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FREE VIBRATION TESTS OF A STRUCTURE ROCKING ON A FLEXIBLE FOUNDATION

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ABSTRACT

A series of free vibration tests have been conducted on a free standing single degree of freedom structure with a range of elastic natural frequencies; the tests were conducted over a rigid concrete floor and over typical Auckland soil. A typical response was obtained when the structure was rocked over the concrete floor. The response fitted Housner's simple rocking model, however it further emphasised discrepancies in the calculation of the damping factors. Intriguingly, the experimental result showed that the amount of radiation damping, measured as the apparent co-efficient of restitution, is relatively constant irrespective of the structure's geometry contradicting the simple rocking models. The results showed that when the structure rocked on insitu ground, the dynamic behaviour was greatly modified. The time history response demonstrated a change of principle damping scheme from radiation to viscous. The response over the elastic medium failed to correlate with Housner's model due to increased viscous damping and an elongation in the rocking period for a given amplitude was observed.

Introduction

Evidence suggests the deliberate use of a rocking mechanism for seismic protection may date back to the ancient Roman times (Pampanin 2005). However, it is not until recently that its application has been fully investigated by modern structural engineers. One of the first scholarly records of rocking mechanics was by George W. Housner who investigated the mechanics and seismic response of a rigid rocking block (Housner 1963). Following this, numerous researchers have studied the rocking problem. The rocking block was found to have five modes of response: rest, rocking, sliding, rocking-sliding and free-flight (Shenton and Jones 1991). The rocking problem was initially thought to have a simple response, however research later showed that the problem is highly non-liner and with periodic excitation the response could be sometimes chaotic (Yim and Lin 1991). Extensive testing of precast concrete walls has been carried out as part of the Precast Seismic Structural System (PRESSS) research program (Priestley et al. 1999). Although there have been numerous efforts to model the dynamic response of rocking walls, shake table testing of post-tensioned concrete masonry (PCM) walls has revealed the shortcomings in these current analytical efforts (Wight et al. 2004). Additional studies carried out at the University of Auckland emphasised the inaccuracy of modelling the damping factor of these systems using standard finite element procedures (Ma et al. 2006a; Ma et al. 2006b). Current efforts are underway at the University to improve these methods, generate scarce dynamic experimental data that is required for verifying the methods, as well as investigating the effects of the interface material on a rocking structure's dynamic

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response (ElGawady et al. 2006a; ElGawady et al. 2006b).

A rocking mechanism provides seismic protection by allowing uplifts to occur. Uplifts reduce the structure's lateral stiffness and hence in effect cause the structure to be isolated from an earthquake, preventing irreparable damage. The South Rangitikei Railway Viaduct, built in New Zealand in 1981, is an example of a structure which incorporates the use of a rocking mechanism for seismic protection (Skinner et al. 1993). Here the piers are designed to step on the pile cap. Additionally, research has found that when structures are allowed to rock on an elastic medium, such as soil, the seismic performance can be further enhanced. Shake table and centrifuge testing has been conducted at the University of California, Berkeley and the University of California, Davis to investigate the behavior of bridge piers allowed to rock on their foundations (Espinoza et al. 2006). Experimental testing demonstrated that an underlying elastic medium can further reduced the force demands on bridge piers and increase the energy dissipation with minimal damage. Research into shallow foundations at the University of California, Davis also found that during rocking subsoil can be forced into a nonlinear range (Gajan et al. 2005). This soil structure interaction further complicates analytical efforts to model these structures.

A series of free vibration tests were carried out on a single degree of freedom column with various elastic natural frequencies. The tests were conducted under three restraint conditions on the concrete pad footing:

1. Footing rigidly fixed, no rocking was allowed.
2. Footing allowed to rock on a rigid concrete strong floor.
3. Footing allowed to rock on an elastic medium overlying insitu ground.

These restraint conditions were designed to allow preliminary investigation into the behavior of structures that are allowed to rock on their foundations as opposed to a rigid interface medium. Additionally, it was intended to investigate the suitability of simple rocking models (SRM), similar to that proposed by Housner, in predicting the response of such structures.

Experimental Setup

A series of free vibration experiments were conducted on an approximately single degree of freedom column. This 1.9m high column was constructed from 100UC14.8 steel section with steel plates welded on the top and bottom. A 300x300x100mm concrete pad footing was cast and bolted to the steel column with two M20 bolts. The concrete footing was trimmed by steel angles which prevented any damage to the edge of the footing when rocking. Five concrete mass blocks were cast to simulate the seismic weight acting at the top of the column. These blocks provided a mass of up to 87kg, but were rearranged to give various mass options. The blocks were secured to the top of the steel column with a threaded bar though the centre. The test rig is as shown in Fig. 1.

The first series of free vibration tests were conducted on the column with the footing rigidly fixed to the strong floor. A hydraulic hand jack was used to displace the top of the column by 30mm. A sacrificial wire loop connecting the jack and the column was cut, releasing the column to freely vibrate. In the second series of tests, the footing was released from the strong floor to allow uplift to occur, and hence a rocking response ensured. Two steel plates were installed either side of the footing to prevent the footing from sliding or walking, this ensured a simple rocking motion. In this series of tests, the column was manually displaced by 120mm and released. Finally, the test rig was relocated outside the laboratory and the footing situated on actual soil. The free rocking tests were repeated to allow comparisons between the two interface materials. In all three experimental scenarios, the tests were repeated with five concrete mass blocks (87kg) and three concrete mass blocks (52kg). This allowed the effect of modifying the seismic mass and hence elastic natural frequency under rocking motion to be investigated.

To record the response of the column, various instrumentation was setup. Two Linear Variable Differential Transformer (LVDT) Displacement Transducers were installed at the top and bottom of the seismic mass to measure the horizontal displacements. These LVDTs recorded displacements which allowed the angular rotation θ to be calculated. An accelerometer was attached to the top of the rig and accelerations in the rocking plane of the column were measured, these were later converted to a horizontal acceleration. Lastly, two strain portal gauges were mounted on the rocking edges of the footing to record the uplift and rotation as the test rig was vibrated.

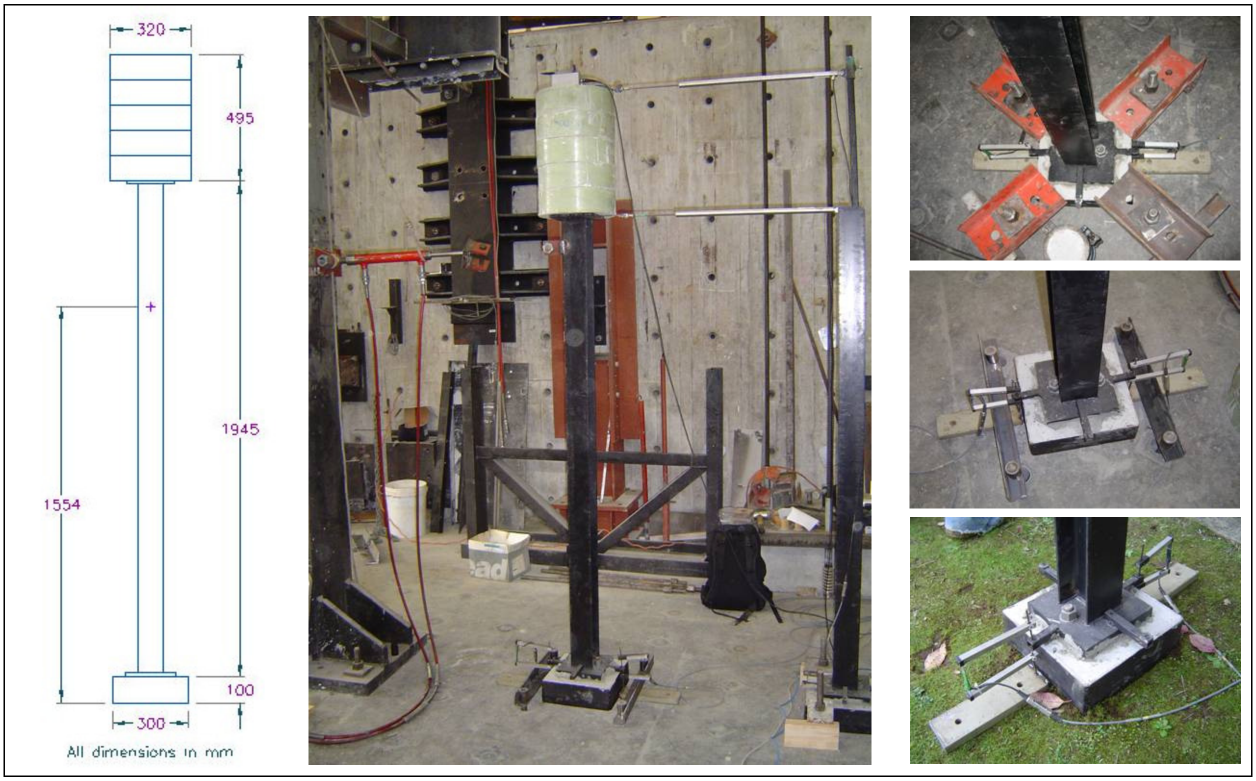

Figure 1. The column test rig used to conduct free vibration/rocking experiments and the three restrain conditions rigid footing, rocking on concrete and rocking on soil.

Experimental Results

1. Free vibration tests with footing rigidly fixed (elastic column tests)

A typical displacement time history measured at the top of the column, below the mass blocks, under free vibration is shown in Fig. 2A. Analyses found that the natural period of vibration for the rig was 0.274s with five seismic mass blocks (87kg). Additionally, the viscous damping was calculated to be 1.4%. This compared well with a 10 element finite element model, created using SAP2000, which predicted a first mode natural period of 0.249s. The 10% discrepancy can be attributed to the flexibility of the baseplate connection. The SAP model applied a rigid restraint at the column base whereas the experimental data from the portal gauges attached to the baseplate/footing, showed that a rotation of up to 0.2 degrees occurred at this connection.

2. Comparisons of Rocking Interface Material

The time history responses for the results of the column rocking on concrete (an ideally rigid surface) and soil (elastic medium) were found to be very different. A plot showing the rotational time history of two tests with the same initial displacements but dissimilar interface materials is shown in Fig. 2B. It can be seen that the response on the elastic medium (soil) showed a significantly increased rate of decay and hence increase amount of energy dissipation. Simplistic analyses equated the results to an average equivalent viscous damping ratio of 11% for the tests on concrete and 22% for the soil. These damping results reaffirmed the suitability of a rocking mechanism in dissipating energy during an earthquake.

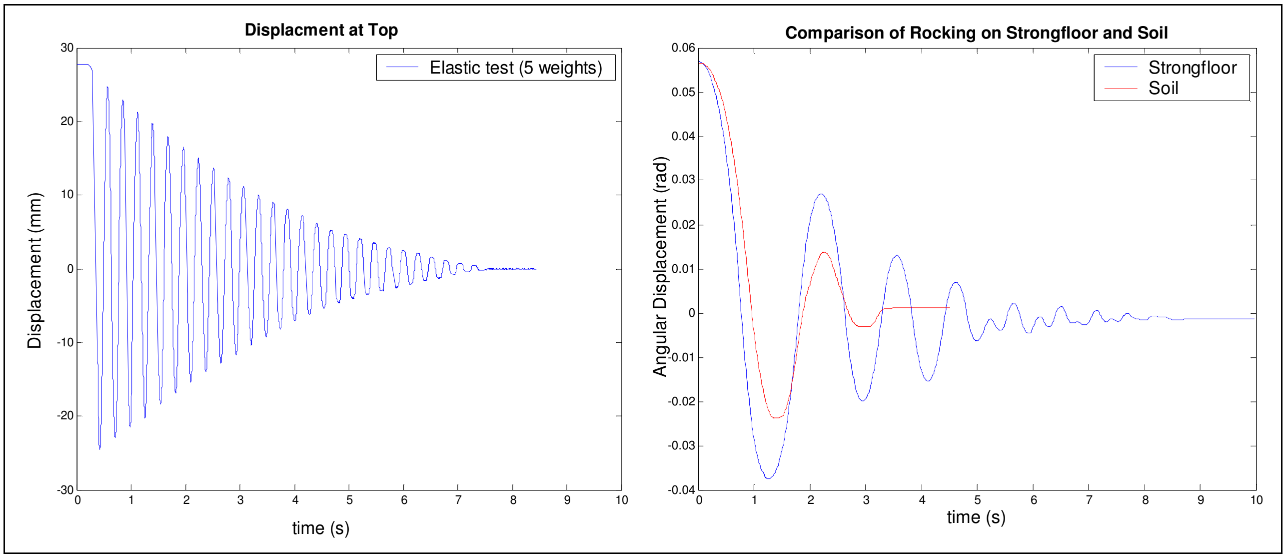

Figure 2. A) Typical displacement time history response for elastic column test.
B) Typical displacement time history response for free rocking tests on concrete and soil.

Additionally, the slope of the response curve during the first quarter cycle in Fig. 2B showed that the soil interface in fact has an additional viscous damping source that acts over the whole cycle, and not just at the time of impacts as with the concrete interface. This phenomenon changes from the principle damping source from radiation damping to viscous damping, and is attributed to the deformations in the elastic medium as the footing rotates. Initial observation suggests that there is a slight elongation in the rocking period from the concrete to soil responses. This confirms previous research that allowing structures to rock on an elastic medium under seismic conditions can be beneficial due to increased damping and an elongation of the natural period (Gajan et al. 2005; Espinoza et al. 2006).

By observing the uplift occurring at the footing edges the effect of the soil deformations were investigated. Fig. 3 shows a plot of the uplift at the critical footing edges. It can be seen that the critical edge in contact with the ground dips below the x axis implying that the soil was deforming under the rocking edge. This is obviously the source of the additional damping postulated in previous time history analyses.

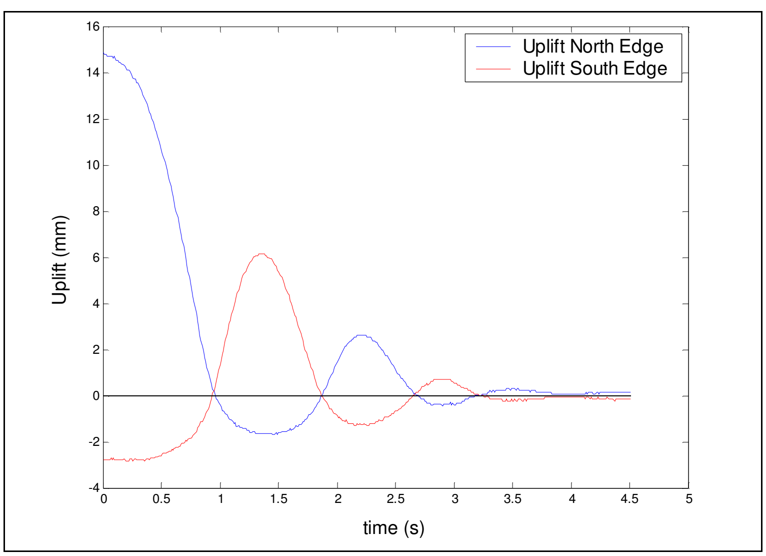

Figure 3. Vertical displacements of critical edges of the footing when rocking on soil.

The benefits of the elastic medium can further be confirmed by comparing the acceleration data from the two rocking experiments. The acceleration data captured was converted to horizontal accelerations by removing the gravity component and accounting for the angle of rotation (θ). This horizontal acceleration, plotted in Fig. 4 for rocking on both concrete strong floor and soil, showed that pulses of large accelerations created when the column impacts the concrete interface were greatly minimised when rocking on soil. This can be reasoned to the absence of viscous damping when rocking on a concrete surface, as this resulted in much of the energy in the impact being reflected back into the structure. Whereas the smooth soil response was a result of viscous damping caused by nonlinear behaviour of the soil and the absorption of any reflective energy at impacts. For similar initial displacements, Fig 4 showed a remarkable reduction of the peak accelerations from 8 m/s^2 to 1.5m/s^2, confirming the additional benefits of a soil interface.

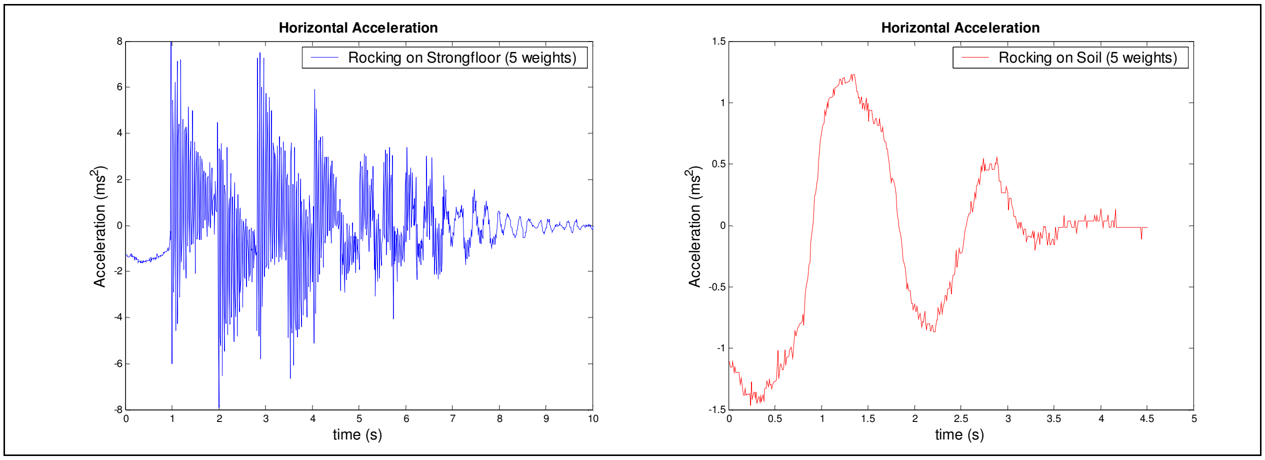

Figure 4. Horizontal component accelerations for the column rocking on (a) concrete and (b) soil.

3. Effect of Variable Seismic Mass

The experiments were repeated with two seismic mass conditions, five weights (87kg) and three weights (52kg). The typical time history response for both setups, freely rocking from the same initial displacement, on the concrete strong floor is shown in Fig. 5. Furthermore, the peak amplitudes and half periods were calculated for both setups and were plotted for comparison in Fig. 6. This comparison showed that there is a shortening in the rocking period for a given initial displacement, as the seismic mass is reduced. This was expected given the lowering of the centre of gravity of the column rig.

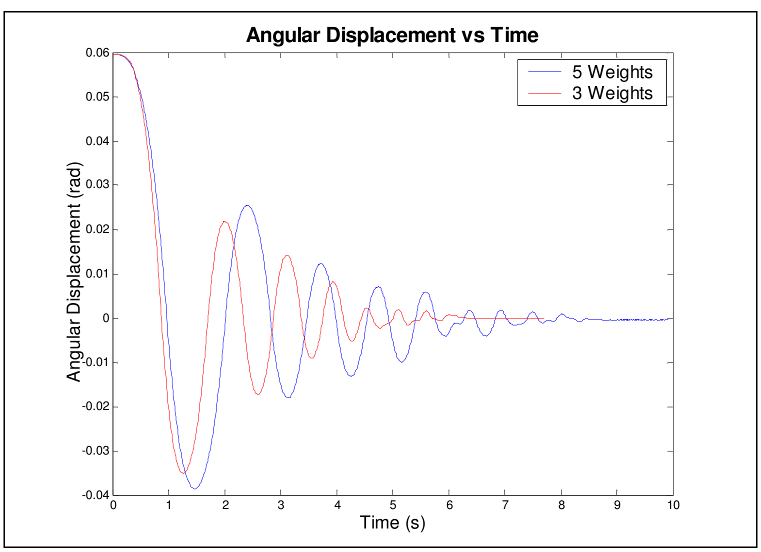

Figure 5. Angular rotation time histories of 3 and 5 weights test rig rocking on concrete.

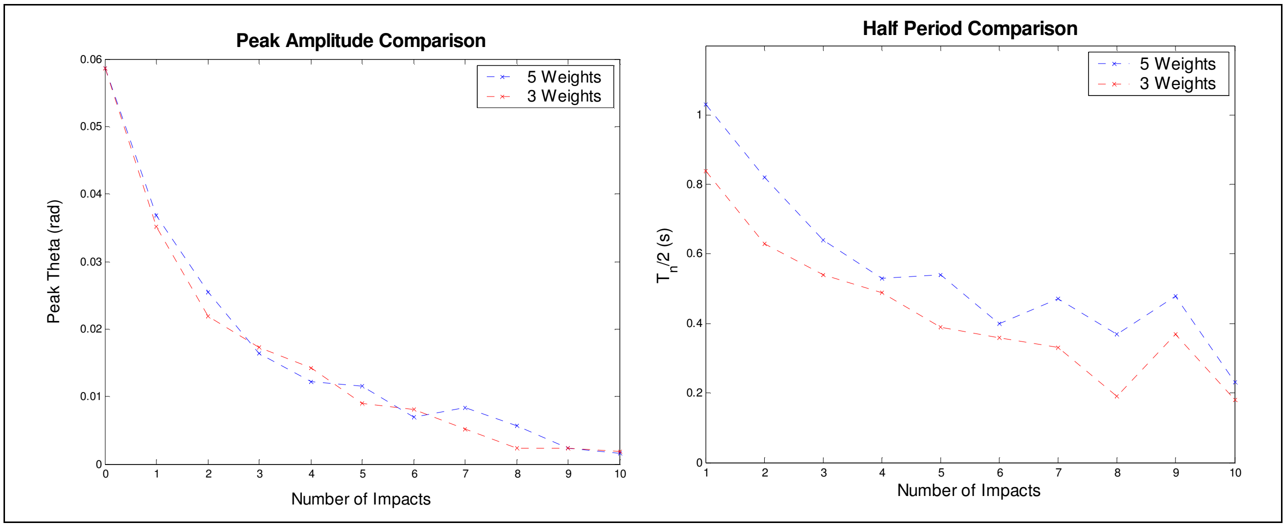

Figure 6. Amplitude and period comparison of variable seismic mass rocking on concrete.

With a lowered centre of gravity due to the reduced seismic mass, the rotational inertia of the rig is significantly reduced. Simple Rocking Model (SRM) predicts that the response of such a rocking structure is dependent on these properties and thus confirm the reduction in period (Housner 1963). However, the relative constant nature of the peak amplitudes indicates that the energy dissipation for both setups is similar. This suggests that the damping term (coefficient of restitution) is independent of the rig properties W, R and I, which contradicts traditional SRM originally proposed by Housner (Housner 1963).

Numerical Simulation

The single degree of freedom rocking column was compared with simple rocking models (SRM) proposed by Housner (Housner 1963). Housner analysed the mechanics of a rigid rocking block, shown in Fig. 7. By equating overturning and the restoring moment due to gravity, he proposed a differential equation of motion for free rocking response, Eq. 1.

$$I_o \frac{d^2\theta}{dt^2} = -WR\sin(\alpha - \theta) \tag{1}$$

where

- I_o = moment of inertia about origin 0 (kgm^2)
- θ = angular displacement (rad)
- W = weight of the block (N)
- R = distance from 0 to centre of gravity (m)
- α = slenderness angle (rad)

Then by applying conservation of momentum he predicted the coefficient of restitution, Eq. 2, which allows for the reduction in velocity caused at impacts.

$$r = \left(\frac{\dot{\theta}_2}{\dot{\theta}_1}\right)^2 = \left[1 - \frac{mR^2}{I_o}\left(1 - \cos(2\alpha)\right)\right] \tag{2}$$

where

- θ = angular velocity (rad/s)
- m = mass of the block (kg)

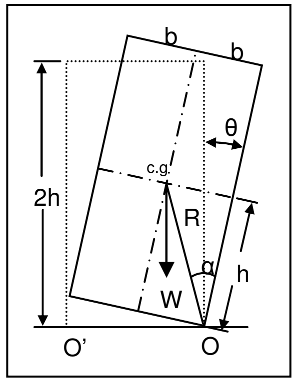

Figure 7. Housner's Rocking Block.

By calculating the properties W, R, I and α for the column rig the equation of motion given in Eq.1 can be solved using MATLAB's ordinary differential equation solver, ode45. Additionally the velocity reduction factor or coefficient of restitution, r, is applied when the footing impacts with the interface surface. Using this approach a simulated time history response can be obtained.

Finally, by assuming a tall slender structure and small angular displacements Housner developed a prediction for the rocking period given in Eq. 3.

$$T = \frac{4}{p}\cosh^{-1}\left(\frac{1}{1-\theta_0/\alpha}\right) \qquad (3)$$

where

T = rocking period (s)

p = √WR/I

θ_0 = initial angular displacement (rad)

1. SRM applied to rocking on strong floor

A comparison of the simulated and actual displacement time history response for tests when the column was freely rocking on the concrete strong floor is shown in Fig. 8. The simulation was first conducted using the apparent coefficient of restitution, r_H, as predicted by Housner in Eq. 2, which results in a value of 0.973. Fig. 8 shows that this approach gave a very poor estimation. Subsequently, the r value was tweaked to achieve a 'best possible fit'. It was found that when the r value was empirically set to 0.757 the simulation best matched the experimental data. This r value required is 22% below that predicted by Housner (r_H), implying that more energy was dissipated at each impact above the assumption of a fully plastic impact essential in the development of the governing differential equation. This confirms previous research which by Ma et al. which highlighted the shortcomings of current dynamic numerical procedures for predicting the damping coefficient for rocking systems (Ma et al. 2006b).

Additionally, it can be observed from Fig. 8 that during the first quarter cycle the simulated responses were identical for all r values. This was because no impact has yet occurred for the first quarter cycle and thus during this time the response is independent of the r factor.

Furthermore, Fig. 9 compared the peak amplitudes and half periods for both the SRMs and the actual test data. These plots further emphasised the suitability of the empirically chosen r value of 0.757. Additionally, it can be seen that after the forth impact the response became a little erratic and unpredictable. This was mainly due to imperfections in the footing surface which caused the rig to develop an out of plane rocking motion at low amplitudes.

The SRM simulations were repeated for the tests with three seismic mass blocks (52kg). It was found that

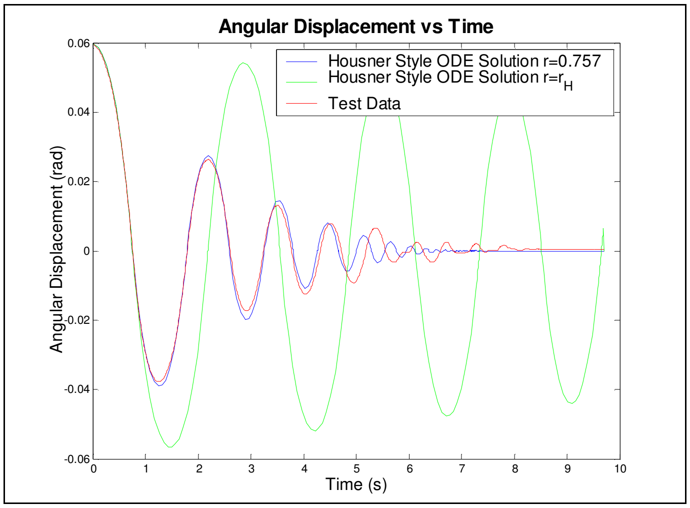

Figure 8. Time history response of SRM simulations compared with test data on concrete strong floor.

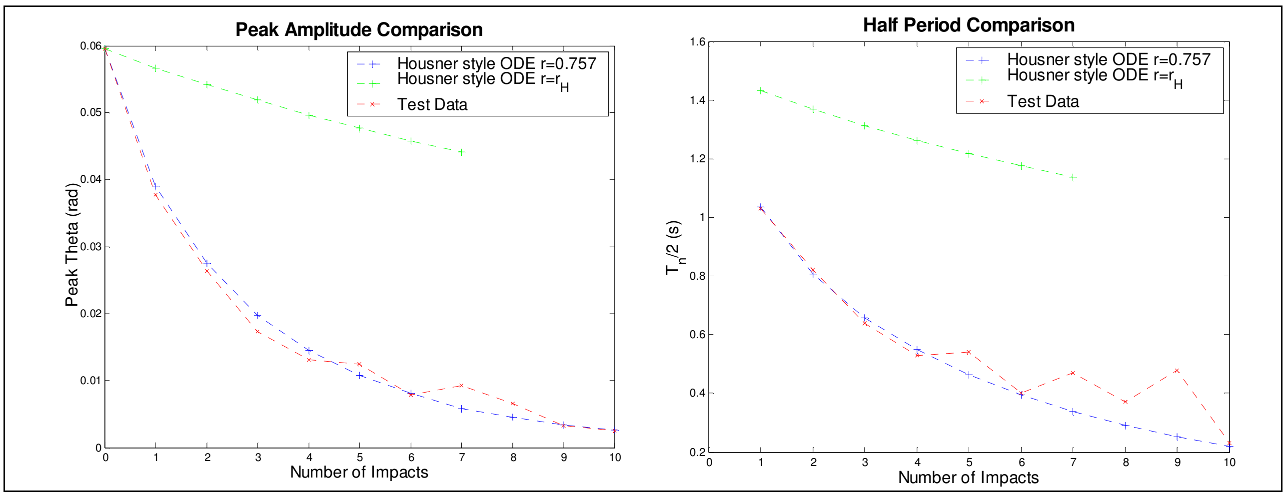

Figure 9. Amplitude and period comparison of SRM simulations and test data on concrete strong floor.

for these tests, the empirically best fit r value required was 0.723. This value is similar to that found for the five weights (87kg) setup of 0.757. This suggests that the damping factor, r, is relatively independent of the structures properties, W, I, R and α. This again is not supported in the original SRM predictions for the r values, as in Eq. 3. However, further investigations are recommended to confirm this finding.

2. SRM applied to rocking on soil

SRM simulations were again conducted to match the column rocking response on the elastic soil medium. Fig. 10 shows angular displacement time history plots of the simulation using an empirically chosen r value of 0.55, and the actual measured response. As expected, the results did not correspond well. While the r value of 0.55 gave approximately the correct peak amplitudes at each cycle, the rocking period was severely elongated. This suggest that a SRM, which only allows for radiation damping at impacts, is not suitable for modelling the dynamic behaviour of a rocking system in presence of a nonlinear elastic rocking interface.

3. SRM period prediction

The rocking data for tests on the concrete strong floor and on soil are further compared to the period prediction proposed by SRM in Eq. 3 . Fig. 11 compares quarter periods from given initial cycle

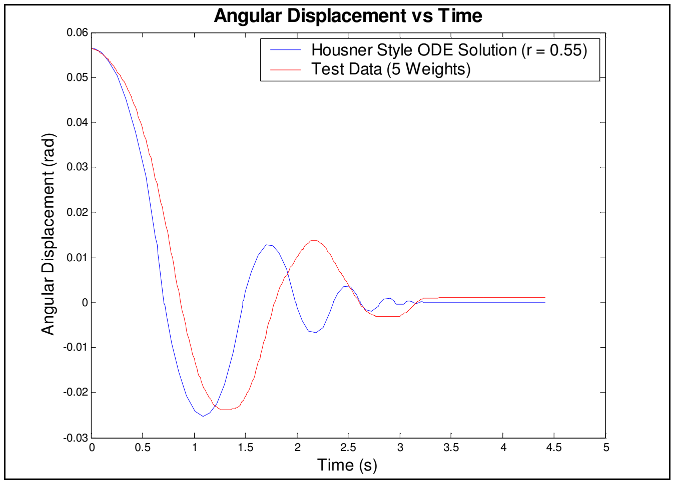

Figure 10. Time history response of SRM simulation and actual test data rocking on soil.

amplitudes against a curve derived from Eq. 3. These plots showed that Eq. 3 captures the rocking structures very well when it is in motion. It should be noted that the quarter period comparison is independent of the r value as they are measured from peak amplitude to impact. Additionally, we see that the period results for the column rocking on soil are clearly elongated from that predicted by the SRM. This reiterates that the equation of motion from SRM is not valid for structures rocking on an elastic medium as there is an additional damping source that acts during the rocking phase.

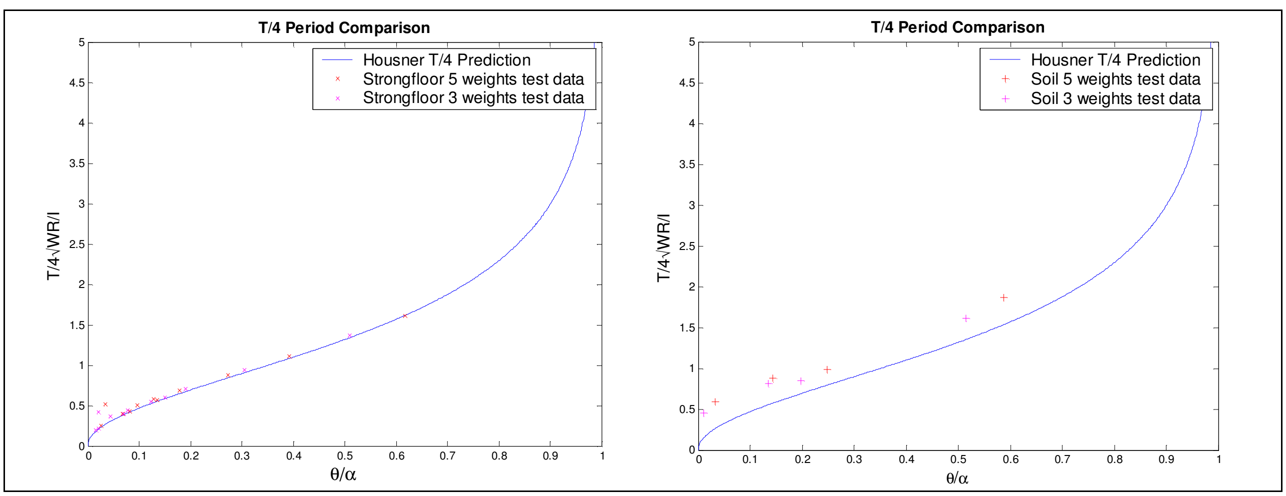

Figure 11. T/4 period comparison of Eq. 3 and the test data (a) on concrete and (b) on soil

Conclusions

This paper confirms the benefits of the use of a rocking mechanism for seismic protection. Furthermore, benefits of a rocking structure being situated on an elastic medium such as soil are demonstrated. It was found that when the restraint conditions were progressively relaxed, from rigidly fixed to rocking on a rigid medium to rocking of an elastic medium, these was a corresponding increase in the equivalent viscous damping as well as an elongation in the natural period of vibration.

Experimental data for the single degree of freedom column freely rocking on the rigid concrete strong floor was found to match closely with conventional simple rocking models. Although as found previously, there are numerous deficiencies in the current methods of predicting the damping factor. When an apparent coefficient of restitution, r, was used to represent the energy dissipation, a smaller r value than the Housner prediction is required, implying that more energy is dissipated. Additionally, the variation of seismic mass and hence structural properties resulted in little change in this r value. This suggests that the amount of radiation damping is not dependent on such properties as previously predicted. Although, given the limited experimental data, further experiments are recommended to confirm this finding.

Also, experimental data for the single degree of freedom column, freely rocking on an elastic medium, showed a dramatic change in the response to that rocking on a rigid concrete surface. The approximate amount of the energy dissipation, expressed as an equivalent damping ratio was doubled. The principle energy dissipating mechanism shifted from radiation to viscous damping. Attempts to simulate the response using a simple rocking model to this system proved unsuccessful. This clearly demonstrated that the SRM is ineffective when modelling rocking structures over an elastic medium.

Acknowledgments

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References

ElGawady, M. A., Q. Ma, J. Butterworth and J. M. Ingham, 2006a. The effect of interface material on the dynamic behaviour of free rocking blocks. Eighth U.S. National Conference on Earthquake Engineering (8NCEE), San Francisco, USA.

ElGawady, M. A., Q. Ma, J. Butterworth and J. M. Ingham, 2006b. Probabilistic analysis of rocking blocks. New Zealand Society for Earthquake Engineering Conference 2006, Napier, New Zealand.

Espinoza, A., S. Mahin, B. Jeremic, B. Kutter and J. Ugalde, 2006. Rocking of bridge piers subjected to multi-directional earthquake loading. Fifth National Seismic Conference on Bridges and Highways, San Francisco, CA.

Gajan, S., B. L. Kutter, J. D. Phalen, T. C. Hutchinson and G. R. Martin, 2005. Centrifuge modeling of load-deformation behavior of rocking shallow foundations. *Soil Dynamics and Earthquake Engineering* 25(7-10), 773-783.

Housner, G. W., 1963. Behavior of inverted pendulum structures during earthquakes. *Seismological Society of America -- Bulletin* 53(2), 403-417.

Ma, Q. T., J. W. Butterworth and B. J. Davidson, 2006a. An alternative mathematical model for a controlled rocking system. New Zealand Society for Earthquake Engineering Conference 2006, Napier, New Zealand.

Ma, Q. T., G. D. Wight, J. W. Butterworth and J. M. Ingham, 2006b. Assessment of current procedures for predicting the in-plane behaviour of controlled rocking walls. Eighth U.S. National Conference on Earthquake Engineering (8NCEE), San Francisco, USA.

Pampanin, S. (2005). Controversial aspect in seismic assessment and retrofit of structures in modern times: understanding and implementing lessons from ancient heritage. The New Zealand Concrete Industry Conference, Auckland, New Zealand.

Priestley, M. J. N., S. S. Sritharan, J. R. Conley and S. Pampanin, 1999. Preliminary results and conclusions from the PRESSS five-story precast concrete test building. *PCI Journal* 44(6), 42-67.

Shenton, H. W. and N. P. Jones, 1991. Base excitation of rigid bodies. I. Formulation. *Journal of Engineering Mechanics* 117(10), 2286-2306.

Skinner, R. I., W. H. Robinson and G. H. McVerry, 1993. *An introduction to seismic isolation*. Wiley, Chichester, New York.

Wight, G. D., J. M. Ingham and M. Kowalsky, 2004. Shake table testing of post-tensioned concrete masonry walls. New Zealand Concrete Industries Conference, Queenstown, New Zealand.

Yim, S. and H. Lin, 1991. Nonlinear impact and chaotic response of slender rocking objects. *Journal of Engineering Mechanics* 117(9), 2079-2100.