AN EFFICIENT METHOD FOR OPTIMUM COMBINATION OF MODES REQUIRED FOR PUSHOVER ANALYSIS

K. Shakeri[1], M.A. Shayanfar[2] and A.S. Moghadam[3]

ABSTRACT

During the last decade, a great deal of attention has been paid to nonlinear static procedures (NSP). These procedures have been used widely in performance based seismic design of structures; since it provides essential information on response parameters that cannot be obtained using conventional elastic methods. However NSP suffers from different drawbacks including restriction with a single - mode response and its inability to include higher mode order effects in the analysis. To overcome these drawbacks, some improved procedures have been recently proposed by several researchers. In order to account for higher modes effects, an optimum method of direct modal combinations for a 12 storey steel building is presented in this paper. The proposed method has been inspired by modal pushover analysis (MPA) proposed by Chopra but accounts for participation of each mode. The results show that the inter-storey drift profiles from optimum combination has minimal error with drift profiles from nonlinear time history analysis (NTHA). The genetic algorithm (GA) is used for optimization of the modal combination and minimization of the error function.

Introduction

Referring to the philosophy of the seismic design and inelastic behaviour of structure at low performance levels such as life safety and collapse prevention, it is clear that the damageability of structures under earthquake is controlled by the inelastic deformation capacities of the structural elements. Therefore, changing the base of codes from force control to displacement control and more explicitly, design consideration to damage control has been widely recognized (see Fajfar 1997 and Priestley 2000). This aim can be achieved only by introducing some kind of nonlinear analysis into the seismic design methodology. Regarding the nature of seismic loads in the form of base acceleration, nonlinear time history analysis (NTHA) is the most rigorous procedure to compute seismic demands. Because of complexity of this method, during the last decade the nonlinear static procedure (NSP) so-called pushover analysis as a practical tool to estimate the inelastic response, has been developed (ATC, 1996, FEMA, 1997-2000).

---

[1]PhD. Student, Dept. of Civil Engineering, Iran University of Science & Technology, Tehran, K_Shakeri@iust.ac.ir
[2]Associate Professor, Dept. of Civil Engineering, Iran University of Science & Technology, Tehran, Shayanfar@iust.ac.ir
[3]Associate Professor, Dept. of Structural Engineering, International Institute of Earthquake Engineering and Seismology(IIEES), moghadam@iiees.ac.ir

In the pushover analysis, a structural model that directly incorporates nonlinear material characteristics is subjected to monotonically increasing invariant lateral force pattern until a predetermined target displacement is reached. Define of the load pattern and target displacement are based on the assumption that the response of a multi-degree-of-freedom (MDOF) structure is directly related to the response of an equivalent single-degree-of-freedom(SDOF) system with a fundamental mode shape and that the mode shape remains unchanged, throughout the time-history, regardless of the level of deformation. While the invariant load pattern according to fundamental mode may be adequate for regular and low-rise structures whose response is effectively dominated by fundamental mode (krawinkler,1998), It can misleading for irregular and high-rise building with significant higher mode contribution.

The major drawback of NSP in its existing form (ATC, 1996, FEMA, 1997-2000) lies in the fact that it is basically restricted with a single-mode response, and it can't account for the contributions of higher modes and changing of modes shape because of structural yielding. Recently to eliminate this drawback several method have been proposed that they can be categorized in two major classes:

1. modal-adaptive pushover
2. modal- combination pushover

In modal-adaptive methods according to the vibration properties of the structure at every step the load pattern is changed e.g., (Gupta, 2000, Antoniou, 2002 and Aydinoglu, 2003). These adaptive load pattern estimate responses better than invariant load pattern but they may be complicated for engineering practices. While in the modal combination pushover methods the simplicity of the conventional methods is kept and the seismic responses are estimated by combining the results of some previously achieved pushover analysis. Each of the pushover analysis is conducted by using a mode shape as its load pattern e.g., (Moghadam 2002 and Chopra, 2002). In the approach proposed by Moghadam, pushover results combination, (PRC), the proposed combination rule is shown in Eq.1,

$$R = \sum_{i=1}^{N} B_i R_i \qquad (1)$$

Where,
R : final estimation of responses;
B_i : mass participating factor of mode i ;
R_i : value of response resulted from pushover analysis using mode shape i as load pattern.

In the modal pushover analysis (MPA) proposed by Chopra in spite of PRC, the target displacement for every mode of pushover analysis is not the same and determined from NTHA of equivalent SDOF system. The responses of the pushover analysis are combined according to the square-root-of-sum-of-squares (SRSS), Eq.2,

$$R = \sqrt{\sum_{i=1}^{n} R_i^2} \qquad (2)$$

Where, R and R_i are the same terms as defined for Eq.1. The approximate MPA procedure estimates the floor displacements and storey drifts better than conventional pushover analysis. Nevertheless, the MPA procedure has error in comparison with NTHA.

The main issue in this paper is to answer this question: can be estimated the seismic responses exactly by using another combination rules in the MPA procedure and how can be found this optimum combination rule? Therefore, a 12 storey moment resistant steel frame as a case study is investigated in this research program. The proposed combination is to add the participation of each mode response directly, in an algebraic fashion, without removing any sign. Participation coefficient of each mode for this

case study obtained by using genetic algorithm (GA) optimization method. While the inter-storey drift profiles from proposed combination has minimal error in comparison with drift profiles from NTHA.

Proposed Procedure

Since the goal of this procedure is to define an optimal combination for MPA relevant to the undertaken steel frame, all the established steps of MPA procedure except the SRSS combination step are included in the proposed method. The error between responses of NTHA and sum of factored responses resulted from each pushover analysis is defined as an error function that must be minimized and the quantity of each factors be determined. Every factor shows the participation of each mode and the GA method is used for minimization of error (objective) function.

The procedure is implemented in a sequence of steps as fallow:

1. Compute the natural frequencies, ω_n and mode shapes, $\varnothing_n$, for linearly elastic vibration of the structure.
2. For a selected number of first modes (4 modes), develop the base shear-roof displacement pushover curve, $V_{bn} - U_{rn}$ for lateral force distribution $S_n = m\varnothing_n$, where m is the mass matrix of the structure.
3. Idealize each pushover curve as a bilinear curve.
4. Convert the idealized pushover curve to the force – displacement relation, $F_{sn}/L_n - D_n$, (Fig.4) for the "n^{th} mode" inelastic SDOF system by utilizing

$$\frac{F_{sny}}{L_n} = \frac{V_{bny}}{M_n^*} \qquad\qquad D_{ny} = \frac{U_{rny}}{\Gamma_n \varnothing_{rn}} \tag{3}$$

In which, $M_n^* = L_n \Gamma_n$ is the effective modal mass , $\varnothing_{rn}$ is the value of $\varnothing_n$ at the roof level,

$$\Gamma_n = \frac{\sum_{k=1}^{N} m_k \varnothing_{kn}}{\sum_{k=1}^{N} m_k \varnothing_{kn}^2} \qquad \text{and} \qquad L_n = \sum_{k=1}^{N} m_k \varnothing_{kn} \tag{4}$$

5. Compute the peak deformation of the nth inelastic SDOF system through NTHA. The elastic period of vibration of the nth system is:

$$T_n = 2\pi \sqrt{\frac{L_n D_{ny}}{F_{sny}}} \tag{5}$$

6. Calculate the peak roof displacement associated with the nth mode from

$$u_{rn} = \Gamma_n \varnothing_{rn} D_n \tag{6}$$

7. From the pushover database (step 2), extract values of the desired responses R_n (floor displacements, storey drifts, etc.) at the peak roof displacement.
8. Perform NTHA of the structure under the desired acceleration record (in this study El Centro 1940) and determine the envelop of inter-storey drift profiles.
9. Define the combination rule as:

$$R = aR_1 + bR_2 + cR_3 + dR_4 \tag{7}$$

Where, R is final (total) response, R_1, R_2, R_3 and R_4 is obtained at step 7 and a, b, c and d are participation factors of each mode that obtained from minimized objective function at step 12.

10. Since inter-storey drift is important factor in damage of structure, so it is desired that the differences between drift resulted from NTHA and proposed combination rule is minimized and the error vector is defined as:

$$\vec{\Delta}_{error} = \vec{\Delta}_{NTHA} - \left(a\vec{\Delta}_1 + b\vec{\Delta}_2 + c\vec{\Delta}_3 + d\vec{\Delta}_4 \right) \tag{8}$$

Where, $\vec{\Delta}_{NTHA}$ is vector of storey drift profile (at every storey) resulted from NTHA and $\vec{\Delta}_1, \vec{\Delta}_2, \vec{\Delta}_3$ and $\vec{\Delta}_4$ is vector of storey drift profile (at every storey) extracted from each pushover analysis according with every mode.

11. To minimize the error vector, all its components and sum of them must be minimized, so the objective function is defined as:

$$norm(\vec{\Delta}_{error}) = \sqrt{d^2_{error1} + d^2_{error2} + d^2_{error3} + ... + d^2_{error12}} \tag{9}$$

Where, $d_{error\,i}$ is i^{th} component of the error drift storey vector, $\vec{\Delta}_{error}$.

12. Use GA optimization method to minimize the objective function defined at step 11 and obtain the optimum quantities of a, b, c and d.

Genetic Algorithms (GAs)

The GA is a stochastic global search method that mimics the metaphor of natural biological evolution. GAs operates on a population of potential solutions applying the principle of survival of the fittest to produce (hopefully) better and better approximations to a solution. At each generation, a new set of approximations is created by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using operators borrowed from natural genetics. This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals that they were created from, just as in natural adaptation. Individuals, or current approximations, are encoded as strings, chromosomes, composed over some alphabet(s), so that the genotypes (chromosome values) are uniquely mapped onto the decision variable (phenotypic) domain. The most commonly used representation in GAs is the binary alphabet {0,1} although other representations can be used, e.g. ternary, integer, real-valued etc.

Real-Coded Genetic Algorithms

The use of real-valued genes in GAs is claimed by Wright (Wright, 1991) to offer a number of advantages in numerical function optimization over binary encodings. Efficiency of the GA is increased as there is no need to convert chromosomes to phenotypes before each function evaluation; less memory is required as efficient floating-point internal computer representations can be used directly; there is no loss in precision by discretisation to binary or other values; and there is greater freedom to use different genetic operators. The use of real-valued encodings is described in detail by Michalewicz (Michalewicz, 1992) and in the literature on Evolution Strategies.

The Objective and Fitness Functions

The objective function is used to provide a measure of how individuals have performed in the problem domain. In the case of a minimization problem, the fit individuals will have the lowest numerical value of the associated objective function. This raw measure of fitness is usually only used as an intermediate

stage in determining the relative performance of individuals in a GA. Another function, the fitness function, is normally used to transform the objective function value into a measure of relative fitness (De Jony, 1975), thus:

$$F(x) = g(f(x)) \tag{10}$$

Where, f is the objective function, g transforms the value of the objective function to a non-negative number and F is the resulting relative fitness. This mapping is always necessary when the objective function is to be minimized as the lower objective function values correspond to fitter individuals. In many cases, the fitness function value corresponds to the number of offspring that an individual can expect to produce in the next generation. A commonly used transformation is that of proportional fitness assignment. The individual fitness, $F(x_i)$, of each individual is computed as the individual's raw performance, $f(x_i)$, relative to the whole population, i.e.,

$$F(x_i) = \frac{f(x_i)}{\sum_{i=1}^{N_{ind}} f(x_i)} \tag{11}$$

Where N_{ind} is the population size and x_i is the phenotypic value of individual i. whilst this fitness assignment ensures that each individual has a probability of reproducing according to its relative fitness, it fails to account for negative objective function values.

Selection

Selection is the process of determining the number of times, or trials, a particular individual is chosen for reproduction and, thus, the number of offspring that an individual will produce. The selection of individuals can be viewed as two separate processes:
1. Determination of the number of trials an individual can expect to receive, and
2. Conversion of the expected number of trials into a discrete number of offspring.
Stochastic universal sampling (SUS) is used in this paper. It is a single-phase sampling algorithm with minimum spread and zero bias. Instead of the single selection pointer employed in roulette wheel methods, SUS uses N equally spaced pointers, where N is the number of selections required. The population is shuffled randomly and a single random number in the range [0 Sum/N] is generated, ptr. The N individuals are then chosen by generating the N pointers spaced by 1, *[ptr, ptr+1, ..., ptr+N-1],* and selecting the individuals whose fitnesses span the positions of the pointers. An individual is thus guaranteed to be selected a minimum of *[et(i)]* times and no more than *[et(i)],* thus achieving minimum spread. In addition, as individuals are selected entirely on their position in the population, SUS has zero bias.

Crossover (Recombination)

The basic operator for producing new chromosomes in the GA is that of crossover. Like its counterpart in nature, crossover produces new individuals that have some parts of both parent's genetic material.
Intermediate Recombination is used in this paper. Given a real-valued encoding of the chromosome structure, intermediate recombination is a method of producing new phenotypes around and between the values of the parent's phenotypes (Muhlenbein, 1993). Offspring are produced according to the rule,

$$O_1 = P_1 + \alpha(P_2 - P_1) \tag{12}$$

Where α is a scaling factor chosen uniformly at random over some interval, typically [-0.25, 1.25] and P_1 and P_2 are the parent chromosomes. Each variable in the offspring is the result of combining the variables in the parents according to the above expression with a new α chosen for each pair of parent genes. In geometric terms, intermediate recombination is capable of producing new variables within a slightly larger

hypercube than that defined by the parents but constrained by the range of α.

Mutation

In order to maintain the variability of the population, a mutation should be performed in certain individuals. In GAs, mutation is randomly applied with low probability, typically in the range 0.001 and 0.01, and modifies elements in the chromosomes (Goldberg, 1989).

With non-binary representations, mutation is achieved by either perturbing the gene values or random selection of new values within the allowed range. Wright (1991) and Janikow et al (Janikow, 1991). demonstrate how real-coded GAs may take advantage of higher mutation rates than binary-coded GAs, increasing the level of possible exploration of the search space without adversely affecting the convergence characteristics. Many variations on the mutation operator have been proposed. For example, biasing the mutation towards individuals with lower fitness values to increase the exploration in the search without losing information from the fitter individuals or parameterising the mutation such that the mutation rate decreases with the population convergence (Fogarty,1989). Mühlenbein (Mühlenbein, 1993) has introduced a mutation operator for the real-coded GA that uses a nonlinear term for the distribution of the range of mutation applied to gene values. It is claimed that by biasing mutation towards smaller changes in gene values, mutation can be used in conjunction with recombination as a foreground search process. This method is used in this paper.

Case Study

Investigated structure is a 12 storey structure presented in (Kalkan, 2004). It is a 12 storey moment resisting frame that conforms to the requirements of the UBC (1997) provisions. The building designs are based on a configuration presented in the SEAOC seismic Design Manual (SEAOC, 2000). The building's lateral force resisting system is composed of steel perimeter moment resisting frames (MRF). The floor-plan and elevation view of the investigated frame are illustrated in Fig.1. Masses assigned for this frame at the storey floors and roof level are 550 and 510 tons, respectively. The yield strength of steel is assumed to be $F_y = 345\ Mpa\ (50\ Ksi)$ for all structural members.
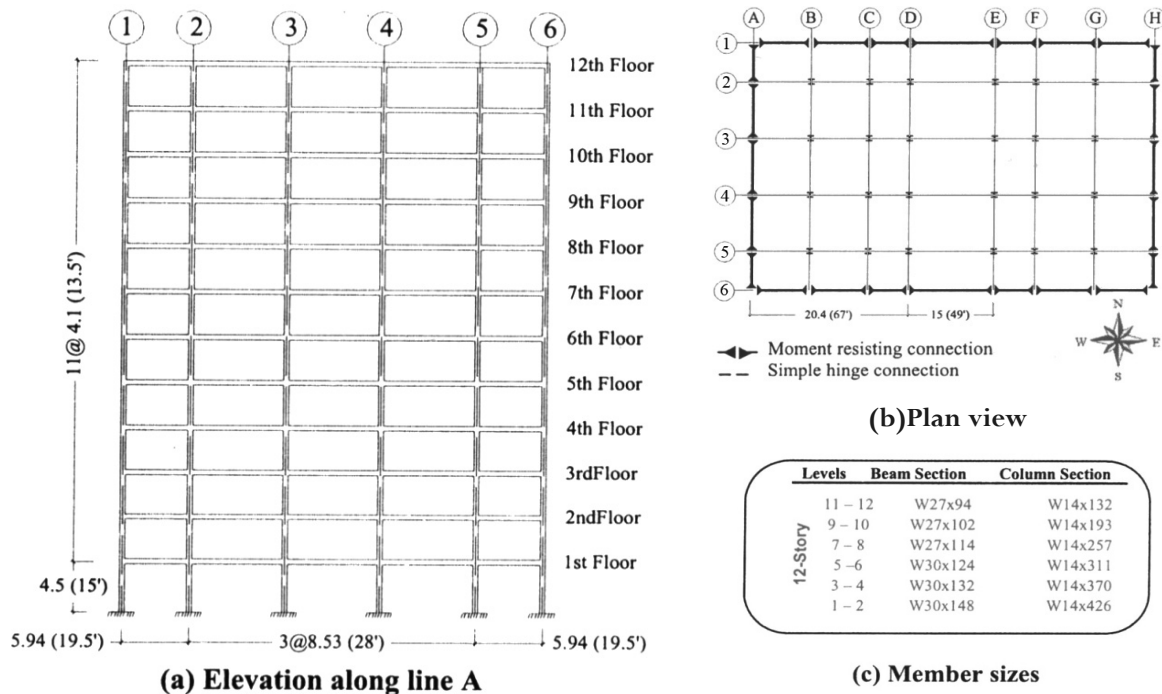

Figure 1. Structural details of case study (units in meters).

Comparison and Analysis of the Results

The proposed procedure was implemented for the 12 storey steel frame subjected to El Centro 1940 ground acceleration record (PGA= 0.348g, at T=2.12 sec) and obtained the optimum combination of modes (OCM), the nonlinear analysis were carried out using DRAIN-2DX computer program (Prakash, 1993). The properties and shape of four first modes is illustrated in Table 1 and Fig. 2, respectively. The base shear-roof displacement relations, $V_{bn} - U_{rn}$ and $F_{sn}/L_n - D_n$ resulted from each pushover analysis using $S_n = m\emptyset_n$ as load pattern are presented in Figs. 3, 4 and Table 2. The maximum displacement of the equivalent SDOF system subjected to El Centro ground motion, D_{n-NTHA}, and its associated maximum roof displacement, $U_{rn-NTHA}$, are also shown in Table 2. In the second mode the value of U_{n-NTHA} is less than D_{ny}, which means the equivalent SDOF system associated with these modes is not yielded.

Total responses profile of pushover analysis according to MPA procedure, PRC procedure and the peak responses resulted from NTHA and NSP with single mode (1th mode) are illustrated in Fig 6. (Note: in PRC method in spite of MPA method, all models are pushed until the same displacement, U_{r-NTHA} is achieved while in this study, because of instability of the model in mode 4 when pushed until $U_{r-NTHA} = 0.41$ meter, only first three modes are included in the combination process).
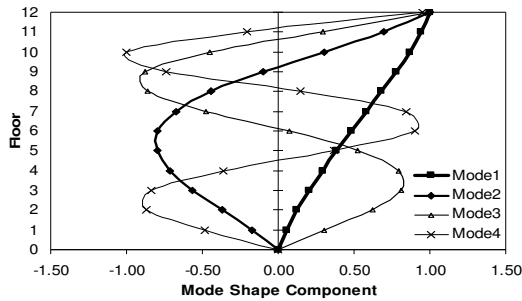

Figure 2. The first four mode shapes.

Table 1. Properties of the first four modes.

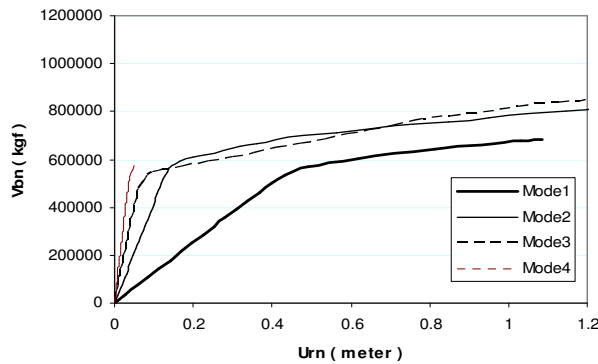|	Mode 1	Mode 2	Mode 3	Mode 4
ω_n	1.89	5.25	9.10	13.28
L_n	355595.11	-151523.41	95089.52	-75634.70
Γ_n	1.40	-0.61	0.34	-0.23
M_n^*	498055.64	92374.26	32255.65	17140.85
B_n	0.74	0.14	0.05	0.03

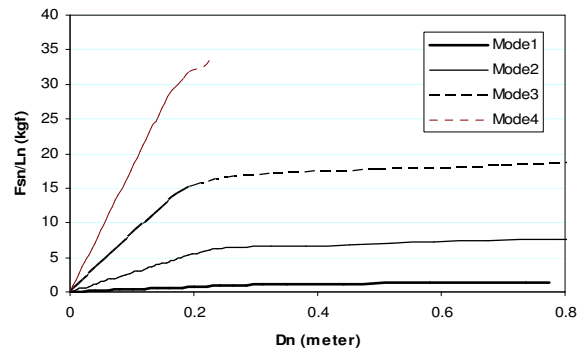Figure 3. Modal pushover curves.

Figure 4. Equivalent SDOF pushover curves.

Table 2. Properties of equivalent nonlinear SDOF systems

Mode	V_{bny}	U_{rny}	F_{sny}/L_n	D_{ny}	D_{n-NTHA}	$U_{rn-NTHA}$
1	578192.80	0.46	1.16	0.33	0.28	0.39
2	629604.46	0.15	6.82	0.25	0.27	0.16
3	539795.10	0.07	16.73	0.20	0.13	16.73
4	527634.69	0.04	30.78	0.17	0.14	0.03

To determine the participation of each mode in optimum combination method (OCM), the GA procedure is employed. The factor of each mode (a, b, c and d) are chosen as variable parameters while their upper and lower bound values are [-1,1]. The parameters of GA to be used in this study are taken as follows: population size = 50; number of generation = 500 and mutation rate = 0.05. After performing the GA it is found that the optimum value of factors are a =0.6651, b=0.3267, c=0.580 and d=0.4023.The evolving best fitness generation can be seen in Fig. 5.
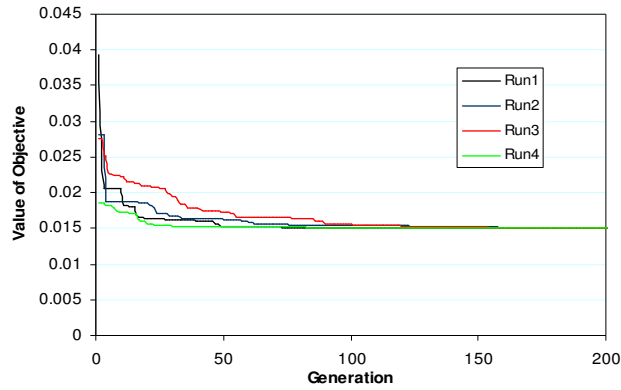

Figure 5. Evolving best fitness generation.

Total responses of pushover analysis according to OCM with obtained factors from GA procedure are also illustrated in Fig.6.
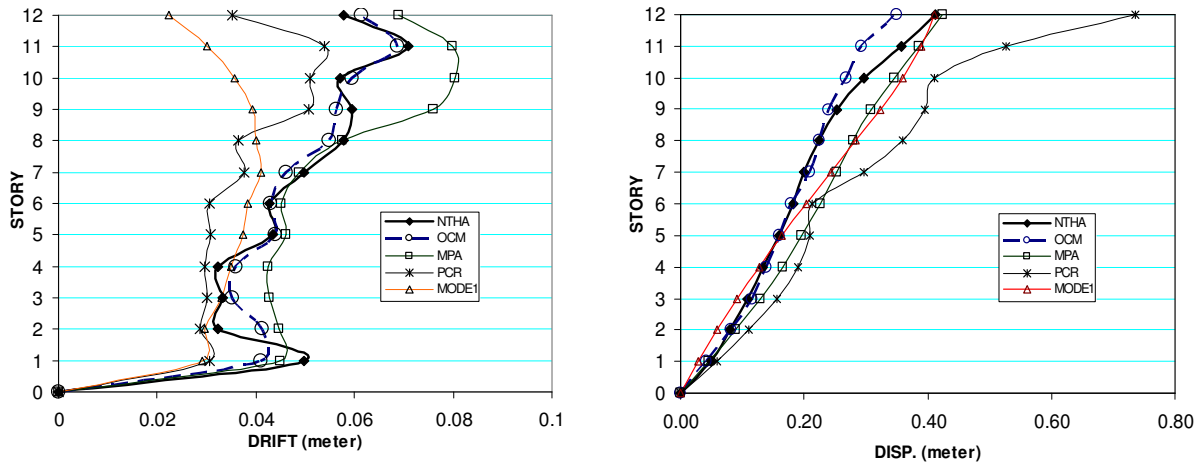

Figure 6. Comparison of drift and displacement profiles from different method.

To compare the accuracy of different methods, an error index is defined as presented by (Lopez, 2004)

$$Error(\%) = 100 \times \frac{1}{n}\sqrt{\sum_{i=1}^{n}\left(\frac{d_{i-NTHA} - d_{i-P}}{d_{i-NTHA}}\right)^2} \qquad (13)$$

Where d_{i-NTHA} is the peak response of drift at a given level i from the NTHA, d_{i-P} is the corresponding drift from the pushover analysis and n is the number of stories. Whatever the error index is close to zero, the pushover response approaches the NTHA response and the error index is calculated for different methods as presented in Table 3.

Table 3. Value of Error Index (%) for different methods.

Methods	OCM	MPA	PRC	Mode 1
Error Index (%)	3.115	6.650	7.343	9.519

Conclusions

In this study, it is shown that an optimum combination rule of pushover analysis, using $S_n = m\varnothing_n$ as a load pattern, could be found while it estimates the seismic responses satisfactorily with minimum error index. It is expected that by using this procedure for different groups of structures, the participation of each mode for every group of structures can be defined statistically.

References

Antoniou, S., A. Rovithakis, and R. Pinho, 2002. Development and verification of a fully adaptive pushover procedure, *Proceedings Twelfth European Conference on Earthquake Engineering*, London, UK, Paper No. 822.

Applied Technology Council, ATC 40, 1996. *Seismic evaluation and retrofit of concrete buildings*.

Aydinoğlu, M.N., 2003. An incremental response spectrum analysis procedure based on inelastic spectral displacements for multi-mode seismic performance evaluation, *Bulletin of Earthquake Engineering,* 1, 3-36.

Chopra, A. K., and R. K. Goel, 2002. A modal pushover analysis procedure for estimating seismic demands for buildings. *Earthquake Engineering and Structural Dynamics,* 31, 561-582.

De jong, K. A., 1975. Analysis of the behaviour of a class of genetic adaptive systems, *PhD Thesis*, Dept. of Computer and Communication Sciences, University of Michigan, Ann Arbor,

Fajfar, P., and H. Krawinkler, 1997. Seismic design methodologies for the next generation of codes, *Balkema*, Rotterdam.

Federal Emergency Management Agency, FEMA 273, 1997. *NEHRP guidelines for the seismic rehabilitation of buildings,* and FEMA 274, *NEHRP Commentary on the guidelines for the seismic rehabilitation of buildings,* Washington, D.C.

Federal Emergency Management Agency, FEMA 356, 2000. *Prestandard and Commentary for the Seismic Rehabilitation of Buildings,* Washington, D.C.

Fogarty, T. C., 1989. Varying the Probability of Mutation in the Generic Algorithm, *Proc. ICGA 3*, 104-109.

Goldberg, D. E., 1989. *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison Wesley Publishing Company.

Gupta, B., and S. K. Kunnath, 2000. Adaptive spectra-based pushover procedure for seismic evaluation of structures. *Earthquake Spectra,* 16 (2), 367-391.

International Conference of Building Officials (ICBO), 1997. *Uniform building code,* Whittier, CA.

Janikow, C. Z., and Z. Michalewicz, 1991. An Experimental comparison of binary and floating point representations in Genetic Algorithms, *Proc. ICGA 4*, 31-36.

Kalkan, E. and S. K. Kunnath, 2004. Method of modal combinations for pushover analysis of buidings. *13th* *world Conference on Earthquake Engineering*. Paper No.2713.

Krawinkler, H., and G.D.P.K. Seneviratna, 1998. Pros and cons of a pushover analysis of seismic performance evaluation. *Engineering Structures*, 20(4-6), 452-464.

Lopez Menjivar, M. A., 2004. A review of existing pushover methods for 2-d reinforced concrete buildings. *PhD Thesis*, Rose School, Universita degli Studi di Pavia, Itally.

Michalewic, Z., 1992. Genetic Algorithms + Data Structures = Evolution Programs. *Springer Verlag*.

Moghadam, A. S., and W. K. Tso, 2002. A pushover procedure for tall buildings. *Proceedings of the 12th* *European Conference on Earthquake Engineering,* Paper 395. Elsevier Science Ltd.

Mühlenbein, H., and D. Schlierkamp-Voosen 1993, Predictive models for the Breeder Genetic Algorithm, *Evolutionary Computation,* 1 (1) 25-49.

Prakash, V., and G. H. Powell, 1993. *DRAIN-2DX Base Program Description and User Guide, Version 1.10, Report No. UCB/SEMM-93/17*, University of California at Berkeley.

Priestley, M. J. N., 2000. Performance based seismic design. *Proceeding of 12th World Conference on Earthquake Engineering*, New Zealand

Structural Engineers Association of California, SEAOC, 2000. *Seismic design manual,Volume III, - Building design examples: Steel concrete and cladding*, Sacramento, CA,

Wright A. H., 1991. Genetic Algorithms for real parameter optimization, Foundations of Genetic Algorithms, J. E. Rawlins (Ed.), Morgan Kaufmann, 205-218.