applied.

The design was repeated by varying two parameters: eccentricity between center of mass (CM) and center of stiffness (CS) and moment of inertia (I) of floor mass about a vertical axis through the CM. Since the geometric configuration of the frame and the cross-section of the elements were considered symmetric, the asymmetry was due only to the position of the center of mass. Therefore the selected structures were mass-eccentric systems. When the design was repeated, only the reinforcements of the elements were varied, while the cross-sections of the elements were maintained the same for all structures. Three values were considered for the eccentricity in the y direction: 10% (e10), 20% (e20) and 30% (e30) of the total length of the span. These structures were then subjected to seismic action in the x direction. For each value of eccentricity two values of moment of inertia of floor mass were adopted: the first was calculated considering a mass distribution consistent with the position of CM, the second was calculated by increasing the first value by a factor of 6.0. In this way two group of structures were designed, one (LI) characterized by a low value of the moment of inertia of floor mass, the other (HI) characterized by a high value of the same parameter. The Table 1 shows the labels of the different



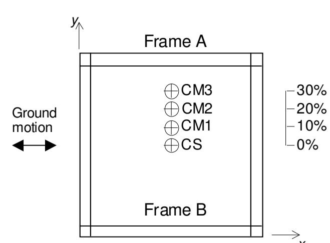


Figure 1. Structures under study: view (a) and plan with considered positions of CM (b).

structures together with the values of the eccentricity and of the ratio I/M, where M is the floor mass. In addition to the six structures indicated in Table 1 also a plan-symmetric structure called LI-e0 was designed for comparison purposes.

Table 1. Characteristics of the considered structures.

Structure	LI-e10	LI-e20	LI-e30	HI-e10	HI-e20	HI-e30
e/L [%]	10	20	30	10	20	30
I/M [m^2]	3.92	3.21	2.58	23.52	19.26	15.48

Table 2. Natural vibration period and ratio of effective modal mass for the first two modes.

Structure	LI-e10	LI-e20	LI-e30	HI-e10	HI-e20	HI-e30
T_1 [sec]	0.57	0.58	0.61	0.72	0.69	0.69
M^*_{x1}/M_{tot} [%]	85.8	84.8	84.2	5.0	24.8	46.6
T_2 [sec]	0.29	0.19	0.22	0.55	0.52	0.47
M^*_{x2}/M_{tot} [%]	0.7	1.6	1.6	81.5	61.7	39.8

Fig. 2 shows the first two modes of the considered structures and Table 2 gives the natural vibration periods T_n and the ratios of effective modal mass M^*_n in the x direction to the total mass of the structure M_{tot}, which is equal to the sum of floor masses. The effective modal mass for a mode n is determined using $M^*_n = L_n \Gamma_n$, where $\Gamma_n = L_n / M_n$ and

$$L_n = \boldsymbol{\phi}_{xn}^T \mathbf{m1} \tag{1}$$

$$M_n = \boldsymbol{\phi}_n^T \mathbf{M} \boldsymbol{\phi}_n \tag{2}$$

In equation 1 L_n is calculated considering the ground motion in the x direction and ϕ_{xn} is the component in the x direction of the n-th natural vibration mode ϕ_n. **M** is a diagonal mass matrix of order 3N, where N is the number of stories, and **m** is the submatrix of order N of the floor masses. The modal properties are calculated after application of gravity loads and before application of seismic action. It is evident (Fig. 2) that for the first group of structures (LI) the lateral displacements are dominant in the first mode, while the torsional rotations are dominant in the second mode. This result is confirmed also by the low values of the effective modal mass in the second mode (Tab. 2), and it means that there is not significant coupling between the two modes. For the LI structures the eccentricity has a slight influence on the degree of coupling of the first two modes. For the second group of structures (HI) there is instead a more significant coupling than for the first group of structures (LI). This indicates the importance of the moment of inertia of floor mass in the torsional response. For the systems HI-e10 and HI-e20 the first mode is dominated by torsional rotations, while for the system HI-e30 the first mode is characterized by significant lateral displacements. The effective modal mass in the x direction of the mode dominated by torsion increases with the eccentricity. For the system HI-e30 there is a strong degree of coupling between the two modes, and the values of rotation of the two modes are similar.

The non-linear static and dynamic analyses of the selected structures were performed through the Opensees software (McKenna and Fenves 2005). Each structural member, column or beam, was modeled with a single distributed plasticity finite element. Five control sections were adopted, two located at the ends and the other along the element. Their response was studied through a fiber model. The control sections were divided into a number of 14x14 fibers representing confined and unconfined concrete, and a number of steel fiber equal to that of the longitudinal bars. The effect of confinement was considered in the stress-strain law relationship of concrete according to the model of Mander et al. (Mander et al. 1988). Stiffness degradation was included for the loading and reloading branches of the concrete law in case of cyclic loading. A bilinear stress-strain law with a hardening ratio equal to 0.01 was adopted for the reinforcement steel. The geometrical non-linearity was considered both in dynamic and pushover analyses in terms of P-delta effects.

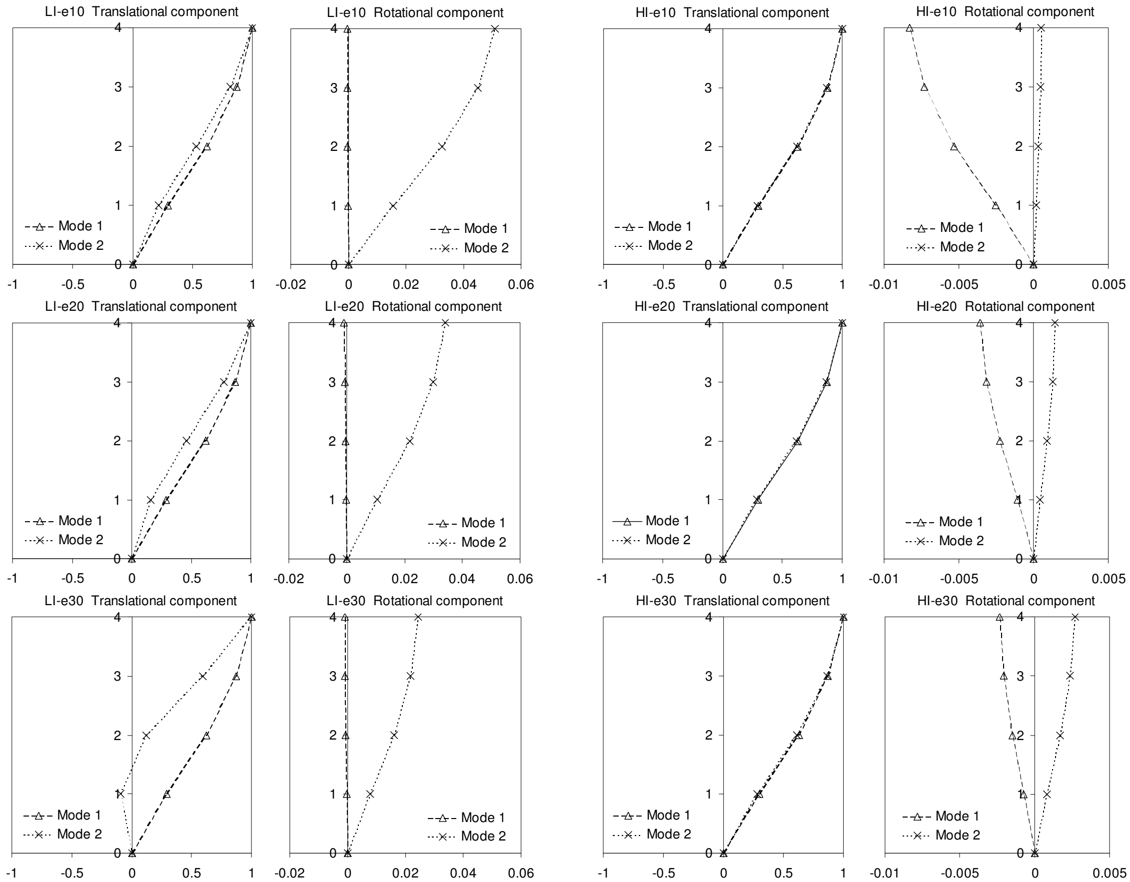


Figure 2. First two modes of vibration of the considered structures.

The Modal Pushover Analysis for Plan-Asymmetric Buildings

The modal pushover analysis (MPA) is a non-linear static procedure which was developed for the purpose of including all modes of vibration which affect the seismic response. In this procedure different pushover analyses are performed, one for each considered mode. The lateral load distribution used for each mode is characterized at the story level by forces proportional to the mass multiplied by the corresponding modal deformation. The non-linear static analysis is carried out after application of gravity loads. The modal deformations are assumed to be invariant during the analysis. Once the pushover curves are determined it is necessary to calculate for each mode the displacement demand due to the seismic action. Then the response parameters associated to the displacement demand are extracted. Finally the response parameters of a proper number of modes are combined using SRSS or CQC rule.

In case of plan-asymmetric buildings the lateral load distribution for the *n*-th mode includes at each story lateral forces in both x and y directions and torques. The vector of lateral forces, called $\mathbf{s}^*_n$, is given by:

$$\mathbf{s}_n^* = \mathbf{M}\boldsymbol{\phi}_n = \begin{Bmatrix} \mathbf{m}\boldsymbol{\phi}_{xn} \\ \mathbf{m}\boldsymbol{\phi}_{yn} \\ \mathbf{I}\boldsymbol{\phi}_{\boldsymbol{\theta}n} \end{Bmatrix} \tag{3}$$

where $\mathbf{I}$ is the submatrix of order N of the moments of inertia of floor mass. With the force distribution of Equation 3 the pushover analysis is carried out for the *n*-th mode, and the curve of base shear versus roof displacement is obtained. The roof displacement is evaluated at the center of mass and the floor diaphragms are assumed to be rigid. In general the analysis gives two pushover curves, one for *x* direction, the other for y direction. However it is recommended to consider the curve in the dominant direction of motion of the mode.

The determination of the displacement demand requires the idealization of the pushover curve as a bilinear curve and the transformation to the force-displacement curve of the equivalent single degree of freedom (SDF) system. In this work the displacement demand at the top of the building is calculated directly by using the displacement coefficient method proposed in the document FEMA 440 (BSSC 2005):

$$\delta_t = C_0 C_1 C_2 S_a \frac{T_e^2}{4\pi^2} \tag{4}$$

where the target displacement δ_t corresponds to the roof displacement of the mode n, T_e corresponds to the effective period of the mode n and S_a is the elastic spectral acceleration at the period T_e. Here it is considered the elastic spectral acceleration of the individual earthquake records used in the non-linear dynamic analysis. The effective period is calculated considering the elastic stiffness of the bilinear idealization of the pushover curve. The coefficient C_0 relates the spectral to the top displacement. It is given by $C_0=\Gamma_n\phi_{rn}$, where ϕ_{rn} is the modal deformation of the n-th mode at the roof and Γ_n depends on the direction of the ground motion. The coefficient C_1 relates the inelastic to the elastic spectral displacement and the coefficient C_2 represents the effect of pinched hysteretic shape, stiffness degradation and strength deterioration. These coefficients may be calculated according to provisions given in FEMA 440.

Since the selected structures, as mentioned earlier, were subjected to ground motions in the x direction, the number of modes considered for the modal pushover analyses was defined considering the effective modal mass in the same direction. For all the structures the first two modes allow to obtain a sum of the effective modal mass larger than 85% of the total mass, as required by Italian Seismic Code and EC8. Therefore the modal pushover analyses were performed considering two modes, i.e. the first translational mode and the first rotational mode. For each structure the pushover analysis was repeated by applying the modal load pattern of the first two modes. The resulting pushover curves are illustrated in Fig. 3. Due to P-delta effect the curves derived with translational modes are characterized by a negative slope of the post-elastic branch.

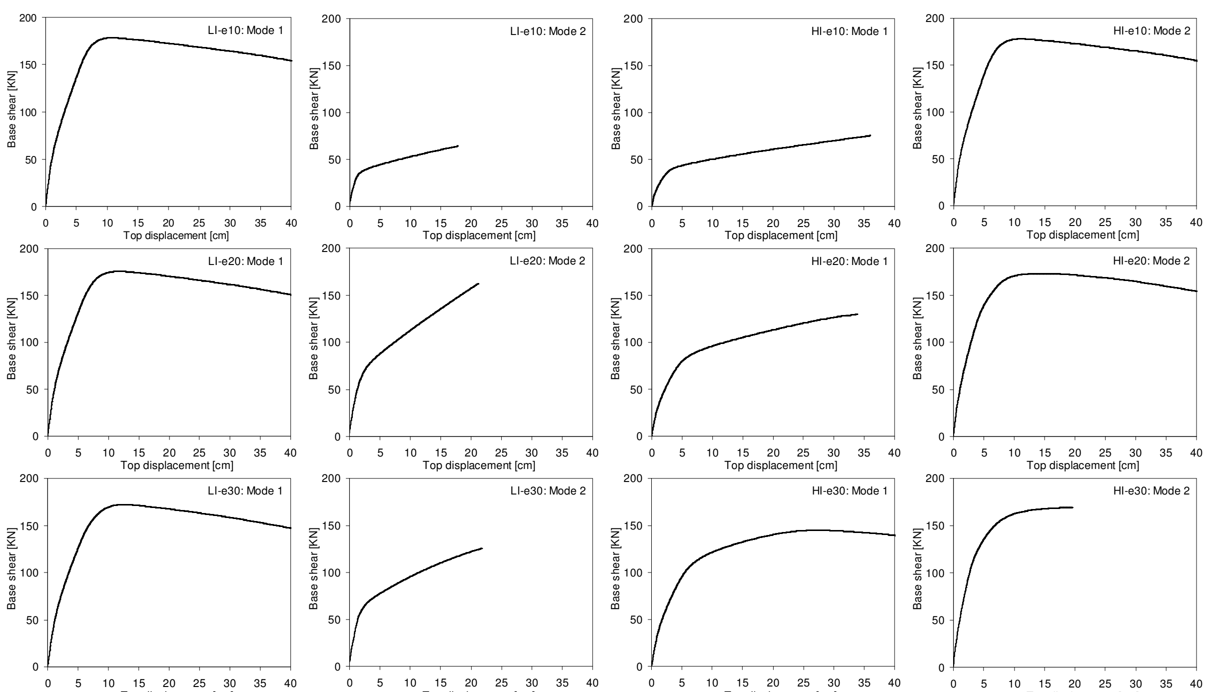


Figure 3. Modal pushover curves of the considered structures.

Non-Linear Dynamic Analyses

Non-linear dynamic analyses were performed for comparison with pushover analyses. Each designed structure was subjected to six recorded ground motions. The selected records are listed in Table 3. The accelerations were scaled in such a way that the peak ground acceleration was equal to the design value. The seismic action was applied only in the *x* direction and the centers of mass, characterized by an eccentricity in the *y* direction, experienced displacements only in the x direction. In general the response of the structures under study was characterized by displacements of the center of mass in the *x* direction and by torsional rotations. Except for the structure without eccentricity, the response was influenced by the value of moment of inertia of floor mass. As expected, the values of floor rotations were larger for the HI group of structures than for the LI group of structures. However the peak values of displacement of the HI structures were associated with low values of rotation. While the response of LI structures was dominated by one mode, the response of HI structures was characterized by the contributions of two modes, and the peaks of lateral and torsional vibrations were not coincident. In several cases the floor rotation of the LI structures at the time of maximum displacement of the center of mass was larger than that of the HI structures. As an example the time-histories of displacement of the center of mass and of floor rotation of the LI-e20 and HI-e20 structures subjected to the El Centro earthquake record are illustrated in Fig. 4.

Table 3. Selected earthquake records for non-linear dynamic analyses.

Earthquake location	Imperial Valley	Kern	Tabas	Loma Prieta	North-ridge	Kobe
Year	1940	1952	1978	1989	1994	1995
Station	El Centro	Taft	Tabas	Berkeley	Saticoy	KJMA
PGA [g]	0.341	0.178	0.836	0.529	0.477	0.821

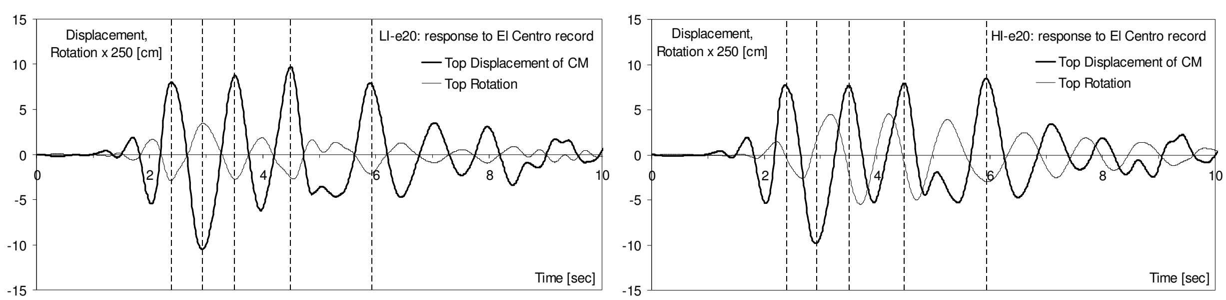


Figure 4. Time-histories of top displacement and floor rotation.

Comparison between Results of Non-Linear Dynamic and Static Analyses

Fig. 5 shows, for each structure, the story displacement of the center of mass (CM) along the height, the story displacements and the inter-story drifts of the frame A (close to CM) along the height, and the top displacement along the plan from frame B to frame A. The values illustrated are the average of the maximum values obtained during the response to the earthquake records. With regard to the pushover analyses, it is assumed that, for each earthquake, the state of damage is the one associated with a top displacement of the CM equal to that obtained with the expression (4) of the displacement coefficient method. In Fig. 5 the results of pushover analyses performed with first mode (M1) and second mode (M2) lateral load distributions and of modal pushover analysis performed considering the first two modes are compared with those of non-linear response history analyses (RHA).

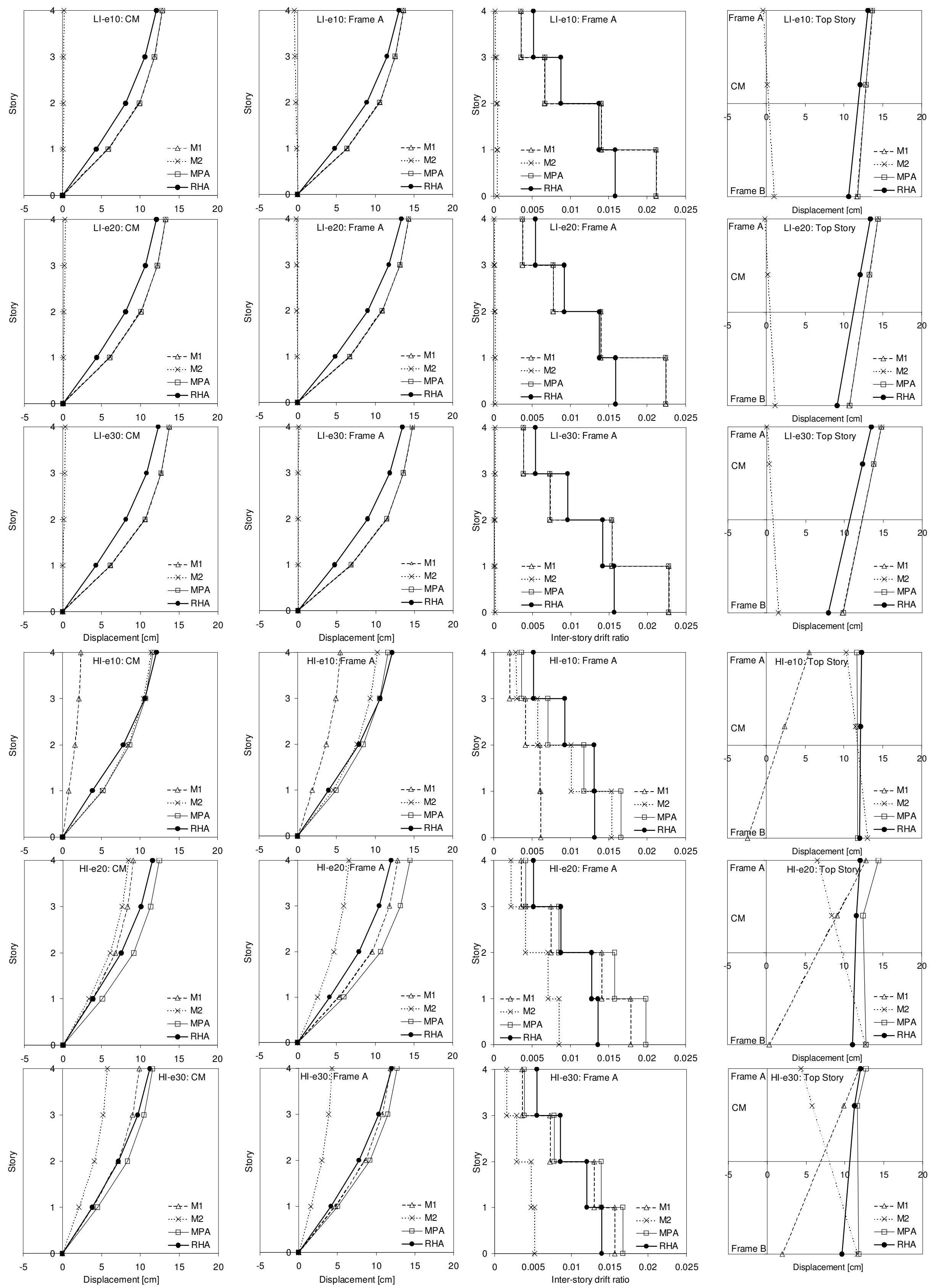


Figure 5. Story displacements and inter-story drifts along the height, top displacements along the plan.

In general the MPA provided a good prediction of the maximum displacements along the plan. It was able to estimate with good accuracy the torsional response of the buildings, even in the cases of large values of eccentricity and of moment of inertia of floor mass. With MPA the estimate of the displacement demand were conservative in almost all examined cases. The largest values of inter-story drift of frame A were obtained at the lower story. The pushover analyses, in comparison with non-linear dynamic analyses, overestimated the inter-story drift of frame A at the lower story and underestimated it at the higher story.
By examining the results regarding the LI structures, it is evident that the second mode, dominated by torsion, did not affect the response. The values obtained with MPA are practically coincident with those obtained with the first mode pushover. The rotation of the floor is larger for the structures with the larger eccentricity. This tendency was well captured by the first mode pushover and by MPA.

With regard to the HI structures, characterized by a strong coupling of the modes dominated by torsion and displacement, the influence of the second mode on the results of MPA was quite prominent. The direction of the floor rotation in the second mode was opposite to that in the first mode. For all the HI structures the envelope of the displacement along the plan obtained with RHA was characterized by low rotations, with similar values of displacement for frame A and B. This result is correlated to the non coincidence of the times of peak displacement and rotation. The first mode pushover provided a good estimate of the displacement of frame A but a large underestimate of the displacement of frame B, except for the HI-e10 structure. For this structure, in fact, the first mode is dominated by torsion and the first mode pushover underestimated the displacements of both frames. The opposite trend was obtained with the second mode pushover, which provided a good estimation of displacement of frame B, and a large underestimation of displacement of frame A. Only the MPA pushover was able to provide a good prediction of the displacement of both frames and of the displacement envelope along the plan.

For selected response parameters the difference between the value from pushover analysis and the value from non-linear dynamic analysis was evaluated as a percentage of the latter value. The average over all earthquake records was then calculated. Fig. 6 shows the results obtained for the top and story displacement of CM, for the top displacement of frames A and B, and for the story displacement and inter-story drift of frame A. The average over all stories was considered for the story displacement and inter-story drift. The same figure illustrates the results obtained by applying MPA according to two different ways. In the first way the results of the pushover analyses correspond to a top displacement of the CM estimated through the displacement coefficient method (DCM), in the second way they correspond to a top displacement of the CM equal to that obtained from non-linear dynamic analysis (ED). The purpose of the second way of applying MPA was to identify the portion of the error due only to the pushover procedure and to the assumed load distribution, and not to the method for estimating the seismic demand.

In the evaluation of the results first the performance of MPA is examined. The error in the estimation of the top displacement of CM was quite low for all structures, ranging from 6 to 15%. The values of top displacements of frame A and B are representative of the capability of the procedure of predicting the torsional effects. The error in the prediction of top displacements of frame A and B ranged between 12 and 25%, being not much larger than those related to the top displacement of CM. Since frame A is the one closer to CM, the values obtained for frame A reflect those obtained for CM. Due to torsional effects, the error in the estimation of the top displacement of frame B was slightly larger than that of frame A. The error in the estimation of story displacement and inter-story drift of frame A ranged between 20 and 30% for all structures. In general the error of MPA was not much influenced by the value of eccentricity and of moment of inertia of floor mass. These results show that the MPA was effective and that it was able to predict with good accuracy the response of the selected plan-asymmetric structures.

The error of MPA procedure is then compared with that of the first mode pushover analysis. This comparison lead to observations correlated to those regarding Fig. 5. For all the LI structures the results of MPA and M1 were practically coincident and the influence of the second mode was negligible. For the HI structures the contribution of each of the first two modes was important. The first mode pushover provided very inaccurate results for both frames A and B of the HI-e10 structure, and for frame B of the other HI structures.

Finally the errors of the two considered ways of applying MPA procedure are compared. As expected, when the top displacement of CM was taken as being equal to that from non-linear dynamic analyses, the resulting error was significantly lower than that obtained with displacement coefficient method. The reduction, which in several cases was about 50%, was dependent on the response parameter and on the structural system. It was more evident for story displacement than for inter-story drift. Due to torsional rotations, this reduction was not noticed for frame B of HI structures. This can be verified by observing the results for the individual earthquake records, rather than the average results.

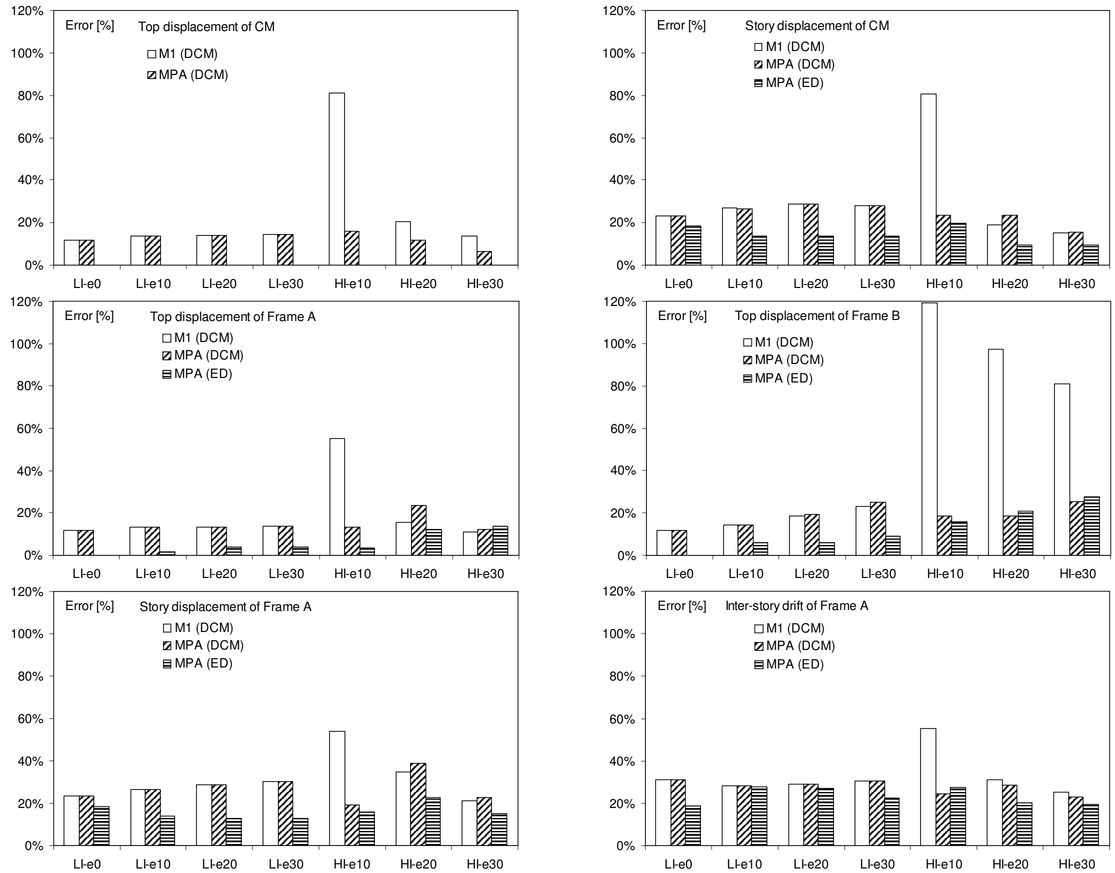


Figure 6. Error in the estimates of the results of non-linear dynamic analyses.

Conclusions

The effectiveness of pushover procedures for plan-asymmetric buildings was investigated. The attention was concentrated on the application of the modal pushover analysis. A set of RC spatial frames was designed considering the same geometric configuration and different values of eccentricity between center of mass and center of stiffness and of moment of inertia of floor mass. The results of modal pushover analyses were compared with those of non-linear dynamic analyses.

In the time-history response the peak values of displacement of the HI structures, characterized by a large value of moment of inertia of floor mass, were associated with low values of rotation. In all considered cases the MPA provided a good prediction of the results of non-linear dynamic analyses and of the torsional effects. For example the average error in the prediction of top displacements of the opposite external frames ranged between 12 and 25%. For the HI structures the MPA allowed a significant improvement of the results when compared with pushover analysis based on a first mode load distribution. For the LI structures, characterized by a low value of moment of inertia of floor mass, the influence of the

mode dominated by torsion was not significant, and this result was independent of the value of eccentricity. By imposing in the pushover analysis a top displacement of CM equal to that of non-linear dynamic analysis, the error in the prediction of response parameters, especially of story displacement, became significantly lower than that obtained by applying the displacement coefficient method.
It is necessary to state that these conclusions are applicable to the selected structural configuration. Further research is needed in order to comprehend other structural configurations, characterized by different heights and various sources of asymmetry. This work focused on the modal pushover procedure, and therefore comparison studies with other procedures are needed.

References

BSSC, 1997. NEHRP Guidelines for the seismic rehabilitation of buildings, FEMA-273, developed by ATC for FEMA, Washington D.C.

BSSC, 2005. NEHRP Improvement of nonlinear static seismic analysis procedures, FEMA-440, developed by ATC for FEMA, Washington D.C.

CEN, 2003. Eurocode 8. Design of Structures for Earthquake Resistance – Part 1: General Rules, Seismic Actions and Rules for buildings. Brussels.

Chintanapakdee, C., and A. K. Chopra, 2003. Evaluation of modal pushover analysis using generic frames, *Earthquake Engineering and Structural Dynamics*, 32 (3), 417-442.

Chopra, A. K., and R. K. Goel, 2002. A modal pushover analysis procedure for estimating seismic demands for buildings, *Earthquake Engineering and Structural Dynamics*, 31 (3), 561-582.

Chopra, A. K., and R. K. Goel, 2004. A modal pushover analysis procedure to estimate seismic demands for unsymmetric-plan buildings, *Earthquake Engineering and Structural Dynamics* 33 (8), 903-927.

Gupta, B., and S. K. Kunnath, 2000. Adaptive spectra-based pushover procedure for seismic evaluation of structures. *Earthquake Spectra*, 16, 367-392.

Faella, G., and V. Kilar, 1998. Asymmetric multistorey R/C frame structures: push-over versus nonlinear dynamic analysis*, Proceedings of the 11th European Conference in Earthquake Engineering*, Paris.

Kilar, V., and P. Fajfar, 1997. Simple push-over analysis of asymmetric buildings, *Earthquake Engineering and Structural Dynamics*, 26, 233-249.

Mander, J. B., Priestley, M. J. N. and R. Park, 1998. Theoretical stress-strain model for confined concrete, *Journal of Structural Engineering* (ASCE), 114, 1804-1825.

McKenna, F., and G. L. Fenves, 2005. Open System for Earthquake Engineering Simulation, University of California, Berkeley.

Moghadam, A. S., and W. K. Tso, 2000. Pushover analysis for asymmetric and set-back multistory buildings, *Proceedings of the 12th World Conference on Earthquake Engineering*, Auckland, paper 1093.

Ordinanza del PCM n. 3274 del 20/03/2003. Primi elementi in materia di criteri generali per la classificazione sismica del territorio nazionale e di normative tecniche per le costruzioni in zona sismica.

Penelis, G. G., and A. J. Kappos, 2002. 3D Pushover Analysis: the issue of torsion, *Proceedings of the 12th European Conference on Earthquake Engineering*, London, paper 015.

Pinho, R., and S. Antoniou, 2004. Advantages and limitations of adaptive and non-adaptive force-based pushover procedures, *Journal of Earthquake Engineering*, 8, 497-522.