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PROBABILISTIC STUDY ON ACCIDENTAL ECCENTRICITY ON BUILDINGS USING THE MONTE CARLO METHOD

J. De-la-Colina[1] and B. Benitez[2]

ABSTRACT

The variability of dead load, live loads and stiffness of columns introduce uncertainties in the calculation of accidental eccentricities for building seismic design. This paper uses distributions of live loads directly obtained from loads on real buildings measured by other researchers. Probability density functions are computed for three uses of structures: apartments, schools and offices. Two models of buildings representative of real structures are studied: one with square plan and the other with rectangular plan. Both models are also proposed with two different heights. Accidental eccentricity is evaluated in each level by using the Monte Carlo method. With the results obtained, probabilities of exceedance of typical values of accidental eccentricity indicated by building codes are computed.

Introduction

The distribution of loads on the slabs of a building affects the behavior of its structure during an earthquake. Taking into account the distribution of weight located on a floor, including people, objects and structural elements, a center of mass can be estimated. Considering the stiffness of columns, beams, and walls, a center or stiffness can also be estimated. For design purposes, it is considered that the forces caused by an earthquake are applied on the centers of mass. The shear force at each story is defined as the summation of all the lateral forces applied above the story being analyzed. The resultant of this shear force passes by the shear center.

Regarding the masses, real data were analyzed and the PDF of the position of the shear center was estimated. The contribution of this paper is that the analysis of real data was included to obtain de PDF's of magnitude and mass positions in schools, offices and apartments, as well as the consideration of normal variation of stiffness of the elements.

Currently, construction codes use the length of the largest dimension of the building plan (*b*) multiplied by a factor β to estimate accidental eccentricity (e_a). For instance, the Uniform Building Code (ICBO 1997) considers β = 0.05; the National Building Code of Canada (NRCC 1985) β = 0.10, similarly to the Mexico City Construction Code (Gobierno 2004).

[1] Assistant professor, School of Engineering, Autonomous University of the State of Mexico, Toluca, Mexico 50000. E-mail: jaime_delacolina@yahoo.com
[2] Graduate student, Autonomous University of the State of Mexico, Toluca, Mexico 50000

The main purpose of this investigation is to study the variation of the β factor of accidental eccentricity, depending on the slab system and the use of the structure. Live load distribution on building was analyzed, considering three uses of structures: schools, offices and apartments, combined with three slab systems: solid slab, waffle slab and steel deck. More precise probability density functions of both the position of the center of mass and the shear center were obtained. A PDF with normal distribution for the stiffness of the elements was considered, similarly to what De-la-Llera and Chopra (1994) as well as Ramsay et al. (1979) did. With the results obtained, accidental eccentricity recommendations are given for the different combinations of slab system and live load used.

Eccentricities

Usually, the center of stiffness and the shear center do not have the same location. The distance between the shear center and the center of stiffness is called eccentricity. Tso (1990) distinguishes two eccentricities. One of them is *floor eccentricity*, defined as the horizontal distance between the center of stiffness and the center of mass of the story. The other one is *story eccentricity* (e_s), which is defined as the distance between the shear center of the story and the center of stiffness. *Story eccentricity* is studied in this paper.

Torsion on Buildings

If the position of the center of stiffness and the position of the shear center are the same ($e_s = 0$), there are no seismic torsion effects on the structure. If the positions of such centers are not the same, there is a twisting moment on the building. It is said that the building is torsionally unbalanced.

At the stage of structural design, it is not possible to precisely know the position of the center of stiffness and the position of the shear center during an earthquake. It is difficult to locate the center of stiffness because of the variations of stiffness on the structural elements. Such variations are due to uncertainties on both the properties of the materials and the dimensions of the structural elements. The shear center depends on the distributions of weights on each story and presents also many variations. Thus, to calculate the torsional moment it is necessary to consider an *accidental eccentricity* (e_a) to take into account these uncertainties.

The Mexico City Building Code (Gobierno 2004) indicates that accidental eccentricity must be calculated as 10% of the largest dimension of the building plan (b), measured orthogonally to the direction of the earthquake. This is specified for all values of live loads and types of slab systems, light or heavy.

In the estimation of the design eccentricity (e_d), the code does not take into account the use of the structure. However, live loads for a school are quite different from live loads for offices or apartments. The effect of these differences also depends on the type of slab (solid slab, waffle slab or steel deck slab). The dead load to live load ratio is different.

Previous Research

Ruiz (2001a, 2001b) as well as Ruiz and Guillén (2003) performed studies of the variation of the dead load magnitude on buildings. Recommendations for live load magnitudes as a function of the tributary area were given. In such study the distributions of live loads on different stories analyzed were presented for classrooms, apartments, and offices. The authors did not estimate a probability density function (PDF) for the position of the shear center nor for the distribution of live loads. Such functions are useful to perform a probabilistic study on accidental eccentricity.

De-la-Colina and Almeida (2004) performed an investigation on accidental torsion on buildings. In their study, accidental eccentricity was studied using a probabilistic approach based on Monte Carlo simulations. A model with lateral bilinear resisting elements with fixed plan distribution was used. The distribution function of loads was assumed to be triangular, due to lack of information. Probabilities of exceedance of the demands of ductility were evaluated. It is clear that triangular distribution for live loads may not accurately represent the live load distribution.

The study of the spatial variation of the center of stiffness has several variables. Ramsay et al. (1979) found out that the probability density function of the stiffness of resisting elements has a normal distribution. De-la-Llera and Chopra (1994) performed a study that used the normal distribution for the stiffness of lateral resisting elements. Heredia et al. (2001) studied accidental eccentricity on symmetric systems considering uncertainties on stiffness and analyzed the structural response as a function of both the coefficients of variation and the correlation of the rigidity among resisting axes, using Monte Carlo simulation. Given the nominal characteristics of symmetry of the analyzed systems, the mass center was assumed to have the same position as the geometric center. The authors also modeled lateral stiffness as the product of a random variable with lognormal probability density function with the nominal value of the stiffness.

Scope of the Study

Three structure uses were considered. For schools, live loads of 34 plans were analyzed, giving a total of 13,700.19 m^2. For offices, 14 plans, with a total of 14,890 m^2. For apartments, 15 plans, 2624 m^2. Three slab systems were considered: solid (flat) slab, waffle slab and steel deck. For apartments, only flat slab was considered, because this is the only type of slab system used in this case. For each structural use, both rectangular and a square plan distributions of columns were proposed, all columns being symmetric. These plans are regarded as representative for most commonly used distributions. As for the height, 5 and 10-story buildings were analyzed for the use of offices, 3 and 5-story buildings for apartments, and 2 and 3-story buildings for schools. These numbers of stories are the most commonly found in Mexico.

The definition of the PDF´s of column stiffness is not studied in this paper. For the simulation, previous results of Ramsay (1979) and of De-la-Llera and Chopra (1994) were used, which considered normal PDF´s. Only framed structures are considered in this study. Reliability analysis is out of the scope of this paper.

Probabilistic Study of Accidental Eccentricity

For the calculation of the PDF´s of live loads, histograms like the one shown in Fig. 1 were analyzed. All this information was obtained from the investigations of Ruiz (2001a, 2001b) and Ruiz and Guillén (2003). Chi-square and Kolmogorov-Smirnov tests were performed. Results are shown in Table 1, along with mean and standard deviation values.

y	s1	s2	s3	s4	s5	s6	s7	s8
1	20.38	34.9	91.48	34.9	33.43	78.44	44.22	26.18
2	40.72	73.63	75.19	64.27	58.19	65.12	68.01	37.05
3	32.47	74.06	50.32	100.55	121.25	58.15	81.3	31.17
4	16.71	40.76	63.12	52.43	56.59	73.14	37.25	8.99
				a)			total	1744.37

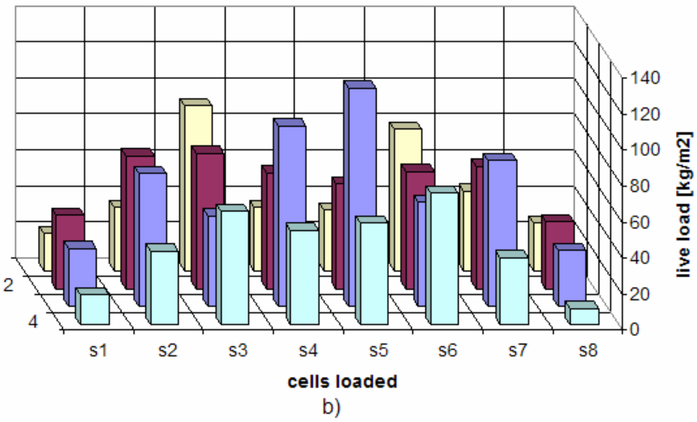

Figure 1. An example of the histogram of live loads used to calculate PDF´s.

Table 1. Results of goodness-of-fit tests performed to live load data.

Structural use	Mean of live load center-of-mass normalized position	Standard deviation of live load center-of-mass position	Chi square	Mean of live load Intensity [kg/m2]	Standard deviation of live load intensity [kg/m2]	Kolmogorov Smirnov
Schools	0.495	0.030	didn´t pass	72.4	19.8	didn´t pass
Offices	0.498	0.037	OK	64.89	18.94	OK
Apartments	0.508	0.039	OK	46.37	9.25	OK

In the case of schools, empirical distributions were used, because they did not pass the goodness-of-fit tests. For the calculation of dead load PDF´s the deterministic value of the nominal weight of each slab system was taken as the mean values of the dead loads, and the variation coefficient of 0.10 recommended by Melchers (1987) for this variable. This reference has been used by other investigators, such as De-la-Llera and Chopra (1994). The position of the center of mass of the slab system, that is, dead load, was considered deterministic, located at the geometric center of the slab. Then the position of the center of mass of both live load and dead load was calculated. A PDF for the magnitude of mass and one for the position of the center of mass were obtained for each combination of slab system and use of structure.

In order to calculate the center of stiffness, two plans were proposed: rectangular and square, distributing columns in a symmetric way. Combining these distributions with the normal PDF´s of stiffness of columns used by Ramsay et al. (1979) and De-la-Llera and Chopra (1994), the PDF for the center of stiffness was obtained. A normalized stiffness of columns was considered, with a standard deviation of 0.11.

Once all the PDF´s necessary were obtained, the Monte Carlo method was used for the simulation. Fishman (1996) found out that the Monte Carlo method provides approximate solutions to a variety of mathematical problems through the performance of statistical sampling experiments. Comparing all numerical methods that produce approximate solutions, Monte Carlo method is the one that decreases the error more rapidly.

Random numbers for the simulation were generated with the prime modulus multiplicative linear congruential generator, recommended by Law and Kelton (2000) for being an acceptable and widely tested generator, and also for being able to be used by almost any computer. Random numbers with a uniform distribution were obtained, and then transformed to obtain random variables. The inverse transform method was used to do so, using a very simple numerical method. The transformation to a standard normal distribution was first performed, and then the specific parameters were given, simply by multiplying by the standard deviation and adding the mean value.

Groups and Number of Runs in the Simulation

A total of 272 cases were analyzed, combining use of structure, plan, slab system, height and direction of analysis. The story height was considered constant and equal to 3 meters in all cases. For schools, 60 cases were analyzed, 180 for offices and 32 for apartments. Regarding the number of runs, 1000 runs were performed for apartments, and an average of 950 for schools and offices. Such number of runs allowed to have confidence intervals equivalent to 0.5% of the mean values, with a confidence interval equal to 0.95. These parameters were adequate for the level of accuracy wanted for this investigation.

Analysis of Results

The mean and standard deviation values of each set of runs were obtained. Goodness-of-fit tests were performed on all of them. Only 14 of the 272 cases analyzed did not pass the tests, which represented only 5.14 % of the cases. A very important point is that the probabilities of exceedance for values of the normalized accidental eccentricity ($\beta = e_a / b$) $\beta = 0.05$ and $\beta = 0.1$ were practically zero. The mean values of probability of exceedance shown in the following figures are the probabilities of exceedance of the value of the normalized accidental eccentricity $\beta = 0.02$. Greater values of accidental eccentricity did not produce significant values of probabilities of exceedance for the studied models.

Fig. 2 shows that the maximum probability of exceedance of the value of the normalized accidental eccentricity equal to 0.02 for schools is 0.035. 2-story height with waffle slab had the largest probabilities of exceedance in both directions. Cases with 3-story height show similar results for all combinations. Y-direction (the largest) presents higher probabilities over all.

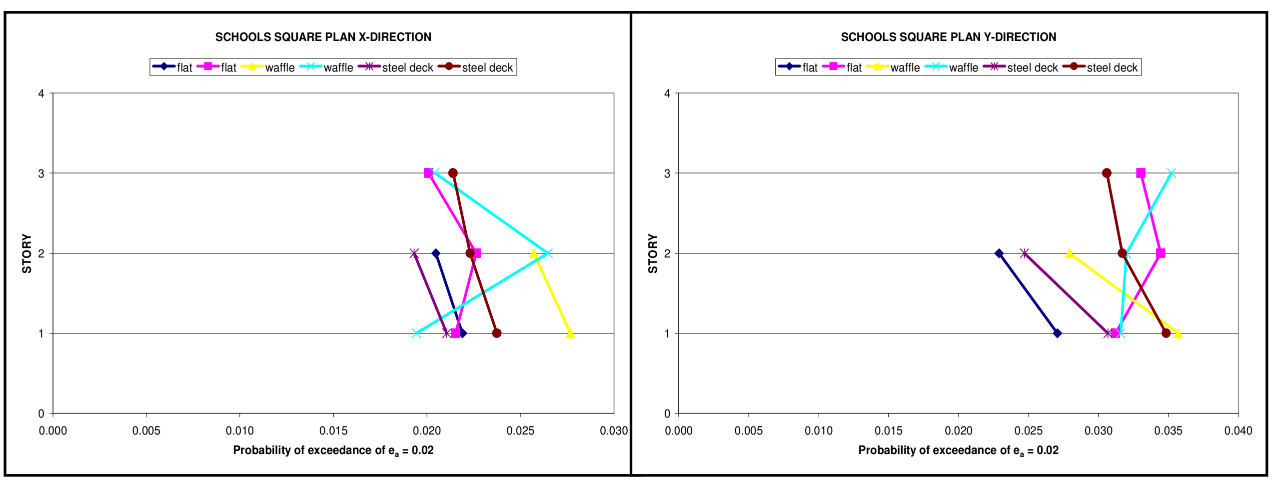

Figure 2. Probabilities of exceedance of the value 0.02 of accidental eccentricity for schools square plan.

For the rectangular plan in schools, the probability of exceedance was 0.01 along the X-direction, and 0.1 along the Y-direction. Flat slab and waffle slab with 2-story height showed the same behavior along the X-direction. Buildings with 3-story height had larger probabilities in the upper stories with flat slab. Y-direction results were similar for all cases, as shown in Fig. 3.

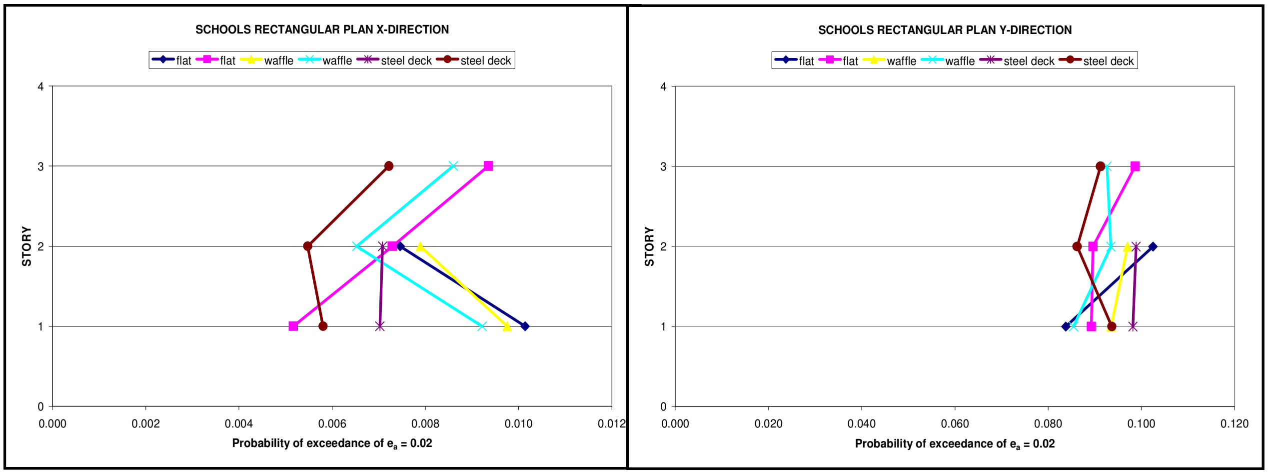

Figure 3. Probabilities of exceedance of the value 0.02 of accidental eccentricity for schools rectangular plan.

For offices, Fig. 4 clearly shows the influence of the height of the structure on the value of the probability of exceedance. In both directions, in both heights and in all slab systems can be seen that the probability of exceedance decreases as long as the height of the story decreases too. The maximum value of the probability of exceedance in this use is 0.01. The difference in slab systems is not very large in 10-story heights, whereas for 5-story heights the difference in slab systems is larger.

For the rectangular plan in offices, the difference in probabilities of exceedance as a function of the height is still clear along the X-direction (large direction), whereas along the Y-direction the variation decreases. The maximum probability of exceedance for the X-direction is 0.045. Along the Y-direction, it is 0.08. The probabilities are larger for the shorter direction. The difference in the slabs tends to be more significant in the upper stories, as shown in Fig. 5.

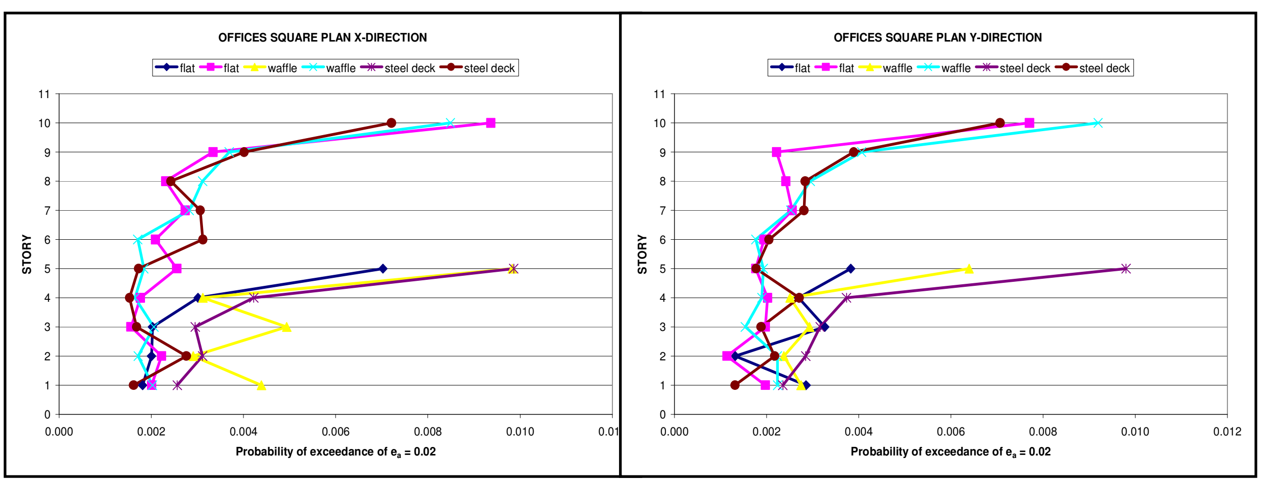

Figure 4. Probabilities of exceedance of the value 0.02 of accidental eccentricity for offices square plan.

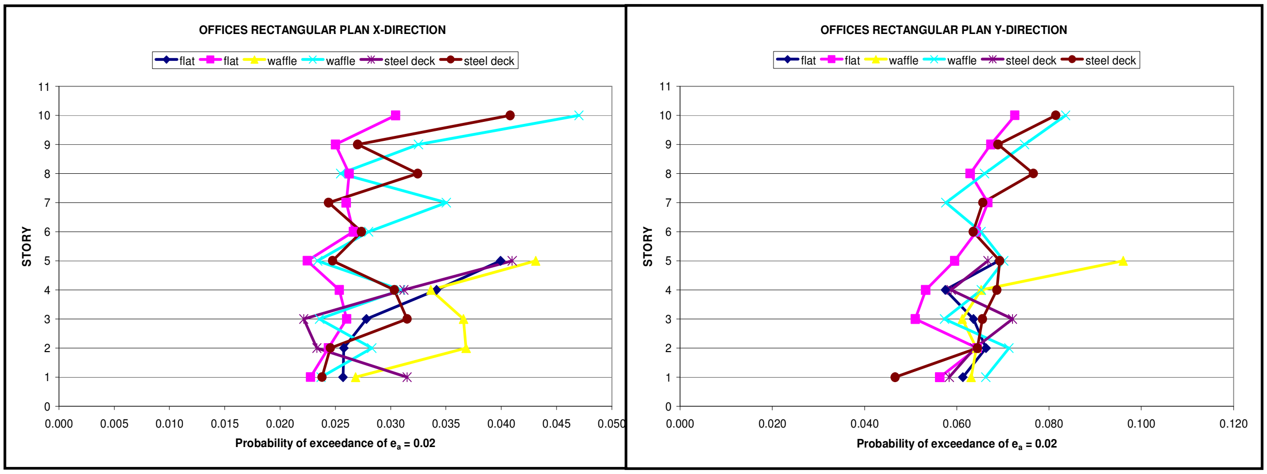

Figure 5. Probabilities of exceedance for the value 0.02 of accidental eccentricity for offices rectangular plan.

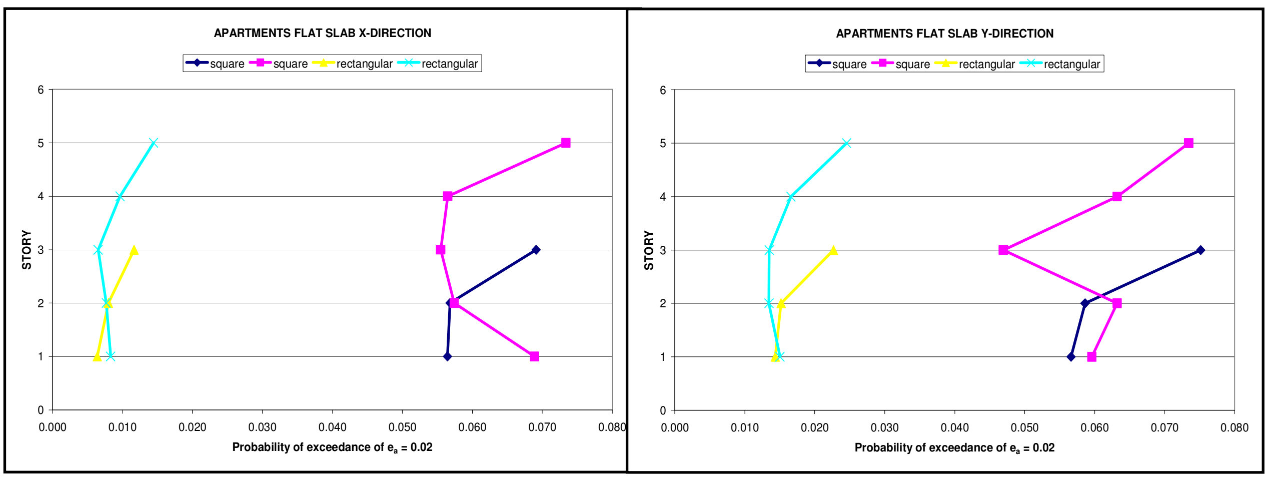

Figure 6. Probabilities of exceedance of the value 0.02 of accidental eccentricity for apartments.

The results for apartments are shown in Fig. 6. For this use, only flat slab was considered. Graphs show both plans and both story heights. The maximum probability of exceedance is similar in both directions. For the square plan, the maximum probability is 0.07, whereas for the rectangular plan is 0.02, being smaller in the X-direction. The increase in the probability of exceedance as the height increases can also be seen in these graphs.

Fig. 7 shows a global comparison of all probabilities of exceedance as a function of use, plan and direction. For offices, the square plan had very similar probabilities of exceedance for both directions, with a maximum value of 0.01. For offices with rectangular plan the values between directions differ approximately in 0.04. The values are larger compared with square-plan offices.

For apartments, probabilities did not vary regarding direction, but they were larger in square plan than in rectangular plan. Square plan had maximum values of 0.075, whereas rectangular plan had maximum values of 0.025.

Schools had large variations between directions in rectangular plan. Y-direction had a maximum of 0.1, whereas X-direction had only a maximum of 0.01. Square plan had an average value of 0.02.

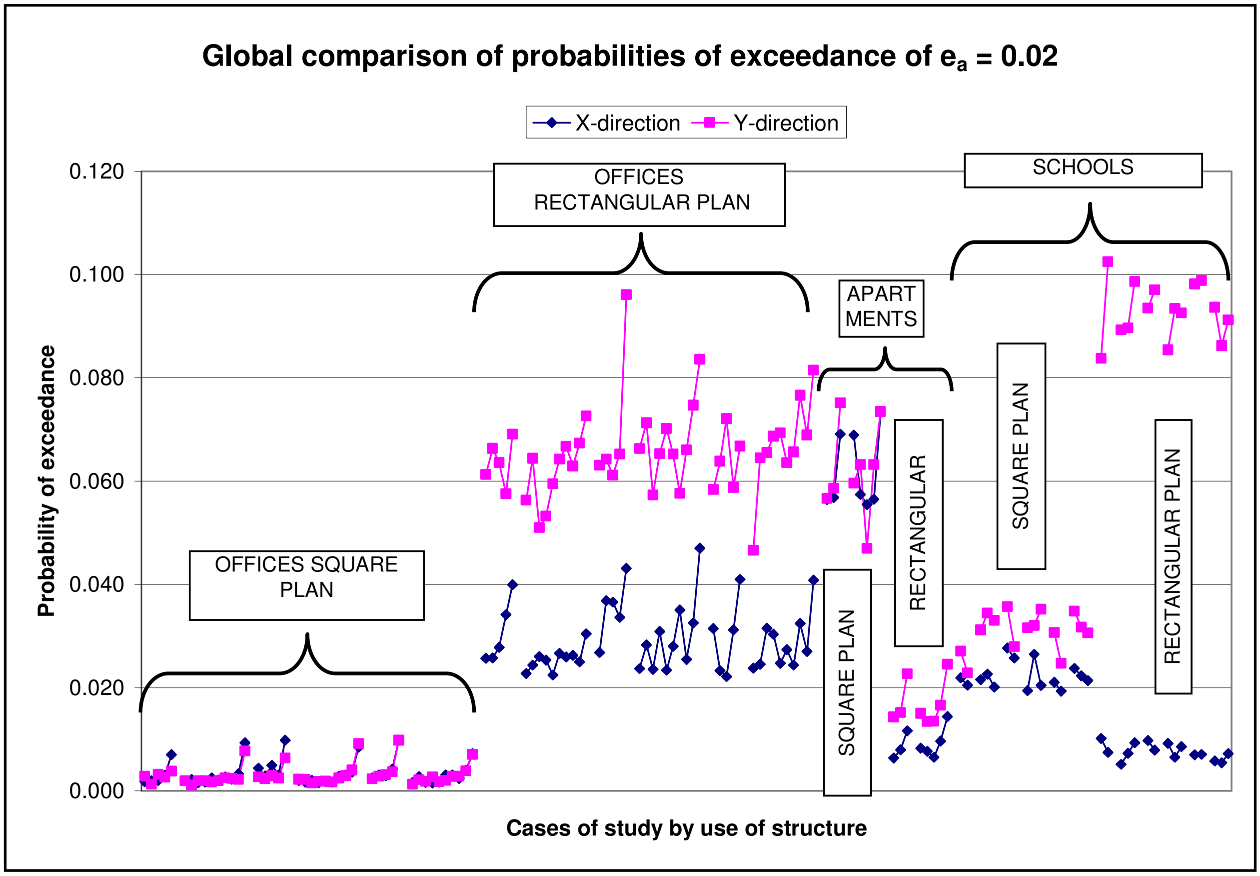

Figure 7. Probabilities of exceedance of the value 0.02 of accidental eccentricity for different use of structure, plan and direction.

Conclusions

Results indicate that the value of the accidental eccentricity recommended by some construction codes of 0.1 and 0.05 times the longest dimension of the plan of the building (*b*) is high. This conclusion is based on the zero probabilities of exceedance estimated for such estimates.

Results also corroborate that upper stories show larger probabilities of exceedance than lower ones, for a given value of accidental eccentricity. This suggests that code recommendations should specify larger coefficients for estimating the accidental eccentricity for supper stories than for lower ones.

Finally, it was observed that computed probabilities of exceedance vary with both the type of structural use (offices, schools, and apartments) and the type of floor system (solid flat slab, waffle slab, and steel deck system).

References

De-la-Colina, J., and C. Almeida, 2004. Probabilistic study on accidental torsion of low-rise buildings. *Earthquake Spectra*, EERI, 20 (1), 25-41.

De-la-Llera, J. C., and A. K. Chopra, 1994. Accidental and natural torsion in earthquake response and design of buildings. Earthquake Engineering Research Center. *Report No. UCB/EERC-94/07*, College of Engineering, University of California at Berkeley.

Fishman, G. S., 1996. *Monte Carlo, Concepts, Algorithms and Applications*. Springer. New York, NY.

Gobierno del Distrito Federal, 2004. *Normas Técnicas Complementarias del Reglamento de Construcciones del Distrito Federal*. México, D. F. México, (in Spanish).

Heredia E. et al., 2001. Torsión accidental en sistemas simétricos por medio de la incertidumbre en la rigidez*. Procedings XII National Conference on Earthquake Engineering*. Guadalajara, Jalisco, México, (in Spanish).

International Conference of Building Officials (ICBO), 1997. *Uniform Building Code*. Whittier, CA.

Law A.M. and W. D. Kelton, 2000. *Simulation modeling and analysis*. McGraw-Hill. New York.

Melchers, R. E., 1987. *Structural Reliability: Analysis and Prediction*. Ellis Horwood, Chichester, W. Sussex, England.

National Research Council of Canada (NRCC), 1985. *National Building Code of Canada*. Ottawa, Ontario.

Ramsay, R., S. A. Mirza, and J. MacGregor, 1979. Monte Carlo study of short time deflections of reinforced concrete beams. *Journal of the American Concrete Institute*. 76, 897-918.

Ruiz, S. et al., 2001a. Cargas vivas máximas de diseño para salones de clase. *Publicación 621, Serie Azul*, Instituto de Ingeniería, UNAM. México, (in Spanish).

Ruiz, S. et al., 2001b. Cargas vivas máximas de diseño para edificios de oficinas en la Ciudad de México. *Publicación 623, Serie Azul*, Instituto de Ingeniería, UNAM. México, (in Spanish).

Ruiz, S. y J. Guillén, 2003. Cargas vivas máximas de diseño para departamentos habitación de interés social. *Publicación 637, Serie Investigación y Desarrollo*, Instituto de Ingeniería, UNAM. México, (in Spanish).

Tso, W. K., 1990. Static eccentricity concept for torsional moment estimations*. Journal of Structural Engineering*, ASCE, 116 (5), 1199-1212.