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A NEW INDEX FOR EVALUATION OF EFFECTIVE INPUT MOTIONS TO STRUCTURES

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ABSTRACT

The response of foundation during earthquakes, which includes the effect of kinematic and inertial interactions, is referred to as an effective input motion (EIM). It expresses total interaction effects during earthquakes and will be a key factor in estimation of seismic responses of structures including the effect of soil-structure interaction. This paper discusses what will be an appropriate measure to evaluate the EIM relative to the free-field motions on the basis of the observed earthquake motions of 19 events recorded on a large-scale shaking-table-foundation and at the surrounding soil. The main conclusions obtained in this study can be summarized as follows. (1) An index for the EIM, which is defined as a ratio of square root of the integrated-squared motions, is proposed and the theoretical basis is shown that the index is related to the transfer function of the soil-foundation system. (2) As the EIM, translational and rotational components of earthquake motions are to be applied to a superstructure. (3) The translational EIM decreases with increase of higher frequencies of the ground motion, and the rotational EIM shows opposite in tendency. (4) The index for the EIM corresponding to equivalent predominant frequency of the soil fits well to the transfer function of soil-foundation system evaluated from the observed records. (5) A conventional simple index evaluated by peak ratios of acceleration motions observed on a foundation and on a free-field is effective to measure the EIM.

Introduction

Seismic motions which will be applied to a superstructure are different from ground motions at the site due to effects of soil-structure interaction. The response of a foundation during earthquakes will be input motions to the superstructure, which are referred to as effective input motions (EIM). Although the EIM can be extracted from simultaneous earthquake records observed on the foundation and on the surrounding soil, there have not been well documented studies in terms of EIM. For evaluation of the change of the EIM comparing to the reference motions on the ground surface, a simple index has been used, such as a ratio between the peak acceleration motions on the foundation (PFA) and on the ground (PGA).

Yasui et al. (1998) revealed 30% reduction of the EIM for a horizontal component with use of the peak ratio PFA/PGA by analyzing seismic records observed in the heavily damaged area during the 1995

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Hyogoken-Nanbu earthquake in Kobe, Japan. Iguchi et al. (2000) indicated that the reduction of EIM is significantly dependent on the frequency component included in the surface ground motions. Kojima et al. (2004) studied the reduction effect of the EIM comparing with the ground motions for various structures with different embedded depth of foundation and with different size. Stewart et al. (1998), on the other hand, evaluated EIM with an index of response spectrum ratio of 5% damping for the seismic motions recorded on the foundation and on the soil surface.

Few studies have been presented about what index is appropriate to evaluate the EIM. As the EIM is dependent on the frequency component included in the ground motions, the index is preferable to be such that corresponds to the transfer function expressing the relation between the foundation responses and the ground motions. This paper proposes an index to measure EIM in comparison to the free-field motions and discusses the effectiveness of the index on the basis of earthquake records observed simultaneously on an embedded foundation and the surrounding soil. The significance of a rotational component of EIM, which has received few attentions so far, is also discussed.

A New Index of Effective Input Motions

The theoretical basis is developed in this section to present an appropriate index for evaluation of the EIM that is related to a transfer function of a soil-foundation system. Let $a_g(t)$ and $a_f^{\Delta}(t)$ be the time histories of acceleration recorded on the ground and on the foundation (translation), and let $A_g(\omega)$ and $A_f^{\Delta}(\omega)$ be the Fourier transforms of $a_g(t)$ and $a_f^{\Delta}(t)$, respectively. Denoting the transfer function of soil-foundation system by $H_{ef}^{\Delta}(\omega)$, we will have following equality.

$$\int_{-\infty}^{\infty} \left| A_f^{\Delta}(\omega) \right|^2 d\omega = \int_{-\infty}^{\infty} \left| H_{ef}^{\Delta}(\omega) \right|^2 \left| A_g(\omega) \right|^2 d\omega \tag{1}$$

From the Parseval's theorem, the left-hand side of Eq.(1) can be expressed by

$$\int_{-\infty}^{\infty} \left| A_f^{\Delta}(\omega) \right|^2 d\omega = 2\pi \int_{0}^{\infty} \left| a_f^{\Delta}(t) \right|^2 dt \tag{2}$$

Denoting the maximum and minimum values of the transfer function by $| H_{ef}^{\Delta}(\omega) |_{\min}$ and $| H_{ef}^{\Delta}(\omega) |_{\max}$, respectively, the right-hand side of Eq. (1) may be expressed by following inequality.

$$\left| H_{ef}^{\Delta}(\omega) \right|_{\min}^2 \int_{-\infty}^{\infty} \left| A_g(\omega) \right|^2 d\omega < \int_{-\infty}^{\infty} \left| H_{ef}^{\Delta}(\omega) \right|^2 \left| A_g(\omega) \right|^2 d\omega < \left| H_{ef}^{\Delta}(\omega) \right|_{\max}^2 \int_{-\infty}^{\infty} \left| A_g(\omega) \right|^2 d\omega \tag{3}$$

With taking into consideration of Eq.(3) and introducing an appropriate value α_a^{Δ} which satisfies $| H_{ef}^{\Delta}(\omega) |_{\min} < \alpha_a^{\Delta} <| H_{ef}^{\Delta}(\omega) |_{\max}$, the right-hand side of Eq. (1) can be expressed as follows.

$$\int_{-\infty}^{\infty} \left| H_{ef}^{\Delta}(\omega) \right|^2 \left| A_g(\omega) \right|^2 d\omega = 2\pi (\alpha_a^{\Delta})^2 \int_{0}^{\infty} \left| a_g(t) \right|^2 dt \tag{4}$$

Substituting from Eq. (4) into Eq. (1) and taking into consideration of Eq.(2) leads to

$$\alpha_a^{\Delta} = \sqrt{\int_{0}^{\infty} \left| a_f^{\Delta}(t) \right|^2 dt} \Big/ \sqrt{\int_{0}^{\infty} \left| a_g(t) \right|^2 dt} \tag{5}$$

Thus defined α_a^{Δ} is a coefficient expressing a ratio of the square root of the integrated squared-motion of the ground to that of the motion observed on the foundation. We refer to α_a^{Δ} as an effective-input-coefficient (EIC) of a translational motion. It will be noticed that the EIC has a relation to the transfer function of soil-foundation system, as being appreciated from above formulation.

In the same manner, we define the EIC for translational velocity motion, α_v^{Δ}, by

$$\alpha_v^{\Delta} = \sqrt{\int_0^{\infty} \left| v_f^{\Delta}(t) \right|^2 dt} \Bigg/ \sqrt{\int_0^{\infty} \left| v_g(t) \right|^2 dt} \tag{6}$$

where $v_g(t)$ and $v_f^{\Delta}(t)$ are time histories of translational velocity motions on the ground and on the foundation, respectively. In addition, the EIC for rotational angular acceleration is defined as

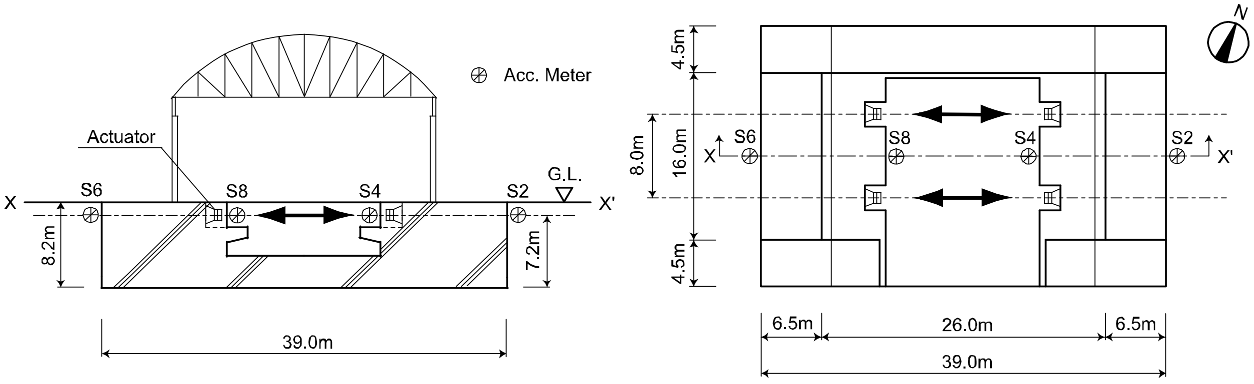
$$\alpha_a^{\Phi} = \sqrt{\int_0^{\infty} \left| L \cdot a_f^{\Phi}(t) \right|^2 dt} \Bigg/ \sqrt{\int_0^{\infty} \left| a_g(t) \right|^2 dt} \tag{7}$$

where a_f^{Φ} is a time history of rotational angular acceleration motion of the foundation, and L denotes the reference length introduced to make α_a^{Φ} dimensionless. For the value L, L =1m will be used in this paper.

Outline of Shaking Table Foundation and Observed Records

Outline of Foundation and Earthquake Observation

The plan and section of the shaking table foundation are shown in figure 1. In this paper, the longitudinal and transverse directions will be called X direction and Y direction, respectively. The details about the foundation and the soil profile at this site can be found elsewhere (Minowa et al. 1991; Iguchi et al. 2000).



(a) Section

(b) Plan

Figure 1. Outline of shaking-table-foundation and location of accelerometers.

Free-field ground motions have been observed by accelerometers at the depths of 1m and 40m, and 100m away from the foundation. We refer to the free-field motions observed at -1m as surface motions. As for the foundation, the responses of foundation have been observed at several points with accelerometers as shown in figure 1. Translational responses of foundation are specified by acclerograms recorded at S4, which is located almost at the center of foundation and at the same level as the observation point of the free-field motions. Rotational responses of the foundation with respect to Y-axis are evaluated using acclerograms recorded at S2 and S6. It has been confirmed that the foundation behaves as a rigid body within frequencies less than10Hz. In analyzing the observed data, azimuthal modification is made for the free-field motions as the azimuthal being different from the longitudinal and transverse directions of the foundation by 58 degrees.

Observed Earthquake Records

Records of about 30 earthquakes had been observed for six years from 1991 to 1996. As some being omitted because of incomplete records, 19 events were analyzed. The earthquake parameters of 19 Earthquakes are shown in table 1. In order to analyze the characteristics of the EIM in relation to frequency component of the ground motion, each earthquake was categorized into 3 groups (groups A, B and C) in accordance with previous paper (Iguchi et al. 2000). The grouping was made according to frequency components included in the free-field motions; earthquake records containing predominantly lower frequencies are categolized into the group A; the records containing predominantly higher frequency are grouped into the group C, and group B is characterized by the motions having intermediate frequency components between the groups A and C. Thus categorized groups for each earthquake are shown in table 1. In the table, shaded earthquakes are referred to as the representative motions selected for each group.

Table 1. Parameters of earthquakes.

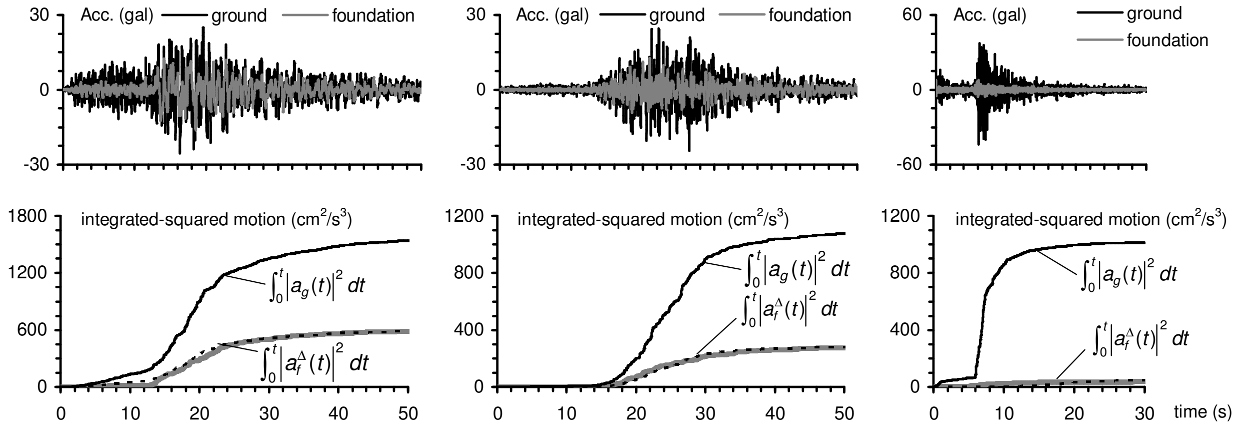
Eq	Date			Hypocenter		Depth	Mag.	Max. of Acc.		Group
No.	Y	M	D	N.L.	E.L.	(km)	(M)	X(gal)	Y(gal)	
1	91	8	6	35.87	141.15	26	5.8	18.10	9.80	A
2		10	19	36.08	139.92	59	4.3	44.00	37.89	C
3		11	19	35.60	140.02	81	4.9	11.03	15.44	C
4		12	12	36.46	140.66	48	4.6	15.95	13.55	B
5	92	2	2	35.23	139.79	92	5.9	22.39	24.12	B
6		5	11	36.53	140.54	56	5.6	26.02	26.82	B
7		6	1	36.67	141.27	44	5.7	22.36	20.90	B
8		8	30	33.20	138.34	325	6.6	21.01	19.37	B
9	93	6	7	36.02	141.76	28	5.9	7.31	8.12	B
10		9	18	36.18	140.88	35	5.0	15.97	15.14	B
11		10	12	32.02	138.24	390	7.0	24.59	30.93	B
12	95	1	7	40.18	142.32	30	6.9	9.72	8.02	A
13		1	7	36.17	139.59	70	5.4	68.83	123.57	B
14		1	8	36.19	139.58	72	4.6	13.77	15.67	C
15		1	10	35.56	141.26	45	6.2	10.39	9.87	A
16		4	12	36.27	140.37	52	4.6	16.19	15.85	B
17		7	3	35.06	139.30	120	5.6	6.37	5.83	B
18		7	30	35.54	140.36	50	5.0	21.57	24.84	B
19	96	9	11	35.07	141.03	30	6.6	25.58	26.90	A

Figure 2 shows time histories of the translational acceleration recorded on the foundation and on the ground surface (above), and the time histories of their integrated-squared motions (bottom). In these figures the time histories of the integrated-squared motions of the free-field multiplied by the ratio of their total cumulative values of the squared motions $(\int_0^\infty | a_f^\Delta(t) |^2 dt \big/ \int_0^\infty | a_f^\Delta(t) |^2 dt)$ are drawn by dashed lines. The ratio of the total cumulative values corresponds to the squared value of the EIC. It should be noted that dashed lines lie over the integrated-squared motions of the foundation regardless of the difference of the group. These results indicate that a certain rate of the ground motions moves the foundation as effective inputs during the whole duration of earthquakes.

Relation between Free-Field Motions and Effective Input Motions

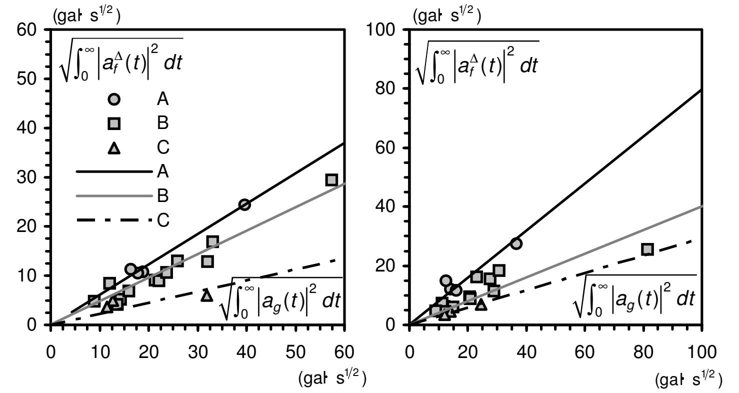
Effective Input Motions of Translational Component

Relationships between the square root of the integrated squared-motion recorded on the foundation and that on the free-field are plotted in figure 3. The straight lines in these figures were drawn for respective groups based on the least-square method, and the slopes may be interpreted as an average EICs for the



(a) Group A (b) Group B (c) Group C

Figure 2. Time histories of translational acceleration (above) and their integrated-squared motions (below) for each group's representative motion.



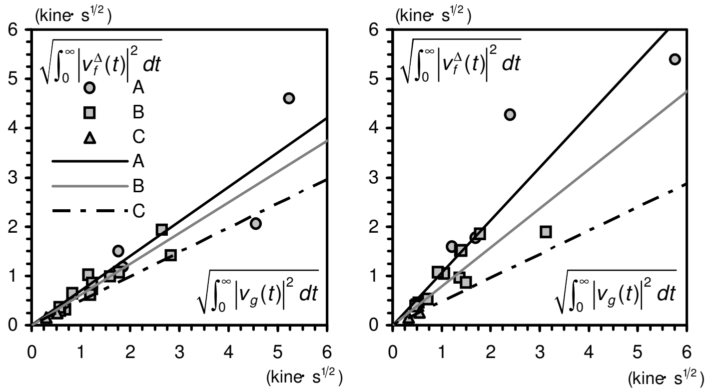
(a) X-direction (b) Y-direction

Figure 3. Relationships between square root of integrated-squared motion on the foundation and that on the free-field (Translational Acceleration).

translational acceleration. In the same manner, relationships between translational velocity motions are plotted in figure 4. The average EIC of translational accelerations and those of translational velocities are shown in table 2, together with the correlation coefficient (C.C.). The results of EIM evaluated by peak values of free-field motions and foundation responses are shown in table 2 for reference (Iguchi et al. 2000).

These results indicate that the EIMs of velocity motions evaluated by average EICs and peak ratios are larger than those of acceleration. This tendency is coincide with the results obtained for the records during the 1995 Hyogo-ken Nanbu Earthquake (Yasui et al. 1998). In addition, table 2 shows that average EICs decrease in the order of groups A, B and C. In other words, the input loss tends to be remarkable, as the higher frequencies contained in the free-field motions become more pronounced.

However, taking a careful look at the results shown in table 2, opposite tendency is detected, which might be found in slopes of regression results evaluated based on peak values of motions, for example, and peak values of acceleration between B and C groups in Y direction. Such mismatches can be seen in the peak values of velocity motions of X direction. These results seem to indicate that the EIC proposed in this paper will be a stable index corresponding to frequency components of free-field motions.



(a) X-direction (b) Y-direction

Figure 4. Relationships between square root of integrated-squared motion on the foundation and that on the free-field (Translational Velocity).

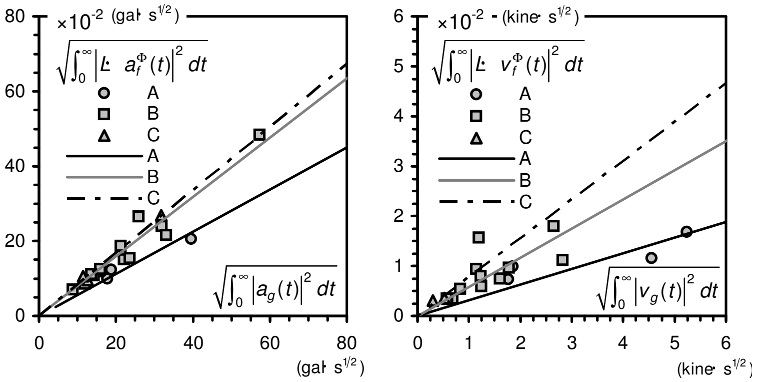
Table 2. Averages of translational effective input coefficients and ratios of peak values.

	Acceleration								Velocity							
	X direction				Y direction				X direction				Y direction			
	Ave. of EIC		Max of Acc.		Ave. of EIC		Max of Acc.		Ave. of EIC		Max of Vel.		Ave. of EIC		Max of Vel.	
Group	Slope	C.C.	Slope	C.C.	Slope	C.C.	Slope	C.C.	Slope	C.C.	Slope	C.C.	Slope	C.C.	Slope	C.C.
A	0.62	0.99	0.56	0.88	0.80	0.95	0.74	0.98	0.70	0.83	0.66	0.96	1.07	0.88	1.30	0.92
B	0.48	0.97	0.53	0.96	0.40	0.89	0.31	0.96	0.62	0.93	0.69	0.99	0.79	0.85	0.52	0.94
C	0.22	0.86	0.20	0.99	0.29	0.99	0.37	1.00	0.48	*	0.55	*	0.48	*	0.37	*
Total ave.	0.48	0.89	0.47	0.87	0.45	0.76	0.34	0.91	0.67	0.92	0.68	0.98	0.97	0.90	0.77	0.75

* Because of small number of records, the correlation coefficients are not shown.

Effective Input Motions of Rotational Component

It should be noted that a rotational component of EIM is applied to superstructure in addition to the horizontal component. In order to investigate the EIC of rotational component defined by Eq.(7), relationships between integrated-squared rotational angular accelerations on the foundation and integrated translational accelerations on the free-field are plotted in figure 5. The lines shown in these figures were drawn for respective groups based on the least-squares method. Averages of EICs for rotational angular acceleration and those of rotational angular velocities are shown in table 3, together



(a) Acceleration (b) Velocity

Figure 5. Relationships between square root of integrated-squared rotational motions and that of ground motions (Rotational Component).

Table 3. Averages of rotational effective input coefficients and ratios of peak values.

	Angular Acceleration				Angular Velocity			
	Ave. of EIC		Max of Acc.		Ave. of EIC		Max of Vel.	
Group	Slope($\times 10^{-2}$)	C.C.	Slope($\times 10^{-2}$)	C.C.	Slope($\times 10^{-2}$)	C.C.	Slope($\times 10^{-2}$)	C.C.
A	0.56	0.97	0.54	0.97	0.31	0.88	0.23	0.90
B	0.79	0.97	0.78	0.96	0.58	0.73	0.50	0.93
C	0.84	1.00	1.46	1.00	0.78	*	0.82	*
Total ave.	0.75	0.93	0.88	0.87	0.41	0.73	0.39	0.80

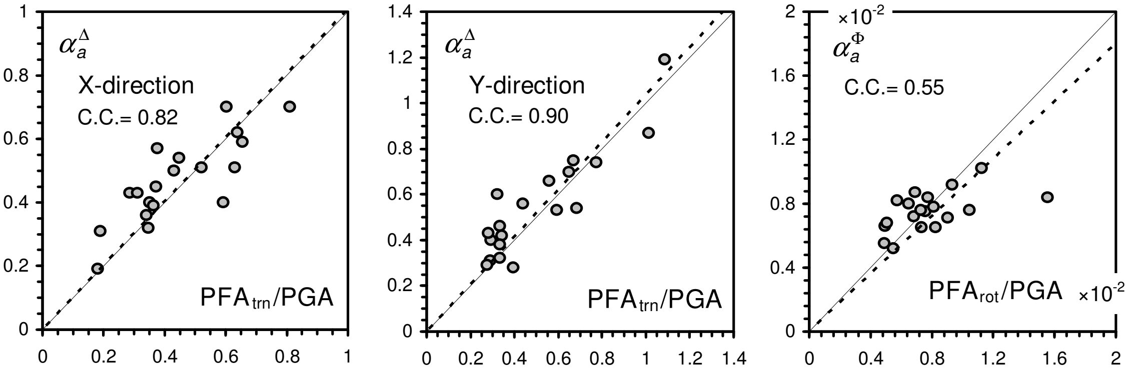
* Because of small number of records, the correlation coefficients are not shown.

with results of EIM evaluated by peak values of the free-field motions and foundation responses.

These results indicate that the average EICs increases in the order of groups A, B and C, thus, the rotational EIM becomes larger as the higher frequencies contained in the free-field motions become pronounced. This tendency is different from that of the translational component of EIM. It should be also noted that average EIC of angular velocity is relatively larger than those of angular acceleration. It may be resulted from that the distribution of velocity motions along the depth is almost uniform (Iguchi et al. 2000), whereas rotational motions of an embedded foundation would be mainly induced by nonuniform horizontal motions applied on the sides of the embedded foundation.

Comparison of Effective Input Coefficients and Ratios of Peak Values

It was shown in Table 2 and 3 that EICs were closely related to peak ratios of the foundation response to those of the free-field motions. To clarify the tendencies, compared results between the EICs and the ratios of peak values of acceleration (PFA/PGA) for each earthquake are shown in figure 6 together with regression results in dashed line. In these figures, $\mathrm{PFA}_{\mathrm{trn}}$ and $\mathrm{PFA}_{\mathrm{rot}}$ denote translational and rotational peaks of foundation acceleration motions, respectively, and PGA denotes the peak ground acceleration. Though somewhat differences may be recognized, the slopes of these lines are nearly 1 in average. This indicates that the EICs of translational acceleration are approximately equal to the results of EIM evaluated based on peak values of acceleration. As for the EIC of rotational acceleration, on the other hand, less correlation with the results evaluated based on peak values of acceleration is detected as shown in figure 6(c).



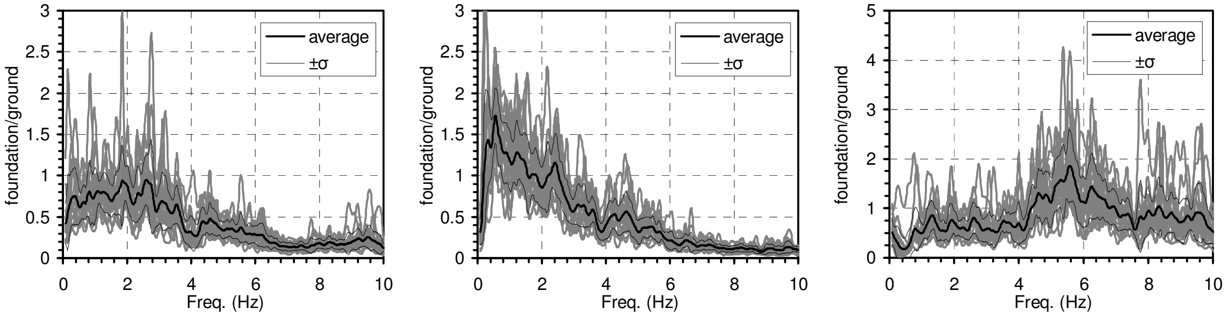
(a) α_a^Δ vs. $\mathrm{PFA}_{\mathrm{trn}}/\mathrm{PGA}$ (b) α_a^Δ vs. $\mathrm{PFA}_{\mathrm{trn}}/\mathrm{PGA}$ (c) α_a^Φ vs. $\mathrm{PFA}_{rot}/\mathrm{PGA}$

Figure 6. Comparison of effective input coefficients and ratios of peak values.

Correspondence between Effective Input Motions and Transfer Function

As described earlier, there is a theoretical basis for the EIC having relation to the transfer functions of soil-foundation system. We confirm in this section the correspondence between the EICs and the transfer functions of the system on the basis of observed records.

The transfer function of the soil-foundation system can be extracted from Fourier spectral ratio between the foundation response and the free-field motions. Thus obtained transfer functions for two translational and rotational components are shown in figure 7, in which Fourier spectra are smoothed by using the Parzen's window with a bandwidth 0.2Hz. Gray lines represent results of respective earthquakes, and the heavy and light lines represent mean and mean±standard variation (σ) of 19 events, respectively. As being detected from the results shown in figure 7(a) and (b), the translational transfer functions tend to decrease with increase of frequencies and become less than one in magnitude. This indicates that the earthquake ground motions would be less effective to move the foundation with increasing of frequencies especially for higher frequencies. The transfer function for the rotational component shown in figure 7(c), on the other hand, shows a different tendency with respect to frequencies. The transfer function of the rotational component has a peak around 6Hz, which may be considered to be the natural frequency of the rocking mode of the soil-foundation system, and tends to decrease for lower frequencies.

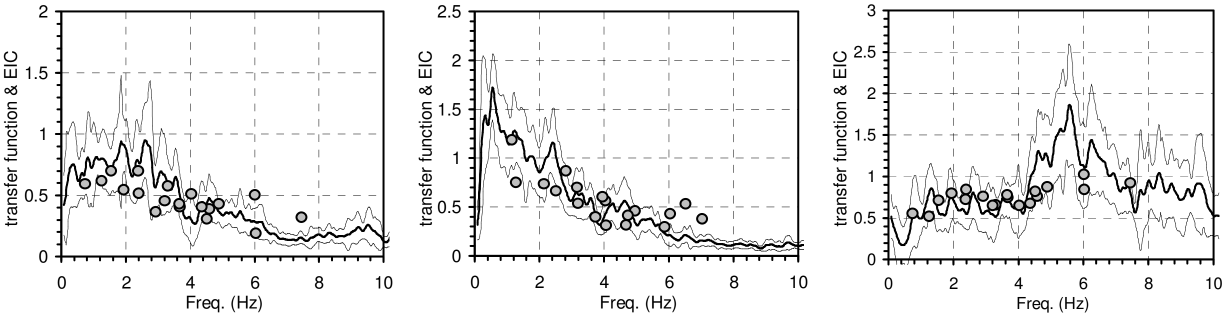


(a) Translation (X-direction) (b) Translation (Y-direction) (c) Rotation

Figure 7. Estimation of the transfer function.

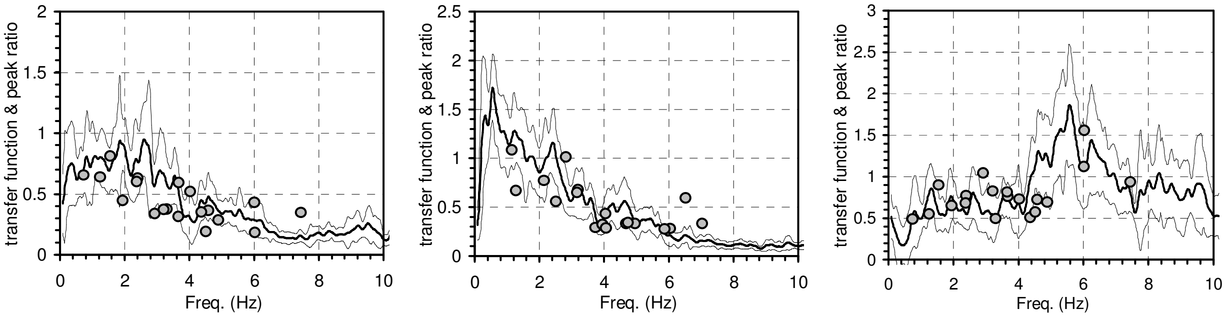
In figure 8, relationships between EICs of translational and rotational acceleration motions and the transfer functions of soil-foundation system are demonstrated. In the figure, the values of EIC are plotted on horizontal axes corresponding to the equivalent predominant frequency of the ground motion, which is defined by PGA/PGV/2π (Kojima et al. 2004). These results show close correlation between EICs and the transfer functions. The amplitude of the transfer function in X-direction has peaks around at 2-3Hz and in Y-direction translational motion at around 0.5Hz, and amplitudes of these transfer functions tend to decrease with increase of frequencies. These tendencies coincide with the tendencies that EICs decrease for the higher frequencies contained in the free-field motions. On the other hand, the amplitude of transfer function of the rotational motion increase as the frequency increases, and has a peak at 5-6Hz, this tendency corresponds to the relationship between the frequency component of ground motions and EIC of rotational motions.

In the same manner, figure 9 shows relationships between ratios of peak values of acceleration motion of foundation to the ground (PFA/PGA) and the transfer function, as in the same fashion as Kojima et al. (2004). Though correlation is slightly weaker than in case of the EIC shown in figure 8, ratios of peak values of acceleration show correspondence to the transfer functions. The theoretical basis of this correspondence can not be found out, but the ratios of peak values correspond to the transfer function as a result. This may be understood from the fact that the correlation between ratios of peak values of



(a) Translation (X-direction) (b) Translation (Y-direction) (c) Rotation

Figure 8. Correspondence of effective input coefficients and transfer function



(a) Translation (X-direction) (b) Translation (Y-direction) (c) Rotation

Figure 9. Correspondence of ratios of peak values and transfer function.

acceleration and EICs is high as shown in figure 6. Thus, the ratio of peak values of the foundation acceleration response and the ground acceleration can be used as a simple index to evaluate EIM.

Concluding Remarks

The objective of this paper is to present an appropriate index to measure effective input motions (EIM) to structures comparing to the reference motions on the free-field. As the EIMs are very much dependent on frequency component included in the ground motions, the index is preferred to reflect the frequency component. This paper proposed an index to measure the EIM and validated the effectiveness on the basis of earthquake records observed on the embedded foundation and surrounding soil. Main findings obtained in this study can be summarized as follows:

1) An index for the EIM, which is defined as a ratio of square root of the integrated-squared motions, is proposed and the theoretical basis is shown that the index is related to the transfer function of the soil-foundation system.

2) As the EIM, translational and rotational components of earthquake motions are to be applied to a superstructure.

3) The translational EIM decreases with increase of higher frequencies of the ground motion, and the rotational EIM has an opposite tendency regarding frequencies.

4) The index for the EIM corresponding to equivalent predominant frequency of the soil fits well to the transfer function of soil-foundation system estimated from the observed records.

5) A conventional simple index evaluated by peak ratios of acceleration motions observed on a foundation and on a free-field is effective to measure the EIM.

It would be worth describing that the above conclusions are found to be valid in tendency for another soil-foundation-structure system evaluated based on earthquake observation. The details will be shown in a separate paper.

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