Ninth Canadian Conference on Earthquake Engineering
Ottawa, Ontario, Canada
26-29 June 2007

APPLICATION OF FIBER ELEMENT IN THE ASSESSMENT OF THE CYCLIC LOADING BEHAVIOR OF RC COLUMNS

R. Sadjadi[1] and M. R. Kianoush[2]

ABSTRACT

In reinforced concrete (RC) structures, columns are the most important elements in terms of maintaining the stability of the structure. Therefore, failure of the column in the event of an earthquake is of main concern. The seismic performance and the capacity assessment of RC columns are usually performed in the form of the lateral load-lateral displacement relationship. The objective of this paper is to study the reliability of an analytical tool for predicting the lateral load-deformation response of RC columns while subjected to lateral cyclic displacements and axial load. The prediction of the analytical tool will be assessed by comparison to experimental data for columns of different cross-sectional shape and configuration. The analytical tool in this study is based on a model where the element is discretized into smaller units and the overall behavior of the element is captured in terms of the behavior of those smaller units. This concept is implemented into the program DRAIN-2DX (fiber element). The program is capable of analyzing different cross sections from rectangular to hollow circular cross section with different arrangement of reinforcing bars. In the fiber element, material nonlinearity can spread through the region which is expected to experience inelastic deformation; therefore the length of the plastic hinge zone is of main importance. The response of RC column under cyclic displacement is defined by the behavior of plain concrete, and reinforcing steel under general reversed-cyclic loading. This study only considers the behavior of columns with flexural dominant mode of failure. It is concluded that with the implementation of appropriate constitutive material models and the plastic hinge zone, the described analytical tools can predict the response of the columns with reasonable accuracy as compared to experimental data

Introduction

The determination of the structural properties of a reinforced concrete building is essential in the evaluation of its seismic response. The initial stiffness, ultimate capacity, and different global and local ductility demands are some of the parameters included in this assessment. Due to complex interaction between the various components of real structures, their dynamic characteristics up to failure cannot be identified solely from dynamic tests of scale models. The cost of such tests is often substantial, particularly for large specimens. Historically these difficulties have been resolved by static tests on components and on reduced scale sub-assemblages of structures under cyclic load reversals. Results from these tests are then used in the development and calibration of hysteretic models, which permits the extrapolation of the available test information to other cases and to the dynamic response of the complete structures (Taucer 1991). Structural columns are the most important elements in reinforced concrete structures. Columns are not only subjected to axial loads due to gravity, but they can be subjected to

[1] Graduate Research Assistant, Dept. of Civil Engineering, Ryerson University, Toronto, Ontario, Canada. E-mail: rsadjadi@ryerson.ca
[2] Professor, Dept. of Civil Engineering, Ryerson University, Toronto, Ontario, Canada. E-mail: kianoush@ryerson.ca

combined axial and shear forces as well as bending moments due to lateral loads such as seismic forces. Based on earthquake resistant design philosophy, the seismic energy input should be dissipated through largest possible number of inelastic regions within the structure. For implementation of the strong column-weak beam mechanism in the moment resisting frames, they are designed such that plastic hinging develops at the girder ends. To avoid the formation of a side-sway collapse mechanism, columns of moment resisting frames are expected to remain elastic during the earthquake response, except at the base of the building where hinging is desired. Attention is thus focused on understanding and predicting the seismic behavior of first story columns. Behavior of reinforced concrete columns has been the subject of many investigations. These researches may be divided into three main categories (Esmaeily 1999). The first category includes those studying the effect of amount, size and arrangement of reinforcement. The second category includes all the investigations regarding the material properties, such as the strengths of concrete. The third category includes all the research, in which different loading conditions such as the effect of variable axial load is investigated which is also investigated in this paper.

Analytical Models

Much research has been conducted for the development of nonlinear models of inelastic response of RC elements subjected to cyclic reversals. These models range from the simple two-component model with bilinear hysteretic rule to refined finite elements models based on comprehensive cyclic stress-strain relationships for concrete and reinforcing steel.

There are two major categories for the nonlinear dynamic analysis of reinforced concrete structures. One is to present the overall behavior of each structural component in terms of a macro-model. These macroscopic models are based on approximations of the physical behavior of RC members and have been most widely used for investigation of the response of multistory buildings because of their simplicity. However the accuracy of these models is questionable as there in no true representation of the controlling parameters in their nonlinear behavior. The second option is to discretize each structural component into smaller units and then capture the overall behavior of the component in terms of the behavior of those smaller units. Although these models are more accurate based on the degree of their complexity, their implementation in dynamic response analyses of large structures with several members is prohibitively expensive.

A major aspect in modeling the nonlinear behavior of the components is how to consider the regions which yield and the regions which remain elastic in the structural elements during the dynamic analysis. In the lumped plasticity model it is assumed that the yielding of the element is localized in the zero length regions in the elements' ends, which are called plastic hinges. In the spread plasticity model the plastic hinges form in the members but parts of the length of the element can experience inelastic deformation. Modeling the behavior of reinforced concrete with lumped-plasticity idealization may not be accurate, since inelastic deformation has been observed over finite length of the member. The fiber model corresponds to a large level of discretization, where each structural member is modeled as a single element and the stress-strain relationships for steel and plain concrete are evaluated during the analysis at several cross-sections comprising the element. The fiber model has the ability to represent the behavior of columns particularly in the critical regions of the element as the cross-sections, the material, and spacing and amount of reinforcement can be varied along the member length. In fiber model, the element is divided into a discrete number of cross sections (segments). The model assumes constant fiber properties over each segment length, based on the properties of the monitored slice at the center of each segment. The non-linear behavior of the element is monitored at these control sections, which are in turn discretized into longitudinal fibers of plane concrete and reinforcing steel. The non-linear behavior of the section is then captured from the integration of the non-linear stress-strain relationship of the fibers. This feature permits the modeling of any type of structural element including irregular cross-sections or cross-sections with different material properties. In the fiber model, the main assumption is that plane sections will remain plane and perpendicular to the longitudinal axis of the element throughout the deformation history. More information on the development of fiber element can be found in (Taucer 1991) and (Maekawa 2003). Since the formulation is based on the assumption of complete bond between all fibers, phenomenon such as shear failure, and bond deterioration between steel and concrete can not be explicitly modeled and such effects are not covered in this study. The present study is based on fiber formulation concept which is implemented in the computer program DRAIN-2DX (fiber element) (Allahabadi 1988). DRAIN-2DX is a nonlinear structural analysis program that can be used to investigate the nonlinear static or dynamic behavior of two dimensional structural frames. Fiber models can be classified into two main groups: stiffness-based and flexibility-based models. The conventional stiffness-based models are unable to satisfy equilibrium along member length when softening occurs (Khaloo 2002). The fiber element in this study, incorporated in the program DRAIN-2DX, is a distributed plasticity-based element with a flexibility-based formulation (Varma et al. 2005). Comprehensive details about mathematical formulation can be found in (Kurama 1996).The element is assumed to be elastic in shear. This paper is focused on the investigation of the columns with flexure dominant mode of failure.

Plastic hinge zone

It is expected that the bases of all columns in the first story at the connection to the foundation experience plastic hinging. As the column experiences cyclic displacement, plastic deformation spreads into the member which should be captured for the simulation of the real inelastic behavior of the RC member. In reinforced concrete columns, the equivalent plastic hinge length is determined based on the experimental curvature distribution and deflection. Therefore the effect of longitudinal reinforcement yield penetration and the cracking due to shear are included in the equivalent plastic hinge length (Sakai 1989).

In fiber element, material nonlinearity can spread through the entire length of the element. Therefore, the plastic hinge zone where element nonlinearity can spread is considered as a fiber element whereas the rest of the element length is modeled as an elastic element. The length of the plastic hinge used in this study for the calculation of the fiber segment length is calculated as $l_p = 0.08l + 0.022d_b{\times}f_y$ as suggested by Paulay and Priestly (Paulay 1992), where l is the total length of the element, d_b is the longitudinal bar diameter, and f_y is the yield stress of steel. The cross section of the fiber element is then divided into a number of fibers of concrete and steel. Regardless of the high memory demand and increasing computational cost, the accuracy of the model increases with the number of fibers in each cross section. It was observed that for the number of fibers of more than 20, which is used as a minimum in this research, there is a negligible difference between the analytical value of the moment of inertia obtained from DRAIN-2DX, and the theoretical value. For smaller number of fibers, the results may become inaccurate (Sadjadi 2004).

Material Models

The nonlinear behavior of the fiber element derives entirely from the nonlinear behavior of the constituent material fibers. Thus, the validity of the analytical results depends on the accuracy of the stress-strain relationship curves for the concrete and steel. With accepted level of accuracy, the three dimensional behavior of each material can be simplified into uniaxial stress-strain relationship. The effect of concrete confinement by reinforcement is considered by using appropriate uniaxial monotonic envelope of concrete which incorporates such effect including arrangement and mechanical properties of the transverse reinforcement. This is of great importance since it is assumed that the concrete monotonic stress-strain curve represents the envelope for the cyclic stress strain branches.

The material model for concrete in DRAIN-2DX considers cracking and crushing and tension stiffening. The concrete material properties are defined as points in the stress-strain curve shown in Fig. 1. There is a maximum of five points for defining the stress-strain relationship in compression, and two points for defining the stress-strain relationship in tension. The point with coordinates (σ_{1C}, ε_{1C}) refers to the cracking of the concrete and the point (σ_{2C}, ε_{2C}) refers to the maximum compression strength of the concrete. The point (σ_{3C}, ε_{3C}) defines the ultimate strength of concrete under high strains. The horizontal branch shows the ability of concrete to sustain some strength at very large strains. The slope of descending branch which is highly dependant on the confinement condition of the cross section has a significant effect on the ductility of the element during cyclic loading demanding an accurate material model for the concrete incorporating the volume and properties of transverse reinforcement. Although several models have been proposed for the stress-strain relationship envelope for plain concrete in compression with consideration of the effect of the confinement, the model proposed by Hoshikuma et al.

(Hoshikuma 1997) is selected for defining the material model for core concrete for both circular and rectangular sections in this study.

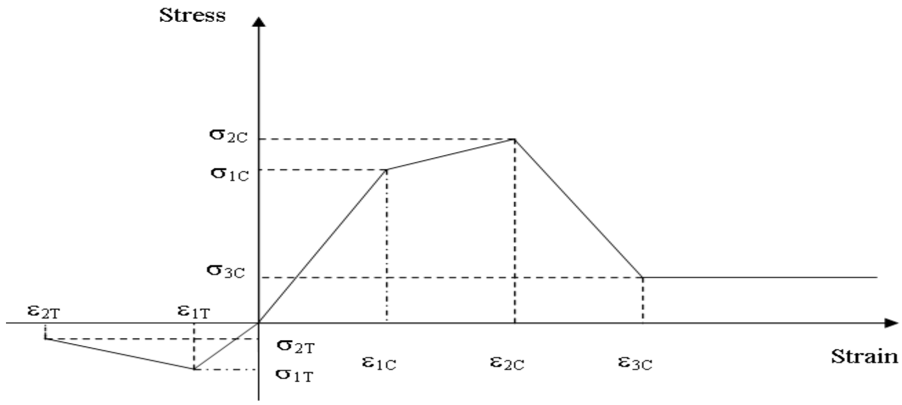


Figure 1. Material model for concrete in DRAIN-2DX.

The model is basically proposed for stress-strain model for confined reinforced concrete in bridge piers considering different mechanical properties, configuration and volumetric ratio of the hoops and cross ties. The model has the advantage of reproducing very comparable results for tests with a wide range of volumetric steel ratios ranging from 0.19 to 4.66%. The models proposed by Kent and Park (Kent 1971), and Mander (Mander 1988) are selected to represent the behavior of the unconfined concrete in the cover concrete for rectangular and circular sections, respectively. For concrete in tension, the relationship developed by Vebo and Gali (Vebo 1977) has been adopted to represent the effect of concrete in tension under cyclic loading.

The material model for steel bars in DRAIN-2DX is shown in Fig. 2. The program assumes that the steel behavior is identical in tension and compression.

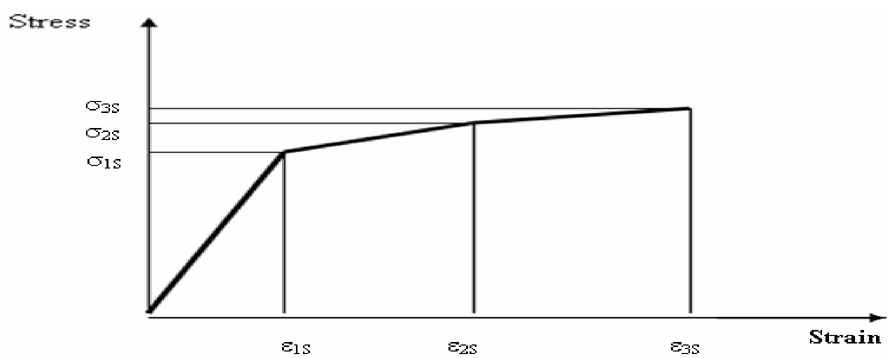


Figure 2. Material model for steel in DRAIN-2DX.

As depicted in Figure 2, DRAIN-2DX steel model assumes the steel modulus in different stages to reduce consequently. Thus, it is not possible to model the yield plateau behavior after yielding of the steel as it has a lower modulus than the ensuing strain hardening state. It is also not possible to implicitly model post-peak phenomenon such as necking and rupture of the steel. Several models have been proposed for defining the stress-strain relationship for steel. These models range from a simple elasto-plastic idealization to more complex models such as that proposed by Menegotto-Pinto for the steel as shown in Fig. 3. In this study it is aimed to propose a simple model to satisfy the requirements of the DRAIN-2DX material model while incorporating essential characteristic of the cyclic behavior of steel as can be observed from Fig. 3. Bauschinger (Bauschinger 1887) reported that the modulus of elasticity of steel reduces at subsequent cycles after the steel has been strained beyond the elastic limit. The analysis of the test data has confirmed that the unloading modulus decreases, and that the rate of decrease is especially rapid after yielding but stabilizes at larger strains. Dodd and Restrepo-Posada (Dodd 1995) proposed a relationship between the maximum plastic strain and the unloading modulus of steel as:

$$E_u = E_s[0.82 + \frac{1}{5.5 + 1000\varepsilon_M^{'}}] \tag{1}$$

E_u, E_s, and $\varepsilon_M^{'}$ are the unloading modulus of elasticity, the initial modulus of elasticity of steel, and the maximum plastic strain, respectively.

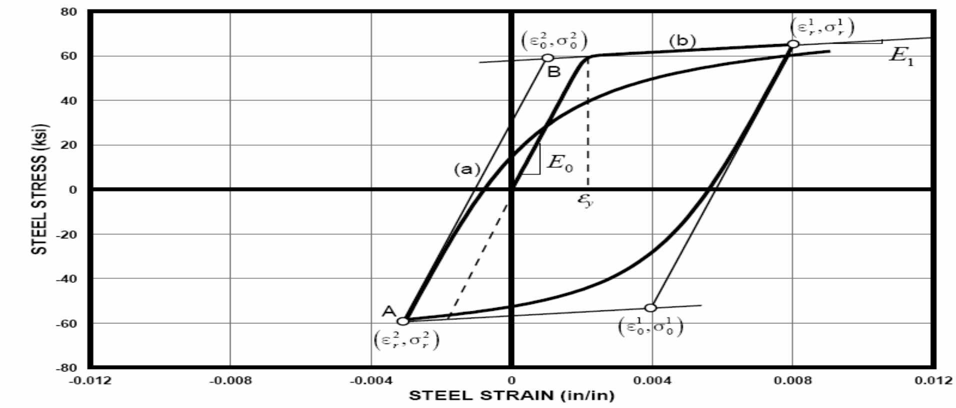


Figure 3. Menegotto-Pinto model for steel (Taucer 1991).

When reversed cyclic loading is applied to a steel bar, stress-strain curve becomes nonlinear at a stress lower than the initial yield strength. This phenomenon is known as Bauschinger effect. Mander (Mander 1983) has proposed a softened branch as shown in Fig. 4 to simulate this effect on the stress reversals. This is an important aspect in the cyclic behavior of the steel which should be considered in modeling the behavior of the element under load reversals.

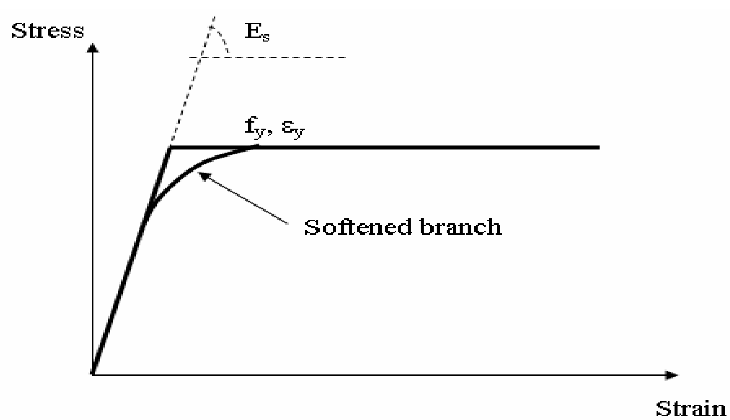
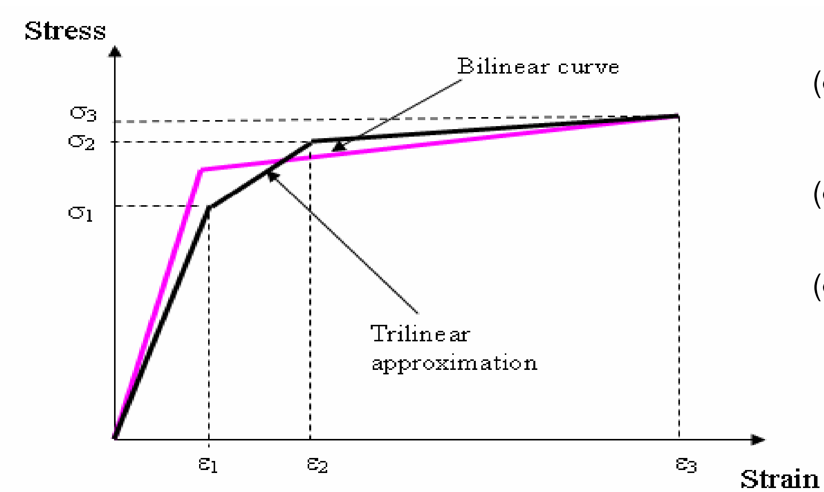


Figure 4. The softened branch in the stress-strain behavior of steel.

To add all the aforementioned effects to the fiber model in DRAIN-2DX, a trilinear approximation of the bilinear curve is introduced as is depicted in Fig. 5. The coordinates of this trilinear approximation in terms of the stress-strain relationships is also shown in the same figure as a function of mechanical properties of steel and the reduced modulus of elasticity (E_u) as obtained from Eq. 1 for each cycle. The analysis is divided into different segments based on the loading history and the maximum strain in the steel. Each cycle is analyzed based on the unloading modulus of elasticity that corresponds to the maximum plastic strain of the steel in the previous cycle. The tri-linear approximation, using corresponding reduced modulus of elasticity, will replace the bilinear relationship wherever the steel layer strain has exceeded the yielding value. The model is able to result in more realistic representation of the dissipated energy as can be observed from comparison of analytical and experimental hysteretic loops in Figs. 7, and 8. The implementation of the proposed steel model in the analysis starts by dividing the loading history to a number of complete cycles for each drift level. An initial analysis will be performed with bilinear model data to find the maximum strain in the steel at the end of the cycle in which yielding is experienced. This maximum strain is then used to obtain the reduced modulus of elasticity *(E_u)* as defined by Eq. 1 to be implemented in the trilinear approximation for the next cycle. The procedure is followed and the maximum strain is obtained at the end of each cycle to be implemented into the trilinear stress-strain relationship of the next cycle. Therefore several complete analyses will be performed with the complete loading history of the test, where each complete analysis has a certain trilinear properties such as the reduced modulus of elasticity of steel for the entire load history. The desired complete load-deformation response consists of several load-deformation loops; one for each cycle (i.e. level of drift) that is selected from the corresponding cycle analysis, analyzed using appropriate trilinear properties and then added to the total load-deformation history. The improvement in the analytical results was observed for several tests as can be observed in the samples which are presented in this paper. However it should be mentioned that the tri-linear curve is obtained by finding the best simulation of the experimental results of a limited number of tests in this research. More research is needed for better idealization of such curve for different condition of bar arrangement and mechanical property for steel provided its simplicity is maintained.

To observe the improvement of such modeling, analytical results using the above mentioned model are compared to the experimental results of the cyclic loading test of two different experiments. In the first case, the result of an experiment conducted by Park (Park 1990) on a rectangular cantilever column is used for evaluation of the fiber model analysis. Because of the high memory demand of the output data, only one cycle per drift level is performed in analytical simulations. In the second case, the ability of the fiber model incorporating the trilinear model is evaluated for simulating a test of a cantilever column with circular cross-section and under variable axial load (Esmaeily 1999).



$$(\sigma_1, \varepsilon_1) = (0.8 f_y, 2.3\frac{f_y}{E_u})$$

$$(\sigma_2, \varepsilon_2) = (1.05 f_y, 9.25\frac{f_y}{E_u})$$

$$(\sigma_3, \varepsilon_3) = (f_u, \varepsilon_u)$$

Figure 5. The proposed trilinear material behavior for steel.

For the first case, the RC column has a height of 1784 mm and is made of concrete with compressive strength of 26.9 MPa. It is reinforced with 10 longitudinal bars of Grade 380 (f_y = 432 MPa) with a diameter of 24 mm.The transverse reinforcement had a yield strength of 305 MPa and diameter of 12 mm placed at spacing of 80 mm. The clear cover to the transverse reinforcement is 24 mm.The column is under constant axial load of 646 kN while lateral cyclic load is applied on the top of the column parallel to the larger side of the cross-section. The cross section of the column is shown on Fig. 6.

Fig. 7(a) illustrates the experimental result of the test in term of horizontal force vs. top displacement of the column. Figs. 7(b) and (c) show the analytical results of the test using bilinear model, and the trilinear approximation, respectively.

It can be observed that the trilinear model is more successful in simulating the test than the bilinear model in terms of the area enclosed by each hystersis loop which is representative of the dissipated energy through that cycle. The bilinear model overestimates the response of the initial cycles with small top displacements.

Table 1 compares the maximum responses of the experimental test and the analytical simulations corresponding to a specific value of displacement at each cycle. Because the DRAIN-2DX analyses were performed for one cycle per drift level, the average value of the experimental test for each drift level is presented. It is observed that the trilinear model is very successful in yielding very comparable results to the experiment. As was mentioned the bilinear model overestimates the response in the first drift level.

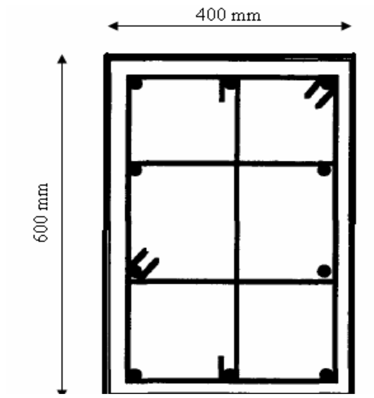


Figure. 6. Cross section of the specimen.

Table 1. Comparison of the maximum response for each cycle

Top Displacement (mm)	**7**	**21**	**42**	**63**	**84**	**105**
Experiment (kN)	257	340	365	374	373	368
Bilinear model (kN)	335	370	380	378	375	371
Trilinear model (kN)	251	340	358	367	370	360

For the second case, cantilever column with circular cross-section and under variable axial load (Esmaeily 1999), the column has a cross sectional diameter of 406.4 mm and a height of 1829 mm. It is made of concrete with the compressive strength of 49.3 MPa, and is reinforced with 12 Grade 410 (f_y = 489.5, f_u= 579.2 MPa) 13 mm longitudinal bars. W2.5 at 32 mm is used for transverse reinforcement with a yield stress of 468.8 MPa. The clear cover to the transverse reinforcement is 13 mm. The column is under variable axial load of *[tan (47.32º) ×lateral force]* while lateral cyclic load is applied on top of the column.

After defining the material models using the trilinear approximation for steel behavior and the sectional properties of fibers, a nodal load pattern consisting of a unit horizontal load and a 1.08 (*tan 47.32º*) vertical load is defined in the program input data to be applied on the column's top throughout the entire analysis. This will ensure that the proportional axial force of is applied simultaneously with the lateral load throughout the loading history. Comparison of the results as shown in Fig .8 indicates a very successful simulation of the analysis particularly before drift ratio of 6%. The axial force was proportional to the horizontal force and its value had opposite signs in two opposite directions. Therefore the cyclic behavior of the column was different in the pull and push directions. This difference is successfully captured in the analytical response as can be observed from maximum responses in both directions in Fig .8(b).

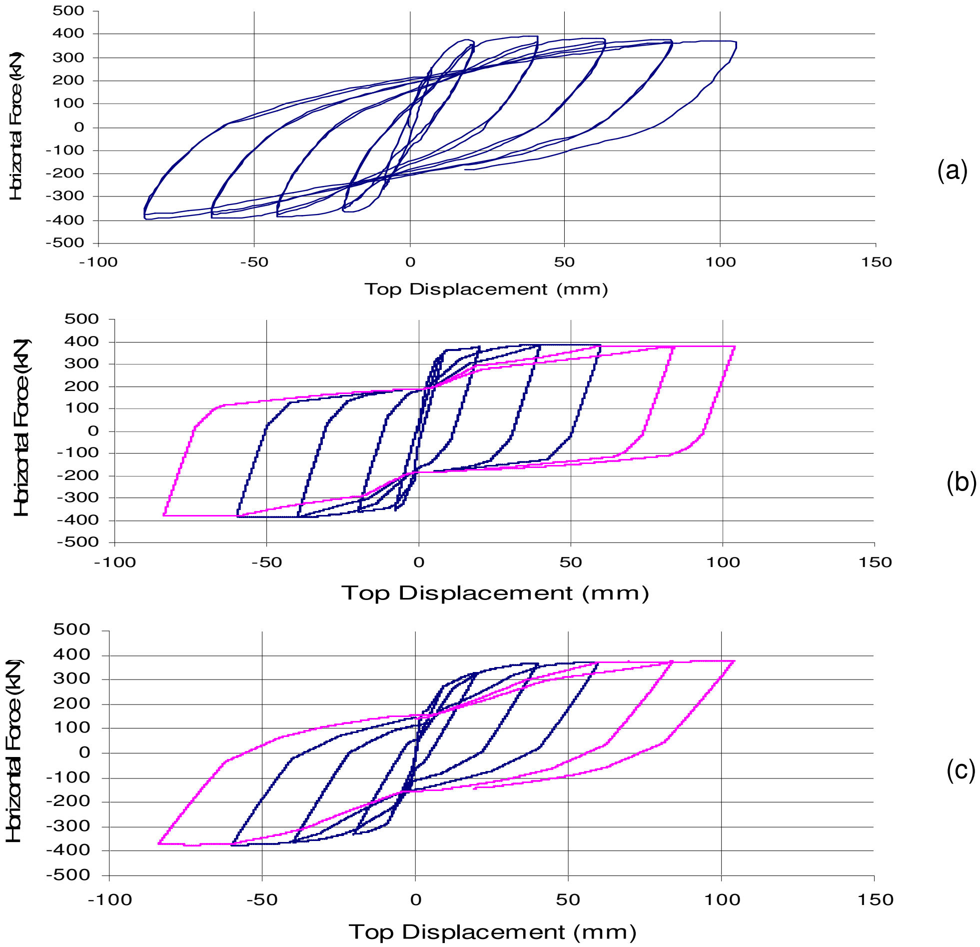


Figure 7. Horizontal force-top displacement response for the column.
(a) experimental, (b) bilinear model, (c) trilinear approximation.

The observation of the experimental test indicated that the furthermost rebar on the push side buckled at the second cycle of 6% drift ratio. The same behavior was observed on the opposite side of the column when the load reversal was applied. At the third cycle of 6% drift ratio, the two adjacent spirals ruptured followed by the buckling of the nearby rebars. The test was continued for the next step, at a drift ratio of 8%, at which the two buckled rebars ruptured. The specimen is reported to have failed in flexural mode. It is not intended to address the buckling and rupturing loads of the column in this study, but rather to check the ability of the model to simulate the behavior using experimental values. Using an approach similar to the one suggested by Lee (Lee 2001) for modeling the buckling of reinforcement bars, it is decided to define a very small value for modulus of elasticity after the stress in the longitudinal steel exceeds the buckling stress of the rebar as reported in the experiment. In this study, this was done manually by defining a very small stiffness for steel after reaching the strain value corresponding to the strain in the highest strained rebar at the end of the first cycle of 6% drift ratio. For simulating the rupture of the rebars, the corresponding steel fiber sections are removed from the beginning of the analysis of cycle of 8% drift ratio, the result of which is shown in Fig. 8(b) as a degraded loop. It is important to note that the validity of the results of the fiber element is based on the assumption that the plane sections remain plane and perpendicular to the longitudinal axis of the element throughout the deformation history.

Conclusions

This study shows that the fiber element incorporating the trilinear approximation of reinforcing bar behavior and appropriate models for concrete leads to very comparable analytical results to the experimental data. The validity of the fiber model is based on the assumption that plane sections remain plane and perpendicular to the longitudinal axis of the element throughout the deformation history. Therefore phenomena such as debonding and shear failure can not be implicitly considered in the model. The model shows comparable results for several different experiments and two of them are presented in this paper. Further studies are being conducted to model and evaluate the analytical results of cyclic loading behavior of high-strength concrete specimens with maximum compressive strength in excess of 80 MPa incorporating well-known material models for high strength concrete into the fiber element.

References

Allahabadi R, and Powell, GH., 1988. DRAIN-2DX User Guide, *Technical Report UCB/EERC* 88/06, University of California, Berkeley.

Bauschinger, J., 1887. Variations in the Elastic Limit of Iron and Steel, *The Journal of the Iron and Steel Institute* 12 (1), 442-444.

Dodd, L., and Restrepo-Posada, J., 1995. Model for Predicting Cyclic Behavior of Reinforcing Steel, *J. Struct. Eng.* 121(3), 433-445.

Esmaeily-Gh, A. and Xiao, Y., 1999. Behavior of reinforced concrete columns under variable axial loads, *Report No. USC-SERP 99/01*, Department of Civil Engineering University of Southern California, Los Angeles, California.

Hoshikuma, J., Kawashima, K., Nagaya, K., and Taylor, A.W., 1997. Stress–strain model for confined concrete in bridge piers. *ASCE Journal of Structural Engineering* 123(5), 624–633.

Khaloo AR, Tariverdilo S., 2002. Localization analysis of RC members with softening behaviour, *ASCE Journal of Structural Engineering* 128(9), 1148-1157.

Kent, D.C., and Park R., 1971. Flexural Members with Confined Concrete, *Journal of the Structural Division, ASCE* 97, No. ST7, 1969-1990.

Kurama, Y., Pessiki, S. P., Sause, R., Lu, L.-W., and El-Sheikh, M., 1996. Analytical Modeling and Lateral Load Behavior of Unbonded Post-Tensioned Precast Concrete Walls, *Earthquake Engineering Research Report No. EQ-96-02*, Department of Civil and Environmental Engineering, Lehigh University, Bethlehem, Pa.

Lee, T. K., and Pan, A. D. E., 2001. Analysis of Composite Beam-Columns under Lateral Cyclic Loading. *ASCE Journal of Structural Engineering* 127(2), 186-193.

Maekawa, K., Pimanmas, A., Okamura, H., 2003. *Nonlinear Mechanics of Reinforced Concrete*. Spon Press, New York.

Mander, J. B., 1983. Seismic Design of Bridge Piers, *Ph.D. thesis*, Dept. of Civ. Engrg., University of Canterbury, Christchurch, New Zealand, Chapter 8.

Mander, J.B., Priestley, M. J. N., and Park R., 1988. Theoretical Stress – Strain Behavior of Confined Concrete, *ASCE Journal of Structural Engineering* 114(8), 1804-1826 and “Observed Stress – Strain Behavior of Confined Concrete, *ASCE Journal of Structural Engineering* 114(8), 1827-1849.

Park, R., and Paulay, T., 1990. Use of Interlocking Spirals for Transverse Reinforcement in Bridge Columns, *Strength and Ductility of Concrete Substructures of Bridges*, RRU (Road Research Unit) Bulletin 8. 1, 77-92.

Paulay, T. and Priestly, M.J.N., 1992. *Seismic design of reinforced concrete and masonry building*, Wiley, New York.

Sadjadi, R. and Kianoush, M. R., 2004. Effect of Modeling Features on Response of Reinforced Concrete Frames, *Proc., 13th World Conference on Earthquake Engineering*, Vancouver, B.C., Canada

Sakai, K. and Sheikh, S.A., 1989. What do we know About Confinement in Reinforced Concrete Columns? *ACI Structural Journal*, 86(2), 192-207.

Taucer, F.F., Spacone, E. and Filippou, F.C., 1991. A Fiber Beam-Column Element for Seismic Response Analysis of Reinforced Concrete Structures, *Report to the National Science Foundation and the California Department of Transportation*, Earthquake Engineering Research Center, University of California, Berkeley, California,

Varma, A.H., Sause, R., Ricles, J.M. and Qinggang, L., 2005. Development and Validation of Fiber Models for High Strength Square CFT Beam-Columns, *ACI Structural Journal* 102 (1).

Vebo, A. and Gali, A., 1977. Moment-Curvature relation of Reinforced Concrete Slabs, *Journal of the Structural Division, ASCE* 103, No. ST3, 515-531.

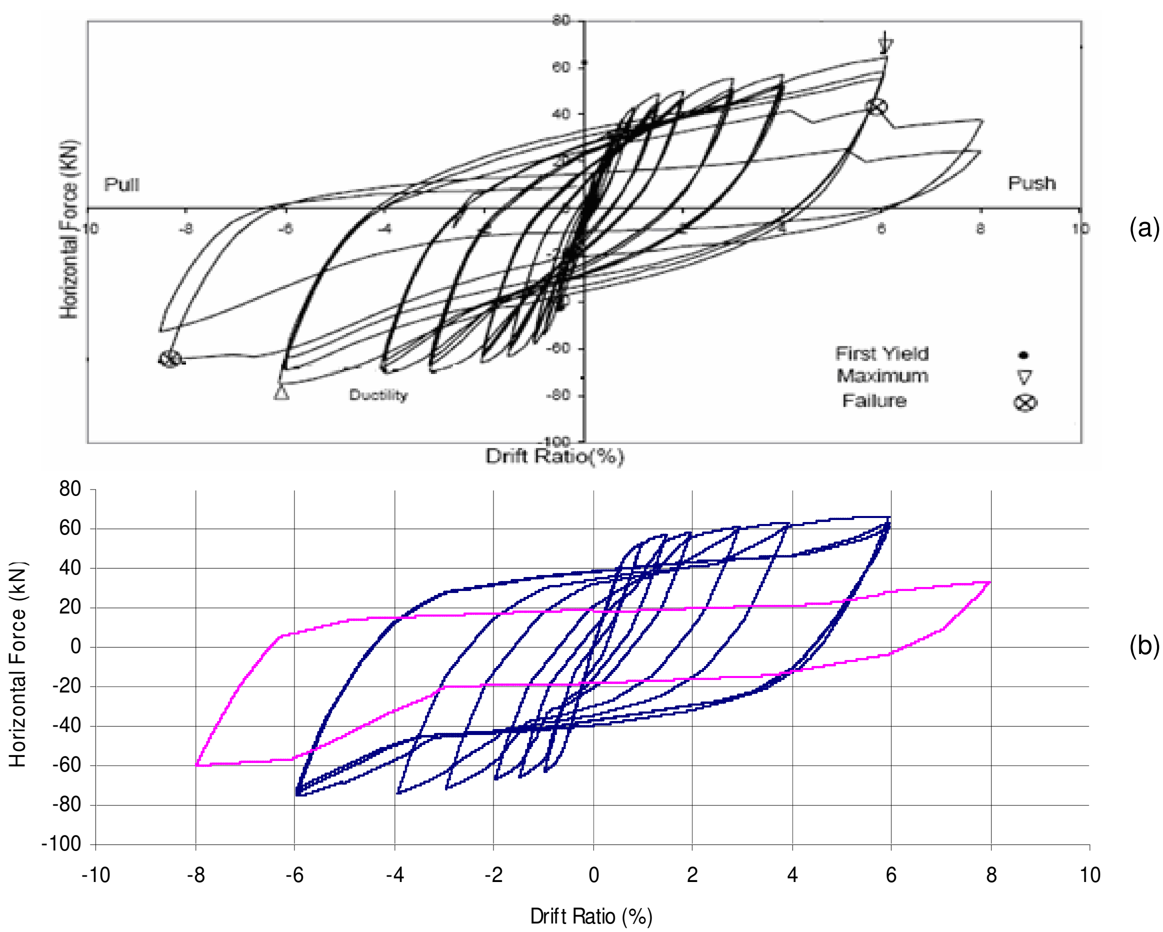


Figure 8. Horizontal force-top displacement response for the column.
(a) experimental (Esmaeily 1999), (b) trilinear model.