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SEISMIC SHEAR MODEL FOR NON-LINEAR DYNAMIC ANALYSIS OF CONCRETE SHEAR WALL BUILDINGS

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ABSTRACT

Nonlinear dynamic analysis has become a requirement for the seismic design of high-rise concrete shear wall buildings on the US west coast. Current models for nonlinear flexural analysis are simple, rational and can accurately predict the response of concrete walls. In comparison, models for seismic shear are very primitive - a linear response until brittle failure is often assumed. Concrete design codes such as CSA-A23.3 and ACI-318 do not include any guidance on the parameters needed to model concrete walls in a non-linear analysis: cracked-section stiffness, shear strain at yielding and ultimate shear strain.

The authors have previously developed a simple, rational model for membrane elements subjected to seismic shear. The model separates deformations due to cracks from deformations due to concrete between cracks. Simple monotonic stress-strain relationships for concrete are combined with a crack-closing function to accurately predict the complex seismic shear response observed in experiments. The membrane element model has been extended to include the influence of bending strains typical of shear walls. The proposed shear model is compatible with the shear strength provisions of the 2004 Canadian concrete code CSA-A23.3-04.

The shear strength of a concrete wall is combined with the cracked-section shear stiffness and ultimate shear strain to define the envelope of shear response. Simple hysteretic rules provide the complete cyclic response. The resulting shear wall model defines the response of a “shear spring” element that can be used independently of (in parallel with) the flexural model, or can be combined with a nonlinear flexural model to capture the complex interaction between shear and flexure. The nonlinear shear and flexure models are combined by using the average vertical strain in the wall determined from the flexural model in determining the shear envelope.

Introduction

Nonlinear dynamic analysis has become a requirement for the seismic design of high-rise concrete shear wall buildings on the US west coast. While such analyses are very sophisticated, the accuracy of the results is limited by the accuracy of the models used to simulate concrete walls. While fibre models for nonlinear flexural analysis are simple, rational and can accurately predict the response of concrete walls, models for seismic shear are very primitive - a linear response until brittle failure is often assumed.

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Concrete design codes such as CSA-A23.3 and ACI-318 provide the necessary equations to calculate the shear strength of concrete walls, including the influence of axial loads and flexure. They do not include any guidance on the parameters needed to model concrete walls in a non-linear analysis: cracked-section stiffness, shear strain at yielding and ultimate shear strain. Seismic guidelines such as FEMA 356 define a non-linear shear model, but take the simplified approach of using the gross concrete shear stiffness (un-cracked section) until the shear strength is reached.

The nonlinear shear response of reinforced concrete is complicated by the influence of bending strains. The simplest shear problem is a membrane element, which is subjected to uniform normal strains (no bending) and has uniformly spaced reinforcement in two orthogonal directions. The authors have developed a simple, rational model for membrane elements subjected to seismic shear (Gérin and Adebar, 2004, n.d.). The model separates deformations due to cracks from deformations due to concrete between cracks. Simple monotonic stress-strain relationships for concrete are combined with a crack-closing function to accurately predict the complex seismic shear response observed in experiments.

This paper presents a non-linear model for the shear response of concrete shear walls that is an extension of the rational model for membrane elements. The model defines the response of a "shear spring" element that can be combined with a single or multi-element flexural model (Fig. 1). The proposed shear model is compatible with the shear strength provisions of the 2004 Canadian concrete code CSA-A23.3-04. The shear strength of a concrete wall, combined with the cracked-section shear stiffness and ultimate shear strain, define the envelope of shear response. Simple hysteretic rules provide the complete cyclic response.

General Model for Seismic Shear

To develop a rational model for reinforced concrete subjected to seismic shear, Gérin and Adebar (2004, n.d.) first studied test results from membrane elements subjected to reverse-cyclic shear. Membrane elements represent the simplest shear problem: a panel (or wall element) subjected to uniform normal strains (no bending) with uniformly spaced reinforcement in two orthogonal directions. Close examination of the relationships between stress and strain components led to identification of key mechanisms of the cyclic shear response:

- Shear strain of the element is controlled primarily by strain of the weaker reinforcement (first reinforcement to yield), e.g., yielding of the reinforcement causes yielding in shear
- Pinching of the hysteresis loops in the typical shear response is primarily governed by plastic strain accumulated in weaker reinforcement.
- The principal strain angle lags the principal stress angle through the reversal stage. This lag is a function of plastic strain in the reinforcement.
- Strains due to applied shear are primarily due to deformations at the cracks; concrete between the cracks contributes very little to the total strains.
- The monotonic response forms an envelope to the cyclic response.

These key mechanisms were implemented in a general model where deformations at the cracks are separated from deformations of concrete between cracks (Gérin and Adebar, 2004, n.d.). While the concrete and reinforcement strains are directly related to applied loads, crack deformations are a consequence of maintaining strain compatibility between concrete and reinforcement. Separating deformations at cracks from deformations in concrete between cracks enables shear strain to be explicitly linked to reinforcement strains. This approach is a significant departure from existing models where cracked concrete is treated as a single homogeneous material.

Figure 2 shows the typical response of a membrane element subjected to reverse-cyclic shear as well as a prediction from the general model. The general model uses simple material stress-strain relationships, which when coupled with deformations at the cracks, capture the complex behaviour including pinching of hysteresis loops. These material models are compatible with typical flexure models. The general model is suitable for a step-wise implementation where stresses and strains are updated at each increment of applied loads (in-plane shear and normal axial loads) to predict the full cyclic load-deformation response.

The general model explicitly considers the influence of crack angle, concrete tension, and axial load in two directions. This allows the importance of each parameter to be evaluated and simplifications can be made on a rational basis. For example, for seismic design of typical walls – with reinforcement ratios within modern code limits – concrete tension can be ignored, and the crack angle can be assumed equal to the principal concrete stress angle. A simpler model, more easily implemented in a non-linear analysis, was developed, where shear strain can be defined directly from shear stress, avoiding the need for an iterative solution (Gérin, 2003).

Model for Seismic Shear with Non-Yielding Flexure

In many regions of a high-rise concrete wall, bending moments are relatively small and have little influence on shear response. This is true, for example, above the plastic hinge region where flexure is not sufficient to yield the vertical reinforcement. In these cases, the shear response can be modelled independently from flexural response; however the influence of axial loads must be accounted for. The membrane element model is the ideal model for shear response in this case.

The basic shear model is shown in Fig. 3. It consists of a tri-linear envelope and two simple rules to define unloading and reloading. The model is defined by shear stress at cracking, shear stress at yielding, shear strain at yielding and ultimate shear strain. Only vertical axial load is included, and concrete tension is ignored. The wall is assumed to have at least minimum reinforcement uniformly distributed in both directions.

Shear Stress at Cracking

The shear stress at cracking may be estimated using empirical equations from concrete design codes for the concrete contribution V_c. Alternatively, the following simple equation for the shear stress at cracking, neglecting flexure and assuming a 45 deg. principal stress angle, can be used:

$$v_{cr} = f_{cr}\sqrt{1+\frac{n_v}{f_{cr}}} \quad (1)$$

where n_v is the vertical axial stress – the axial force over the gross concrete area – and f_{cr} is the principal tensile stress at cracking, which can be estimated from the specified concrete compression strength, f'_c, as follows:

$$f_{cr} = 0.33\sqrt{f'_c} \quad (2)$$

Before cracking, the concrete section shear stiffness can be taken as 0.4 times the concrete modulus of elasticity, $G = 0.4E_c$.

Shear Stress at Yield

The shear at yielding is assumed equal to the shear strength of the wall. The shear strength equations from the 2004 Canadian concrete code CSA A23.3-04 are used here, although other strength equations may be used. For consistency with other aspects of the model, the equations are expressed in terms of shear stress instead of shear force.

The shear stress at yield can be assumed as the sum of a concrete contribution, v_c, and a reinforcement contribution, v_s, i.e.:

$$v_y = v_c + v_s \tag{3}$$

and

$$v_c = \beta\sqrt{f_c'} \tag{4}$$

$$v_s = \rho_h f_y \cot\theta \tag{5}$$

where β is defined in the Canadian concrete code, ρ_h is the horizontal reinforcement ratio, f_y is the reinforcement yield strength (typically 400 MPa), and θ defines the orientation of the principal compression stress with respect to the vertical. In the Canadian concrete code CSA-A23.2-04, β and θ are defined as a function of the average axial strain of a member. For a concrete wall, this would be the vertical strain at the mid-point of the wall. As a simplification, the Canadian concrete code allows values of β = 0.18 and θ = 35 deg to be used for any axial strain values. Therefore, these values can be used in a simplified shear model for a concrete wall.

Under monotonic loading, additional strength may be gained after the horizontal reinforcement yields and the vertical reinforcement resists a larger portion of the applied shear. Under reverse-cyclic loading, very little of that strength gain may be realized, therefore, the response after yielding is assumed to be perfectly plastic.

To ensure the reinforcement yields before a concrete compression failure occurs, the shear stress at yielding should be limited to:

$$v_y \le 0.25 f_c' \tag{6}$$

Shear Strain at Yield

The shear strain at yield is used to define the cracked-section shear stiffness and the ultimate shear strain. The following simple equation can be used to estimate shear strain at yield:

$$\gamma_y = \frac{f_y}{E_s} + \frac{v_y - n_v}{\rho_v E_s} + \frac{4v_y}{E_c} \tag{7}$$

with the condition that: $0 \le \frac{v_y - n_v}{\rho_v E_s} \le \frac{f_y}{E_s}$

Equation 7 is based on the assumption that the horizontal reinforcement yields first, which is the case when the flexural strains are relatively small. Equation 7 is also based on the conservative assumption of a principal stress angle of 45 deg, which gives a minimum value for shear strain at yield.

Cracked-section Shear Stiffness

With the shear stress at yielding (shear strength) and shear strain at yielding defined, the cracked-section shear stiffness is simply the ratio of these:

$$G_{cr} = \frac{v_y}{\gamma_y} \tag{8}$$

It is interesting to note that while the uncracked section shear stiffness is entirely a function of the concrete (E_c), the cracked-section shear stiffness is governed primarily by the quantity of reinforcement.

Ultimate Shear Strain

The ultimate shear strain is defined as the maximum shear strain without significant loss of shear strength. It is defined as a function of the shear strain at yield and the demand on the concrete, which is expressed as the ratio of shear strength to concrete compression strength. A concrete shear failure – failure due to excessive shear deformations at the cracks – is assumed to occur once the shear strain exceeds the limit:

$$\frac{\gamma_u}{\gamma_y} = 4 - 12\frac{v_y}{f_c'} \tag{9}$$

Equation 9 is compared with experimental data in Fig. 3.

Hysteretic Rules

The simple hysteretic model shown in Fig. 2 assumes that yielding occurs at v_y during each cycle and unloading occurs at a constant slope equal to G_{cr}. The plastic shear strain, γ_p, remaining at the end of each unloading segment is assumed to be cumulative from one direction of loading to the other. The reloading curve accounts for the closing of diagonal cracks in one direction and the simultaneous opening of diagonal cracks in the other direction in a simple way.

The shear strain at any applied shear stress level is given by:

$$\gamma = \gamma_e + k\gamma_p \tag{10}$$

where γ_e is the elastic shear strain, γ_p is the accumulated plastic shear strain and the coefficient k controls the reversal of plastic strain from one direction to the other. These parameters are defined as:

$$\gamma_e = \frac{v}{G_{cr}} \tag{11}$$

$$k = 2e^{\left(\frac{-2|v|}{1+0.4|v|}\right)} - 1 \tag{12}$$

$$\gamma_p = \gamma_{\max} - \frac{v_{\max}}{G_{cr}} \tag{13}$$

where v is the applied shear stress at a given time-step. In Eq. 13, γ_{max} and v_{max} are the maximum shear strain and shear stress reached at that point. Unlike the other two parameters, the plastic shear strain γ_p is only updated when unloading from a cycle where yielding has occurred, i.e., the parameter is not updated continuously.

Model for Seismic Shear with Significant Flexural Yielding

If the strains due to bending are significant, they need to be accounted for in the shear model. This is certainly the case in the flexural hinge region of a concrete wall as shown in Fig. 6. The following is a summary of the changes to the model above in order to account for bending deformations. Those aspects of the model that are unchanged are not repeated.

In the plastic hinge region of a wall, the rotational demand on the wall is the best indicator of average deformations. The rotational demand can be determined directly from the nonlinear flexural model for the wall. Approximate procedures for estimating inelastic rotation from global drift are given elsewhere (Adebar 2006).

Shear stress at yield

Due to "fan action" in the lower portion of the wall shown in Fig. 6, there is a concentration of shear stress in the compression zone at the bottom right corner of the wall. There are no requirements in non-seismic provisions to account for this concentration of shear stress because the increased diagonal compression stresses are compensated for by increased diagonal compression strength of concrete due to reduced tension strains in this highly compressed region.

When there is reverse cyclic loading, the concrete compression zone will be damaged from previous cycles during which the compression zone is the tension zone. The limit on the shear stress given by Eq. 6 to ensure the horizontal reinforcement yields before a concrete diagonal compression failure must be reduced depending on the rotational demands on the wall. The factor 0.25 in Eq. 6 should be reduced to 0.10 unless the inelastic rotation demand is less than 0.015. When the inelastic rotation demand is less than 0.005, the 0.25 factor need only be reduced to 0.15. For inelastic rotations between 0.005 and 0.015, linear interpolation should be used.

A conservative model to determine the quantity of horizontal reinforcement needed to avoid diagonal tension failure results from assuming $\beta = 0$ in Eq. 4 and $\theta = 45$ deg in Eq. 5 for the critical diagonal failure plane shown in Fig. 6. As this approach is conservative for walls with significant damage, it is correspondingly too conservative for walls with low levels of damage. It is common practice to use non-seismic shear design provisions for members with low levels of seismic damage. Thus β can be taken as 0.18 when the inelastic rotational demand on the plastic hinge region of a concrete wall is less than 0.005. The value of β should be taken as zero ($V_c = 0$) when the inelastic rotational demand on the wall is equal to or greater than 0.015. When the inelastic rotational demand is between 0.005 and 0.015, the value of β in Eq. 4 should be determined by linear interpolation between the values of 0.18 and 0.

For a non-seismic shear model, axial tension strain is a good indicator of critical diagonal crack inclination in the uniform stress regions of members prior to yielding of longitudinal reinforcement. Unfortunately, this approach is not suitable for disturbed stress regions such as the base of a concrete wall, particularly after the vertical reinforcement yields. The level of axial compression force in the wall divided by $f_c'A_g$ was found (Adebar, 2006) to be a good indicator of the critical crack angle at the boundary of the fan. The compression stress angle θ in Eq. 5 is assumed to be 45° unless the axial compression force acting on the wall is greater than $0.1f_c'A_g$. When the axial compression is greater than or equal to $0.2f_c'A_g$, the value of θ can be taken as 35°. For axial compression between these two limits, linear interpolation should be used.

Cracked-section shear stiffness

The influence of bending strain on the cracked section shear stiffness can be included as:

$$G_{cr}^{*} = \frac{G_{cr}\rho_h f_y}{G_{cr}\varepsilon_v + \rho_h f_y} \tag{14}$$

where ε_v represents the average vertical strain at the mid-point of the wall over the plastic hinge height. It can be determined from the flexural model or estimated from:

$$\varepsilon_v = \frac{M / jd - P/2}{2E_s A_{sT}} \geq 0 \qquad (15)$$

where A_{sT} is the flexural tension (vertical) reinforcement concentrated in the zone on the flexural tension end of the wall. Note that Eq. 14 does not include a shear force term because the influence of shear force is already accounted for in the shear stiffness equation (Eq. 14).

Shear strains

The state of strain within the plastic hinge region shown in Fig. 6 is very complex, and can really only be modelled using a complex finite element model. However, a reasonable estimate of the strains can be made in a very simple way. The plastic hinge region can be divided into two parts: the fan region (triangular portion shown shaded in Fig. 6), and the uniform stress field region (non-shaded portion). The fan region has large vertical tension strains from the flexural tension, and significant diagonal compression strains along the compression struts which are inclined at varying angles. A reasonable estimate of shear strains can be made by assuming the shear strain in the uniform stress portion is the average strain across the entire plastic hinge region. Thus the membrane shear model described above can be used as the shear element in the plastic hinge region if the shear stress at yield and cracked-section shear stiffness are adjusted as described above. Eqs. 7 and 9 can be used to estimate the shear strain at yield and the ultimate shear strain capacity, respectively.

Conclusions

This paper describes a simple non-linear shear model for concrete walls. The shear strength of a concrete wall, combined with the shear strain at yielding of the horizontal reinforcement, defines the cracked-section shear stiffness of the wall. The ultimate shear strain capacity of the wall depends on the shear stress ratio. With the envelope of shear response accurately defined, simple hysteretic rules provide the complete cyclic response.

The shear model presented here defines the response of a "shear spring" element that can be used independently of (in parallel with) the flexural spring, or can be combined with a nonlinear flexural model to provide a model that captures the complex interaction between shear and flexure. The nonlinear shear and flexure models are combined by using the average vertical strain in the wall determined from the flexural model in determining the shear envelope.

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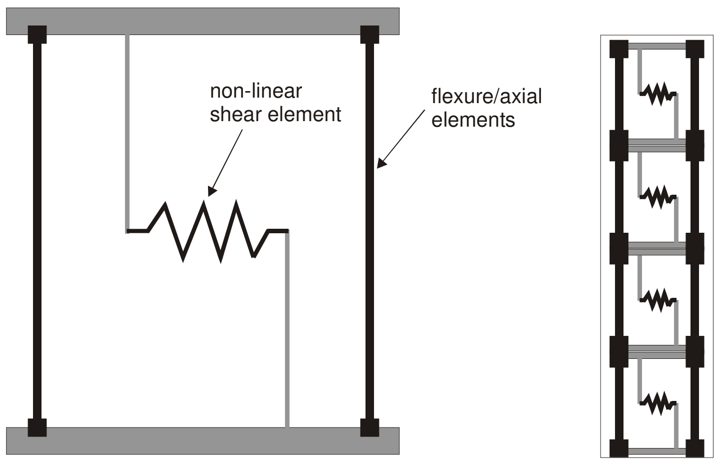

Figure 1. Simple model for concrete shear wall.

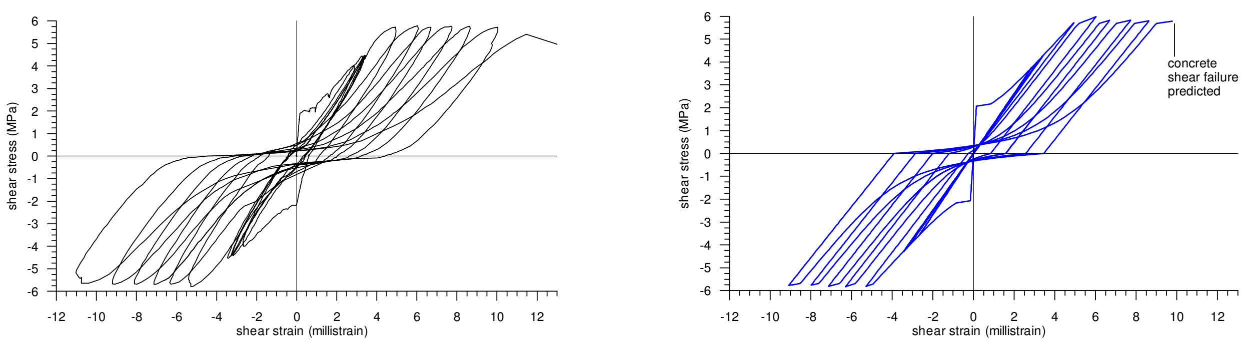

Figure 2. Example membrane shear response: (a) experimental results, (b) general model prediction.

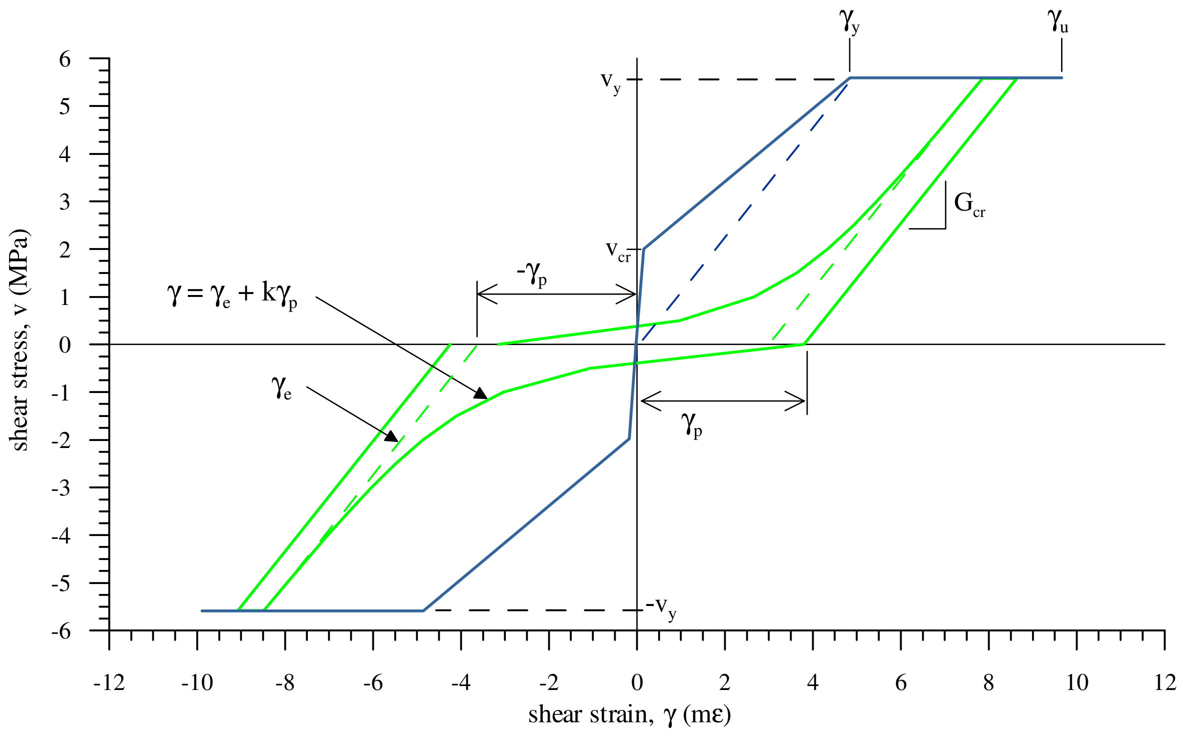

Figure 3. Proposed shear model for non-yielding flexure.

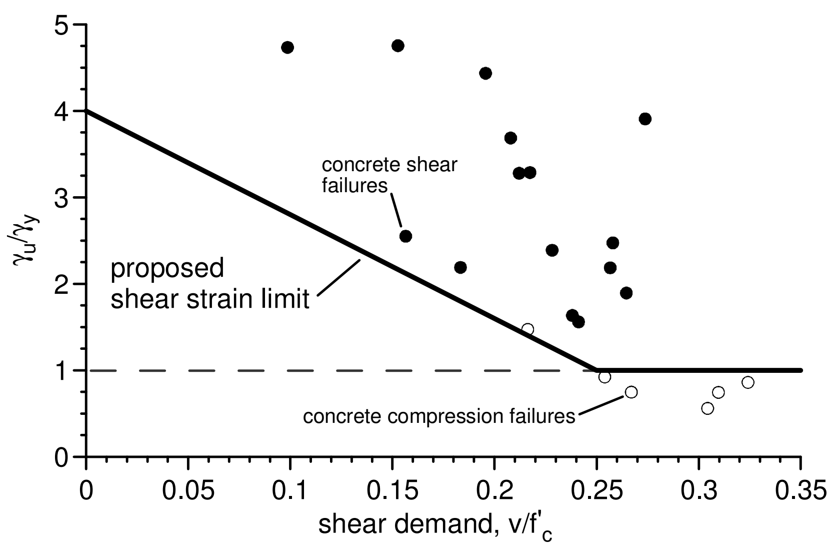

Figure 4. Influence of shear stress ratio on ultimate shear strain.

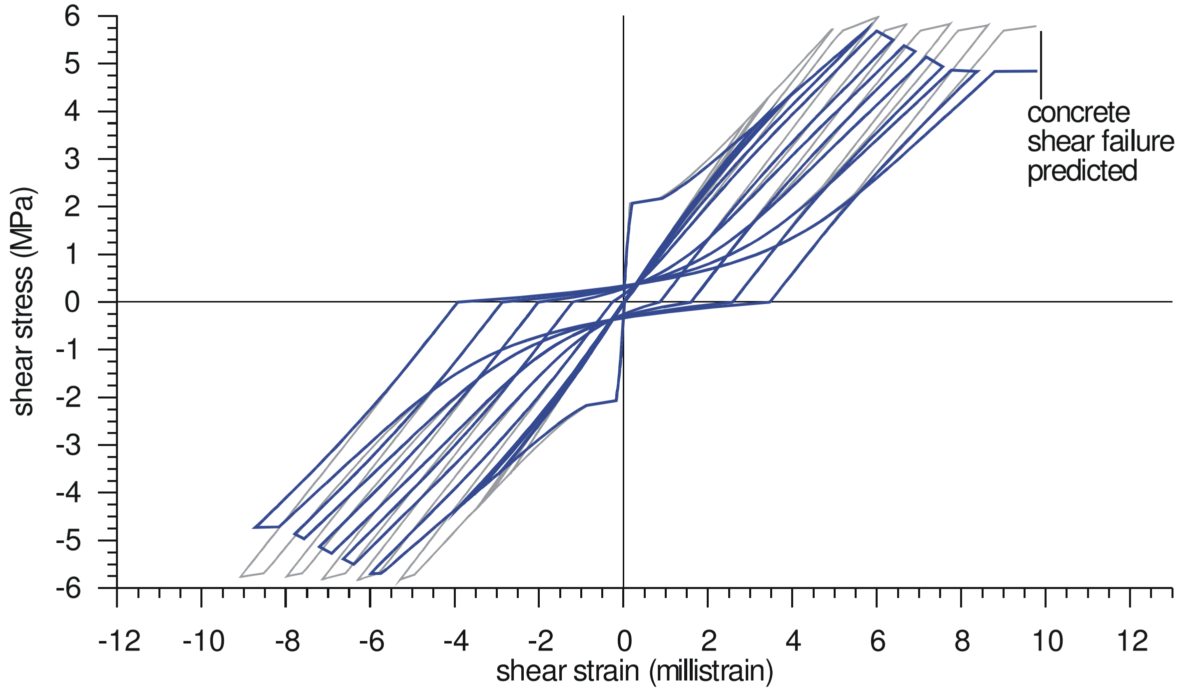

Figure 5. Example shear model with flexure.

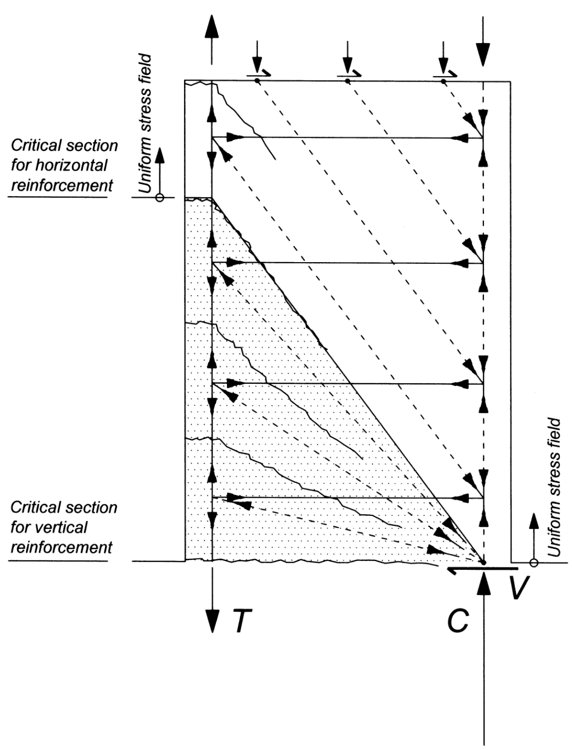

Figure 6. Strut-and-tie model showing internal force flow due to stirrup contribution of shear resistance in the plastic hinge region of a concrete wall.