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GENERALIZED SDF SYSTEM FOR ANALYSIS OF CONCRETE RECTANGULAR LIQUID STORAGE TANKS

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ABSTRACT

This paper presents a simplified method using the generalized single degree of freedom (SDF) system for seismic analysis and design of concrete rectangular liquid storage tanks. In most of the current design codes and standards for concrete liquid storage tanks, the hydrodynamic pressures are determined assuming that the tank wall is rigid. However, it has been shown that the flexibility of the tank wall increases the hydrodynamic pressure calculated, based on the rigid wall boundary condition. For the proposed method, the consistent mass and the effect of the flexibility of the tank wall by hydrodynamic pressures are considered. Five prescribed vibration shape functions representing the first mode shape for the cantilever wall boundary condition are studied. A case study for a tall tank is presented and compared with that using the finite element method from previous investigations. It is concluded that the flexibility of the tank wall should be considered in the dynamic analysis of concrete rectangular liquid storage tanks. Also, the proposed method presents accurate results and can be used in the structural design for liquid containing structures.

Introduction

Liquid storage tanks, as part of environmental engineering facilities, are primarily used for water and sewage treatment plants and other industrial wastes. Normally, they are constructed of reinforced concrete in the form of rectangular or circular configurations. Early investigations of dynamic response of liquid storage tanks subjected to earthquake was conducted by Housner (1963). An approximate method was proposed to include the effect of hydrodynamic pressures for a two fold-symmetric-fluid container subjected to horizontal acceleration. The fluid response was represented for impulsive and convective components. The fluid was assumed to be incompressible and the container was assumed to have rigid walls. Yang and Veletsos (1976) used Flűggle's shell theory to analyze circular tanks. It was found that for tanks with realistic flexibility, the impulsive forces are considerably higher than those in rigid wall. Veletsos et al. (1984) considered the effect of the wall flexibility on the magnitude and distribution of the hydrodynamic pressures and associated tank forces. They assumed that the tank-fluid system behaved like a single degree of freedom system and the base shear and moment were evaluated for several prescribed modes of vibration. Most of the research conducted on liquid storage tanks, as mentioned above have been of circular configurations made of structural steel. For rectangular tanks, Haroun (1984) presented a very detailed method of analysis in the typical system of loadings. The hydrodynamic

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pressures were calculated by classical potential flow approach. The formula of hydrodynamic pressures only considered the rigid wall condition. Park et al. (1990) studied the dynamic behaviour of rectangular tanks using boundary element modeling for the fluid motion and finite element modeling for the solid walls. The time history analysis method was used to obtain the dynamic response of fluid storage tanks subjected earthquakes. Subsequently, they presented an analytical method for calculation of the hydrodynamic pressures based on three-dimensional analysis of tanks. They applied Rayleigh-Ritz method using assumed vibration modes of rectangular plate with boundary conditions as admissible functions (Kim et al., 1996).

Chen and Kianoush (2005) developed a procedure referred to as the sequential method for computing hydrodynamic pressures based on a two-dimensional model for rectangular tanks in which the effect of flexibility of tank wall was taken into consideration. Also, Kianoush and Chen (2006) investigated the response of concrete rectangular liquid storage tanks subjected to vertical ground acceleration in which the importance of the vertical component of ground motion on the overall seismic behavior of liquid storage tanks was evaluated. It was concluded that the response of tank wall due to vertical ground acceleration can be significant and should be considered in the design. Kianoush et al. (2006) and Ghaemian et al. (2005) applied the staggered method to solve the coupled liquid storage tank problems in three-dimensional space. The staggered method is general and applicable on any shape of storage tanks and taken into account both convective and impulsive components. Also, the seismic excitation can be applied in any direction to the system. It is worth noting that both the sequential method and the staggered method adopt the sequential analysis algorithm which means the two domains transfer the data between each other through a sequential procedure.

As part of the ongoing research effort, in this paper a simplified method using the generalized SDF system is proposed to study the dynamic response of liquid storage tanks. The consistent mass and the effect of flexibility of tank wall on impulsive hydrodynamic pressure are considered using the prescribed shape functions. A case study for a tall rectangular tank is presented to illustrate the application of the proposed method.

Generalized SDF System

Fig. 1(a) shows a 3-D rectangular tank. It is assumed that the liquid storage tank is fixed to the rigid foundation. A Cartesian coordinate system (x, y, z) is used with the origin located at the center of the tank base. Furthermore, it is assumed that the width of tank $2L_z$ is sufficiently large so that the unit width of tank can represent the tank wall and the corresponding 2-D model as shown in Fig. 1(b).

Fig. 2 shows a cantilever tank wall with the distributed mass m(y) and stiffness EI(y) per unit height subjected to the earthquake ground acceleration $\ddot{u}_g(t)$. The wall exhibits an infinite number of degrees of freedom for flexural mode of response. If there are some predetermined shapes to approximate the vibration of the system, then the motion of the system can be described by a single variable, or generalized coordinate in which only one DOF exists. The system idealized in this manner is referred to as generalized SDF systems. In this study, the generalized SDF system is applied to solve the dynamic response of liquid storage tanks subjected to earthquakes. The equation of motion for a generalized SDF system is that:

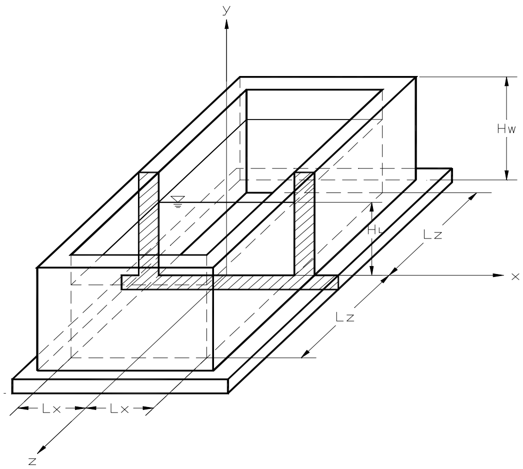
$$\tilde{m}\cdot\ddot{u}+\tilde{c}\cdot\dot{u}+\tilde{k}\cdot u=\tilde{p} \tag{1}$$

Where $\tilde{m}$, $\tilde{c}$, $\tilde{k}$, $\tilde{p}$ are defined as the generalized system of mass, damping, stiffness and force respectively.

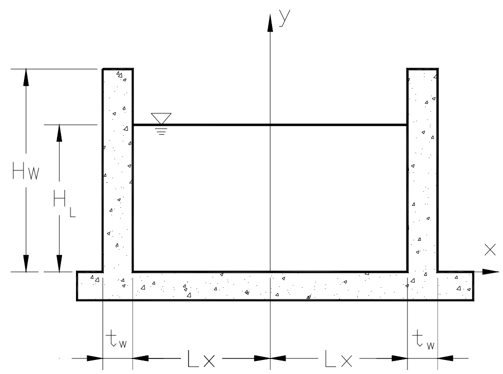
The generalized properties in Eq.1 are associated with the selected generalized displacement $u(y,t)$ as defined below:

$$u(y,t)=\psi(y)u(t) \tag{2}$$

where $u(t)$ is the defined time function related to a single generalized displacement, and $\psi(y)$ is the assumed shape function.



(a) 3 –D model of rectangular tank



(b) 2 –D model of rectangular tank

Figure 1. Schematic of rectangular tank.

When using generalized SDF system, it is critical to choose an appropriate shape function to estimate the natural frequencies of tank walls. In principle any shape function may be selected if it satisfies the displacement boundary conditions at the supports. However, a shape function that satisfies only the geometric boundary conditions does not always ensure an accurate result for the natural frequency. In this study, five shape functions are selected for analysis. They are listed below and referred to as SF1 to SF5 as follows.

$$\text{SF1(y)}= \psi 1(y)=\frac{1}{2}\frac{y^2}{H_W^2}+\frac{1}{2}\frac{y}{H_W} \tag{3}$$

$$\text{SF2(y)}=\psi 2(y)=\frac{y^2}{H_W^2} \tag{4}$$

$$\text{SF3(y)}=\psi 3(y)=\frac{3}{2}\frac{y^2}{H_W^2}-\frac{1}{2}\frac{y^3}{H_W^3} \tag{5}$$

$$\text{SF4(y)}=\psi 4(y)=1-\cos(\frac{\pi\cdot y}{2\cdot H_W}) \tag{6}$$

$$\text{SF5(y)}=\psi 5(y)=\sin(\frac{\pi\cdot y}{2\cdot H_W}) \tag{7}$$

Fig. 3 shows the normalized height versus normalized deformation based on the above shape functions. Shape functions SF3 and SF4 approximate the cantilever wall boundary condition that it is fixed at bottom and free at top. Compared with SF3 and SF4, SF1 and SF2 represent the more flexible and more rigid tank wall condition respectively. Therefore, the shape functions SF1 and SF2 can be used to study the effect of flexibility of tank wall on dynamic response of liquid containing structures. The shape function SF5 is introduced to represent the shear dominated deformation function.

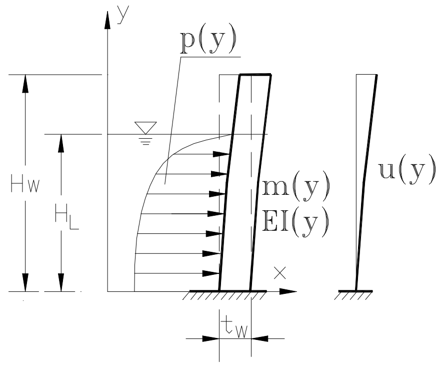


Figure 2. Concrete Rectangular Tank in Generalized SDF System.

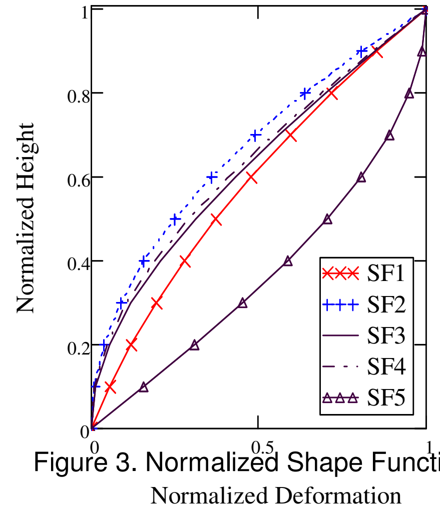


Figure 3. Normalized Shape Functions.

Hydrodynamic Pressure

The fluid filled in the rectangular tank as shown in Fig. 2 is of height, H_L above the base. The fluid is considered to be ideal, which is incompressible, inviscid, and with a mass density ρ_l. The response of the body of fluid to an earthquake can be treated as gravity waves on its free surface, which is irrotational in most instances.

The hydrodynamic pressure is analyzed using the velocity potential method, which satisfies the boundary conditions. This can be solved by the method of separation of variables introduced by Currie (1973). The hydrodynamic pressure distribution on the flexible wall related to the velocity potential can be expressed as:

$$p = \sum_{i=1}^{\infty} \frac{2 \cdot \rho_l \cdot \tanh(\lambda_i \cdot L_x)}{\lambda_i \cdot H_L} \cdot \cos(\lambda_i \cdot y) \cdot \int_0^{H_L} \cos(\lambda_i \cdot y) \cdot \ddot{u}(t) dy \tag{8}$$

Where $\lambda_i = (2i-1)\pi/2H_L$. The detailed derivation of the above equation is discussed by Chen (2003). As the series in the above equation convergence very fast, only the first term of the series may be used for practical application.

For the rigid tank $\ddot{u}(t) = \ddot{u}_g(t)$ which means that the acceleration along the height of the wall is the same as the ground acceleration, then Eq.(8) becomes:

$$p_{rigid} = \sum_{i=1}^{\infty} \frac{2 \cdot (-1)^i \cdot \rho_l}{\lambda_i^2 \cdot H_L} \tanh(\lambda_i \cdot L_x) \cdot \cos(\lambda_i \cdot y) \cdot \ddot{u}_g(t) \tag{9}$$

The above equation is the same as that derived by Haroun (1984) for the rigid wall boundary condition.

Coupling Analysis

For the coupling analysis between the structure and the contained liquid, the direct coupling method is used in the analysis. This means the responses of liquid and structure can be directly solved using the equation of motion.

For the liquid containing structural system, the generalized system of mass, stiffness and force in terms of the generalized coordinate and assumed shape function can be obtained as following:

$$\tilde{m}=\int_0^{H_W} m(y)\cdot[\psi(y)]^2\cdot dy+\int_0^{H_L} p_1(y)\cdot[\psi(y)]^2\cdot dy \tag{10}$$

$$\tilde{k}=\int_0^{H_W} EI(y)\cdot[\ddot{\psi}(y)]^2\cdot dy \tag{11}$$

$$\tilde{p}=u_g(t)\cdot\hat{p}=u_g(t)\cdot\left(\int_0^{H_W} m(y)\cdot\psi(y)\cdot dy+\int_0^{H_L} p_1(y)\cdot\psi(y)\cdot dy\right) \tag{12}$$

It is worth noting that the inertial mass of tank wall is considered as consistent mass rather than lumped mass in the generalized SDF system. In addition, $p_1(y)$ is the distribution function for impulsive hydrodynamic pressure which can be expressed as that:

$$p_1(y)=\sum_{n=1}^{\infty}\frac{2\cdot\rho_l}{\lambda_{i,n}\cdot H_L}\tanh(\lambda_{i,n}L_x)\cos(\lambda_{i,n}y)\int_0^{H_L}\cos(\lambda_{i,n}y)\cdot dy \tag{13}$$

The above equation is similar to the inertial mass of tank wall and can be treated as the added mass of hydrodynamic pressure in the liquid containing system. Therefore, the generalized mass in Eq.10 is separated into two parts; the inertial mass of tank wall m_w and the effective added mass for impulsive component of hydrodynamic pressure m_L.

Therefore, the equation of motion for coupling the structure and the contained liquid subjected to earthquake is obtained by substituting the Eq.10 to Eq.12 into Eq.1. Then by dividing both sides of equation by $\tilde{m}$, the following relationship is obtained:

$$\ddot{u}+2\cdot\zeta\cdot\omega_n\cdot\dot{u}+\omega_n^2\cdot u=-\hat{q}\cdot u_g(t) \tag{14}$$

Where $\omega_n=\sqrt{\tilde{k}/\tilde{m}}$ is the natural frequency of liquid containing structural system and $\hat{q}=\tilde{p}/\tilde{m}$ is the factor of external load applied. If an estimated damping ratio ζ is assumed, all the unknown parameters i.e. $u,\dot{u},\ddot{u}$ can be determined by one assumed shape function. Therefore the infinite degrees of freedom system for liquid containing structures can be simplified to a generalized SDF system.

It is worth noting that the model using the generalized SDF system in this study is not the same as the Housner's model (Housner, 1963) based on SDF system. In Housner's model, the entire inertial mass is lumped at an equivalent height above the base of the tank wall. In this paper, one generalized coordinate is used to approximate the vibration mode. As a result, the predefined shape function can reduce the infinite degrees of freedom system into a SDF system. The efficiency of the generalized SDF system used for dynamic response of liquid containing structures is presented in the following case study.

Analysis of a Rectangular Tank

To demonstrate the efficiency of generalized SDF system for dynamic analysis of liquid containing structures, a tall tank that was studied previously (Chen and Kianoush (2005) and Kianoush and Chen (2006)) is used in this study. Both empty as well as full tank is considered. The dimensions and the properties of the tank are as following:

ρ_w = 2300 kg/m^3 ρ_l = 1000 kg/m^3 E = 20776MPa   v = 0.17
H_w =12.3 m H_L =11.2 m L_x = 9.8 m t_w = 1.2 m L_z = 28 m

In a previous study (Chen and Kianoush (2005)), six models were presented using the finite element method (FEM). The mode superposition method was used in Model 4 in which the distributed added mass of hydrodynamic pressure was considered. The distribution function for the added mass of hydrodynamic pressure was calculated based on the rigid wall boundary condition. In Model 5, the time history analysis method including the sequential procedure was used. The effect of flexibility of tank wall for dynamic response for both the tank wall and hydrodynamic pressure was considered. As Models 4 and 5 presented the most accurate results in previous study, those results are shown in Table 1 for comparison with those obtained from the generalized SDF system which are described subsequently.

A summary of results using the generalized SDF system is presented in Table 1. The generalized mass of tank wall m_W for the first mode based on the selected shape functions is presented and compared to the total mass of tank wall M_W. It can be seen that except shape function SF5, the mass percentages obtained from the shape functions SF1 to SF4 are in the range of 20% to 26%. As expected, because there are infinite degrees of freedom for the tank wall, the participation of generalized mass for the first mode using the consistent mass is less than that using the lumped mass based on the rigid wall boundary condition. As the first mode represents the most critical mode for dynamic analysis, the analysis from the first mode in this study shows sufficiently accurate results compared to the previous results obtained using the FEM.

Table 1. Summary of Dynamic Response of Tall Tank.

Items		SF1	SF2	SF3	SF4	SF5	Model 4*	Model 5*
Empty	m_W (10^3kg)	8.77	6.79	8.00	7.70	16.97	-	-
	% of M_W	25.8	20.0	23.6	22.7	50.0	-	-
	K_W(kN/m)	1608	6431	4823	4894	4894	-	-
	T1 (sec)	0.464	0.204	0.256	0.249	0.370	0.262	-
	A_a (m/sec^2)	0.837g	0.647g	0.840g	0.831g	0.674g	-	-
	d_{max} (mm)	72.2	11.2	21.7	20.5	29.2	21.8	17.2
	V_B (kN)	187.3	119.6	166.7	161.0	181.8	167.3	153
Full	m_L (10^3kg)	7.13	4.21	5.70	5.27	20.23	-	-
	% of M_L	11.9	7.0	9.5	8.8	33.8	-	-
	T1 (sec)	0.625	0.260	0.335	0.323	0.548	0.341	-
	A_a(m/sec^2)	0.629g	0.845g	0.633g	0.665g	0.756g	-	-
	d_{max} (mm)	117.7	29.0	33.7	33.4	78.7	32.9	26.9
	V_B (kN)	365.1	380.6	310.5	316.3	538.1	314.8	338.1
	P_i (kN)	196.7	189.2	159.7	160.8	314.6	-	-

* Chen and Kianoush (2005)

For the full tank, the generalized mass based on the first mode shape function representing the effective added mass for impulsive component of hydrodynamic pressure m_L is calculated. In addition, it is assumed that the generalized mass based on the rigid wall boundary condition M_L represents the total effective added mass resulting in hydrodynamic pressure in the liquid containing structural system. A special shape function ψ(y)=1 can be applied to evaluate the rigid wall boundary condition. Accordingly, the total effective added mass M_L for impulsive component of hydrodynamic pressure is 59.86x10^3 kg. However, only part of the effective added mass for the first mode m_L participates in the dynamic analysis compared to the total effective added mass M_L. The same trends can be found in the generalized inertial mass of tank wall for the first mode shape function m_W as discussed above. The mass participation is about 23% of the total inertial mass and 9% of total effective added mass for impulsive component of hydrodynamic pressure for the shape functions SF3 and SF4. As SF1 and SF2 represent the more flexible and the more rigid walls respectively, it can be concluded that a larger portion of inertial mass of tank wall and effective added mass for hydrodynamic pressure participate in the first mode when the wall is more flexible.

The stiffness of structure is calculated using Eq.11. Based on a unit load applied at the top of the wall, the wall stiffness can also be determined using the following simple relationship:

$$\tilde{k} = \frac{E_c}{4} \cdot \left(\frac{t_W}{H_W}\right)^3 \tag{15}$$

Based on the above equation, the stiffness of tank wall is 4823 kN/m. This agrees well with the results obtained for shape functions SF3, SF4 and SF5.

Based on the previous study as discussed above, the fundamental natural frequency of empty tank is 0.262 sec as indicated in Table 1. It is shown that shape functions SF3 and SF4 provide the most accurate results in this respect. For shape functions SF1 and SF2 which represent the more flexible and the more rigid tank wall condition respectively, the actual fundamental natural frequency is expected to be between the values of these two limits.

The periods of the first mode for the full tank are 0.335 sec and 0.323 sec for shape functions SF3 and SF4 respectively. The values are similar to those obtained using the FEM.

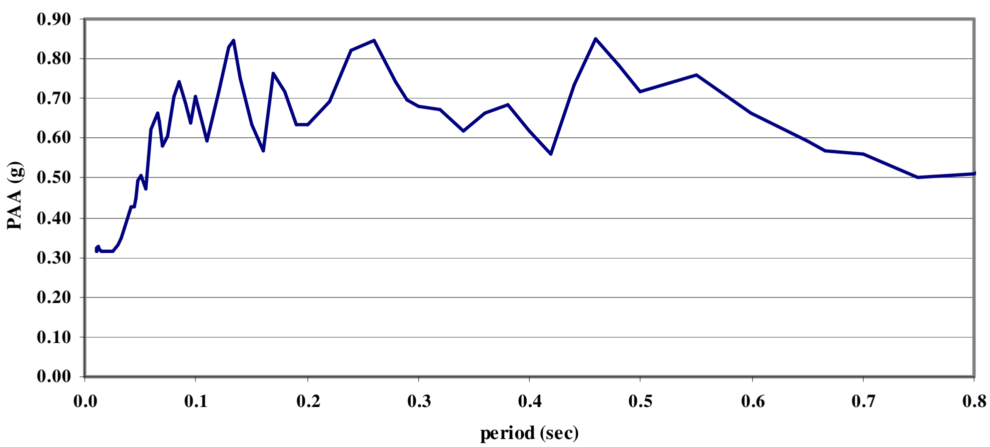


Figure 4. Response Spectrum - 1940 El Centro Earthquake.

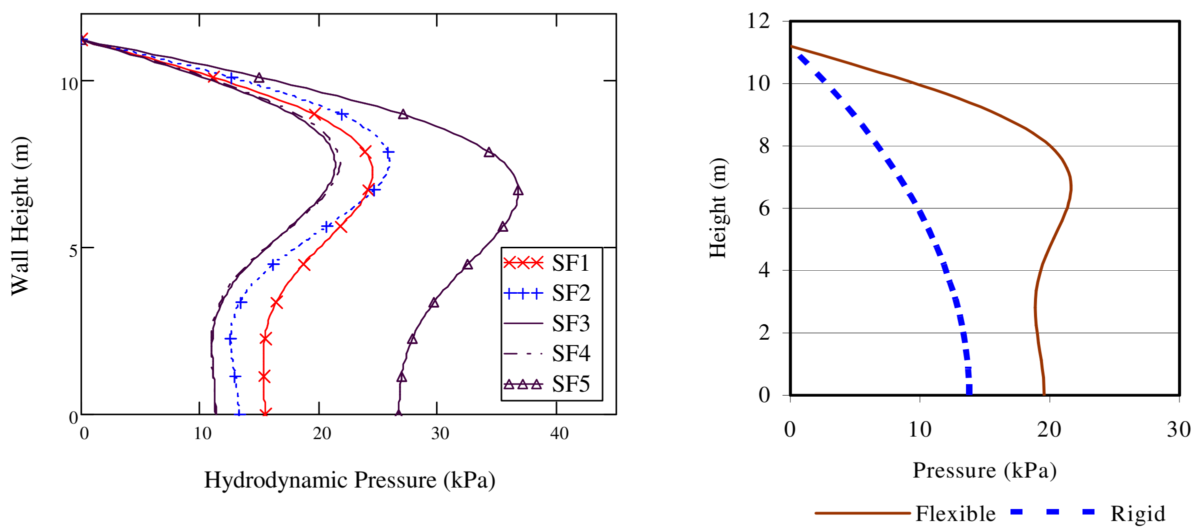
The maximum response of structure can be obtained using the pseudo-ground acceleration of the response spectrum. The El Centro 1940 Earthquake used in the previous investigation is also used in this study. Fig. 4 shows the response spectrum for such a record based on a 5% damping ratio. The pseudo-ground accelerations A_a corresponding to the periods for different shape functions are listed in Table 1. It should be noted that the actual response spectrum rather than the design response spectrum is used in this study. This is because the previous study was based on time history analysis using the El Cento record which is used as the basis for comparison.

Based on the response spectrum curve, the maximum wall displacement d_{max} and the maximum base shear V_B are calculated. For the empty tank, the maximum displacement at the top of the concrete wall is 21.8 mm and the maximum base shear is 167.3 kN using the FEM. It can be observed that the results using the shape functions SF3 and SF4 match the FEM results very well.

For the full tank, the maximum displacement at the top of the concrete wall is about 33 mm for both SF3 and SF4. This is similar to the result obtained for Model 4 in the previous study. The maximum displacements based on the previous study are 32.7 mm for Model 4 but it is 26.7 mm for Model 5. This difference in results may be attributed to the response within the small range of period in the response spectrum curves. However, it can be concluded that the shape functions SF3 and SF4 provide the most accurate results based on the maximum displacements.

The maximum base shears are 310.5 kN and 316.3 kN for SF3 and SF4 respectively. The maximum base shears from the previous study are 314.8 kN and 338.1 kN for Models 4 and 5 respectively. Again, the generalized SDF system can provide accurate results in this respect.

The hydrodynamic pressure is calculated by substituting $\hat{q}\cdot A_a$ into Eq.8 where $\hat{q}=\tilde{p}/\tilde{m}$. The total hydrodynamic pressures P_i are shown in Table 1. The distribution of hydrodynamic pressure along the height of wall is demonstrated in Fig. 5. The overall response from this study compares very well with that obtained using Model 5 in which the effect of wall flexibility was considered in the analysis. However, the lower portion of hydrodynamic pressure distribution obtained from this study is less than that of Model 5. This can also be due to the variable response in the small range of period in the response spectrum curve.



(a) Generalized SDF System (b) Model 5 (Chen and Kianoush, 2005)

Figure 5. Hydrodynamic Pressures Distribution along Height of Wall.

Conclusions

A simplified method using the generalized SDF system is presented to study the dynamic response of concrete rectangular liquid storage tanks. Five prescribed shape functions representing the first mode shape are used for analysis. The consistent mass and the effect of flexibility of tank wall on hydrodynamic pressures are considered. A tall liquid storage tank studied previously is analyzed to demonstrate the efficiency of the generalized SDF system applied for the dynamic analysis of liquid storage tanks. Comparing the results obtained using the generalized SDF system proposed in this study with those obtained using the finite element method from the previous investigation show that the proposed method can provide sufficiently accurate results. It is concluded that the proposed shape functions SF3 and SF4 are the appropriate shape functions to approximate the response of liquid storage tanks for the cantilever wall boundary condition. This study also recommends that the effect of flexibility of tank wall to be considered in the calculation of hydrodynamic pressures for concrete rectangular tanks. It is also recommended to use a design spectrum when using the generalized SDF system for dynamic analysis of liquid storage tanks.

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