



## **RESPONSE OF SANDS UNDER IMPOSED LINEAR AND NONLINEAR STRAIN PATHS**

P. Logeswaran<sup>1</sup> and S. Sivathayalan<sup>2</sup>

### **ABSTRACT**

A comprehensive experimental study was carried out in the laboratory to investigate the behaviour of Fraser River sand subjected to simultaneous changes in pore pressure and pore volume. Boundary conditions anticipated in-situ following earthquakes were simulated in the laboratory using strain path controlled tests. Two distinct types of strain paths, named herein as (i) constant strain increment ratio paths, and (ii) variable strain increment ratio paths, representing different volumetric to axial strain ratio were imposed under triaxial conditions. Expansive volumetric deformation during shear loading increased the domain of strain softening at a given initial state. The test results support the contention that undrained state is not the most damaging scenario under field loading conditions. Much smaller minimum shear strength compared to the undrained strength were measured when the boundary conditions resulted in expansive volume changes. These results suggest that the boundary conditions should be considered an important parameter in liquefaction susceptibility assessment.

### **Introduction**

Depending on the magnitude and duration of ground shaking, the soil may lose a significant portion of its initial effective stress, and in the extreme case it may reach a state of zero effective stress. The zero effective stress state under earthquake loading is transient in nature, and is responsible for the development of large strains. The term liquefaction in soil mechanics has been used to refer to a state of zero effective stress, or to the development of large deformation (Castro, 1969; Casagrande, 1975; Seed, 1983; Vaid & Chern, 1985).

Liquefaction susceptibility in current geotechnical practice is evaluated assuming undrained condition during cyclic loading. Undrained conditions are presumed to represent the worst case scenario in the field based on the understanding of typical soil behaviour and shear induced pore pressure generation. Recent research on the mechanics of pore fluid migration, and the associated void redistribution clearly indicates that simultaneous changes in pore volume and pore pressure following an earthquake may lead to conditions weaker than undrained. Generation of excess pore water pressure during earthquake loading is one of the key causes of strength reduction in soils. Spatial redistribution of pore water pressure may contribute to further strength reduction depending on the variation of in-situ pore pressure.

Published case histories indicate that several instances of liquefaction ground failures, such as the Lower San Fernando Dam, and Mochikoshi Tailings Dam were triggered in a few minutes to several hours after the cessation of earthquake (Okusa, 1979; Seed, 1987; Berrill et al. 1997). The failure of San Fernando dam

---

<sup>1,2</sup> Department of Civil and Environmental Engineering, Carleton University, Ottawa, ON, Canada

occurred 30 seconds after, and the failure of Mochikoshi tailings dam occurred almost 24 hours after cessation of earthquake shaking. These failures lead to the conclusion that the excess pore pressure generation due to dynamic loading alone cannot be a sole cause of this failure. The delayed triggering of liquefaction is a post seismic event that is responsible for failure. Such failures occur under the influence of gravitational loads.

The excess pore pressure generation due to undrained conditions is generally fully responsible for soil liquefaction in homogeneous soil deposits, and therefore the presumption of undrained cyclic shear during cyclic loading is valid in such soils. However, other mechanisms may lead to failure in heterogeneous, and layered soil deposits. A vivid demonstration using model studies has clearly shown the potential for catastrophic consequences, when a low permeability soil layer is present in an otherwise uniform soil deposit. (Kokusho, 1999). Void redistribution due to pore-water migration, and the associated increase in excess pore water pressure near the permeability barrier combined with the formation of a water film have led to the collapse of the slope in the experiments conducted by Kokusho (1999). The soil deposit would have been stable, if not for the presence of the thin silt layer which impeded drainage. In addition to layering, natural heterogeneity in soil deposits may also lead to conditions weaker than undrained on the account of pore water migration.

The effects of drainage on the void ratio of the in-situ material has been recognized for several years (Casagrande & Rendon, 1978; NRC, 1985; Gilbert, 1984; Seed, 1987; Boulanger & Truman, 1996), but its effects in triggering of liquefaction have been recognized only recently. It has been experimentally demonstrated in recent years that conditions much weaker than undrained could result due to void redistribution (Vaid & Eliadorani, 1998; Kulasingam et al, 2004; Malvick et al. 2006). It has been shown that small volume inflow into the soil could result in significant loss of shear strength.

These tests clearly suggest that the pore pressure gradients that may exist in-situ following an earthquake (due to spatial variability/heterogeneity) will play a key role on the subsequent response of soil mass. Strain softening and instability may be triggered in soils that would be deemed safe under traditional undrained considerations. (Lade et al, 1988, 1993). Also, a soil that may be dilative under undrained loading conditions may exhibit strain softening behaviour due to volume inflow. Only limited experimental research has been carried out to understand these phenomena, and all experiments conducted to date were limited to idealised linear strain paths (Eliadorani, 2000; Logeswaran, 2005), which have been termed constant strain increment ratio paths herein. This type of strain path facilitates a fundamental understanding of the effects of pore pressure gradients in-situ, but it is not realistic given the field conditions. The rate of water migration/volume change is governed by piezometric head gradient and hydraulic conductivity. The piezometric gradients will gradually change with volumetric deformation, and hence nonlinear strain paths are expected in-situ.

Typical experimental assessment of the behaviour of soils is either drained or undrained considering the rate of loading and permeability of the soil. A conventional drained triaxial test yields a linear stress path (in  $p$ - $q$  space), and a non-linear strain path ( $\varepsilon_v$ - $\varepsilon_a$  space). An undrained test on the other hand results nonlinear stress path, and a linear strain path. A sample deforms with no change in pore pressure in a truly drained test. In this case, loading rate is slow and pore space is not adequate to allow the free unhindered drainage. No volume change occurs in an undrained test. Such will be the case if the loading rate is fast and permeability is relatively very low. However, the deformation during pore water migration in-situ involves simultaneous changes in both volume and pore pressure. Fig. 1 shows a schematic illustration of the various strain paths described above in a loose sand under triaxial compression loading. Corresponding stress paths are also shown in the Figure. Type 1 represents typical fully drained response in a conventional triaxial test, where the total and effective lateral stresses remain constant throughout the test. Type 2 response corresponds to undrained shear, where the specimen is sheared under constant volume. Type 3 corresponds to partially drained response where the behaviour is bounded by the drained and undrained responses. Types 4 and 5 represent behaviour on either extreme of the drained and undrained paths as illustrated. These possible stress/strain paths that a soil element can be subjected have been termed (1) Normal drainage/Full drainage, (2) Zero drainage, (3) Partial drainage, (4) Expansive drainage, and (5) Excessive drainage respectively.

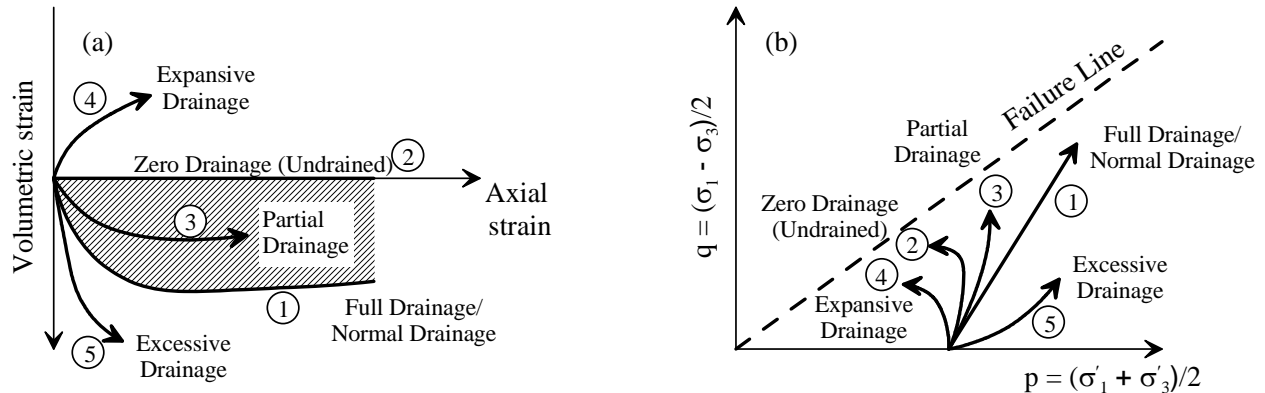


Figure 1. Schematic Illustration of stress, and strain path for loose sand under conventional triaxial compression loading.

### Experimentation and Material

All tests were carried out using a stress/strain path capable triaxial device in the geotechnical laboratory at Carleton University and performed under displacement controlled loading mode. This test device permits extreme flexibility in testing soils along various strain paths under axisymmetric loading conditions. State-of-the-art data acquisition system, and high quality transducers enable confident, and repeatable measurements of loads and displacements. Two electro-pneumatic transducers enable computerised control of deviatoric stress and cell pressure, and a Digital Pressure/Volume Controller (DPVC) enables desired strain path testings. Both cell and pore water pressures are measured with a resolution of 0.05 kPa, and the deviatoric stress with a resolution of about 0.1 kPa. Resolution of axial strain measurement is better than  $10^{-5}$ , and that of volumetric strain is between  $10^{-4}$  and  $10^{-5}$  depending on the technique used to measure the volumetric strain. Membrane penetration corrections on the sample volume were determined using the methodology proposed by Vaid & Negussey (1984).

A multi-threaded data acquisition program that was developed in-house was used to acquire the data, and control the system. Multiple execution threads within a single process enable continuous and smooth operation of the control hardware, and proper sampling of the input channels without interruption or delay. The desired strain paths were imposed by relating the stepping of the DPVC that was connected to the pore space with the stepping of the motor that controls axial deformation. Repeatability of the test device, and that of the specimen preparation technique were directly checked by testing identical specimens under similar boundary conditions. Measured deviatoric stress, volumetric deformation and pore pressure all showed excellent repeatability of the test device and the sample preparation technique (Logeswaran, 2005). Fig. 2 shows a schematic diagram of the triaxial system used for carrying out the test program.

The sand tested was dredged from the Fraser River near Abbotsford, British Columbia. The natural material was wet-sieved through #200 sieve (0.075mm) to remove most of the fine particles passing #200 sieve. This resulted in a fairly uniform test sand with a mean diameter of 0.3mm, uniformity coefficient of 2.9 and coefficient of curvature of 1.3. Such uniform material is essential for fundamental laboratory studies that require several repeatable, homogeneous specimens to be reconstituted in the laboratory. Similar material has been used in several past studies reported in the literature (Vaid & Thomas, 1995; Vaid & Sivathayalan, 1996; Wijewickreme et al., 2005; Logeswaran, 2005). The maximum and minimum void ratios of this batch of Fraser River sand determined according to the ASTM test standards (ASTM D4253, D4254) are 0.806 and 0.509 respectively. While the mineral composition of this sand is similar to the various batches of Fraser River sand discussed in the literature, the differences in the geographical origin and particle gradation cause fairly significant changes in the maximum and minimum void ratios. Similar changes in the maximum and minimum void ratios between different batches of Fraser River sands can be noted in the data reported in the literature (Vaid & Thomas, 1994; Vaid & Sivathayalan, 1996; Vaid & Sivathayalan, 2000; Wijewickreme et al., 2005).

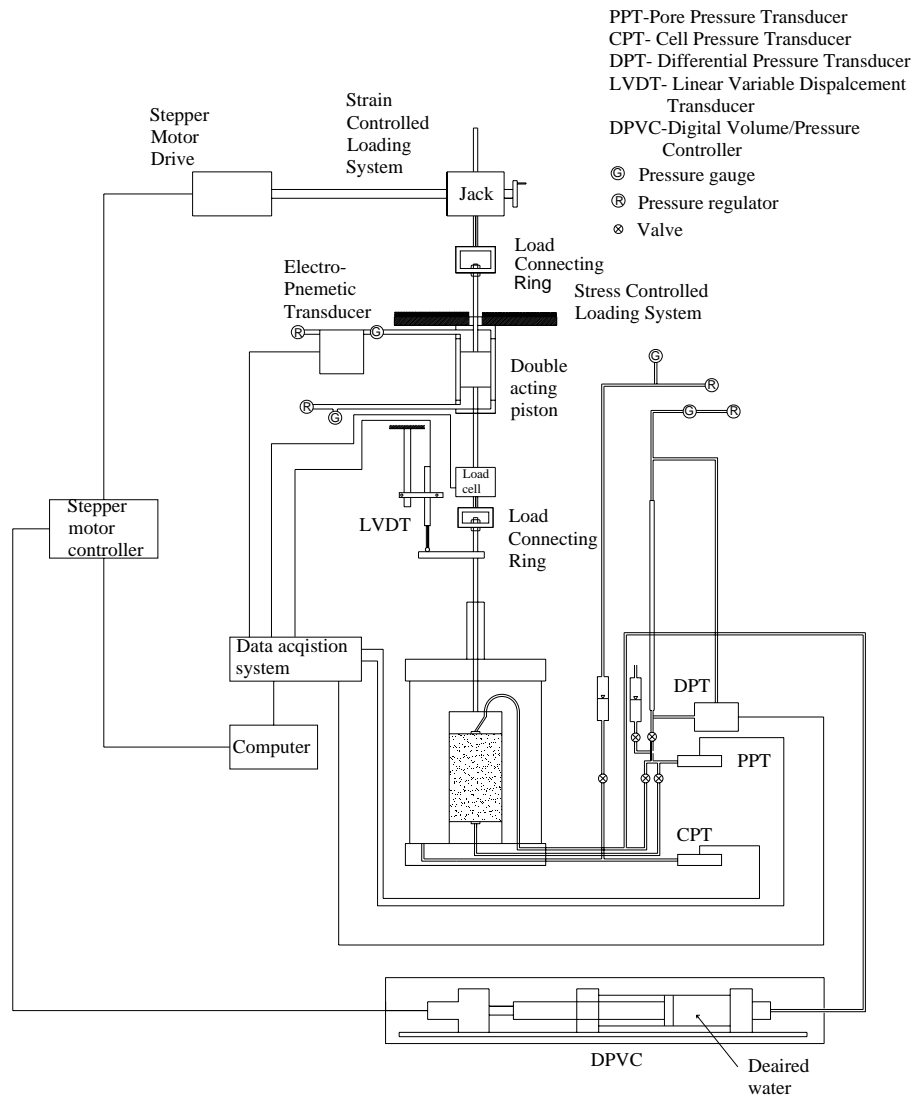


Figure 2. Schematic illustration of strain path controlled triaxial device used in the study.

Samples were reconstituted in the laboratory using the water pluviation technique (Vaid & Negussey, 1988). This specimen reconstitution method mimics the natural deposition process in alluvial/fluviial soil deposits, and hence yields a fabric similar to that of several natural sands. As a result, the measured laboratory response is expected to be applicable to alluvial soils in-situ. In addition, the very high repeatability of this method permits the reconstitution of several identical samples, which is an essential requirement in fundamental experimental studies. All tests were conducted on specimens reconstituted at the loosest deposited state. The void ratio following consolidation to a hydrostatic effective consolidation stress of 200 kPa was  $e_c = 0.739 \pm 0.003$ , which corresponds to a consolidated relative density of about 23%.

All tests were carried under triaxial compression loading on specimens of approximately 63mm diameter by 125 mm height. The measured Skempton's B-value exceeded 0.99 in all tests. This is clear indication that all samples were fully saturated prior to shear loading. Two way drainage was employed during consolidation, and a significant portion of the volumetric strain during consolidation occurred during the first few minutes.

Regardless, all specimens were allowed to consolidate for 30 minutes to avoid any potential creep effects (Shozen, 2001). Following consolidation to the desired effective stress state, samples were sheared at a constant axial strain rate of about 3% per hour along the prescribed strain paths. This relatively slow rate of loading was chosen to ensure that the velocity head remains negligibly small during the water injection or withdrawal process. The total lateral stress was held constant during shear loading in all tests.

### Imposed Strain Increment Paths

Two distinct strain paths, one proportional (linear) and the other nonlinear, were imposed on the specimen subjected to axial compression loading. The linear strain paths shown in Fig. 3a were characterized by the strain increment ratio  $\zeta$ , which is defined as the ratio of incremental volumetric strain  $\Delta\varepsilon_v$  to incremental axial strain  $\Delta\varepsilon_a$ . Following the commonly adopted sign convention, axial compression and volume outflow were considered positive. Hence, a negative  $\zeta$  value corresponds to expansive volumetric deformation in a triaxial compression test.

The second series of strain paths shown in Fig. 3b represent the potential field behaviour much more closely compared to the linear strain path tests noted earlier. The migration of pore water (and the associated changes in pore water pressure) will gradually decrease the piezometric head gradient in-situ. As a result, regardless of whether the volume flow is contractive or expansive, the volume flow rate will gradually reduce with time. The nonlinear strain paths used in the study capture this phenomenon by employing an exponential function to define the strain increment ratio. The general form of the imposed nonlinear strain paths is given by

$$\varepsilon_v = m.e^{R\varepsilon_a} \tag{1}$$

where  $\varepsilon_v$  and  $\varepsilon_a$  are the volumetric, and axial strains respectively. The constants  $m$  and  $R$  define the nature of the strain path. Constant  $m$  determines the peak volumetric strain imposed in the test, and constant  $R$  dictates the relative rate of volumetric strain increment. Tests were conducted at different  $m$  values, ranging from -0.90% to 0% and  $R$  value was maintained constant for all nonlinear strain paths as 0.75. The  $R$  value chosen implies that 90% of the total volumetric deformation permitted was reached at an axial strain of 3%. A negative  $m$  value represents expansive volumetric deformation (or volume inflow), and a positive value represents contractive deformation. The path corresponding to  $m = 0$  is the undrained test.

### Test Results and Discussion

The stress-strain response of Fraser River sand subjected to the linear strain paths, and the corresponding effective stress paths and pore pressure responses are shown in Fig. 4a. As noted in the literature, Fraser River sand exhibits fully strain hardening response under monotonic undrained compression. Hence, no

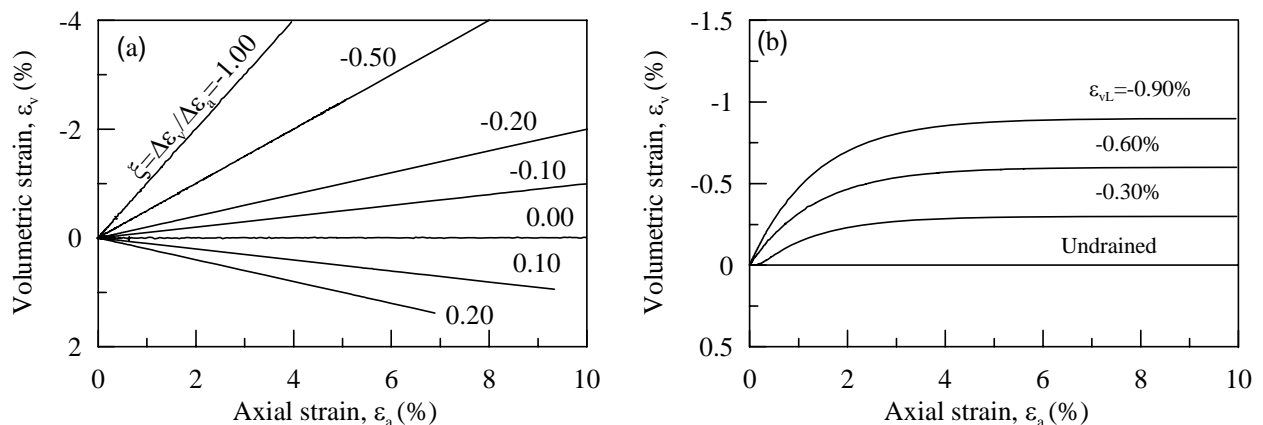


Figure 3. Strain paths imposed in the study. (a) Linear strain increment paths, (b) Nonlinear strain increment paths.

strength reduction occurs in the undrained loading. However, negative constant strain increment ratios (causing expansive volumetric strain) lead to strain softening response as shown in the Figure. The strength reduction was realized in all tests with expansive volumetric deformation. When the imposed volumetric strain path is  $\zeta = -0.10$ , the sand strain softens somewhat, and shear deformation occurs at essentially constant values of shear and effective stresses at large strains. It should, however, be noted that this state of deformation does not correspond to the steady or critical state of deformation discussed in the literature because the volume is continuously changing in this case. As the rate of volume inflow increases ( $\zeta = -0.2, -0.5$  &  $-1.0$ ), the soil generates almost 100% excess pore pressure, and thus reaches a state of zero effective stress. This results in deformation at essentially zero shear strength in tests with significant expansive volumetric deformation. Excessive volume outflow during shear, on the other hand, results in a much smaller pore pressure increase and hence stronger response.

Fig. 4b shows the stress-strain, pore pressure responses and the stress path of identical Fraser River sand specimens subjected to nonlinear strain increment ratio paths illustrated in Fig. 4. Specimens were subjected to expansive volumetric deformation and undrained deformation for the comparison. Expansive volumetric deformation caused the material to strain soften upon reaching the peak state as noted in the linear strain path response. The amount of imposed volumetric deformation does not significantly influence the peak strength (the variation in  $S_{peak}$  as  $\varepsilon_{vL}$  varied from  $-0.3\%$  to  $-0.9\%$  is about 20%), but the degree of strain softening is significantly influenced by  $\varepsilon_{vL}$ . The higher value of  $\varepsilon_{vL}$  results very weaker response and lower minimum undrained strength. The minimum undrained strength mobilized decreased from about 50 kPa for  $\varepsilon_{vL} = -0.3\%$  to about 20 kPa for  $\varepsilon_{vL} = -0.9\%$  (about 60% of reduction in strength). In terms of brittleness index, ratio between loss in strength and peak strength,  $I_B$  (Bishop 1971), this corresponds to a variation from about  $I_B = 0.1$  to  $I_B = 0.7$ . On the other hand, the response of the material becomes progressively stronger depending on the  $\varepsilon_{vL}$  (higher level of  $\varepsilon_{vL}$  results weaker response). As expected, excess pore pressure generation is higher compared to the pore pressure generation in an undrained test. The development of pore increases with the increase of  $\varepsilon_{vL}$ .

These two sets of test results clearly demonstrate that the undrained state may not represent the lowest strength of the soil in the field, and that the static liquefaction potential of a soil deposit may be significantly influenced by the drainage boundary conditions. Spatial distribution of pore pressure in the soil might be the cause of much

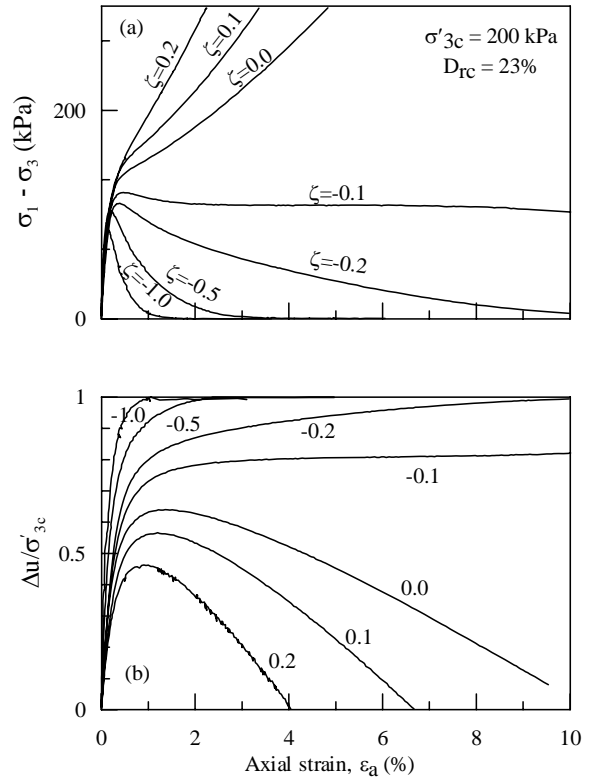


Figure 4(a). Response of sand subjected to linear strain increment paths.

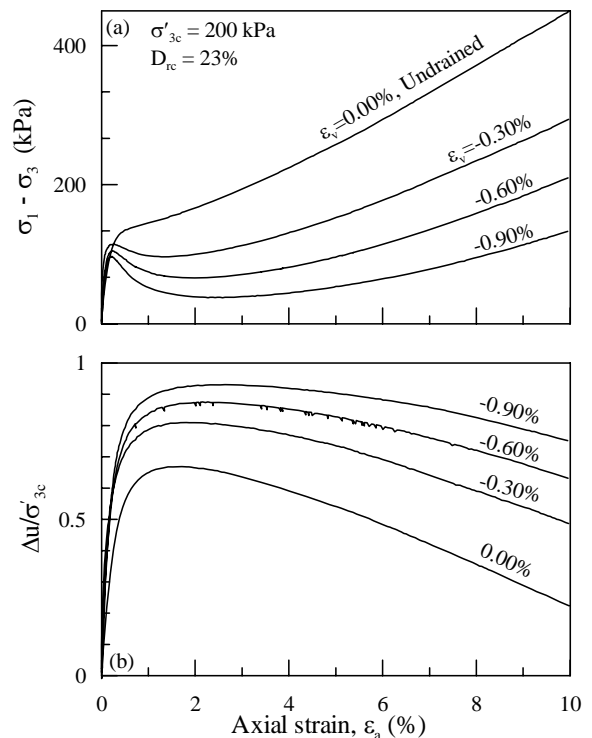


Figure 4(b). Response of sand subjected to non-linear strain increment paths.

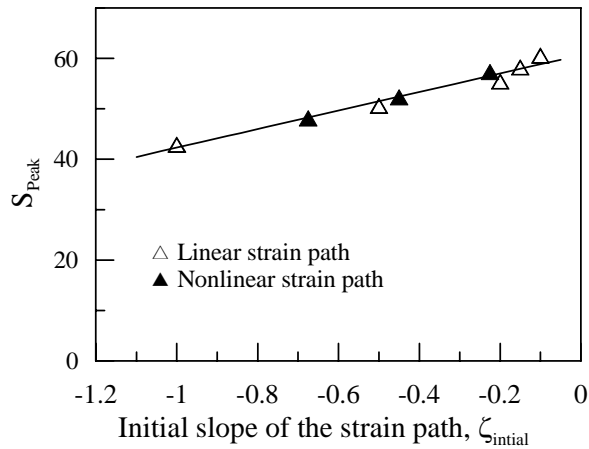


Figure 5. Variation of peak shear strength with the initial slope of the strain path.

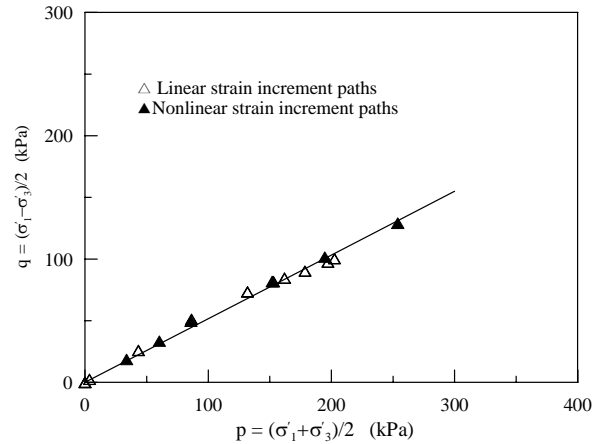


Figure 6. Mobilised friction angle at the state of peak pore pressure.

weaker response than undrained.

Fig. 5 shows the linear variation of the peak shear strength (which corresponds to the instant of the triggering of strain softening) as a function of the initial slope strain paths,  $\zeta_{initial}$ . Peak state is realised only in strain softening materials, and thus limited to test results with initial slope between -1.0 and -0.1 (Linear  $\zeta = -0.1$  to -1.0; Nonlinear  $\varepsilon_{vL} = -0.90\%$  to -0.30%). Marginal reduction in peak strength (60kPa to 40kPa) can be noted as  $\zeta_{initial}$  varies from -0.1 to -1.0.

Fig. 6 shows the effective stress states at the instant of peak pore pressure in various linear, and nonlinear strain path tests carried out in this study. Regardless of the loading path, and the initial state the friction angle mobilized at the state of peak pore pressure is essentially a constant, and is equal to about  $32^\circ$  for the tested Fraser River sand. This angles matches closely with the friction angle at phase transformation in undrained tests reported in the literature.

### Summary and Conclusions

An experimental study was conducted to assess the behaviour of a relatively uniform sand under different strain paths that represent shearing under simultaneous changes in pore volume and pore pressure. The stress-strain response of the soil systematically changes as the volumetric deformation transforms from contractive to expansive. Strain paths leading to expansive volumetric strains may cause strain softening even in sands which are dilative, and stable under undrained conditions. Spatial variation of excess pore pressure in-situ on account of natural heterogeneity is often responsible for such loading. Systematic variations in peak and minimum strengths occurred as the rate of volume inflow varied. Friction angle mobilized at the state of peak pore pressure state is independent of the strain path imposed.

Current practice, which considers drained and undrained responses only, presumes that these form the bounds of all possible responses. This approach has been shown to be not valid, and it may result in unsafe designs, if the pore pressure distribution in-situ leads to expansive volumetric deformation in certain soil elements. This represents a critical risk if such materials are evaluated using traditional undrained considerations.

### Acknowledgments

The research reported herein has been supported by grants from the Natural Sciences and Engineering Research Council of Canada, Canada Foundation for Innovation, and the Ontario Innovation Trust. Technical assistance provided by laboratory technicians Jim Whitehorne, and Stanley Conley, and the financial assistance provided to the first author by Carleton University are gratefully acknowledged

## References

- ASTM, 2001. Standard test methods for minimum index density and unit weight of soils and calculation of relative density (D-4254-00), Annual Book of ASTM Standards, American Society for Testing and Materials, Philadelphia, PA. Vol. 04.08.
- ASTM, 2001. Standard test methods for maximum index density and unit weight of soils using a vibratory table (D-4253-00), Annual Book of ASTM Standards, American Society for Testing and Materials, Philadelphia, PA. Vol. 04.08.
- Berrill, J.B., Christensen, R.J., Keenan, R.J., Okada, W., & Pettinga, J.R., 1997. Lateral spreading loads on a piled bridge foundation, Proc., Intl. Conf. on Soil Mechanics and Geotechnical Engineering, Seismic Behaviour of Ground and Geotechnical Structures: Special Technical Session on Earthquake Geotechnical Engineering, Balkema Publisher, Rotterdam, Netherlands, 173-183.
- Boulanger, R. W., & Truman, S. P., 1996. Void redistribution in sand under post-earthquake loading, Canadian Geotechnical Journal, 33(5): 829-834.
- Casagrande, A., 1975. Liquefaction and cyclic deformation of sands, a critical review, In Proceedings of the 5<sup>th</sup> American Conference on Soil Mechanics and Foundation Engineering, Buenos Aires, 5: 79-123.
- Casagrande, A., & Rendon, F., 1978. Gyratory shear apparatus design, Testing procedures. U.S. Army Corps of Engineers, Waterways Experiment Station, Vicksburg, Miss., Technical Report S-78-15.
- Casro, G., 1969. Liquefaction of sands, PhD, Thesis, Harvard University, Cambridge, Mass.
- Eliadorani, A., 2000. The response of sands under partially drained states with emphasis on liquefaction. Ph.D. thesis, University of British Columbia, Vancouver, Canada.
- Gilbert, P.A., 1984. Investigation of density variation in triaxial test specimens of cohesionless soil subjected to cyclic and monotonic loading. U.S. Army Corps of Engineers, Waterways Experiment Station, Vicksburg, Miss. Technical report No. GL-84-10.
- Kokusho, T., 1999. Water film in liquefied sand and its effect on lateral spread, Journal of Geotechnical and Geoenvironmental Engineering, 125(10): 817-826.
- Kulasingam, R., Malvick, E. J., Boulanger, R. W., & Kutter, B. L., 2004. Strength loss and localization at silt interlayers in slopes of liquefied sand, Journal of Geotechnical and Geoenvironmental Engineering, 130(11): 1192-1202.
- Lade, P.V., Nelson, R.B., & Ito, Y.M., 1988. Instability of granular materials with non-associated flow, Journal of Engineering Mechanics, ASCE, 114(2): 2173-2191.
- Lade, P.V., Bopp, P.A., & Peters, J.F., 1993. Instability of dilating sand, Mechanics of Materials, 16: 249-264.
- Logeswaran, P., 2005. Behaviour of sands under simultaneous changes in volume and pore pressure. MASc thesis, Carleton University, 184p.



- Malvick, E.J., Kutter, B.L., Boulanger, R.W., & Kulasingam, R., 2006. Shear localization due to liquefaction-induced void redistribution in a layered infinite slope, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 132(10): 1293-1303.
- NRC., 1985. Liquefaction of soils during earthquakes, National Research Council, National Academy Press, Washington, D.C.
- Okusa, S., Anma, S., & Maikuma, M., 1984. The propagation of liquefaction pressure and delayed failure of a tailings dam dike in the 1978 Izu-Oshima-Kinkai earthquake, In *Proceedings of the 8th World Conference on Earthquake Engineering*, July 21-28, San Francisco, 8(1): 389-396.
- Seed, H.B., 1987. Design problems in soil liquefaction, *Journal of Geotechnical Engineering*, ASCE, 113(8): 827-845.
- Seed, H. B., Lee, K. L., Idriss, I. M., & Makdisi, F. I., 1975. The Slides in the San Fernando Dams during the earthquake of February 9, 1971, *Journal of Geotechnical Engineering Division*, ASCE, 101(7): 651-688.
- Shozen, T. 2001. Deformation under the constant stress state and its effect on stress-strain behaviour of Fraser River sand, MSc thesis, University of British Columbia, Canada
- Wijewickreme, D., Sriskandakumar, S., & Byrne, P., 2005. Cyclic loading response of loose air-pluviated Fraser River sand for validation of numerical models simulating centrifuge tests, *Canadian Geotechnical Journal*, 42(2): 550-561.
- Vaid, Y. P., & Negussey, D., 1984. A critical assessment of membrane penetration in the triaxial test, *Geotechnical Testing Journal*, 7(2): 70-76.
- Vaid, Y. P., & Negussey, D., 1988. Preparation of reconstituted sand specimens, *Advanced Triaxial Testing of Soil and Rock*, ASTM STP 977: 405-417.
- Vaid, Y.P., & Chern, J.C., 1985. Cyclic and monotonic undrained response of sands, *Advances in the art of testing soils under cyclic loading conditions*, In *Proceedings of the ASCE Convention*, Detroit, 171-176.
- Vaid, Y. P., & Sivathayalan, S., 1996. Static and cyclic liquefaction potential of Fraser Delta sand in simple shear and triaxial tests, *Canadian Geotechnical Journal*, 33(2): 281-289.
- Vaid, Y.P., & Sivathayalan, S., 2000. Fundamental factors affecting liquefaction susceptibility of sands, *Canadian Geotechnical Journal*, 37: 592-606.
- Vaid, Y. P., & Eliadorani, A., 1998. Instability and liquefaction of granular soils under drained and partially drained states, *Canadian Geotechnical Journal*, 35(6): 1053-1062.
- Vaid, Y.P., & Thomas, J., 1995. Liquefaction and post liquefaction behaviour of sand, *Journal of Geotechnical Engineering*, ASCE, 121(2): 163-173.