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RELIABILITY ANALYSIS OF A TOWER STRUCTURE ON RANDOMLY VARIABLE SOIL

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ABSTRACT

In the present study, a Monte Carlo simulation approach is used to investigate the effects of soil spatial variability and earthquake intensity on foundation settlements and rotations. The analyses are performed for a range of seismic acceleration intensities, and incorporate the effects of random soil spatial variability as well as uncertainties related to the actual realizations of the seismic motion. The results from the Monte Carlo simulations are presented in terms of fragility curves, which are a suggestive method for illustrating the probability of exceeding a certain degree of structural damage as a function of load intensity. Generation of a new set of combined damage curves are proposed in this study. Those combined damage curves are constructed based on basic probability theory, expressing the conditional probability that gives the likelihood of exceeding a specific level of damage for a given load intensity. Two types of combined damage curves are generated and described here as Type-I and Type-II damage curves.

Introduction

Tower structures are one of the most commonly used lifeline structures. This type of structure is generally used for communication (e.g. transmission of radio wave, power transmission), water supply, etc. Therefore, the safety of this type of structure during a seismic event is very important. Several centrifuge studies of seismic behaviour of tower structures have been reported. Weissman and Prevost (1989) studied the dynamic behaviour of a tower structure on dry sand; and Madabhushi and Schofield (1993) studied the behaviour on a saturated soil. However, centrifuge tests are costly and time consuming. Finite element modelling, on the other hand, is more economical and can be effectively used. Similar to most observed natural phenomena, the performance of a tower structure during an earthquake contains a certain degree of uncertainty. Due to this uncertainty, the satisfactory performance of the system cannot be absolutely guaranteed. The assurance can only be given in terms of the probability of the structure in satisfying some performance criteria. Therefore, reliability based methods combined with finite element (FE) analyses is a possible approach to evaluate and quantify the probability of damage of the structure during an earthquake.

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One of the most widely used methods in the reliability analysis of a nonlinear model is the Monte Carlo method. In this study, Monte Carlo simulations combined with FE analysis is used for calculating the probability of damage of a tower structure resting on a strip footing upon a liquefiable soil. The finite element analyses were performed using fully coupled solid-fluid equations and a multi-yield plasticity soil constitutive model implemented in DYNAFLOW (Prevost 2002). The results obtained from the Monte Carlo simulations were expressed by a set of fragility curves. A fragility curve is a curve of conditional probabilities giving the likelihood of exceeding a specific level of damage for given load intensities. Fragility curves accounting for various uncertainties in loads and resistance have been used for expressing the probability of damage of various types of structures (e.g. Nielson 2005, Shinozuka et al. 2000; Shinozuka 1998, 2000; Yamaguchi and Yamazaki 2000, Popescu et al. 2005). There are several methods available for constructing the fragility curves (for more information the reader is referred to Nielson 2005). Among them, the method proposed by Shinozuka (1998) is the most widely used method because of its simplicity. The present study will use this method for generating fragility curves for each response variables. Random spatial variation of soil strength and variations in seismic acceleration are the uncertainties built in the fragility curves used here. Uncertainties in seismic ground motion attenuation within crustal and bedrock layers are indirectly addressed by considering two different types of design response spectra (namely UBC type 1 and UBC type 3 (UBC 1994)). Maximum structural settlement and maximum base rotation were considered as the two important damage parameters in this study. A safe foundation design requires the correct understanding of the susceptibility of damage due to the structure exceeding permissible limits of these responses. Three different damage levels were selected for both maximum settlement and maximum base rotation for generating the fragility curves.

The generation of a new set of combined damage curves are proposed in this study. These combined damage curves are constructed based on the conditional probability that gives the likelihood of exceeding a specific level of damage for a given load intensity. Two types of combined damage curves are generated and described as Type-I and Type-II damage curves. The probabilities of exceeding the limiting value of settlements and/or base rotations are expressed by the “Type-I” damage curve while the probabilities of exceeding both the limiting values for settlement and base rotation are expressed by the “Type-II” damage curve.

Finite Element Model and Analysis Method

Finite element analyses

The numerical model and analysis results used in this study have been presented in detail by Popescu et al. (2005). A brief description is provided hereafter. The Monte Carlo simulation (MCS) procedure used in this study combines digital generation of sample functions of a bi-dimensional, bi-variate non-Gaussian random field and deterministic finite element (FE) analyses using as input the sample functions of spatially variable soil properties. For a detailed presentation of the simulation algorithm the reader is referred to Popescu et al. (1998). One hundred sample functions of the bi-dimensional, bi-variate, non-Gaussian stochastic field with probabilistic characteristics described in the next section are simulated. The two components of the bi-variate field are stress-normalized cone tip resistance, q_n, and soil classification index, I_c. A nonlinear dynamic finite element analysis is then performed using DYNAFLOW (Prevost, 2002) for every sample function. The finite element calculations are conducted in terms of effective stress, using fully coupled solid-fluid equations for the treatment of saturated porous media. For each FE analysis, the soil parameters in each finite element are estimated based on the values of q_n and I_c at the element centroid, following the procedure described by Popescu et al. (1997). The structure and adjacent soil are analyzed using the plane strain assumption. The structure (Fig. 1) is idealized as a single degree-of-freedom oscillator with a characteristic frequency of 1.4Hz. The foundation is 8m wide and situated at a depth of 1m. The factor of safety for bearing capacity under static conditions is about 13. A 12m deep, 72m long saturated sand layer under a 1.0m dry sand layer is included in the analysis domain. Free field boundary conditions are prescribed at the lateral boundaries of the analysis domain. The finite element dimensions are selected to be four times smaller than the correlation distances in all spatial directions to accurately capture the soil spatial variability. Smaller finite elements are used directly below the structure to capture the stress gradients. Only the saturated sand is included in the stochastic analysis, the dry soil is assumed uniform.

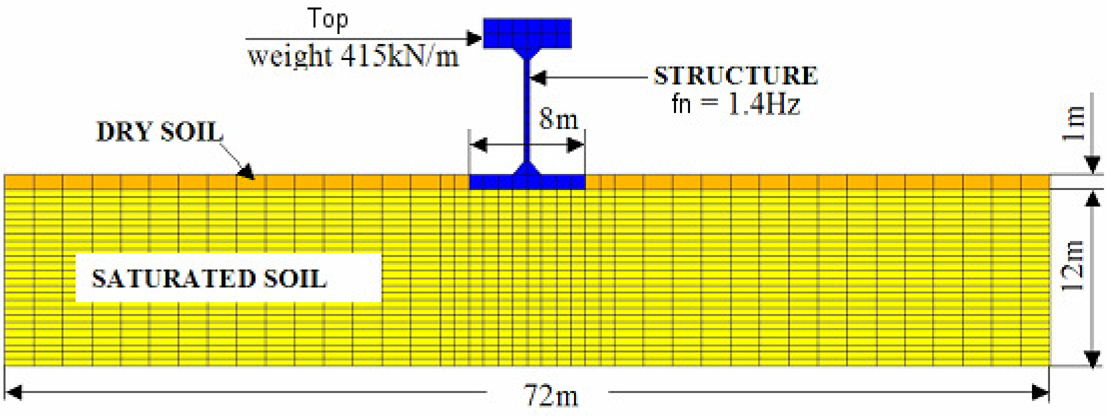

Figure 1. Finite element mesh of soil-structure model (after Popescu et al. 2005).

Soil Properties

The probabilistic characteristics of two index soil properties: q_n (overburden stress-normalized cone tip resistance - related to soil strength) and I_c (soil classification index - related to grain size and hydraulic conductivity) are used in this study to describe the soil heterogeneity. These two properties are considered as two components of a bi-variate stochastic field. Various soil properties are derived from these two parameters and are used in the analysis. The probabilistic characteristics of the soil properties considered in this study are:

a) For q_n: average value=6 Mpa and coefficient of variation=0.5, marginal probability distribution function is gamma with $\eta=4$, $\lambda=0.67$ and lower bound zero.
b) For I_c: average value=2, coefficient of variation=0.15, symmetric beta distribution bounded between 1 and 3.
c) Squared exponential auto-correlation structure, common for both q_n and I_c (see Vanmarcke 1983 for a description of the auto-correlation model). The correlation distances are assumed as: $\theta_h=8$m in the horizontal direction and $\theta_v=2$m in the vertical direction.
d) The cross-correlation coefficient between q_n and I_c is assumed to be $\rho = -0.58$ (Popescu 1995).

A "deterministic" analysis is performed corresponding to each stochastic analysis for comparison using soil properties that are uniform in the horizontal direction and with values equal to the average values of the soil properties in the MCS. The constitutive parameter values used in the deterministic analysis are listed in Table 1.

Combined damage curves

A fragility curve or fragility function is one way of expressing the probability of the degree of structural damage as a function of load intensity. In this study a new way of expressing combined damage probability is proposed. For extremely important lifeline structures such as a nuclear reactor or a communication tower, where one and/or more than one exceeding responses can cause partial or total failure of the structure, this type of damage curve can give a quick assessment of the probability of damage. This method is based on basic probability theory and will be briefly discussed next.

Let the event *S* express the damage of the structure due to maximum settlement and the event *R* express the damage of the structure due to maximum base rotation. It is also assumed that the damage of the structure can be due to maximum settlement, or maximum base rotation, or both maximum settlement and rotation, of the structure being exceeded. This is represented in the Venn diagram in Figure 2. The total sample space is shown by the rounded rectangle. Circles *S* and *R* are the events of failure of the structure due to maximum settlement and maximum base rotation exceeding corresponding permissible limits, respectively. So, the space *C* is the event of no failure. Therefore, The probability of failure due to maximum settlement or maximum base rotation or both exceeding corresponding permissible limits = P ($S \bigcup R$) = P (S) + P (R) – P ($S \bigcap R$), the union of S and R. Where, the probability of failure due to both the maximum settlement and maximum base rotation exceeding permissible limits = P ($S \bigcap R$), the intersection of S and R. If assuming that S and R are independent, P ($S \bigcap R$) = P(S)x P(R).

Table 1. The parameters of the multi-yield plasticity model, and the values used for the saturated soil in the deterministic analysis.

Constitutive parameter	**Symbol**	**Value**	**Type**
Mass density – solid	ρ^s	2660 kg/m^3	State parameters
Porosity	n^w	0.435	
Hydraulic conductivity	k	0.000264m/s	
Low strain elastic shear modulus	G_0	19.4MPa	Low strain elastic parameters
Poisson's ratio	ν	0.35	
Power exponent	n	0.5	
Friction angle at failure	ϕ	37.2^0….43.7^0	Yield and failure parameters
Maximum deviatoric strain (comp/ext)	ε_{dev}^{max}	0.07/0.04	
Coefficient of lateral stress	k_0	0.7	
Stress-strain curve coefficient	α	0.2756	
Dilation angle	ψ	31^0	Dilation parameters
Dilation parameter	X_{pp}	0.035….0.04	

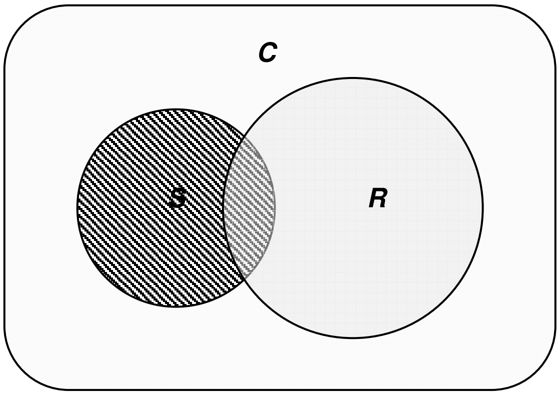

Figure 2. Venn diagram of damages.

From the above, two types of combined damage curves can be generated: Type-I and Type-II damage curves. Where the fragility function in the Type-I damage curve is the conditional probability of ($S \bigcup R$) exceeding specific levels of S or R or both S and R. This conditional probability is expressed by the following equation:

$$\text{Type-I Fragility function} = P\left(LS_{(S>S_t \bigcup R>R)_t} \middle| AI = AI_i\right) \tag{1}$$

Where LS is the limit state of damage level of the structure, S is the maximum foundation settlement at arias intensity=AI$_i$, S$_t$ is the specific value of maximum settlement (10mm, 15mm, 20mm in this study), R is the maximum foundation rotation at arias intensity=AI$_i$, and R$_t$ is the specific value of maximum foundation rotation (0.3 degree, 0.4 degree, and 0.5 degree in this study).

Similarly, the fragility function in the Type-II damage curve is the conditional probability of ($S \cap R$) exceeding both the specific level of maximum foundation rotation and a specific level of maximum foundation settlement for a given load intensity. This conditional probability is expressed by the following equation:

$$\text{Type-II Fragility function} = P\left(LS_{(S>S_t \cap R>R)_t} \middle| AI = AI_i\right) \qquad (2)$$

From the Type-I damage curve we can determine the probability of damage due to maximum settlement exceeding a permissible maximum settlement or maximum base rotation exceeding the permissible maximum base rotation, or both exceeding their maximum permissible limits. On the other hand the Type-II damage curve expresses the probability of exceeding both the permissible maximum settlement and maximum base rotation. This type of fragility curve is important for those structures where significant damage is only possible when both maximum settlement and maximum base rotation exceed the corresponding limiting values. Three different damage levels are considered in this study for each response variables and are shown in Table 2.

Table 2. The damage levels used in the study for generating fragility curves.

Response parameter	Damage level	Value
Maximum permissible settlement (S_t)	Level – 1 damage	10cm
	Level – 2 damage	15cm
	Level – 3 damage	20cm
Maximum permissible base rotation (R_t)	Level – 1 damage	0.3 degree
	Level – 2 damage	0.4 degree
	Level – 3 damage	0.5 degree

Analysis Results

Maximum structural deformation

A loose-to-medium-dense cohesionless soil (relative density=44%) is considered in the study. So there is a high risk of structural damage due to excessive total and/or differential settlement. Therefore, the two important parameters are selected for quantifying the damage of the structure: maximum settlement and maximum rotation of structure base. Computed maximum values of the structure settlements and rotations are shown in Figure 3 for all the cases analyzed (200 each for uniform and variable soil). From the results in Figure 3 it is observed that both the type of input acceleration and the soil variability affects the response of the structure. From both Figures (3a, 3b) it can be concluded that UBC type-3 acceleration has a stronger effect on both maximum settlements and rotations. This is believed to be due to the change in vibration characteristics of the soil-structure system to fundamental frequencies close to the dominant frequency values of UBC type-3 acceleration. This aspect of the seismic response is discussed in greater detail by Chakrabortty et al. (2004) for structures on uniform soils and by Popescu et al (2005) for variable soils. The results for maximum base rotations are more scattered than the results maximum settlements for each set of runs (e.g. variable soil: type 3) because the uncertainty in input acceleration affects the maximum base rotation more than the maximum settlements.

Fragility curves for the response variable

As discussed earlier, a fragility curve or a fragility function is a conditional probability that gives the likelihood of exceeding a specific level of damage for a given load intensity. This conditional probability is expressed by the following equation:

$$\text{Fragility function} = P\,(LS|AI=AI_i) \quad (3)$$

Where LS is the limit state of damage level of the structure, AI is the load intensity (arias intensity in this study) and AI_i is a realization of the load intensity. From eq. 3 it can be seen that given an earthquake of a specific intensity, a prediction of the damage level may be made for a structure for which a fragility function is defined.

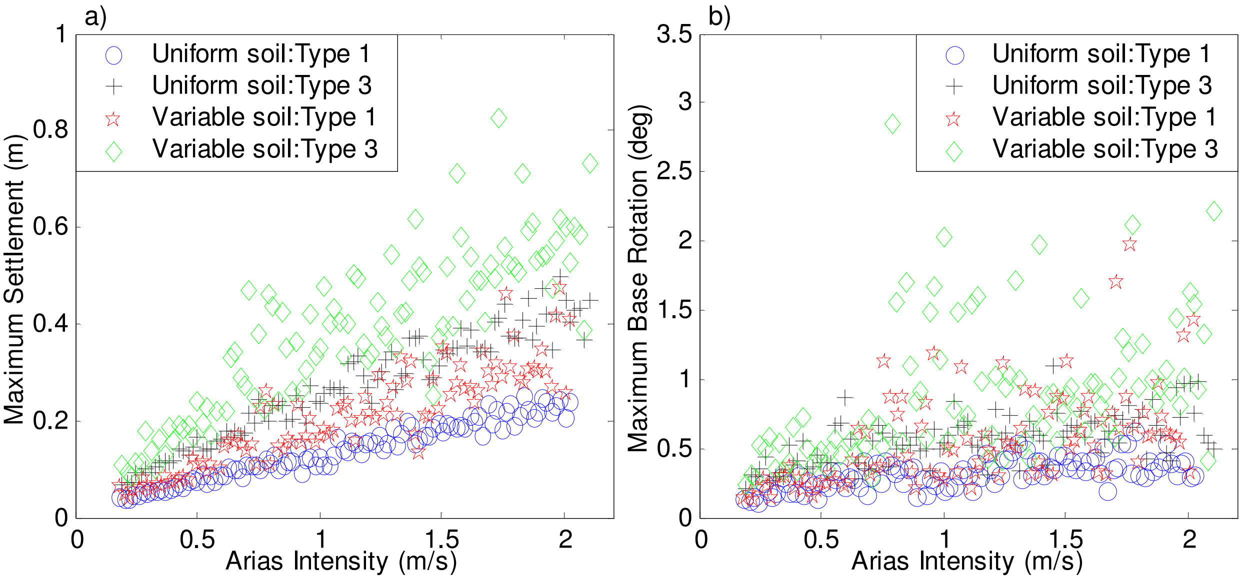

Figure 3. Predicted maximum values of structural a) maximum settlement and b) base rotation, for all cases analyzed (after Popescu et al. 2005).

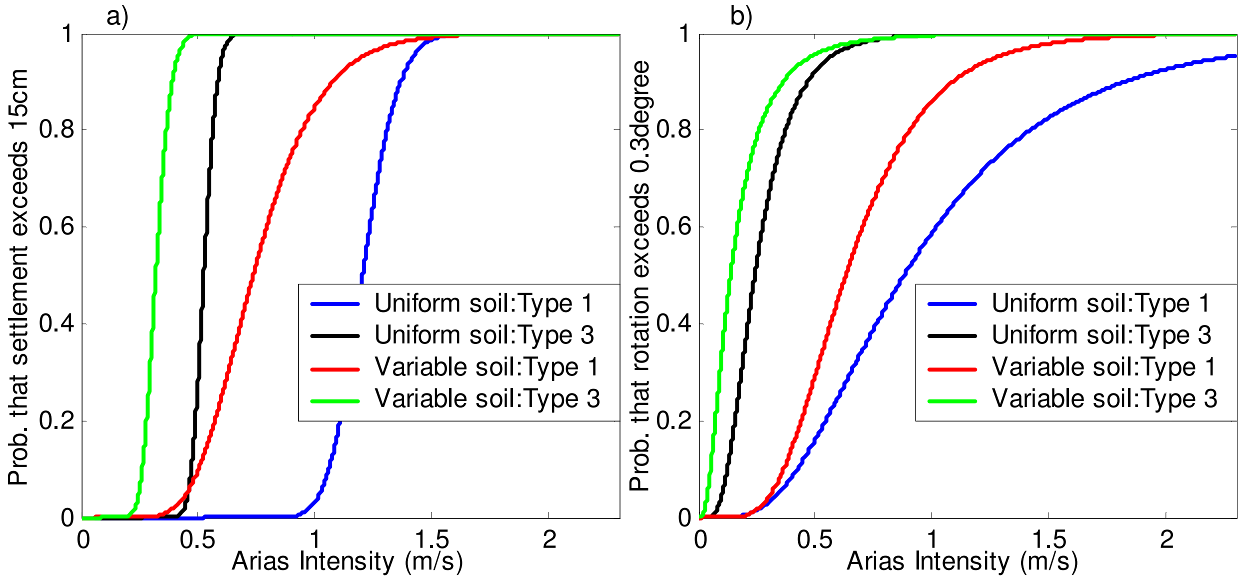

Figure 4: Effect of uncertainty in seismic input and soil variability on building responses a) maximum settlements b) maximum base rotations.

The fragility curves can be expressed in the form of two-parameter lognormal distribution functions. The lognormal distribution is chosen because, when the structural capacity and demand roughly fit a normal or lognormal distribution, using the central limit theorem, it can be said that the composite performance will be lognormally distributed (Shinozuka 1998). The estimations of these two parameters are done by the maximum likelihood method treating each event of damage or no-damage as a realization from a Bernoulli experiment. For more details about this method, the reader is referred to Shinozuka (1998).

Fig. 4 shows the effect of uncertainty in seismic inputs and the soil variability on the structure's response using the fragility curves. Both Figures (4a, 4b) are plotted for level-2 maximum permissible settlement and maximum permissible base rotation of the structure, respectively. From these curves it is observed that soil variability adversely affects both responses (maximum settlement and maximum base rotations) by increasing the probability of damage. Similarly, UBC type-3 inputs caused more damage to the structure than UBC type-1 inputs due to more soil softening after the build-up of excess pore pressures. The characteristic frequency of the soil-structure system goes down to values close to the dominant frequency of UBC type-3 inputs, and therefore considerably more structural damage is predicted for this type of seismic motion. Figure 5a shows the probability of damage of the structure (resting on variable soil) due to maximum settlement exceeding three different permissible (level-1, level-2 and level3) damage levels. Similarly, Figure 5b shows the probability of damage of the structure due to maximum base rotation exceeding different damage levels for UBC type-1 seismic input.

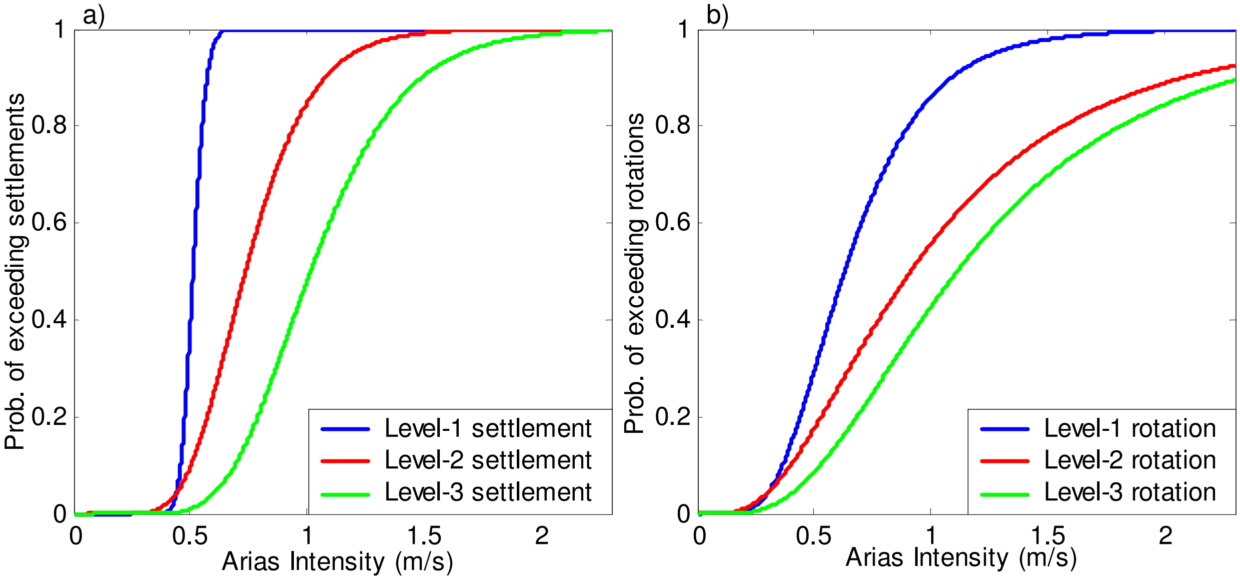

Figure 5: Fragility curves for structure on variable soil subjected to type-1 seismic input motion a) maximum settlements b) maximum base rotation.

Combined damage curves

The combined damage curves are generated using the method described earlier. Figure 6 shows the Type-I combined damage curve. Figure 6a shows the Type-I damage curve for a level-2 damage limit. Similarly, Figure 6b shows the Type-I damage curve exceeding the level-3 permissible limit for settlement and level-2 limit for rotation. This type of curves will be very useful when failure could be possible due to these conditions. Figure 7 shows the Type-II combined damage curves. Figure 7a shows the Type II curve for level-2 damage limits. Similarly Figure 7b shows Type II curve for a level-2 damage limit for settlement and level-1 damage limit for rotation.

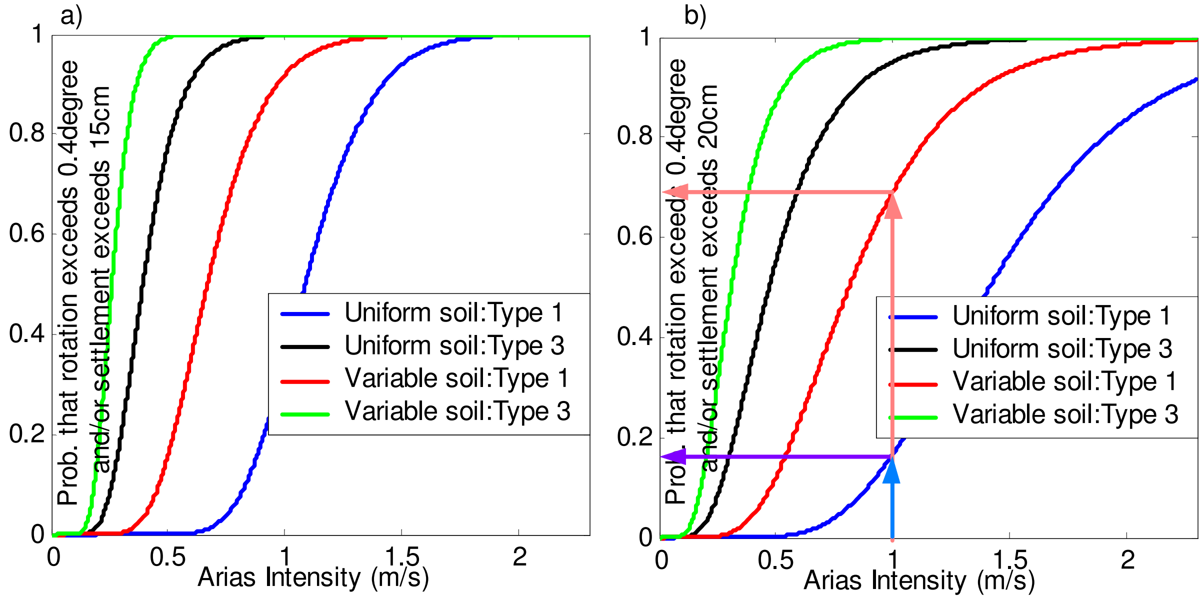

Figure 6: Type-I damage curve: probability of damage due to exceeding a) level-2 maximum settlement and/or level-2 maximum base rotation, b) level-3 maximum settlement and/or level-2 maximum base rotation.

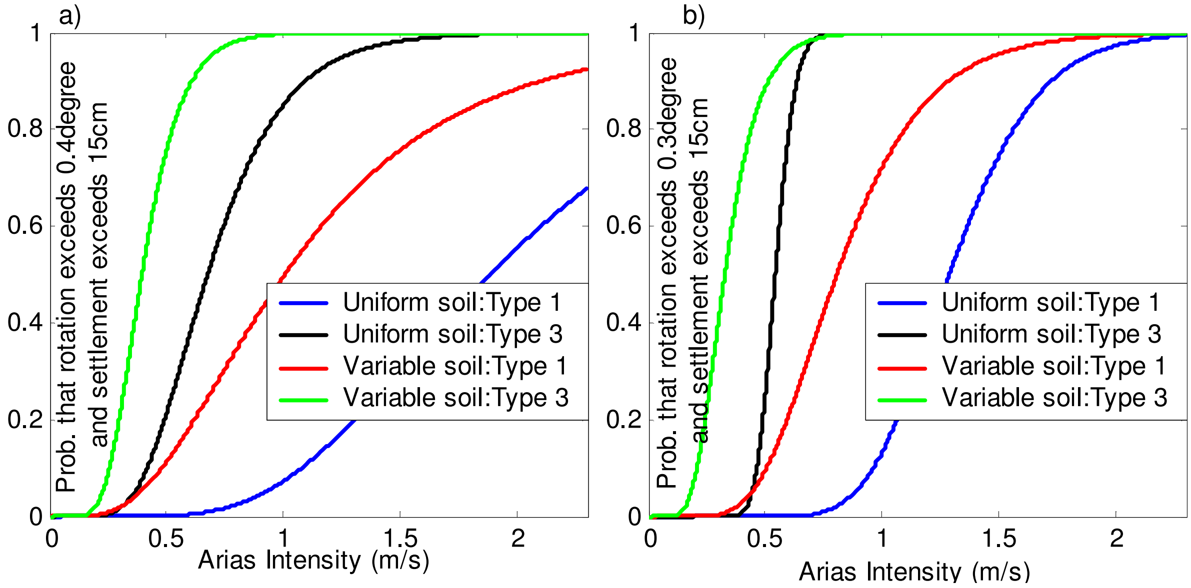

Figure 7. Type-II damage curve: probability of damage due to exceeding a) level-2 maximum settlement and level-2 maximum base rotation, b) level-2 maximum settlement and level-1 maximum base rotation.

How might these curves be useful for design? Suppose, for illustration purposes, that the Arias intensity corresponding to the design earthquake is 1m/s, and that type 1 acceleration is warranted by deeper soil strata. It is also assumed that the permissible maximum settlement value is 20cm and the limiting value of maximum base rotation is 0.4 degrees. A certain degree of damage to the structure is possible when any of those limits are exceeded. Therefore the Type-I combined damage curve (Figure 6b) can be used to calculate the probability of damage. From the damage curve for uniform soil and type-1 input acceleration, the probability of damage is 17%. As soil properties in natural soil deposits vary randomly from one point to another, a fragility curve accounting for this aspect should be considered. For the degree of soil variability assumed in this study, the damage probability for the same structure results about 70%.

Conclusions

From the study it is concluded that UBC type-3 input acceleration time histories caused more damage than type-1 input acceleration to the structure analysed here. It is also concluded that soil variability will adversely affect the performance of the structure. The probability of damage of the structure situated on variable soil will be higher compared to a structure situated on a uniform soil with the soil properties that are the average soil properties of the variable soil. The proposed combined damage curves constructed based on basic probability theory has been shown to be very useful for the initial reliability analysis of lifeline structures.

Acknowledgments

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