

Ninth Canadian Conference on Earthquake Engineering Ottawa, Ontario, Canada 26-29 June 2007

NONLINEAR DYNAMIC BEHAVIOUR OF RC TALL STRUCTURES SUBJECTED TO NEAR-FAULT EARTHQUAKES INCLUDING SOIL STRUCTURE INTERACTION

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ABSTRACT

Characteristics of near-fault earthquakes (NFE) are particularly different from that of the far-field ones. Far-field ground motions are characterized by low peak ground acceleration (PGA) and high frequency; whereas near-fault ground motions have a high peak ground velocity (PGV) and long period pulse. Several recent earthquakes, e.g. 1992 Landers, 1994 Northridge, 1995 Kobe, and 1999 ChiChi earthquake events, have caused substantial damage to near-fault flexible structures.

The nonlinear dynamic behavior of tall reinforced concrete (RC) frame structures subjected to NFE records is influenced by the characteristics of the foundation soil. The assumption of a fixed-base model for this type of structures might not adequately represent their seismic response. Therefore, the seismic performance evaluation analysis should take into account the soil structure interaction (SSI).

In this study, the seismic performance represented by the inter-storey drift of a 20 storey RC frame structure with fixed-base and flexible-base conditions is evaluated. The characteristics of the flexible-base models covers four types of soils, namely; soft soil, medium soil, stiff soil and a rock soil as classified by the International Building Code (IBC). A set of thirteen near-fault acceleration time histories recorded on the four types of soil from major earthquake events is selected for the analysis. Three criteria for scaling the records were considered, namely, same maximum spectral acceleration, same spectral acceleration at fundamental period of fixed-base model, and same spectral acceleration at fundamental period of flexible-base model. The analysis evaluates the effect of SSI on the dynamic behaviour by comparing the response of the flexible-base model to the fixed-base model of the structure when subjected to different earthquakes records on a specific soil type.

It is concluded that for the same type of soil, regardless of the scaling criterion, the nonlinear seismic response of the RC structure is greatly influenced by the acceleration spectrum of the earthquake record. Moreover, the effect of SSI on the inter-storey drift reduces with increasing the shear wave velocity of the foundation soil.

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Introduction

The near-fault ground motions are generally characterized by long duration pulses that subject the structure to very high input energy at the early stage of the record. Several research studies (e.g. Krawinkler et al., 2003; and Alavi and Krawinkler, 2004) have reported that near-fault ground motions, characterized by equivalent pulses (Liao et al., 2001), can cause high values of storey drift in structural members, forcing the structure to behave in the inelastic range (Iwan et al. 2000) and leading to severe damage of the structure. Several investigations have reported that near-fault earthquake (NFE) ground motions caused severe damage to structures. The severity in the structural damage is related to the nature of the ground motion which in turn is related to other parameters among which are the geology and characteristics of foundation soil.

For tall structures, the presence of a flexible foundation soil influences the nonlinear dynamic behaviour of the structure. If the foundation soil is stiff enough, the dynamic response of the structure is not influenced by the soil characteristics and the structure can be assumed as fixed at its base. If the structure is resting on a flexible medium the dynamic response of the structure will be different from the case of a fixed base condition due to the interaction between the soil and the structure. Therefore a complete dynamic analysis to evaluate the performance level of a structure should consider the effect of SSI in the model.

Performance-based seismic engineering is the modern approach to earthquake resistant design. Seismic performance (performance level) is described by designating the maximum allowable damage state (damage parameter) for an identified seismic hazard (hazard level). Performance levels describe the state of a structure after being subjected to a certain hazard level as: Fully operational, Operational, Life safe, Near collapse, or Collapse (Vision 2000, 1995; and FEMA 273/274, 1997). Overall lateral deflection, ductility demand, and inter-storey drift are the most commonly used damage parameters.

In this study, the seismic performance represented by the inter-storey drift of a 20-storey RC frame structure with Fixed-base and Flexible-base conditions is evaluated. The characteristics of the Fixed-base and Flexible-base models cover four types of soils, namely, soft soil, medium soil, stiff soil and a rock soil as classified by the International Building Code (IBC 2000). To simulate the impact of soil-structure interaction on the foundation flexibility, the procedures for kinematics effects of ATC-40 recommendations (1996) are used. A set of thirteen near-fault acceleration time histories recorded on the four types of soil from major earthquake events is selected for the analysis. The criterion for scaling the records was considered, namely, same maximum spectral acceleration. The analysis evaluates the effect of SSI on the dynamic behavior of the Flexible-base model of the structure when subjected to different earthquakes records on a specific soil type.

Model of Soil-Structure System

Super-structure parameters and model

A RC moment resisting frame (MRF) building is used in this study; a 20-storey building to represent a tall structure. The structure was designed according to the National Building Code of Canada (NBCC, 1995) and CSA (1994) using seismicity characteristics of Victoria city in British Colombia, Canada, as a highly seismic zone in comparison to the rest of the country (El-Sheikh, 2002). The building has 3-bays by 3-bays square plan. The floor-to-floor height of the storeys is 3.6 m. The total height for the 20-storey building is 72 m. Fig. 1 shows the elevation of the two MRF and the cross-section properties of the columns and beams. The slab thickness is taken 180 mm to meet the minimum thickness requirement according to CSA (1994). In the original design, the structure models were assumed to be fixed at the base with no consideration of the soil flexibility. The weight of the structure is input as masses concentrated at the member joints. The total weight per floor per frame is 675 kN, which includes the weight of floor slab, exterior and interior partition walls and mechanical services. Thus, the weight at exterior nodes and interior nodes is taken as 112.5 kN and 225 kN, respectively.

The building is idealized as a two-dimensional frame model (Fig. 1). Beam-column joints are assumed to be rigid and are represented by rigid zones at the elements ends. The length of the rigid zone was assumed to be 0.4 of the depth of the columns and beams in each side of the joint. The beams are modelled as linear elastic elements with two inelastic flexural springs (plastic hinges) for flexural deformation at each end as shown in Fig. 1. A trilinear pinching model is used to model the nonlinear rotation at each end. Shear deformation is assumed to be elastic. The columns are modelled as an elastic element with two inelastic multi-spring elements at each end as shown in Fig. 2. The multi-spring elements are capable of simulating the axial and flexural nonlinear behaviours taking into account the coupling effect between flexure and axial force.

The trilinear/bilinear model SS3 shown in Fig. 3a was used to represent the steel bars (Li, 1999). A bilinear skeleton curve with specified hysteretic parameters was used for the current analysis with the stress-strain properties provided for each steel bar. The concrete behaviour was represented using the bilinear model CS2 (Li, 1999) shown in Fig. 3b. The parameters for the steel and the concrete skeleton curves are listed in Tables 1a and 1b respectively. Linear shear deformation was assumed.

The specified compressive strength of concrete was $f_c' = 25$ MPa, the specified yield strength of steel was $f_y = 400$ MPa and the concrete density is $\gamma_c = 24$ kN/m³. The Modulus of elasticity of concrete was calculated according to clause 8.6.2.2 (CSA, 1994).



Figure 1. (a) Elevation and cross-section properties of the studied 20-storey building MRFs (Fixed-base model), (b) Modelling of the foundation supports, (c) Multi-spring model.



Figure 2. Modelling of columns: (a) Hysteresis model for steel, SS3; and (b) Concrete material bilinear model, CS2. (Li, 1999).

Parameter	Value
Skeleton curve parameters v and κ	1.0
Post-yielding parameter β	0.01
Parameter ϕ to direct unloading	0
Unloading stiffness degrading parameter γ	0.2
Unloading control parameter θ	0.75

Table 1a. Parameters used for element SS3 (Li, 1999).

Table 1b. Parameters used for element CS2 (Li, 1999).



Figure 3. Modelling of the: (a) vertical; and (b) horizontal and rotational springs (Li, 1999).

Soil structure interaction (SSI) representation

The soil-structure model should be chosen cautiously in order to represent adequately the overall system. The soil physical characteristics are assumed as a lumped parameter system at the foundation level represented by the stiffness coefficients. Thus, a set of two translational and rotational springs (Fig. 1b) are placed at the base of the ground floor columns to model the foundation soil. The spring stiffness values are determined according to the shear wave velocity and Poisson's ratio of the geological medium that supports the structure.

In representing the effect of SSI, the spring models used for the foundation soil are assumed to have compression resistance with no tension. The nonlinear behaviour of soil under dynamic cyclic strain can be represented by complex hysteretic models that have several control parameters which are difficult to evaluate. In this study, the nonlinear behaviour of soil is simplified by assuming a degrading bilinear model with no tension for the vertical springs, as shown in Fig. 3a. The yielding force of the springs, F_y , is defined by the ultimate soil bearing capacity multiplied by the effective foundation area. The estimated values of the ultimate bearing capacity are obtained from ATC-40 (1996). The post-yield stiffness is assumed as 0.1 of the initial stiffness. The horizontal and rotational springs are assumed to have a linear elastic behaviour with not tension (Fig. 3b).

Due to the effect of SSI, the damping of the structure will be different from that of a Fixed-base condition. The reason for this is that the presence of soil introduces additional damping in the soil-structure system in addition to the radiation damping. The current study is mainly concerned with the investigation of the effect of soil conditions and the characteristics of the ground motions on the nonlinear dynamic performance of the structure through the estimation of the maximum inter-storey drift (ID_{max}). Thus, the effects of radiating damping on the response calculations are not included herein and it is assumed that only the material damping is present in the system. This is accounted for by using Rayleigh damping as a combination of the stiffness matrix and the mass matrix.

The structure is assumed to be resting on a mat foundation of plan dimensions $21x21 \text{ m}^2$ with thickness of 1 m and embedment of 2.5 m. The flexibility of the foundation soil is accounted for by using concentrated equivalent horizontal, vertical and rotational springs placed at the ground floor level of the structure as shown in Fig. 1b. This representation is found to be a satisfactory model for soil structure interaction (Wolf, 1985; FEMA 440, 2005). The procedure outlined in FEMA 440 (2005), developed by Gazetas (1991), is applied for the estimation of springs' stiffnesses.

The values of the stiffnesses of the springs are dependent on the mechanical characteristics of the soil material, the dimensions of the foundation, and its embedment length. The mechanical characteristics of the foundation soil medium are represented by the effective shear modulus *G*, the mass density ρ_o , and Poison's ratio *v*. At low strain, the maximum shear modulus G_0 is related to the shear wave velocity C_s

according to $C_s = \sqrt{\frac{G_0}{\rho_0}}$ and $G_0 = \frac{\gamma}{g}C_s^2$ (ATC-40, 1996), where γ is the unit weight of the soil and g is

the gravity acceleration.

Input Ground Motion

A set of thirteen NFE acceleration time histories recorded on different soil types is used to assess the nonlinear dynamic response of the 20-storey frame structure including SSI. The records were retrieved from the PEER web site (PEER, 2005). Table 2 shows the properties of the NFE records.

The records were divided into four groups with respect to the type of soil. The four soil types; E, D, C and B are specified with respect to a given range of the shear wave velocity, C_s , shown in Table 3. In order to have a unified characteristic of the input motions, the scaling criterion is used for the records. The set corresponds to scaling the earthquake records on the basis of maximum pseudo spectral acceleration ($S_{a,max}$) as given by the International Building Code (IBC 2000). The $S_{a,max}$ of the records are scaled to match those of the IBC code for the corresponding soil profile as shown in Fig. 4. The second set is obtained by scaling the records such that the spectral acceleration of the record matches that of the IBC at the computed period of the Fixed-base (i.e. with no springs for SSI representation) structure model, $S_a(T_{fixed})$. The third set is obtained by scaling the records such that the spectral acceleration of the record matches that of the IBC at the computed period of the structure model with Flexible-base condition (i.e. with springs that represent SSI), $S_a(T_{fixeible})$.

Soil type	Earthquake	Date	Station & M Component		R (km)	PGA (a)	PGV (cm/s)
E	Imperial Valley	1979/10/15	Impall/H-E03140	6.5	9.3	0.266	46.8
	Kobe	1995/01/16	Kobe/TAK000	6.9	0.3	0.611	127.1
	Westmorland	1981/04/26	Westmorl/WLF225	5.8	0-15	0.199	16.4
D	Cape Mendocino	1992/04/25	Capemend/PET090	7.1	9.5	0.662	89.7
	Central California	1960/01/20	Ctrcalif/B-HCH271	-	14.9	0.063	3.6
	Chi-Chi, Taiwan	1999/09/20	Chichi/CHY101-N	7.6	11.14	0.44	115.0
С	Cape Mendocino	1992/04/25	Capemend/RIO270	7.1	18.5	0.385	43.9
	Coyote Lake	1979/08/06	Coyotelk/G06230	5.7	3.1	0.434	49.2
	Duzce, Turkey	1999/11/12	Duzce/1058-E	7.1	0.9	0.111	14.2
В	Anza (Horse Cany)	1980/02/25	Anza/PFT135	4.9	13.0	0.131	5.1
	Cape Mendocino	1992/04/25	Capemend/CPM000	7.1	8.5	1.497	127.4
	Northridge	1994/01/17	Northr/PUL194	6.7	8.0	1.285	103.9
	N. Palm Springs	1986/07/08	Palmspr/WWT180	6.0	7.3	0.492	34.7

Table 2. NFE ground motions recorded on soil types E, D, C and B.

Table 3. Properties of the considered four soil types.

Soil type (IBC)	Soil profile	Range of shear wave velocity <i>C</i> s	Average Shear wave velocity (m/s)	Poisson's ratio v	Initial shear modulus G _o (kN/m ²)x10 ³	Effective shear modulus G=0.42G _o (kN/m ²)x10 ³
E	Soft soil	<i>C</i> _s < 180	160	0.3	4.4	1.84
D	Stiff soil	180 < <i>C_s</i> < 360	270	0.3	13.7	5.75
С	Very dense soil and soft rock	$360 < C_s < 750$	564	0.3	67.5	28.40
В	Rock	$750 < C_s < 1500$	1125	0.3	335.0	141.00

Results and Discussion

Results of the nonlinear time history analysis of the 20-storey structure with Flexible-base models in the form of the fundamental period, the distribution of the shear force and the inter-storey drift were obtained. The following is a discussion of the results.

Free vibration analysis

The eigenvalue analysis is conducted to demonstrate the effect of including the soil parameters in predicting the fundamental period of the structure. The fundamental period of the Fixed-base and Flexible-base models were computed for the structure. Table 4 shows the elongation of the fundamental period of the Flexible-base model for different types of foundation soil. The soft soil condition (soil type E) gives the highest fundamental period compared to the other cases, showing the importance of including soil parameters that lead to a lengthening of the period. The stiff soil profile (type D) and the very dense soil (type C) result in higher values of the fundamental period of the Flexible-base model, whereas the case of rock material (type B) results in an almost identical value of the period compared to the Fixed-base model indicating that the foundation soil medium is rigid enough. This shows that for soil type B there is no soil structure interaction effect and the structure can be modelled using a fixed-base condition.

Soil Type	Fixed base - model (sec)	Flexible base model (sec)	Ratio of periods T _{Fixed} /T _{Flexible}
E	3.28	9.74	0.34
D	3.28	5.70	0.57
С	3.28	3.93	0.83
В	3.28	3.44	0.95

Table 4. Fundamental period, T, of the 20-storey building.

Influence of SSI on Inter-storey Drift and shear distribution

The response of the Fixed-base and Flexible-base models of the 20-storey building when subjected to a set of NFE records having the spectral accelerations shown in Fig. 4 for soils types E, D, C and B as classified by the IBC (2000) code, is evaluated. Figs. 5 and 6 show the distribution of the inter-storey drifts and the shear force of the 20-storey building for Fixed-base and Flexible-base models when subjected to NFE records.

It is interesting to note that the increase in inter-storey drift due to SSI for both Imperial Valley and Chichi records is characterized by a more uniform ID along the height with higher ID values at the lower level storeys when compared to the Fixed-base model. This can be attributed to the lumped rotation mode at the lower floor of the building.

Fig. 6 shows the distribution of the maximum storey shear ($V_{i,max}$) –to– the maximum base shear ($V_{base,max}$) ratio over the normalized height of the 20-storey frame for soil types E, D, C and B. From the figure, it can be seen that the SSI represented by the reduction in the maximum base shear of the Flexible-base model compared to the Fixed-base model reduces with increasing the soil stiffness (i.e. from soil type D to soil type B). For flexible soil types E and D, it can be seen that the SSI effects on the maximum base shear is dependent on the characteristics of the NFE record, even though the records are scaled to the same $S_{a,max}$.

Influence of scaling criteria on SSI effects

From Table 5, it can be seen that SSI effects vary significantly according to the characteristics of the NFE record. This observation is more pronounced in less stiff soil, i.e. soil types E, D and C, and diminishes for soil type B. On the other hand, it can be seen from the table that the SSI is sensitive to both the scaling criterion and the indicator representing the SSI (i.e. ID_{SSI} or V_{SSI} ratios). For example, for soil type E, the Imperial Valley earthquake has the highest SSI effects on the building according to both ID_{SSI} and V_{SSI} ratios, if the $S_{a,max}$ scaling criterion is chosen. Yet, for the same soil type, Westmorland earthquake gives the highest SSI effect if ID_{SSI} ratio is considered and the second highest SSI effect if V_{SSI} ratio is considered. Moreover, it can be seen from the table that there are some cases where considering the SSI can result in a beneficial, i.e. un-conservative, predicted performance in the analysis of a tall building subjected to a particular NFE record. This can be seen in the case of Central California earthquake recorded on soil type D.



Figure 4. Spectral acceleration of the NFE records scaled to match $S_{a,max}$ of the IBC (2000) for soil types E, D, C and B.



Figure 5. Distribution of inter-storey drift ratio over the normalized height of the 20-storey building for Fixed-base and Flexible-base models subjected to NFE records (scaled to match *S_{a,max}* of IBC 2000) for soil types E, D, C, and B



Figure 6. Distribution of maximum store shear-to-maximum base shear ratio over the normalized height for Fixed-base and Flexible-base models subjected to NFE records (scaled to match *S*_{*a*,*max*} of IBC 2000) for soil types E, D, C, and B.

Conclusions

The nonlinear performance of two-dimensional 20-storey reinforced concrete (RC) building frame subjected to a set of thirteen near-fault earthquake (NFE) ground motions having different characteristics has been presented. The seismic performance of the structures was evaluated using the inter-storey drift and shear distribution along the height of the building. To study the effect of soil conditions on the seismic performance of the building with Flexible-base model is compared to that of a Fixed-base model. The characteristics of the Flexible-base model cover four types of soils, namely, soft soil, medium soil, stiff soil and a rock soil as classified by the International Building Code (IBC). In the nonlinear analysis, the criterion for scaling the records was based on the same maximum spectral acceleration. Based on the conducted analysis, the following conclusions and recommendations are reached:

- 1- SSI effects could vary significantly according to the characteristics of the NFE record, the scaling criterion and the indicator representing the SSI.
- 2- The increase in inter-storey drift (ID) due to SSI for some NFE records is characterized by a more uniform ID along the height with higher ID values at the lower level storeys when compared to the Fixed-base model. This can be attributed to the lumped rotation mode at the lower floor of the building.
- 3- In order to evaluate the seismic performance of structures on flexible soil in the near-fault region, it is recommended to conduct a site study to determine a design ground motion record or spectrum for the specific site soil conditions. On the other hand, attention should be paid to whether considering SSI is beneficial or detrimental to the seismic performance of the structure when subjected to a particular ground motion. Consequently, in the former case, other alternative measures, e.g. by considering several design ground motions, might need to be considered.

It should be noted that the above conclusions are based on limited analysis of specific structures and specific NFE records combinations. More analysis covering more cases will need to be conducted to refine or generalize the conclusions.

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Table 5. Influence of scaling criteria of NFE records on the SSI represented by the ratios of the maximum inter-storey drift and base shear of the Flexible- and Fixed-base models of the 20-storey building for soil types E, D, C and B.

Soil	Earthquake	Scaling criteria						
type		S _{a,max} = S _{a,max} of IBC		$S_a(T_{Fixed}) =$ $S_a(T_{Fixed})$ of IBC		$S_a(T_{Flexible}) =$ $S_a(T_{Flexible})$ of IBC		
		ID _{SSI} ratio ^a	V _{SSI} ratio ^b	ID _{ss} ratio _I	V _{SSI} ratio	ID _{SSI} ratio	V _{ssi} ratio	
E	Imperial Valley	1.17	0.51	1.20	0.53	1.11	0.61	
	Kobe	1.07	0.95	1.02	0.97	1.22	1.08	
	Westmorland	1.06	0.64	0.87	0.76	1.25	0.94	
D	Cape Mendocino	1.04	0.71	1.09	0.78	1.00	0.83	
	Central California	0.90	0.82	0.98	0.91	1.05	1.01	
	Chichi	1.61	0.80	1.75	0.84	2.12	0.96	
С	Cape Mendocino	0.98	0.93	0.98	1.02	0.98	1.02	
	Coyote Lake	1.02	0.94	1.01	0.96	1.01	0.97	
	Duzce Turkey	1.11	0.83	1.15	0.81	1.13	0.82	
В	Anza	0.99	0.99	1.00	1.00	1.00	1.00	
	Cape Mendocino	1.02	0.97	1.03	0.98	1.02	0.98	
	Northridge	0.99	0.98	1.00	0.99	1.00	0.99	
	Palm Springs	0.99	0.97	0.99	0.98	0.99	0.97	

^a ID_{SSI} ratio =
$$\frac{ID_{max} \text{ for Flexible-base model}}{ID \text{ for Fixed-base model}}$$

 ID_{max} for Fixed – base model

^b
$$V_{SSI}$$
 ratio = $\frac{V_{base,max} \text{ for Flexible-base model}}{V_{base,max} \text{ for Fixed-base model}}$