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SHEAR MODULUS OF BABOLSAR SAND: MEASUREMENT AND PARAMETER EFFECTS

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ABSTRACT

In this study, the effect of various parameters, especially the excited frequency on the shear modulus of Babolsar sand, which is identified as the standard soil for investigation in Iran, is studied. A series of cyclic strain-controlled simple shear tests with an approximately sinusoidal shape of cyclic loading are performed on the uniform sand. The specimens were cylindrical, 7cm in diameter and 2.3cm in height. The apparatus used in this study is the NGI type cyclic simple shear device. The tests were conducted under constant-volume method equivalent to undrained conditions. The cyclic shear strain amplitudes during the tests were 1.0% and 1.5%. Frequency and the vertical effective consolidation stress varied from test to test.

The variables in this study are cyclic strain amplitude (γ), vertical effective consolidation stress (σ'_{vc}), relative density (Dr) and excitation frequency (f). The shear modulus for all tests was calculated from the hysteresis loop. The shear modulus (G) approximated from the tests was compared to those estimated by the equations proposed by other researchers. Test results indicate that for a given cyclic strain amplitude, G increases with the vertical effective consolidation stress and relative density, but it decreases with the cyclic strain amplitude. The effect of the cyclic strain amplitude becomes smaller if the vertical effective consolidation stress and relative density increase. According to the test results, the shear modulus increases with the increase in the frequency. The rate of change of the shear modulus with the frequency is very low and can be ignored especially at low vertical consolidation stress. This effect is independent of Dr but it becomes smaller if the shear strain amplitude decreases.

Introduction

Reliable and accurate damping ratio and shear modulus of soils are necessary for the solution of many soil dynamics problems such as vibration of machine foundation, response at soil deposits and earth structures to earthquake loads, etc. In these problems, typically involving cyclic loading, the stress- strain behavior of soils is described by the cyclic stress-strain loops (Kramer, 1996).

This comprehensive general stress-strain relation for soils would be very complex simply because of a large number of factors that contribute to the shear modulus and damping of soils (Hardin and Black (1968)).

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These parameters can be grouped into three categories: very important, less important and relatively unimportant. In cohesionless soils, there are four parameters that, in general, are very important, e.g., strain amplitude, effective confining pressure, void ratio and number of cycles of loading (Hardin and Drnevich, 1972). The degree of saturation has been reported as an important influencing parameter only for modulus of cohesive soils (Hardin and Drnevich 1972). Most of the published literature concentrates on sands and clays. Seed and Idriss (1970, 1986) have summarized the available data on dynamic shear modulus and damping ratios for sands and saturated clays.

Numerous dynamic laboratory studies have been conducted over the past 35 years to evaluate the dynamic properties of soils. The vast majority of this work has been directed towards the shear modulus measurements because of the importance of this parameter in determining the response of soils to all types of dynamic loads. Studies and state-of-the-art reviews such as those presented by Hardin and Drnevich (1972), Kokusho (1980), Ishibashi and Zhang (1993), Ishihara (1996), Lanzo et al (1997) and Stokoe (1999) are good examples.

In the past, the shear modulus and damping ratio of soils at the small strain have been investigated in the laboratory with the high-frequency resonant column test (Woods, 1994) while the cyclic shear tests, such as cyclic triaxial, cyclic torsional shear, and cyclic simple shear have been used to investigate the properties at large cyclic strain amplitudes.

The cyclic tests conducted by various researchers on sand have shown that increasing confining pressure and relative density increases shear modulus but increase in the strain amplitude decreases the shear modulus (Kokusho, (1980), Ishibashi (1993), Lanzo (1997), Stokoe (1999) etc). An increase of the stiffness for an increase of the strain rate is generally recognized. Tatsuoka and Shibuya (1992) consider that, at small strains, the soil behaviour is essentially elastic and hence rate independent. Therefore, the same elastic initial tangent modulus (or small strain stiffness) is expected from dynamic and static (cyclic or monotonic) tests. At larger strains, when the influence of strain rate on the stress-strain curve becomes more and more important, rate effects can makes the soil non-linearity moving up the elastic limit. Therefore, the stress-strain relationship, in a typical quasi-static test performed at constant frequency, can be distorted, as experimentally observed by Isenhower and Stokoe (1981). According to results of studies taken previously, G is independent of frequency in the approximated range $\gamma_c \leq 0.002\%$, so these tests are conducted at medium to large strains (Stokoe, 1999).

This paper presents the result of a series of constant-volume equivalent-undrained simple shear cyclic tests. The purpose of this study is to investigate the effects of cyclic shear strain amplitude, effective vertical consolidation stress, σ'_{vc}, relative density, Dr and especially frequency, f, on the shear modulus of Babolsar sand.

Soil Tested

The soil tested in this study is Babolsar Sand, and due to a lack of a standard soil in the country to conduct various tests, has been described as standard Iranian sand by researchers of Sharif University of Technology (Khosravi, 2005). This natural and uniform sand was provided from southern shores of the Caspian Sea near Babolsar City. This sand is classified as poorly graded sand, SP, according to the Unified Soil Classification System. The basic physical properties and classification characteristics of the soil testes are listed in Table 1. The grain-size distribution curve of this sand is shown in Fig. 1.

Testing Apparatus and Procedure

The testing apparatus used in this research is the cyclic simple shear device of NGI type. As shown in Fig. 2 the specimens tested in the device are cylindrical, 0.70 cm in diameter and 2.30 cm high.
At the beginning of any test, the soil specimen was consolidated by applying loads to the loading rod. After consolidating, an equivalent undrained shear test was performed by applying a horizontal shear strain to the specimen at a predetermined rate. The specimen has a rubber membrane placed and secured with

"O" rings. To maintain a constant diameter throughout the test, the sample is supported by a series of rings. During shear, the rings slide across each other.

Table 1. Physical Properties and Classification Characteristics of Babolsar Sand.

Parameters	**Values**	**Unit**
G_s	2.724	-
e_{max}	0.76	-
e_{min}	0.54	-
$\gamma_{\mathbf{max}}$	17.03	kN/m^2
$\gamma_{\mathbf{min}}$	14.80	kN/m^2
D_{50}	0.23	mm

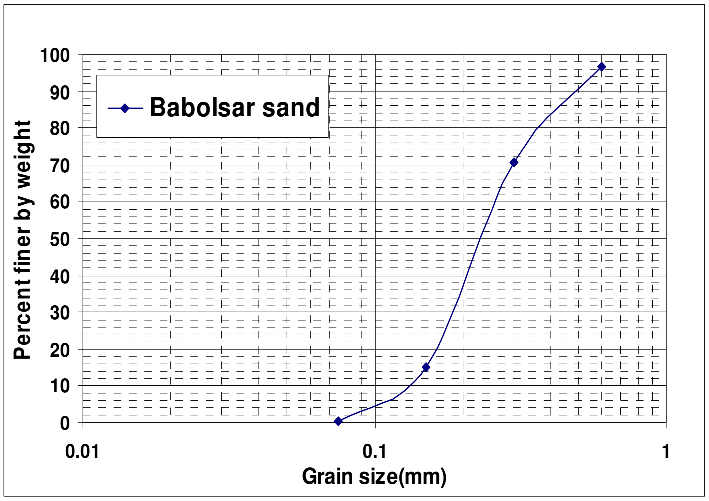

Figure 1. Grain Size Distribution Curve

During the shearing stage of the test, the specimen was drained and test conditions were selected such that the pore pressures in the specimen were zero throughout the tests. The vertical height of the sample is maintained at a constant height by the vertical actuator in a closed control loop with the vertical displacement transducer. The rings maintain a constant sample diameter. During shear only the length of the sample side changes. The test is undrained so we maintain a constant volume. The constant-volume test is equivalent to an undrained test and the change of applied vertical stress on the specimen is equivalent to the change in pore pressure would have occurred in the specimen if the specimen had been prevented from draining for a condition of constant apply vertical stress. Therefore, the results presented in this paper are applicable to undrained conditions (Bjerrum et al. 1966).

Testing Program

Several tests were conducted during this study on Babolsar sand, which was collected from different sites in the North of Iran. The samples tested in this study were distributed and prepared by moist placement method (wet tamping) for desired density.

The influencing parameters that were investigated in the consecutive cyclic strain-controlled tests were, the effective vertical consolidate stress, σ'_v, the level of constant cyclic shear-strain amplitude, γ_c, relative density, Dr, frequency, f, and the number of cycles, N.

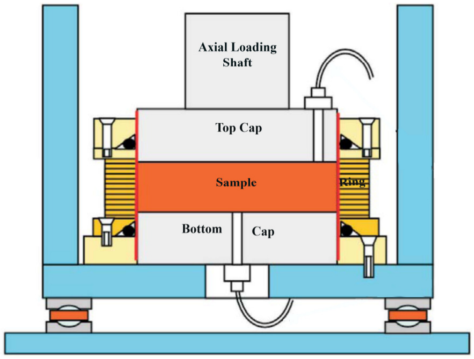

Figure 2. NGI-type direct simple shear device at Soil Dynamics Laboratory of the Sharif University.

Specimens were tested at two effective vertical consolidate stresses, 50 and 150 kPa. The shear strain amplitudes γ_c were 1% and 1.5% and the relative density was 30% and 70%. During shear tests, the frequencies used were 0.1, 0.5 and 1.0 Hz. The number of cycles was varied from 50 to 1000 cycles based on the specimen specifications. The testing program is summarized in Table 2. It should be mentioned that the shear modulus is identified as the slope of a line through the end points of the hystresis loop as can be seen in Fig. 3.

Finally, maximum shear modulus (G_{max}) was estimated from the equation proposed by Seed and Idriss (1970) and the normalized shear modulus approximated from the tests were compared to those estimated by Ishibashi and Zhang (1993).

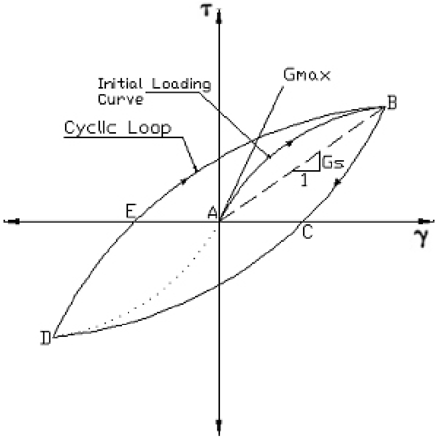

Figure 3. Calculation of shear modulus from hysteresis loop (Ishihara, 1996).

Test Results

In this study, the values of shear modulus, G, at different cycles, for all the specimens are calculated and the effects of the number of cycles, the cyclic strain amplitude (γ), vertical effective consolidation stress (σ'_{vc}), relative density (Dr) and frequency, f, on the shear modulus (G) were investigated. Maximum shear modulus (G_{max}) was estimated from the equation proposed by Seed and Idriss (1970) and normalized shear modulus approximated from the tests were compared to those estimated by Ishibashi and Zhang (1993).

Table 2. Summary of Testing Program.

σ'$_{vc}$ (kPa)	f (Hz)	e	Dr (%)	γ_c %	Test No
50	0.1	0. 6964	29.0	1.0	1
50	0.5	0.6918	31.0	1.0	2
50	1.0	0.6916	31.1	1.0	3
150	0.1	0.6676	42.0	1.0	4
150	0.5	0.6786	37.0	1.0	5
150	1.0	0.6841	34.5	1.0	6
50	0.1	0.6962	29.0	1.5	7
50	0.5	0.6931	30.4	1.5	8
50	1.0	0.6964	28.9	1.5	9
150	0.1	0.6977	28.3	1.5	10
150	0.5	0.6852	34.0	1.5	11
150	1.0	0.6896	32.0	1.5	12
50	0.1	0.6082	69.0	1.0	13
50	0.5	0.6012	72.2	1.0	14
50	1.0	0.5998	72.8	1.0	15
150	0.1	0.5994	73.0	1.0	16
150	0.5	0.5895	77.5	1.0	17
150	1.0	0.6022	71.7	1.0	18

Evaluations of Maximum Shear Modulus, G_{max}

The value of G_{max} was estimated by the relation developed by Seed and Idriss (1970):

$$G_{\max} = 1000\, K_{2\max}\, (\sigma'_m)^{0.5} \tag{1}$$

In Eq. 1, σ'$_m$ is the mean effective consolidation stress and equal to $(\sigma'_v + 2\sigma'_h)/3$, where σ'$_v$ is the vertical effective consolidation stress, σ'$_h$ is the horizontal effective consolidation stress and K_{2max} is an empirical factor. In this equation, the units of G_{max} and σ'$_m$ are pounds per square feet. The mean effective stress, σ'$_m$, was estimated by assuming that σ'$_h$ = K_0 σ'$_v$, where K_0 is the coefficient of earth pressure at rest. The values of K_{2max} vary according to the void ratio as presented in Table 3.
The values of shear modulus, G, at 10th cycles, of all the specimens tested and G/G_{max} approximated from the tests are presented in Table 4.

Evaluations of Shear Modulus by the relation proposed by Ishibashi and Zhang (1993)

Ishibashi and Zhang (1993) developed an expression for estimating the G/G_{max} ratio. The two main variables are PI (plasticity index) and confining pressure. The expression was developed for the confining pressure in kPa.

Table 3. Values of K_{2max} (Seed and Idriss 1970).

e	K_{2max}
0.4	70
0.5	60
0.6	51
0.7	44
0.8	39

$$\frac{G}{G_{\max}} = K(\gamma, PI)\,(\sigma'_m)^{m(\gamma, PI)-m_0} \tag{2}$$

$$K(\gamma, PI) = 0.5\left\{1 + \tanh\left[\ln\left[\frac{0.000102 + n(PI)}{\gamma}\right]^{0.492}\right]\right\}$$

$$m(\gamma, PI) - m_0 = 0.272\left\{1 - \tanh\left[\ln\left[\frac{0.000556}{\gamma}\right]^{0.4}\right]\right\}\exp\left[-0.0145\,PI^{1.3}\right]$$

$$n(PI) = \begin{cases} 0.0 & \text{for PI} = 0 \\ 3.37\times10^{-6}\ \text{PI}^{1.404} & \text{for } 0 < \text{PI} \le 15 \\ 7.0\times10^{-7}\ \text{PI}^{1.976} & \text{for } 15 < \text{PI} \le 70 \\ 2.7\times10^{-5}\ \text{PI}^{1.115} & \text{for PI} > 70 \end{cases}$$

As mentioned before, the mean effective stress, σ'_m, was estimated by assuming that $\sigma'_h = K_0\,\sigma'_v$, where K_0 is the coefficient of earth pressure at rest. The values of normalized shear modulus estimated by Ishibashi and Zhang (1993) can be seen in Table 5. It can be clearly seen in Tables 4 (column 8) and 5 that the normalized shear modulus values approximated from the test are in a relatively good agreement with those estimated by Ishibashi and Zhang (1993).

Effect of various parameters on shear modulus of Babolsar sand

A summary of the test results, in which samples were consolidated to the vertical stresses varying from 50 to 150 kPa and subjected undrained to cyclic shear stress, is shown in Figs. 4 and 6. When a soil is subjected to cyclic loads under undrained strain-controlled conditions, shear modulus identified as the slope of a line through the end points of the hystresis loop decreases with increase in the number of cycles due to structural changes and building pore water pressure. It can clearly be seen in Figs. 4 and 5. The test results showed that the shear modulus values obtained at the 4th and 20th cycles differ between 5 to 15% according to the test conditions. Therefore this parameter can be grouped as a relatively important parameter. According to the test results, the vertical confining stress has no significant effect on the influence of number of cycles.

As shown in Fig. 6, the influence of the number of cycles on this parameter, G, becomes more relevant as the shear strain amplitude increases. Influence of the vertical consolidation stress on the shear modulus of Babolsar sand was also investigated.

Table 4. Values of normalized shear modulus approximated from test results.

No	Dr (%)	e	K_{2max}*	σ'$_v$ (kPa)	G_{max} (kPa)	G (kPa)	G/G_{max}
1	21.0	0.714	43.310	50	55251.41	3313.14	0.0599648
2	31.0	0.692	44.574	50	56863.92	3294.90	0.0579436
3	31.1	0.692	44.589	50	56883.57	3313.70	0.0582541
4	42.0	0.668	46.268	150	102234.29	6976.70	0.0682423
5	37.0	0.679	45.498	150	100532.89	6839.00	0.0680275
6	34.5	0.684	45.113	150	99682.18	6732.30	0.0675376
7	29.0	0.696	44.266	50	56471.00	1823.70	0.0322945
8	30.4	0.693	44.482	50	56746.05	2074.90	0.0365647
9	28.9	0.696	44.251	50	56451.36	1748.00	0.0309647
10	28.3	0.698	44.158	150	97572.45	5729.90	0.0587246
11	34.0	0.685	45.036	150	99512.04	6612.80	0.0664523
12	32.0	0.690	44.728	150	98831.48	6430.60	0.0650663
13	69.0	0.608	50.426	50	64329.43	3751.90	0.0583232
14	72.2	0.601	50.919	50	64958.11	3422.70	0.0526909
15	72.8	0.600	51.014	50	65080.07	3314.40	0.0509280
16	73.0	0.599	51.054	150	112809.48	7829.10	0.0694011
17	77.5	0.590	51.945	150	114778.25	7665.00	0.0667809
18	71.7	0.602	50.842	150	112340.60	7340.60	0.0653424

* These values are estimated by interpolation according to void ratio

Table 5. Values of normalized shear modulus estimated by Ishibashi and Zhang (1993).

No	γ_c %	σ'$_v$ (kPa)	σ'$_m$ (kPa)	G/G_{max}
1	0.010	50	33.33	0.061585
2	0.010	150	100.00	0.106078
3	0.015	50	33.33	0.043357
4	0.015	150	100.00	0.075729

According to the test results, the increase in the vertical consolidation stress acts to increase the shear modulus at the given strain level as illustrated in Fig. 4 and varies by the test results (Fig. 6). This fact can be explained by considering the dependency of the vertical consolidation stress on the strength τ_f and the initial shear modulus, G_0. It can be seen that as σ'$_v$ increases, shear modulus increases.

As can be seen in Fig. 5, the shear modulus at the given vertical stress tends to decrease with the decrease in the relative density due to increase in the void ratio and decrease in the soil shear strength. Accordingly, the vertical consolidation stress and the relative density can be classified as very important and less important parameters respectively as have been reported by other researchers.

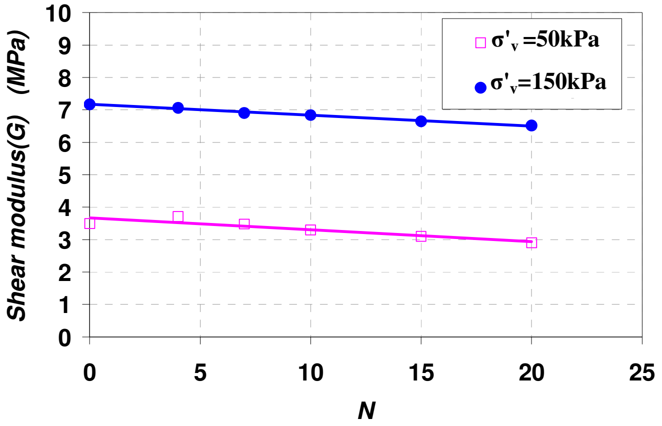

Figure 4. Effect of number of cycles and σ'$_v$.

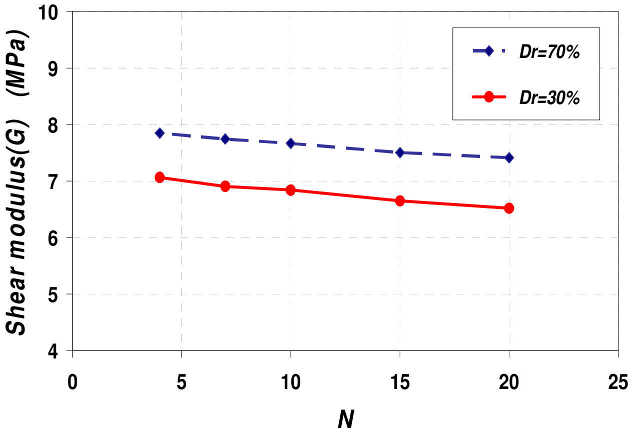

Figure 5. Effect of number of cycles and Dr on shear modulus of Babolsar sand (f = 1 Hz).

Shear modulus decreases with growing γ_c. The effect of the cyclic shear strain amplitude depends upon several factors such as the effective vertical stress and the number of cycles. It is apparent from Fig. 6. This figure indicates that the rate of reduction in shear modulus with strain becomes greater as the vertical consolidation stress decreases.

As shown in figs. 7 and 8, the variation of shear modulus G with loading frequency. The data confirm that for sands the shear modulus significantly affected by σ'$_v$ and Dr as previously indicated. It can be also seen that for a given γ_c and σ'$_v$, G increases with loading frequency and the effect of frequency relatively independent of Dr but it increases as γ_c increases. As shown, the rate of change of shear modulus with frequency is between 2% and 6% that confirm that the effect of frequency on shear modulus of Babolsrar sand is small.

Conclusions

This paper summarizes the results of a laboratory study of shear modulus of Babolsar sand soil that is identified as the standard soil for investigation in Iran. In order to determine this parameter, a series of cyclic simple shear tests using NGI type cyclic simple shear device under cyclic shear strain 1.0% and 1.5%, were carried out. The tests were conducted under constant-volume method that is equivalent to undrained cycle conditions. The effect of the number of loading cycles, vertical effective consolidation stresses, relative density, cyclic shear strain amplitude and loading frequency were investigated. On the basis of the experimental results shown previously, it is possible to draw the following conclusion.

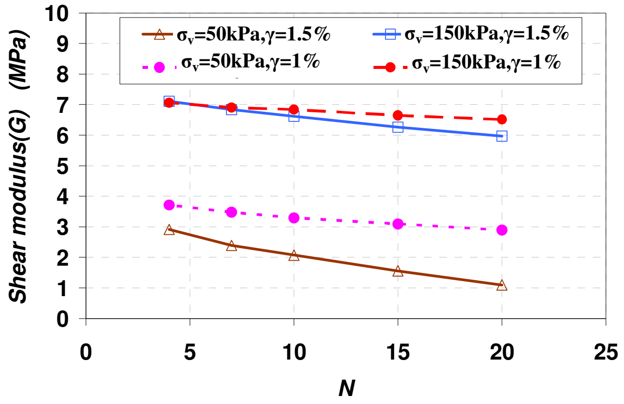

Figure 6. Effect of Shear strain amplitude and σ'_v on shear modulus of Babolsar sand (f= 0.5 Hz).

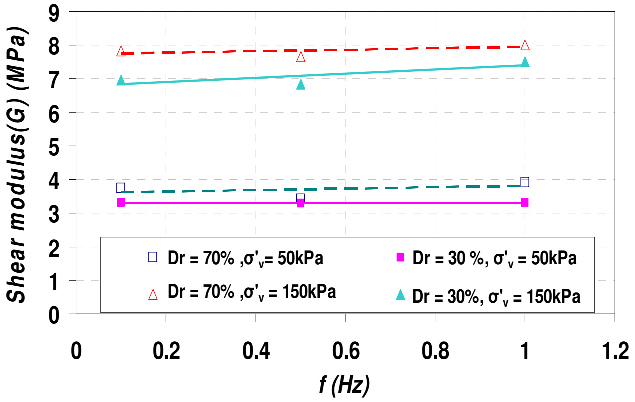

Figure 7. Effect of frequency on shear modulus of Babolsar sand (Dr=30%).

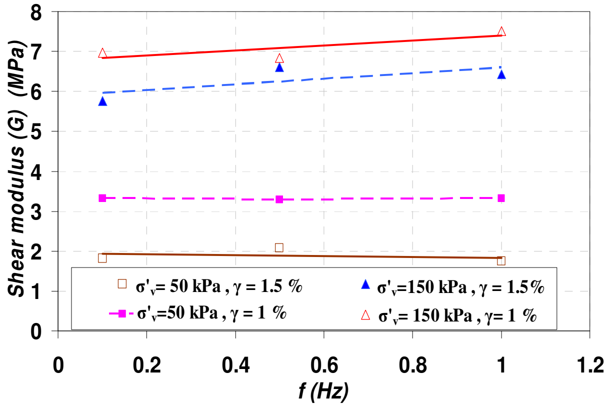

Figure 8. Effect of frequency on shear modulus of Babolsar sand (γ_C=1.0%).

The shear modulus decreases with an increase in the number of cycles due to structural changes and building pore water pressure. This effect becomes more relevant as the shear strain amplitude increases but the vertical confining stress has no significant effect on the influence of the number of cycles on this parameter.

The increase in the vertical consolidation stress acts to increase the shear modulus at the given strain level. The relative density has the same influence on the shear modulus of Babolsar sand.

The shear modulus at the given vertical stress tends to decrease with the decrease in the relative density.

Shear modulus decreases with growing γ_c. The effect of the cyclic shear strain amplitude depends upon several factors such as effective vertical stress and number of cycles and the rate of reduction in shear modulus with the strain becomes greater as the vertical consolidation stress decreases.

According to test results, shear modulus increases with the increase in the frequency. The rate of change of shear modulus with the frequency is very low and can be ignored especially at low vertical consolidation stress. This effect is independent of Dr but it becomes smaller as the shear strain amplitude decreases.

Therefore, the parameters such as vertical consolidation stress, relative density and cyclic shear strain amplitude are grouped as important to very important parameters. In contrast the ground motion parameters such as the number of cycles and frequency are classified as the relatively important to less important parameters as have reported by other researches.

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