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PLASTIC HINGE LENGTH OF CONCRETE WALL SYSTEMS

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ABSTRACT

The objective of the current study was to determine the plastic hinge length in concrete wall systems using nonlinear finite element analysis. The parameters considered were wall length, wall height, level of shear stress, and level of axial load – both compression (due to gravity loads) and tension due to coupling beam shear forces. The analysis was performed using program VecTor2, which uses the Disturbed Stress Field Model to account for shear in reinforced concrete. To calibrate the model, comparisons were made between predictions and the strains measured in a large-scale isolated cantilever wall tested at UBC. The plastic curvatures were observed to vary linearly over about twice the length of the traditional plastic hinge length assuming the maximum plastic curvature is uniform. The length of linearly varying plastic curvatures increase with wall length because larger strains cause the bending moment resistance of the wall to increase due to strain hardening, which results in an increase in height over which the bending moment exceeds the yield moment. Axial compression reduces the length of linearly varying plastic curvatures, while axial tension increases the length of plastic curvatures. Diagonal cracking due to shear increases the plastic hinge length, and a simple model was presented to estimate this increase. An expression was presented for estimating the maximum curvature demand in a more flexible wall that is interconnected by rigid floor slabs to a less flexible (longer) wall.

Introduction

The flexural displacement capacity (ductility) of a concrete wall subjected to lateral seismic force depends on the vertical extent of plastic curvatures near the base of the wall, and the curvature capacity of the wall, which in turn depends on the compression strain capacity of concrete. The plastic curvatures near the base of a wall are assumed to be uniform over a height called the plastic hinge length. Building code requirements, such as in ACI 318 and CSA A23.3, for determining whether confinement reinforcement is required in the ends of a concrete wall assume the plastic hinge length is equal to half the wall length.

Concrete walls in a building are interconnected by slabs at every floor level. Thus the deflected shape of all parallel walls must be identical when displaced in a translation mode, which implies that the plastic hinge lengths of two such walls cannot be very different regardless of the wall lengths. No recommendations currently exist for what plastic hinge length should be used for walls of different lengths in a building. Similarly, no information currently exists for what plastic hinge length should be used for coupled walls where axial forces are the dominant forces applied to the vertical wall segments.

The objective of the current study was to determine the plastic hinge lengths in complex wall systems

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using nonlinear finite element analysis. To calibrate the model, comparisons were made between predictions and the curvatures measured in a recent large-scale isolated cantilever wall test.

Previous Work on Plastic Hinge Lengths of Concrete Walls

Paulay and Uzumeri (1975) modified expressions developed by Sawyer (1964) and Mattock (1967) from beam tests by assuming that the effective depth is equal to 80% of the wall length l_w, and by assuming that the shear span is equal to the height of the wall h_w resulting in the following two equations for plastic hinge length:

$$l_p = 0.2\, l_w + 0.075 h_w \tag{1}$$

$$l_p = 0.4\, l_w + 0.05 h_w \tag{2}$$

The value of h_w/l_w for flexural concrete walls varies from a minimum of about 3 to a maximum of about 12, with most walls having a ratio from 5 to 10. Over this range, Eqs. 1 and 2 give a plastic hinge length l_p varying from about $0.5l_w$ to $1.0l_w$. In his well known paper on the design of ductile reinforced concrete structural walls for earthquake resistance, Paulay (1986) stated that the plastic hinge length l_p in walls varies from $0.5l_w$ to $1.0l_w$.

Paulay and Priestly (1993) presented the following equation for a lower-bound estimate of plastic hinge length in concrete walls:

$$l_p = 0.2\, l_w + 0.044 h_w \tag{3}$$

Sasani and Der Kiureghian (2001) recently re-examined the test data from 29 of the larger beams tested by Mattock (1964) and Corley (1966), which were up to 750 mm (30 in.) deep, and proposed the following expression for mean plastic hinge length in concrete walls:

$$l_p = 0.43 d + 0.077 \frac{\sqrt{z}}{d} \tag{4}$$

where the second term has units of metres (m).

Analytical Model

The current study was conducted using VecTor2, which was developed by Frank Vecchio at the University of Toronto. The program can be used to perform nonlinear finite element analysis of two-dimensional reinforced concrete membrane elements using the constitutive relationships of the Disturbed Stress Field Model (Vecchio 2000), which is a refinement of the Modified Compression Field Theory (Vecchio and Collins 1986). A complete description of the program is given by Wong and Vecchio (2002).

4-node plane stress rectangular elements with eight degrees of freedom were used to model the concrete with smeared reinforcement. A very fine mesh (about 11 elements per metre) were used to model the portion of the concrete walls where plastic hinging may occur. A courser mesh was used above this zone.

To calibrate the model, predictions were compared with the results of a test on a ¼ scale model of a slender concrete shear wall conducted by Adebar et al. (2004). The test specimen was 11.76 m (38.5 ft) high and 1.625 m (64 in.) long resulting in a height-to-length ratio of 7.2. To simulate a portion of the core of a high-rise building, the wall had a flanged cross-section, a low percentage of vertical reinforcement (0.45%), and was subjected to a constant axial compression of $0.1f_c'A_g$. The maximum displacement at the top of the wall was 281 mm (2.4% global drift), and the elastic portion of this displacement was 46 mm. Assuming a compression strain capacity of concrete of 0.003, the total curvature capacity of the wall is 22 rad/km, and the elastic portion of the curvature capacity is 2 rad/km. The height from the base of the plastic hinge to the level of measured wall displacement is 11.33 m. Using these values results in a plastic hinge length l_p = 1.09 m, which is equal to $0.67l_w$.

The curvatures of the wall were measured at numerous locations over the height, and these are compared with predictions from VecTor2 in Fig. 1. The predictions and experimental results, which are in reasonably good agreement, both indicate that the plastic curvatures (curvatures greater than the yield curvature) are not uniform over a plastic hinge length; but vary approximately linearly. A linearly varying plastic curvature distribution gives the same plastic rotation as calculated from the maximum plastic curvature assumed to be uniform over half the height of the linearly varying plastic curvature.

Additional comparisons between predictions made with VecTor2 and other experimental results on concrete walls are presented by Bohl (2006).

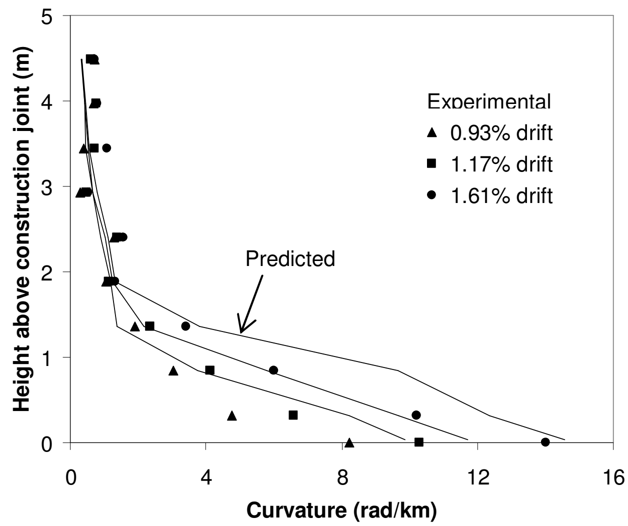

Figure 1. Calibration of analytical model: comparison with results from a large scale test (Adebar et al. 2004).

Plastic Hinging in Isolated Concrete Walls

A parametric study was conducted to investigate plastic hinge length in isolated concrete walls. The parameters that were investigated included: wall length, wall height (shear span), level of axial compression or axial tension, and level of shear stress. Figure 2 summarizes the cross sections of the two walls W1 and W2 that were used for the parametric study on isolated walls. The concrete had a cylinder compression strength of 40 MPa, while all reinforcement had a yield strength of 400 MPa. The shape of the stress-strain relationship for the reinforcing bars was matched to test results from typical reinforcing steel used in ductile walls. Strain hardening was included using a simple linear relationship from a stress of 400 MPa occurring at an average strain of 0.01 to a stress of 650 MPa occurring at an average strain of 0.061. Average strain refers to the strain measured over several cracks and including the influence of concrete tension stiffening.

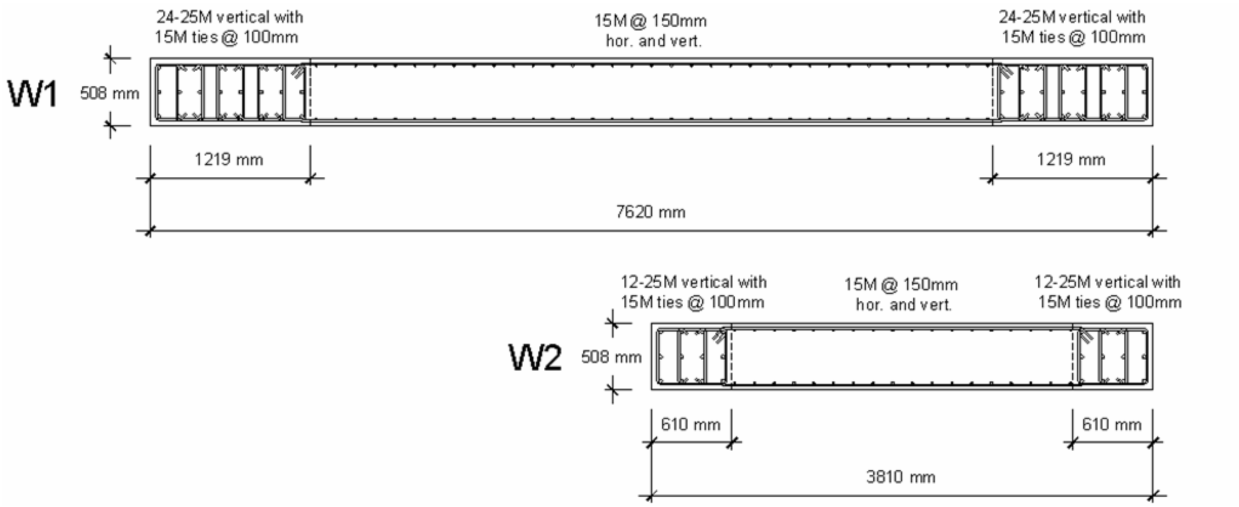

Figure 2. Details of walls used in study.

Fig. 3(a) shows the curvatures in wall W1 and W2 at a global drift of 2% for a wall height of 54.86 m. This drift is close to the displacement capacity of the longer wall. The curvatures vary approximately linearly near the base of the wall. Fig. 3(b) presents the steel strains in the walls at the same drift level. As there are a number of layers of concentrated reinforcement, the steel strains vary. The data points in Fig. 3(b) are for the outside layers of concentrated reinforcement on the tension side of the wall, i.e., the maximum and minimum strains in the concentrated reinforcement on the tension side of the wall. The steel strains clearly show at what level the tension reinforcement is yielding; however this does not define yield curvature. The yield curvature point in Fig. 3(a) is defined as the intersection of the two linear segments of curvature distribution. That is, the yield curvature is a section property that is not defined by any particular layer of reinforcement reaching the yield strain. For Wall W1 the yield curvature occurs at 8.4 m from the base, while for Wall W2 it occurs at 6.2 m from the base. In this paper, these heights are referred to as the length of linearly varying plastic curvatures l_p^*, which are approximately twice the length of the traditional plastic hinge length l_p, which assumes the plastic curvature is uniform.

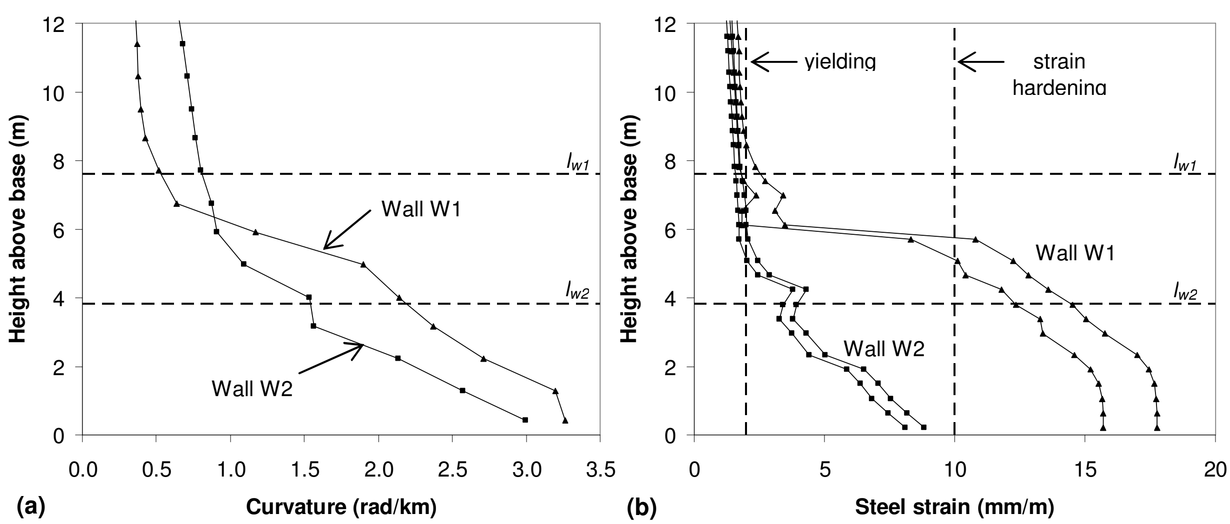

Figure 3. Results from isolated walls at 2% drift: (a) curvature distributions, (b) vertical reinforcing steel strains.

Fig. 3(b) indicates that the reinforcing steel in the longer wall W1 exceeds the strain at which strain hardening starts (0.010), while the reinforcing steel in wall W2, which is half as long, does not strain harden. The reason for this is that for the same curvature, a longer wall has proportionally larger tension strains. Strain hardening of the reinforcement in wall W1 causes the bending moment resistance of the wall to increase, which results in an increase of the height over which the bending moment exceeds the yielding moment. This in turn causes an increase in the height of linearly varying plastic curvatures, and is an explanation of why the plastic hinge length of a wall increases proportional to wall length.

Fig. 4 summarizes the length of linearly varying plastic curvatures l_p* in wall W1 (top) and W2 (bottom) at different levels of axial force; compression is (–) and tension is (+). The results are shown for a number of drift levels up to the drift capacity of the longer wall. Axial compression generally reduces the drift capacity of a wall. The walls with axial tension are relevant for coupled wall systems where the up lift is generated by the shear forces in the coupling beams. The results indicate that axial compression reduces the plastic hinge length, while axial tension increases the plastic hinge length. The reason for this is the shape of the bending moment – curvature relationship. When a wall is subjected to axial tension, the ratio of M_{max} to M_y is significantly increased, while when a wall is subjected to axial compression, the ratio decreases. M_{max} is the maximum bending moment at the curvature capacity of the wall, while M_y is the bending moment at the yield curvature of the wall.

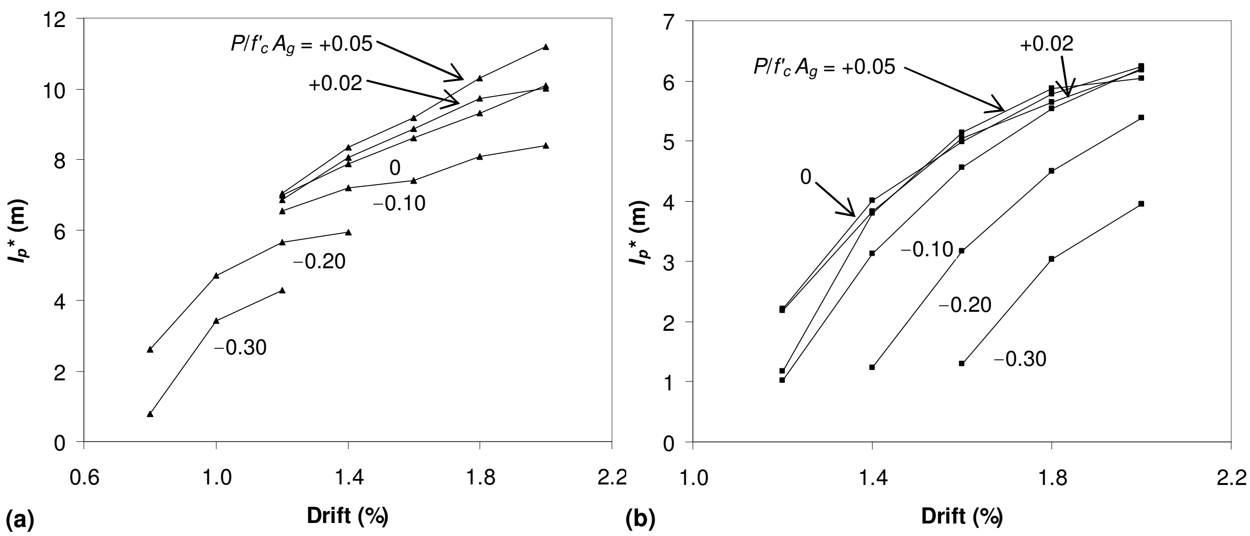

Figure 4. Influence of axial load on length of linearly varying plastic curvatures l_p* at varying drift levels: (a) wall W1, (b) wall W2.

Fig. 5 examines the influence of shear stress on the length of linearly varying plastic curvatures. For both wall W1 and W2, the yield curvature was determined from the bending moment – curvature relationships ignoring the influence of shear. The bending moment at yield curvature, i.e., the yield moment, ignoring the influence of shear was used to predict the height of linearly varying plastic curvatures l_p* from the bending moment diagram, and these were compared to the observed height of linearly varying plastic curvatures in Fig. 5(a). For cases where there was significant diagonal cracking, the length of linearly varying plastic curvatures significantly exceeds the length over which the bending moment exceeds the yield moment by up to a factor of almost 2. The reason for this is the additional tension force demands on the vertical reinforcement in a wall caused by shear in a diagonally cracked member.

A simple model for additional tension force demand due to shear on the vertical reinforcement on the flexural tension side of the wall is $T_v = V/2$. This assumes that the shear is resisted entirely by diagonal compression at 45 degrees, and that there is no significant concrete contribution (concrete tension stresses). The vertical tension force can be converted to a bending moment by multiplying the force by the internal flexural lever-arm normally assumed to be $0.8l_w$ for a wall. Thus the additional bending moment in a wall due to shear is $M_v = 0.4V \cdot l_w$. When this additional bending moment is added to the bending moment due to the applied lateral load, the height over which the bending moment exceeds the yield moment increases.

The predicted height of linearly varying plastic curvatures l_p* based on the adjusted bending moment is compared with the observed height of linearly varying plastic curvatures in Fig. 5(b). The additional bending moment due to shear is only added when there is significant diagonal cracking. For the results shown in Fig. 5, this is when the shear stress is equal to or greater than 1.8 MPa, which corresponds to $0.3\sqrt{f_c'}$. Fig. 5(b) indicates that the simple model used to account for the influence of shear works well at high drift levels; but is conservative (gives short lengths) at lower drift levels. The reason for this is because the influence of concrete tension stress (the concrete contribution) was ignored for simplicity. At lower drift levels, the influence of concrete tension stresses is significant on reducing the tension force demand on the vertical reinforcement.

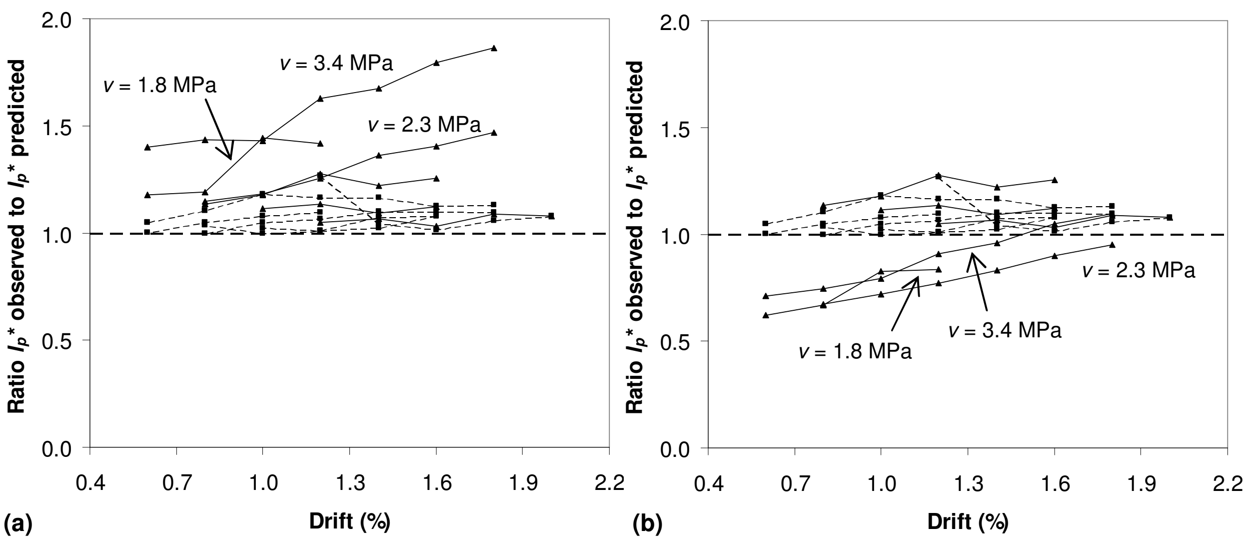

Figure 5. Influence of shear stress on predicted length of linearly varying plastic curvatures: (a) ignoring shear, (b) accounting for shear using proposed simple model.

Plastic Hinging in Inter-connected Concrete Walls

Concrete walls in a building are interconnected by slabs at every floor level. Thus the deflected shape of all parallel walls must be identical when displaced in a translation mode, which implies that the plastic hinge lengths of two such walls cannot be very different regardless of the wall lengths. As no information currently exists regarding what plastic hinge length should be used for walls of different lengths in a building, this was investigated in the current study.

Fig. 6 presents the results for Walls W1 and W2 when they are isolated (denoted in the figure by A for Alone), and when they are combined together (denoted by C). All walls are 54.86 m high, and the displacement at the top of the walls is about 1.10 m (2% global drift) at the deformations shown in Fig. 6. To create the combined case, the walls were interconnected by rigid diaphragms at 20 story levels. The height of each story was 2.743 m. Neither the curvatures of Wall W1 shown in Fig. 6(a), or the maximum steel strains in Wall W1 shown in Fig. 6(b) indicate any significant difference when the wall is isolated or combined with wall W2. It seems reasonable that a more flexible wall (W2 in this case) would not have a significant influence on the deformations of a longer and less flexible wall (W1 in this case).

In the case of Wall W2 however, there does appear to be a very significant effect whether the wall is isolated (Alone) or combined with Wall W1. The length of linearly varying plastic curvatures l_p* does not appear to be significantly influenced, however the maximum curvature and the maximum steel strain at the base of the wall are significantly increased. The expected result was that when the two walls are tied together at numerous floor levels, the curvature distributions would be very similar in order for the displacements of the two walls to match at the floor levels. Clearly the curvature distributions are not similar and thus the displacements of the two walls require further examination.

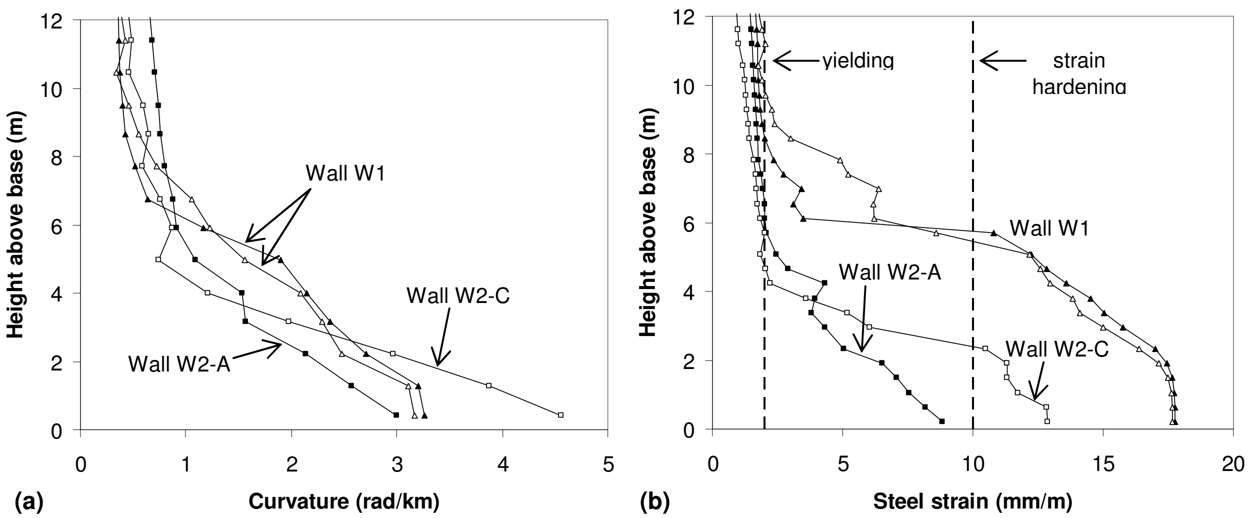

Figure 6. Influence of interconnecting two walls of different lengths by rigid floor slabs: (a) curvature distributions, (b) maximum steel strains.

Fig. 7 summarizes the displaced shapes of the walls resulting in the curvatures and steel strains shown in Fig. 6. Figure 7(a) shows the displacements over four floors, while Fig. 7(b) shows a close up of the displacements in the first story. The total displacements of the two walls are exactly equal at each floor level as the floor slabs were modeled as rigid diaphragms; however the flexural displacements of the two walls are not equal. Wall W1 has significantly more shear deformation than Wall W2. The two walls have the same width; and Wall W1 is twice as long as Wall W2. Thus Wall W1 has twice the shear area and twice the shear stiffness of Wall W2. On the other hand, Wall W1 has a flexural stiffness that is $(2)^3 = 8$ times larger than Wall W2. In the upper floors of the building where flexural displacements dominate, Wall W1 resists much more than twice the shear resisted by Wall W2, which is why W1 has larger shear deformations.

A simple expression was developed for estimating maximum curvature demand in the smaller wall (referred to as Wall 2 in the discussion below). It is based on the assumption that the deformations of the structure are controlled by the longer wall (called Wall 1), and that the flexural slopes of the walls (integrals of the curvatures) are equal immediately above the region of plastic curvatures. Immediately above the region of linearly varying plastic curvatures in the longer wall, the curvatures in both walls are assumed to be equal to the yield curvature of the longer wall φ_{y1}. The curvatures in the longer wall are assumed to vary linearly from φ_{y1} to a maximum value of φ_{m1} at the base of the wall over the length (height) of lp1*. The curvatures in the smaller wall are assumed to vary linearly from the yield curvature of the smaller wall φ_{y2} to a maximum value of φ_{m2} at the base of the wall over the length (height) of l_{p2}*.

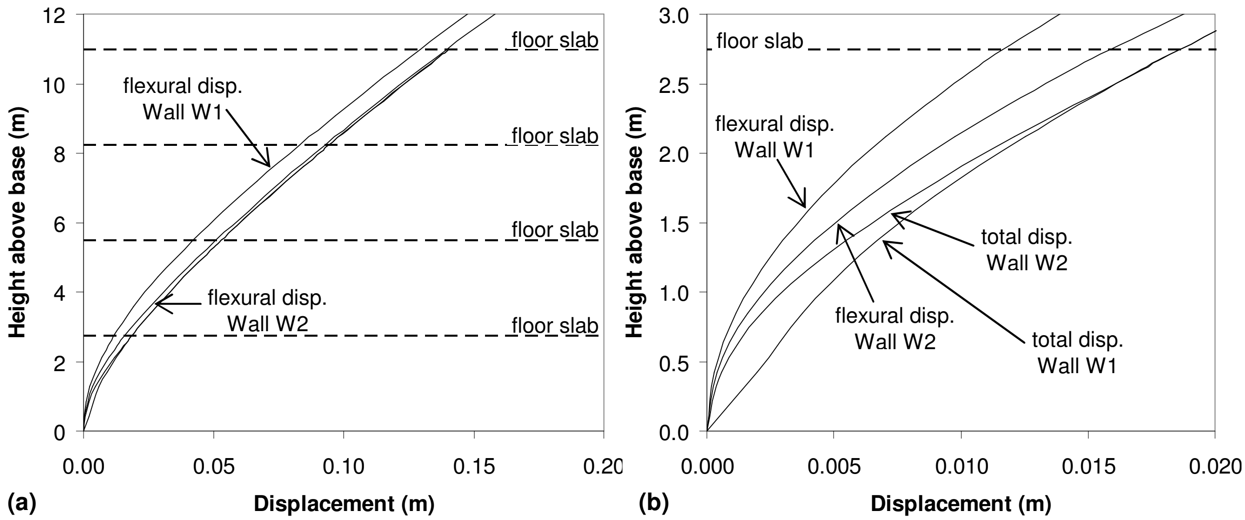

Figure 7. Displacement profiles of interconnected walls: (a) lower four floors, (b) close up of first floor.

From the height of l_{p2}* to l_{p1}*, the curvatures in the smaller wall are assumed to vary linearly from φ_{y2} to φ_{y1}. With these assumptions, the following simple expression results for the maximum curvature at the base of the smaller wall:

$$\varphi_{m2} = (\varphi_{m1} - \varphi_{y2})\left(\frac{l_{p1}*}{l_{p2}*}\right) + \varphi_{y1} \tag{5}$$

For the curvature distributions shown in Fig. 6(a) (combined wall case), the following values are relevant: l_{p1}* = 8.39 m, l_{p2}* = 4.64 m, φ_{y1} = 0.00045 rad/m, φ_{y2} = 0.00090 rad/m, φ_{m1} = 0.00326 rad/m. Substituting these values into Eq. 5, and solving gives: φ_{m2} = 0.00472 rad/m. The actual value was 0.00456 rad/m.

Conclusions

The plastic curvatures were observed to vary relatively linearly from a maximum value at the base of the wall, over about twice the length of the traditional plastic hinge length assuming the maximum plastic curvature is uniform.

The analyses indicate that longer walls have proportionally larger tension strains at the same curvature. Strain hardening of the reinforcement due to this larger strain causes the bending moment resistance of the wall to increase, which results in an increase in the height over which the bending moment exceeds the yield moment. This in turn causes an increase in height of linearly varying plastic curvatures, and is an explanation of why the plastic hinge length of a wall increases with wall length.
Axial compression reduces the length of linearly varying plastic curvatures, while axial tension increases the length of plastic curvatures. The reason for this is the shape of the bending moment – curvature relationship. When a wall is subjected to axial tension, the ratio of maximum moment at the curvature capacity of the wall to the yield moment of the wall is significantly increased, while when a wall is subjected to axial compression, the ratio decreases.

For cases where there was significant diagonal cracking, the length of linearly varying plastic curvatures significantly exceeded the length over which the bending moment exceeded the yield moment. The reason is the additional tension force demand on vertical reinforcement due to shear in a diagonally cracked member. A simple model was presented to account for this effect.

The influence of two different length concrete walls being interconnected by rigid floor slabs at every floor level was investigated. As expected, the more flexible wall did not have a significant influence on the deformations of the longer, less flexible wall; but the longer wall did have a significant influence on the more flexible wall. A simple expression was developed for estimating the maximum curvature demand in the more flexible wall assuming the flexural slopes of the walls are equal immediately above the region of plastic curvatures.

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