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A SIMPLIFIED APPROACH TO OPTIMIZATION OF SEISMIC RESPONSE OF BUILDING STRUCTURES

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ABSTRACT

Seismic response/performance of a building structure depends on the characteristics of the structural system and the seismic event, and can be evaluated by a number of response/performance parameters, viz. (i) total input energy to the system, (ii) energy absorbed by the structural components of the system (i.e. structural distress energy), (iii) maximum inter-story drift, (iv) maximum displacement-at-top, (v) root-mean-square (RMS) displacement-at-top, (vi) maximum acceleration-at-top, (vii) RMS acceleration-at-top, (viii) maximum base shear, and (ix) maximum base moment. Seismic response optimization can be achieved by optimizing one or more of the above parameters, while keeping a check on the remaining ones. This paper elaborates a simplified approach to optimization of seismic response of two structural systems, a 10-story and a 30-story building, subjected to a set of past earthquake records with different characteristics. In this approach, the concept of input-energy-per-unit-mass spectra is used to optimize the structural/seismic design of the two structural systems. The input-energy-per-unit-mass spectra are obtained for a single-degree-of-freedom system for a particular ground motion/acceleration time history and are used, in conjunction with the acceleration response spectra, for selecting an optimum structural configuration for a multi-degree-of-freedom system to be located in an area where such a ground motion is expected. This study shows that optimized seismic design of structural systems can be easily achieved with the proposed approach.

Introduction

The response of a structure to a seismic activity depends on the characteristics of the structural system and the seismic activity. A number of techniques for controlling/improving the seismic response of structures, like base isolation for low-rise to medium-rise buildings and seismic bifurcation (Dua 2004) for medium-rise to tall building structures, aim at reducing the seismic input energy by shifting the natural periods of the structural system and dissipating a part of the input energy through non-structural elements in the system. Seismic response/performance of a structural system can be evaluated by a number of parameters. Some of the performance parameters are: (i) total input energy to the structural system, (ii) energy absorbed by the structural components of the system i.e. structural distress energy, (iii) maximum inter-storey drift, (iv) maximum displacement-at-top, (v) root-mean-square displacement-at-top, (vi) maximum acceleration-at-top, (vii) root-mean-square acceleration-at-top, (viii) maximum base shear, and (ix) maximum base moment.

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The total input energy to the structural system is an important parameter from the viewpoint of damageability of the seismic activity and safety of the structural system, as the structural distress energy is a part of and dependent on the total input energy. Housner (1956) proposed an energy based method for earthquake resistant design of structures. Uang (1990) proposed input-energy-equivalent-velocity spectra for determining seismic energy in structures. This paper proposes a simple method for determining the total input energy to a structural system with the help of input-energy-per-unit-mass spectra, which in turn helps in selecting the optimum structural configuration/properties leading to an optimized seismic response. The proposed method is much simpler and quicker than the prevailing time history method for determining seismic input energy to a structural system during an earthquake.

Structural Systems and Earthquake Motions Chosen for the Study

To demonstrate the usefulness of the proposed optimization technique four different structural systems, viz. 30-story: option 1, 30-story: option 2, 10-story: option 1, and 10-story: option 2, are selected. Modal analyses are carried out to obtain the modal characteristics of the structural systems. The Option 2 structural systems are chosen to exhibit an improved seismic response over their corresponding Option 1 structural systems with respect to the total input energy during a seismic event. This is achieved by selecting the sectional properties and the concrete grade of the Option 2 structural systems to have the desired modal properties for attracting lesser seismic input energy than the Option 1 structural systems. The grade of concrete influences its Young's modulus of elasticity which in turn determines the stiffness or the modal characteristics of a structural configuration. The present study is based on the single-bay modeling of the structure, as Roehl (1971) demonstrated that for a specified stiffness ratio, the natural frequencies and the mode shapes practically do not change with the number of bays.

Four earthquake records are chosen for the study, covering a wide spectrum of earthquake characteristics, e.g. near-field or far-field, on-rock or on-soft-soil etc. The time histories of the selected horizontal ground acceleration records, viz. (i) El Centro NS 1940, (ii) Imperial Valley 1979, (iii) Mexico City 1985, and (iv) Chile 1985, are depicted in Fig. 1.

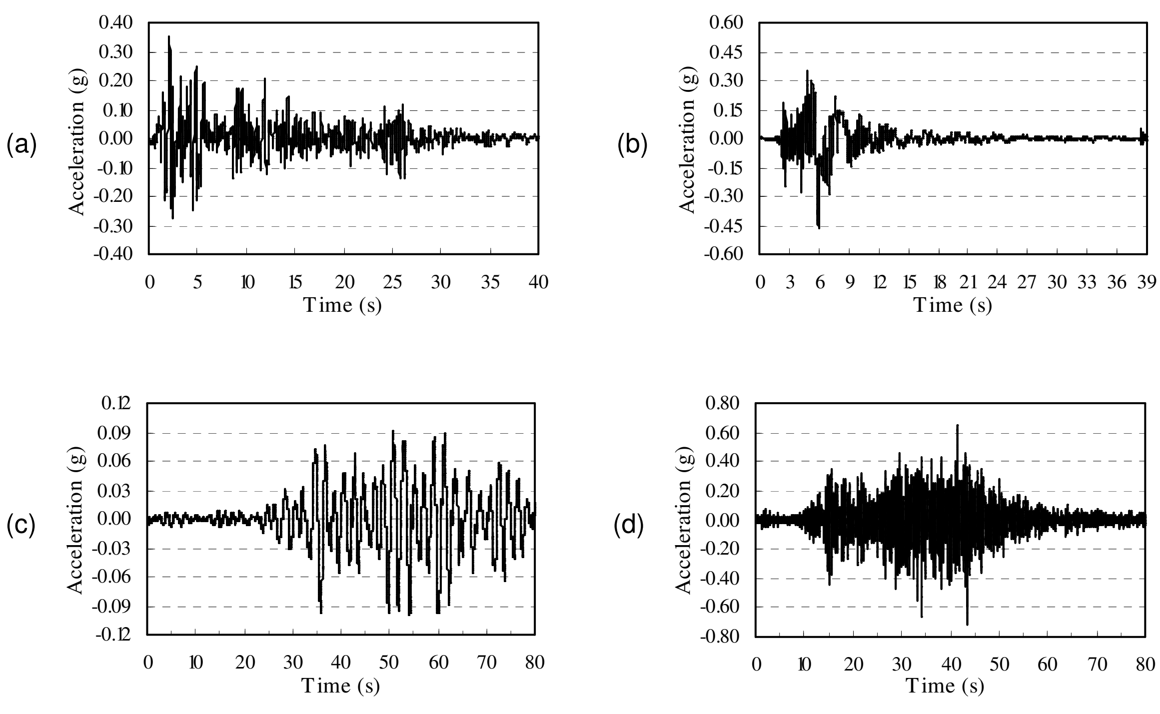

Figure 1. Time histories of horizontal ground acceleration for (a) El Centro NS 1940, (b) Imperial Valley 1979, (c) Mexico City 1985, and (d) Chile 1985 earthquakes.

Salient features of the selected earthquake records are reported in Table 1. Housner (1952) proposed a measure of the "damageability" of a seismic activity by introducing the term "spectrum intensity", defined as the integral of the pseudo-velocity response spectrum over the period range of 0.1 to 2.5 s and for a particular damping ratio. Table 1 shows the Housner's spectrum intensity for a damping ratio of 0.20. A normalized pseudo-acceleration response spectrum represents the response amplification characteristic of an earthquake. Maximum response amplification (MRA) for an earthquake is the highest ordinate of its normalized pseudo-response spectrum for a particular damping ratio. The normalized pseudo-acceleration response spectra are obtained for single-degree-of-freedom structural systems for the four earthquake records, and are depicted in Fig. 2. Table 1 depicts the MRA and the corresponding structural period for a damping ratio of 0.02 for the chosen earthquake records.

Table 1. Features of the horizontal ground acceleration records chosen for the study.

Horizontal Ground Acceleration	Record Length (s)	Peak Ground Acceleration (g)	RMS Value of Record (g)	Housner's Spectrum Intensity (m)	MRA and Corresponding Structural Period	
					MRA	Period (s)
El Centro NS 1940	40	0.348	0.054	0.813	3.61	0.45
Imperial Valley 1979	39	0.468	0.058	1.609	3.59	0.25
Mexico City 1985	80	0.100	0.031	0.882	12.64	2.05
Chile 1985	80	0.712	0.111	0.998	4.81	0.23

The El Centro NS 1940 and Imperial Valley 1979 earthquake records are chosen for the 30-story as well as the 10-story structural systems. In addition, the Mexico City 1985 record is chosen for the 30-story structural systems as their structural periods are close to the period (2.05 s) for which the MRA occurs. Similarly, the Chile 1985 record is chosen for the 10-story structural systems.

Structural Properties of the Structural Systems

Structural properties of the "30-story: option 1" structural system are detailed in Table 2.

Table 2. Structural properties of the "30-story: option 1" structural system.

Structural System			
Single Bay – 2D Frame with Single Diagonal Bracing in Each Story (All Bracings being Parallel) Story Height = 3.5 m; Bay Width = 6.0 m; Mass per Node = $8.25\text{x}10^3$ kg; 2 Nodes per Floor Total Mass on 60 Nodes Above Ground, $M_T = 4.95\text{x}10^5$ kg; Young's Modulus, $E_c = 2.7\text{x}10^7$ kNm^{-2}; Strain Hardening Ratio (Proportion of E_c) = 0.04			
(i) Column Elements			
Story	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-9	$6.0\text{x}10^{-1}$	$5.00\text{x}10^{-2}$	$1.6\text{x}10^{3}$
10-16	$4.8\text{x}10^{-1}$	$2.56\text{x}10^{-2}$	$1.2\text{x}10^{3}$
17-23	$3.6\text{x}10^{-1}$	$1.08\text{x}10^{-2}$	$9.0\text{x}10^{2}$
24-30	$2.4\text{x}10^{-1}$	$3.20\text{x}10^{-3}$	$6.0\text{x}10^{2}$
(ii) Beam Elements			
Floor	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-30	$2.0\text{x}10^{-1}$	$3.60\text{x}10^{-3}$	$5.0\text{x}10^{2}$
(iii) Diagonal Bracing Elements			
Story	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-30	$9.0\text{x}10^{-2}$	$7.00\text{x}10^{-4}$	$3.0\text{x}10^{2}$

The structural properties of the "30-story: option 2" structural system (Table 3) are selected to achieve optimization of the seismic response with respect to the total input energy, while keeping a check on the acceleration response of the system. This is achieved with the help of the input-energy-per-unit-mass spectra (Fig. 3) and the normalized pseudo-acceleration response spectra (Fig. 2). The optimization technique for medium-rise and high-rise structural systems is elaborated in the subsequent sections.

Table 3. Structural properties of the "30-story: option 2" structural system.

Structural System			
Single Bay - 2D Frame with Single Diagonal Bracing in Each Story (All Bracings being Parallel) Story Height = 3.5 m; Bay Width = 6.0 m; Mass per Node = 8.25×10^3 kg; 2 Nodes per Floor Total Mass on 60 Nodes Above Ground, $M_T = 4.95 \times 10^5$ kg; Young's Modulus, $E_c = 3.0 \times 10^7$ kNm^{-2}; Strain Hardening Ratio (Proportion of E_c) = 0.04			
(i) Column Elements			
Story	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-9	7.5×10^{-1}	6.25×10^{-2}	1.8×10^3
10-16	6.0×10^{-1}	3.20×10^{-2}	1.4×10^3
17-23	4.5×10^{-1}	1.35×10^{-2}	1.0×10^3
24-30	3.0×10^{-1}	4.00×10^{-3}	7.0×10^2
(ii) Beam Elements			
Floor	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-30	2.5×10^{-1}	4.50×10^{-3}	6.0×10^2
(iii) Diagonal Bracing Elements			
Story	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-30	1.2×10^{-1}	9.00×10^{-4}	4.0×10^2

To demonstrate the usefulness of the proposed optimization technique to medium-rise structural system, a 10-story structural system (Table 4) is selected.

Table 4. Structural properties of the "10-story: option 1" structural system.

Structural System			
Single Bay - 2D Frame with Single Diagonal Bracing in Each Story (All Bracings being Parallel) Story Height = 3.5 m; Bay Width = 5.0 m; Mass per Node = 7.21×10^3 kg; 2 Nodes per Floor Total Mass on 20 Nodes Above Ground, $M_T = 1.44 \times 10^5$ kg; Young's Modulus, $E_c = 3.0 \times 10^7$ kNm^{-2}; Strain Hardening Ratio (Proportion of E_c) = 0.04			
(i) Column Elements			
Story	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-5	3.2×10^{-1}	1.71×10^{-2}	1.0×10^3
6-10	2.8×10^{-1}	1.14×10^{-2}	8.0×10^2
(ii) Beam Elements			
Floor	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-10	2.0×10^{-1}	4.20×10^{-3}	6.0×10^2
(iii) Diagonal Bracing Elements			
Story	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-10	9.0×10^{-2}	7.00×10^{-4}	3.0×10^2

Table 5 depicts the structural properties of the "10-story: option 2" structural system, which are selected to achieve optimization of the seismic response with respect to the total input energy, while keeping a check on the acceleration response of the system. This is achieved with the help of the input-energy-per-unit-mass spectra (Fig. 3) and the normalized pseudo-acceleration response spectra (Fig. 2).

Table 5. Structural properties of the "10-story: option 2" structural system.

Structural System			
Single Bay - 2D Frame with Single Diagonal Bracing in Each Story (All Bracings being Parallel) Story Height = 3.5 m; Bay Width = 5.0 m; Mass per Node = 7.21×10^3 kg; 2 Nodes per Floor Total Mass on 20 Nodes Above Ground, $M_T = 1.44 \times 10^5$ kg; Young's Modulus, $E_c = 2.7 \times 10^7$ kNm^{-2}; Strain Hardening Ratio (Proportion of E_c) = 0.04			
(i) Column Elements			
Story	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-5	3.2×10^{-1}	1.71×10^{-2}	1.0×10^3
6-10	2.8×10^{-1}	1.14×10^{-2}	8.0×10^2
(ii) Beam Elements			
Floor	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-10	2.0×10^{-1}	4.20×10^{-3}	6.0×10^2
(iii) Diagonal Bracing Elements			
Story	Cross-sectional Area (m^2)	Moment of Inertia (m^4)	Yield Moment (kNm)
1-10	9.0×10^{-2}	7.00×10^{-4}	3.0×10^2

Modal Characteristics of the Structural Systems

The modal characteristics of the four structural systems, as evaluated by the modal analysis, are depicted in Table 6.

Table 6. Modal characteristics of the four structural systems.

Structural System	Parameter	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
30-Story: Option 1	Period (T_n), s	2.712	0.611	0.279	0.175	0.126
	Proportion of Critical Damping (ξ_n)	0.038	0.096	0.205	0.325	0.448
	Effective Modal Mass Ratio (β_n)	0.634	0.210	0.066	0.027	0.016
30-Story: Option 2	Period (T_n), s	2.421	0.541	0.246	0.154	0.111
	Proportion of Critical Damping (ξ_n)	0.039	0.108	0.231	0.368	0.509
	Effective Modal Mass Ratio (β_n)	0.633	0.210	0.067	0.024	0.016
10-Story: Option 1	Period (T_n), s	0.538	0.141	0.069	0.047	0.035
	Proportion of Critical Damping (ξ_n)	0.038	0.134	0.273	0.404	0.537
	Effective Modal Mass Ratio (β_n)	0.696	0.201	0.049	0.022	0.011
10-Story: Option 2	Period (T_n), s	0.750	0.186	0.089	0.059	0.045
	Proportion of Critical Damping (ξ_n)	0.030	0.102	0.188	0.318	0.421
	Effective Modal Mass Ratio (β_n)	0.680	0.210	0.054	0.023	0.011

Pseudo-Acceleration Response Spectra

The pseudo-acceleration response spectra, obtained for a single-degree-of-freedom system, are used in this study to visually assess the acceleration response of the four selected multi-degree-of-freedom structural systems for the chosen earthquake records. These spectra are depicted in Fig. 2.

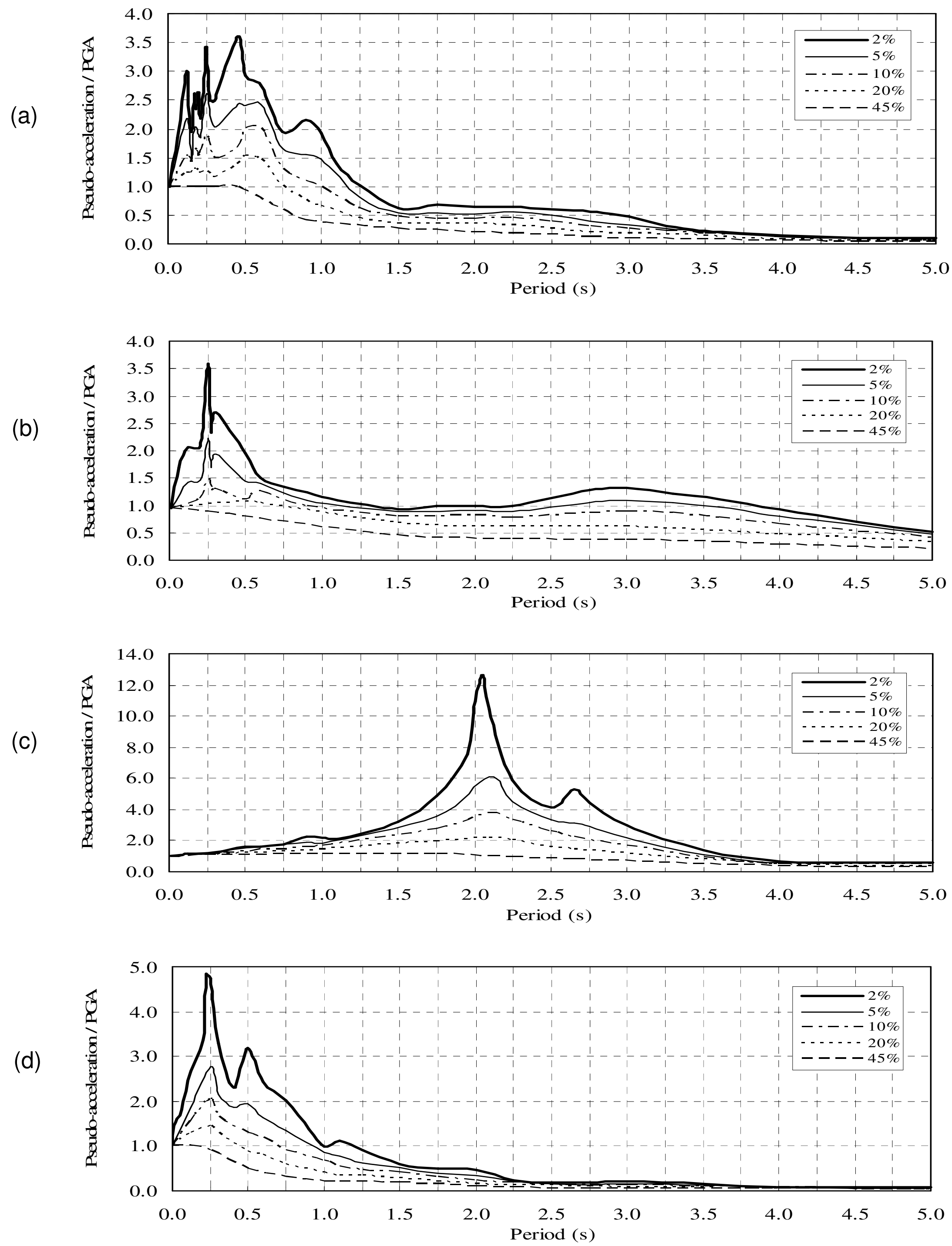

Figure 2. Normalized pseudo-acceleration response spectra for (a) El Centro NS 1940, (b) Imperial Valley 1979, (c) Mexico City 1985, and (d) Chile 1985 earthquake records.

Determination of Total Input Energy using Input-Energy-per-Unit-Mass Spectra

The proposed method uses the input-energy-per-unit-mass spectra (Fig. 3) for determining the total input energy to the system during a seismic event.

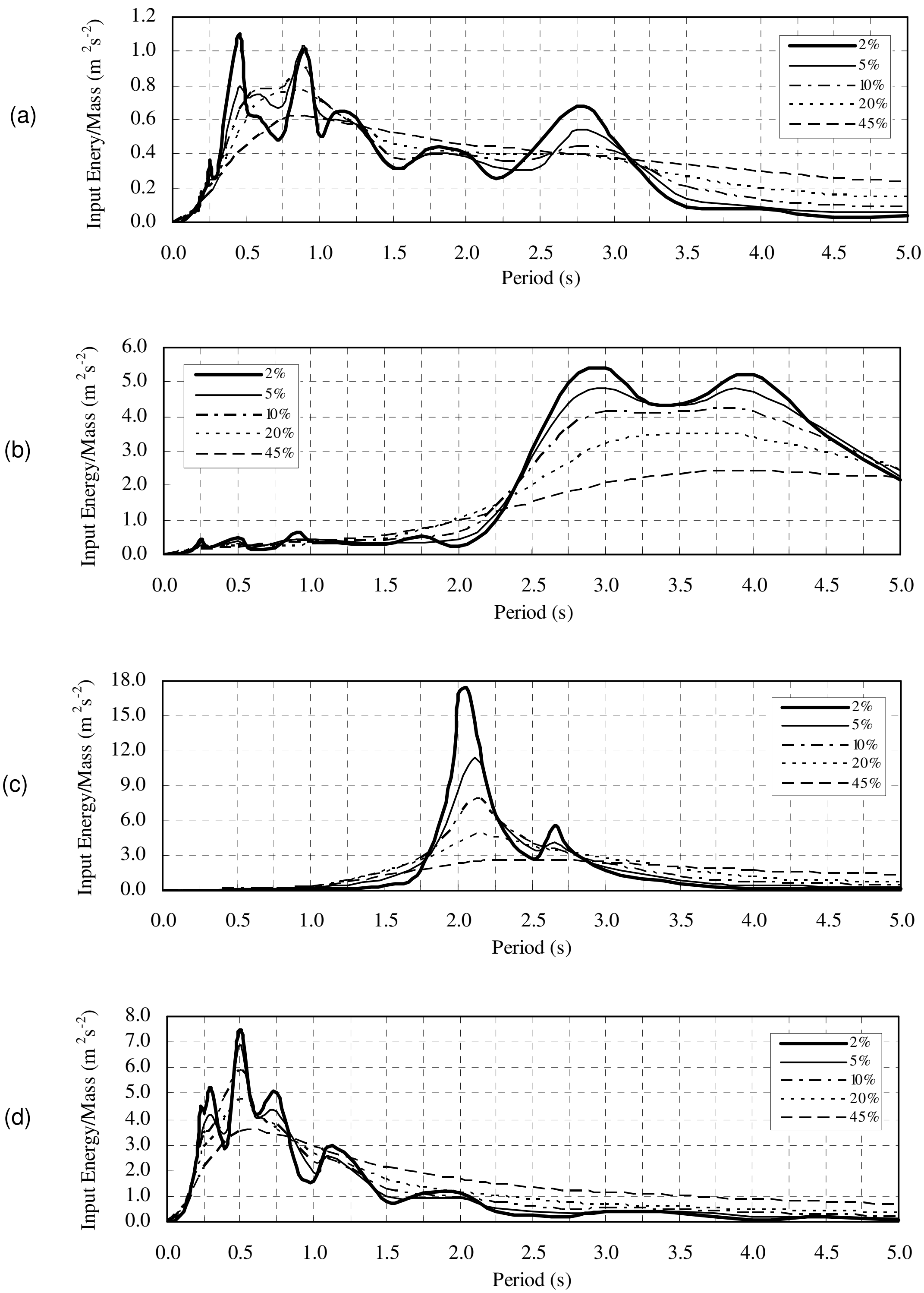

Figure 3. Input-energy-per-unit-mass spectra for (a) El Centro NS 1940, (b) Imperial Valley 1979, (c) Mexico City 1985, and (d) Chile 1985 earthquake records.

The input-energy-per-unit-mass spectra are obtained for single-degree-of-freedom systems for various damping values ranging from 2 to 45% of the critical for the chosen earthquake records, and are depicted in Fig. 3. The spectral values of the input-energy-per-unit-mass corresponding to the first five modes of the structural systems for the four earthquake records are reported in Table 7.

Table 7. Spectral values (S_{ie}) of input-energy-per-unit-mass for the four structural systems.

Structural System	**Horizontal Ground Acceleration**	**Input Energy per Unit Mass (m^2s^{-2})**				
		Mode 1	**Mode 2**	**Mode 3**	**Mode 4**	**Mode 5**
30-Story: Option 1	El Centro NS 1940	0.56	0.78	0.24	0.09	0.04
	Imperial Valley 1979	4.52	0.23	0.21	0.07	0.04
	Mexico City 1985	4.00	0.10	0.05	0.00	0.00
30-Story: Option 2	El Centro NS 1940	0.36	0.75	0.17	0.07	0.04
	Imperial Valley 1979	2.50	0.40	0.30	0.20	0.10
	Mexico City 1985	3.30	0.08	0.04	0.00	0.00
10-Story: Option 1	El Centro NS 1940	0.75	0.08	0.03	0.00	0.00
	Imperial Valley 1979	0.45	0.20	0.12	0.05	0.00
	Chile 1985	7.00	1.00	0.35	0.20	0.10
10-Story: Option 2	El Centro NS 1940	0.56	0.14	0.05	0.00	0.00
	Imperial Valley 1979	0.30	0.20	0.15	0.07	0.00
	Chile 1985	4.75	1.60	0.40	0.25	0.10

The modal contributions to the total input energy are determined with the help of the spectral values of the input-energy-per-unit-mass (Table 7) and the modal characteristics of the structural system (Table 6). The total input energy is determined by combining the modal contributions by the absolute-sum-rule of modal combination.

Total input energies to the "30-story: option 1" structural system during the three seismic events, viz. the El Centro NS 1940, Imperial Valley 1979 and Mexico City 1985, are determined using the proposed method (Table 8). The values of β_n, Sie_n, and M_T for determining the modal contributions to the input energy are as given in Table 6, Table 7, and Tables 2 through 5, respectively. The results of the proposed method are then compared with those of the detailed time history analysis (THA), as depicted in Table 8.

Table 8. Determination of total input energy to the "30-story: option 1" structural system by the proposed method and comparison of results with THA results.

Horizontal Ground Acceleration	**Modal Contributions to Input Energy (kJ)**					**Total Input Energy (kJ)**	
	Mode 1	**Mode 2**	**Mode 3**	**Mode 4**	**Mode 5**		
	$\beta_1 S_{ie1} M_T$	$\beta_2 S_{ie2} M_T$	$\beta_3 S_{ie3} M_T$	$\beta_4 S_{ie4} M_T$	$\beta_5 S_{ie5} M_T$	**Proposed Method**	**THA**
El Centro NS 1940	175.7	81.1	7.8	1.2	0.3	266.1	267.0
Imperial Valley 1979	1418.5	23.9	6.9	0.9	0.3	1450.5	1452.0
Mexico City 1985	1255.3	10.4	1.6	0.0	0.0	1267.3	1278.0

The comparison of the results shows that the values of total input energies as determined by the proposed method are quite close to those determined by THA.

Seismic Response Optimization

The study aims at optimizing the seismic response of a medium-rise (10-story) and a high-rise (30-story) structural system with respect to the total input energy while keeping a check on the acceleration response.

Seismic Response Optimization of High-Rise Structural System

The first natural period of the "30-story: option 1" structural system is 2.712 s (Table 6), which corresponds to peaks in the input-energy-per-unit-mass spectra for the three earthquake records chosen for the high-rise system, viz. El Centro NS 1940, Imperial Valley 1979, and Mexico City 1985. This leads to relatively high values of total input energy for the "30-story: option 1" structural system.

A glance at the input-energy-per-unit-mass spectra (Fig. 3) reveals that a slight decrease in the period or a slight increase in the stiffness of the structural system results in much lesser spectral values of the input-energy-per-unit-mass for all the three earthquake records. Hence, it is aimed to increase the stiffness of the structural system by increasing the sectional areas of various elements, and by using a richer concrete (or higher modulus of elasticity), to achieve a period to around 2.4 s. The structural configuration, i.e. the story height, and the bay-width etc., is maintained. The resulting structural system (30-story: option 2), obtained after a number of trials, has the first period of 2.421 s (Table 6) leading to a lesser total input energy than the original system (30-story: option 1). A comparison of the spectral acceleration values for the two periods, 2.712 s and 2.421s, reveals that the acceleration response remains almost the same. The total input energy to the "30-story: option 2" structural system is determined using the input-energy-per-unit-mass spectra. The results are depicted in Table 9.

Table 9. Determination of total input energy to the "30-story: option 2" structural system.

Horizontal Ground Acceleration	Modal Contributions to Input Energy (kJ)					Total Input Energy (kJ)
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	
	$\beta_1 S_{ie1} M_T$	$\beta_2 S_{ie2} M_T$	$\beta_3 S_{ie3} M_T$	$\beta_4 S_{ie4} M_T$	$\beta_5 S_{ie5} M_T$	
El Centro NS 1940	112.8	78.0	5.6	0.8	0.3	197.5
Imperial Valley 1979	783.3	41.6	9.9	2.4	0.8	838.0
Mexico City 1985	1034.0	10.4	1.7	0.0	0.0	1046.1

Seismic Response Optimization of Medium-Rise Structural System

The "10-story: option 1" structural system reports a period of 0.538 s (Table 6). A glance at the input-energy-per-unit-mass spectra (Fig. 3) reveals that a slight increase in the period or a slight decrease in the stiffness of the structural system results in lesser spectral values of the input-energy-per-unit-mass for all the three earthquake records, viz. the El Centro NS 1940, Imperial Valley 1979, and Chile 1985.

In this case, it is aimed to decrease the stiffness of the structural system by decreasing the sectional areas of various elements, and by using a lower grade of concrete (or lower modulus of elasticity), to achieve a period to around 0.75 s. The resulting structural system (10-story: option 2), obtained after a number of trials, has the first period of 0.750 s (Table 6) leading to a lesser total input energy than the original system (10-story: option 1). A comparison of the spectral acceleration values for the two periods, 0.538 s and 0.750 s, reveals that the acceleration response also improves considerably for all the three earthquake records. The results are depicted in Tables 10 and 11.

The results are compared for the original (option 1) and the optimized (option 2) high-rise and medium-rise structural systems, and are reported in Table 12 as ratios. These show improvement in the seismic response/performance of the optimized (option 2) structural systems over the original (option 1) ones. The input energies reduce to 71.5% and 72.5%, on an average, for the high-rise and medium-rise structural systems, respectively.

Table 10. Determination of total input energy to the “10-story: option 1” structural system.

Horizontal Ground Acceleration	Modal Contributions to Input Energy (kJ)					Total Input Energy (kJ)
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	
	$\beta_1 S_{ie1} M_T$	$\beta_2 S_{ie2} M_T$	$\beta_3 S_{ie3} M_T$	$\beta_4 S_{ie4} M_T$	$\beta_5 S_{ie5} M_T$	
El Centro NS 1940	75.3	2.3	0.2	0.0	0.0	77.8
Imperial Valley 1979	45.2	5.8	1.1	0.2	0.0	52.3
Chile 1985	702.5	29.0	2.5	0.6	0.2	734.8

Table 11. Determination of total input energy to the “10-story: option 2” structural system.

Horizontal Ground Acceleration	Modal Contributions to Input Energy (kJ)					Total Input Energy (kJ)
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	
	$\beta_1 S_{ie1} M_T$	$\beta_2 S_{ie2} M_T$	$\beta_3 S_{ie3} M_T$	$\beta_4 S_{ie4} M_T$	$\beta_5 S_{ie5} M_T$	
El Centro NS 1940	54.9	4.2	0.4	0.0	0.0	59.5
Imperial Valley 1979	29.4	6.1	1.2	0.2	0.0	36.9
Chile 1985	465.8	48.5	3.1	0.8	0.2	518.4

Table 12. Comparison of results of the original and the optimized structural systems.

Horizontal Ground Acceleration	Ratio of Total Input Energy	
	High-rise (30-story)	Medium-rise (10-story)
El Centro NS 1940	0.742	0.765
Imperial Valley 1979	0.578	0.706
Mexico City 1985 (for High-rise) / Chile 1985 (for Medium-rise)	0.825	0.705
Average	0.715	0.725

Conclusions

(a) The proposed method determines the total input energy to a multi-degree-of-freedom structural system during a seismic activity quite accurately while retaining the simplicity of a single-degree-of-freedom analysis.

(b) The proposed method can be used as a check for the THA results with respect to the total input energy.

(c) This simple method helps in selecting an optimal structural configuration at the design stage by getting an idea of contribution of various modes to the total input energy and, in turn, getting an insight into the seismic behavior of a structural system to a seismic event.

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