Ninth Canadian Conference on
Earthquake Engineering
Ottawa, Ontario, Canada
26-29 June 2007

ON DEFORMATION AND ENERGY DISSIPATION CAPACITY OF RC COLUMNS IN FLEXURE

I. Perus[1], K. Poljansek[2] and P. Fajfar[3]

ABSTRACT

The capacity of RC columns is determined in terms of the normalized force – deformation relationship with special consideration of yield and ultimate drifts at cyclic loading, and in terms of dissipated hysteretic energy. The CAE (Conditional Average Estimator) method was applied. The method is based on a special type of multi-dimensional non-parametric regression, and represents a kind of probabilistic neural network. The PEER database, prepared at the University of Washington was used. Due to the limited number of test specimens, only columns which failed in flexure have been considered. Five input parameters were defined and used in calculations: axial load index, index related to confinement, shear span index, concrete compressive strength and longitudinal reinforcement index. The axial load index and the longitudinal reinforcement index exhibit substantial influence on yield drift, ultimate drift and hysteretic energy. The index related to confinement increases the ultimate drift and the hysteretic energy, whereas it does not influence the yield drift. Yield drift, ultimate drift and hysteretic energy first increase and then increase with increasing shear span index. The influence of the strength of concrete is substantial only in the case of the hysteretic energy.

Introduction

Deformation capacity of structural members and whole structures must be known in performance-based seismic design and in assessment of existing structures. Unfortunately, reliable data on deformation capacity of reinforced concrete (RC) structures are still missing. Semi-empirical and empirical approaches are used for determination of the deformation capacity of structural members (e.g. Paulay 1992, Fardis 2003). In many cases, especially for existing structures with poor detailing, the structural elements degrade with increasing number of cycles. The ultimate deformation capacity decreases due to cumulative damage. The energy dissipation capacity could provide valuable information about the extent and importance of the cumulative damage. However, very little data are available for the energy dissipation capacity of RC structural elements.

In the paper the non-parametric empirical CAE method was applied for the determination of the whole normalized force - drift envelope of RC columns, with special consideration of the yield and ultimate drift at cyclic loading, and hysteretic energy dissipation capacity. The authors have already published a paper (Perus 2006), in which the CAE method was summarized and applied for the estimation of the ultimate

[1] Researcher, University of Ljubljana, Faculty of Civil and Structural Engineering, Slovenia
[2] Ph.D. student, University of Ljubljana, Faculty of Civil and Structural Engineering, Slovenia
[3] Professor, University of Ljubljana, Faculty of Civil and Structural Engineering, Slovenia

flexural deformation capacity of rectangular RC columns. This paper presents some results of more recent research aimed at obtaining more complete information on the deformation and energy dissipation capacity of RC columns.

CAE Method, Database, Input and Output Parameters

Description

The CAE method is a non-parametric empirical approach for the estimation of unknown quantities as a function of known input parameters, provided that an appropriate database is available. The detailed description of the method is given in (Perus 2006).

The problem addressed in this paper is the estimation of the normalized force – deformation envelope, defined by discrete points $\delta_i = \delta(f_i)$, where δ_i is drift at normalized force f_i, and some other quantities, for reinforced concrete rectangular columns as a function of known data on column's material, geometrical and loading characteristics. The first and second set of variables are called the output and input parameters, respectively.

As described in (Perus 2006), the results of predictions depend on the smoothness parameter *w* which has to be chosen by the user. All results presented in this paper have been obtained by using the same values of the smoothing parameter as in (Perus 2006), i.e. w_{min}=0.07, w_{max}=0.2. The results represent the best estimates. Measures for dispersion of results are the "local standard deviation", which corresponds to standard deviation in usual statistical studies, and the average prediction error *E*. For details see (Perus 2006).

The CAE approach requires an appropriate database and a numerical analysis for each estimate. This may be a drawback compared to closed form equations derived from traditional regression analysis. On the other hand, the advantage of the CAE approach is its flexibility. There are no fixed functional relations between the input and output parameters. Any number of input parameters (which are contained in the database) can be used, and different databases or different subsets of a database can be employed. Whereas closed form empirical equations may be more convenient for practical applications, the CAE approach can be a very powerful tool in research.

Database

The PEER Structural Performance Database compiled at the University of Washington (PEER 2004) includes data on about 300 rectangular RC column specimens. The basic input data consist of the material and geometrical properties, the mechanical properties of the longitudinal and transverse reinforcing, the confinement details and the test configurations. The test results include force – deflection histories, axial loads, observed damage and failure classification. A subset of the database, which includes 156 test specimens, has been used in our study. For details see (Perus 2006). P-Δ effects were eliminated from the original data in the database.

Input parameters

Different input parameters have been used by different authors for the prediction of element drifts and hysteretic energy. In the presented study, five input parameters were used:

- the axial load index ($P^*=P/P_o$: 0.0 – 0.6),
- the shear span index ($L^*=L/h$: 2.0 – 6.0),
- the concrete compressive strength (f_c': 20 – 120 MPa),
- the confinement effectiveness factor multiplied by conf. index ($\alpha\rho_s^*=\alpha\rho_s f_{ys}/f_c'$: 0.00 – 0.14),
- the longitudinal reinforcement index ($\rho_l^*=\rho_l f_{yl}/f_c'$: 0.04 – 0.45).

Considering the distribution of input parameters in the available database, the bounds indicated above were used (Perus 2006). The bounds were used only for the normalization of the input parameters. All 156 test specimens, including those with input parameters outside of the bounds, were used in calculations.

The axial load index represents the normalized axial load of the column (P is the axial force positive for compression, $P_o = b\, h\, f_c^{'}$, where b represents the width of compression zone and h the depth in the direction of loading). The shear span index represents the shear span-to-depth ratio, where L is the length of the equivalent cantilever defined as the moment-shear ratio at the end section: $L = M/V$. ρ_s is the ratio of the transverse steel parallel to the direction of loading (the ratio between the area of the transverse steel A_{sx} and the area of the confined concrete; $\rho_s = A_{sx}/b\, s_h$, s_h is the spacing of stirrups, the distances are measured from the centerline of the stirrups) and f_{ys} is the yield stress of the transverse steel. The factor α represents the confinement effectiveness factor (Fardis 2003, CEN 2005, Perus 2006). ρ_l is the ratio of the longitudinal reinforcement area and cross-section area and f_{yl} is the yield stress of the longitudinal reinforcement. Note that in the previous paper by the authors (Perus 2006) only the first four input parameters (without ρ_l^*) were used. The first four parameters are used also in the empirical formula for the ultimate drift in Eurocode 8, Part 3 (CEN 2005), which is based on the work by Fardis and co-authors (Panagiotakos 2001, Fardis 2003).

Output parameters

The output parameters are the drifts of the whole normalized force - drift envelope, the effective yield drift (Fig. 1c) and the normalized hysteretic energy. The drift, sometimes called also the drift ratio, which is equal to the chord rotation, is obtained as the displacement at the top of an equivalent cantilever divided by the length of this cantilever (L, also called shear span). The ultimate drift, representing a “near collapse” limit state, is defined as the drift at a 20% drop below maximum strength and represents the last point of the force-drift envelope. Effective yield drift is calculated from the bilinear (elastic – ideal plastic) idealization of the force – drift envelope using the equal area rule (the areas under the envelope until the capping displacement and its bilinear idealization are equal (Fig. 1c)). Different rules for determination of the effective elastic stiffness can be used, and they result in different effective yield drifts. Other drift related output parameters, which have been studied but are not presented in this paper, are capping drift and ductility. The forces in the force – drift envelope are not output parameters. They are normalized to the maximum force, i.e. to the strength of the column, observed in the tests. It is believed that the flexural strength of a column can be estimated quite accurately by the existing approaches.

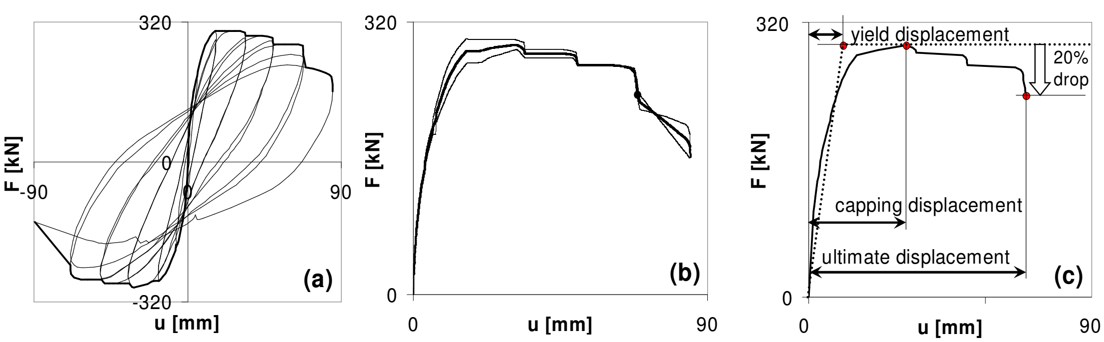


Figure 1. An example of the processing the PEER database: (a) hysteretic force-displacement relationship from the experiment, (b) the force - displacement envelope for both positive and negative loading and the mean envelope, (c) (mean) envelope and idealized force – displacement relation. The effective yield, capping, and ultimate displacement defined at a 20% drop in maximum strength are shown.

The output parameter used in this paper for normalized hysteretic energy is

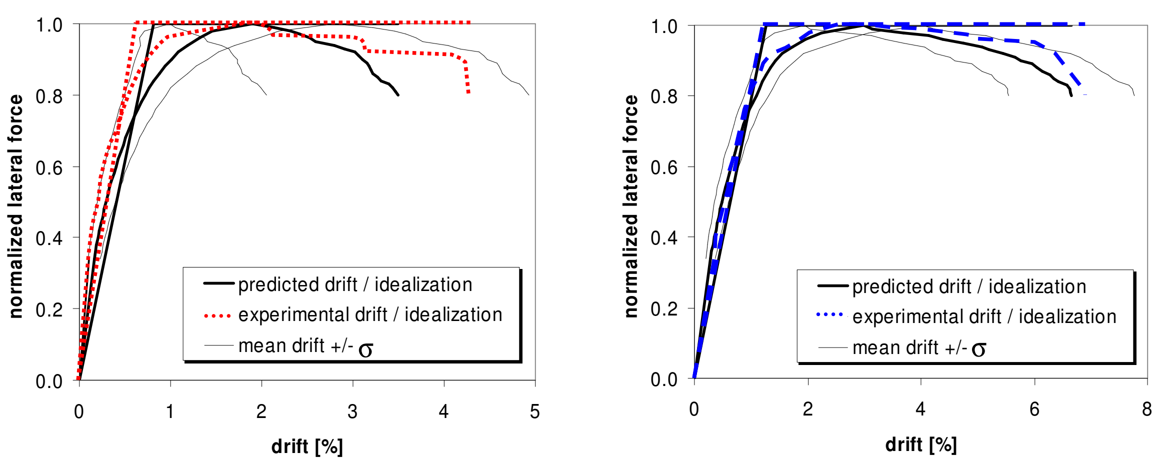
$$E^{*}_{H,L} = \frac{E_H}{F_y\, L} \qquad (1)$$

where F_y, is the maximum force both in the actual and idealized force – drift envelope. $E^{*}_{H,L}$ is a non-dimensional parameter and can be interpreted as the cumulative plastic displacement divided by the shear span, i.e. as a cumulative plastic drift. In figures, $E^{*}_{H,L}$ is shown in percents as all other drift related quantities. Other normalizations of hysteretic energy have been also used but are not presented here.

Results of Analyses

Normalized force - drift envelope

The normalized force - drift envelope provides important data on the capacity characteristics of a RC column. Fig. 2 shows the predicted results for the example from Fig. 1 (including plus/minus one local standard deviation) together with the experimental results. The initial part of the force – drift relation depends very much on the initial state of the (un)cracked test specimen. We believe that it is not appropriate to estimate this part of the force – drift relation from the available data in the database. Therefore, in all cases the prediction of the envelope was made only for normalized forces larger than 0.33. A linear force – drift relation was assumed in the range of normalized forces lower than 0.33. Bilinear idealizations of the experimental and mean predicted force – drift relation are also shown in Fig.2.



$P^{*} = 0.30$, $\alpha\rho_s^{*} = 0.05$,
$\rho_l^{*} = 0.15$, $L^{*} = 4$, $f_c' = 44$ MPa
(column from Figure 1)

$P^{*} = 0.16$, $\alpha\rho_s^{*} = 0.09$,
$\rho_l^{*} = 0.40$, $L^{*} = 3.5$, $f_c' = 27$ MPa

Figure 2. Normalized lateral force – drift envelope for the two RC columns from the database.

Some examples of predicted force – drift envelopes are shown in Fig. 3. Four input parameters were fixed, whereas the fifth input parameter was varied. The idealized bilinear force – drift relations are shown for the two extreme predicted cases. It can be clearly seen that the input parameters substantially influence the envelope, especially the ultimate drift. Ultimate drift increases with decreasing axial load index, and with increasing longitudinal reinforcement index and index related to confinement. The influence of the shear span is more complex as described later in the text. The major influence on the effective yield drift exhibits the longitudinal reinforcement index. Observed trends agree well with trends presented later in this paper. A graphical presentation of the whole force-drift envelope gives a clear image of different drift related parameters and of their relation with input parameters. From the graphs also some ideas about the ductility capacity can be obtained. For example, with better confinement both the increase of the ultimate drift and the decrease of the effective yield contribute to increasing ductility.

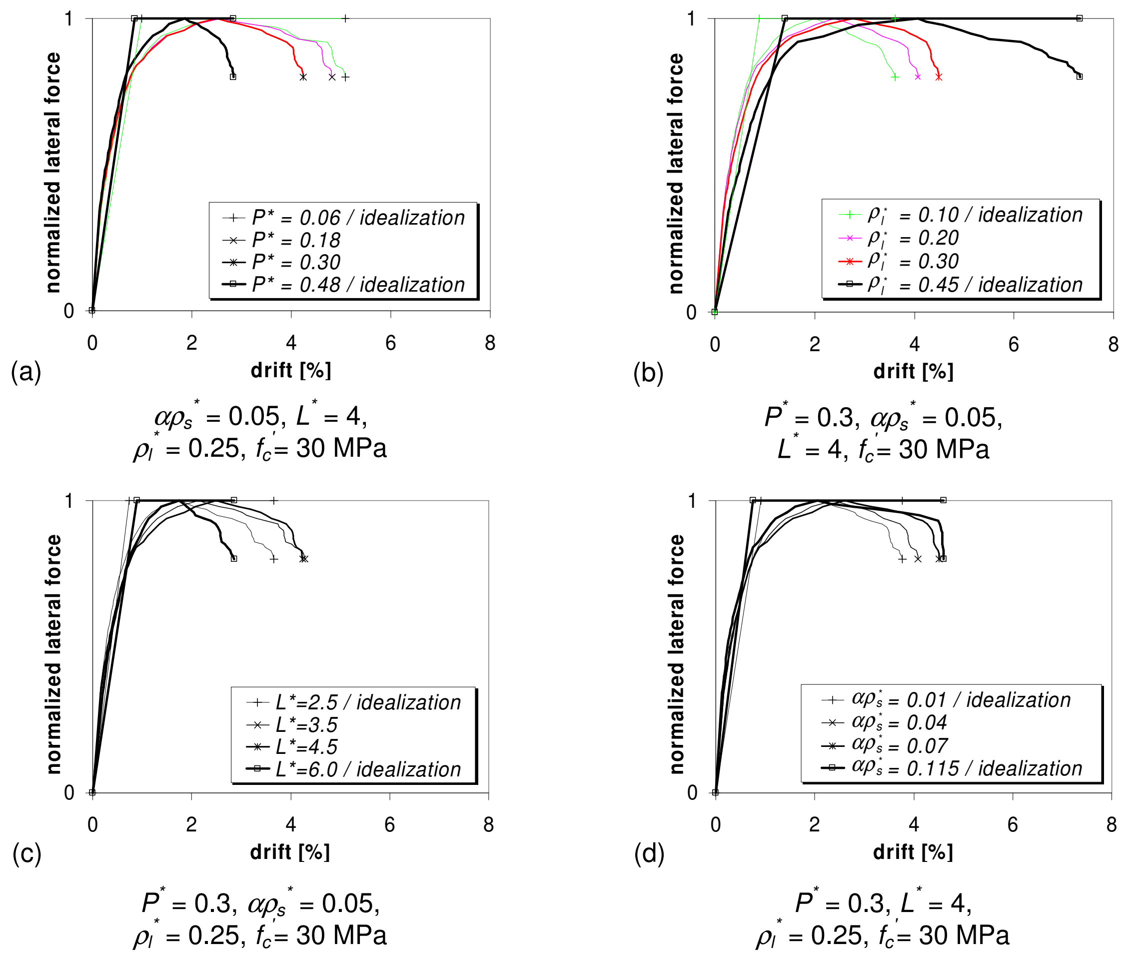


Figure 3. Predicted and idealized force – drift relations for variation of one input parameter: (a) axial load, (b) longitudinal reinforcement index, (c) shear span index, and (d) index related to confinement.

Ultimate and effective yield drifts

In this chapter some estimates of the characteristic drifts related to the force-displacement envelope, i.e. the ultimate and effective yield drift (for definitions see chapter “Output parameters”) obtained by using the CAE method are presented and discussed.

The basic trends are presented in Fig. 4 for input parameters inside the bounds. These results were obtained by using only one input parameter, whereas the other four were not taken into account. The axial load index and the longitudinal reinforcement index substantially influence both effective yield and ultimate drift. The influence of confinement is significant only for the ultimate drift. Both drifts first increase with shear span and then decrease. The compressive strength of concrete exhibits only a minor influence. Note that there are few test specimens outside the bounds, therefore the results near to the bounds may be less reliable.

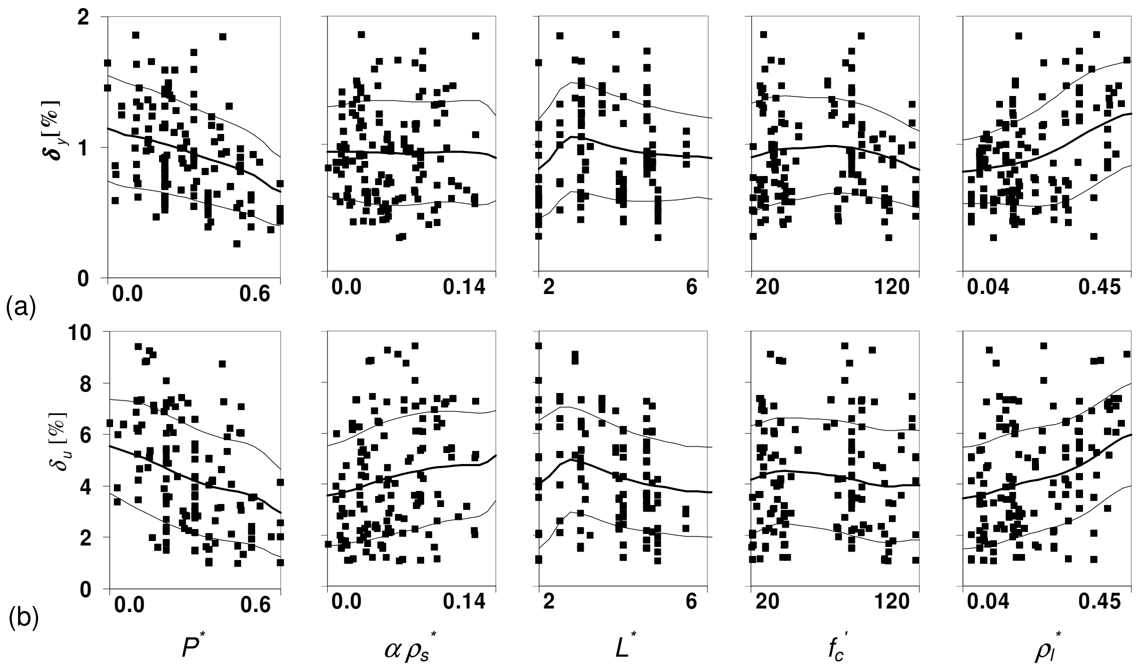


Figure 4. (a) Effective yield drifts and (b) ultimate drifts determined by CAE using only one input parameter. The local standard deviation is also shown.

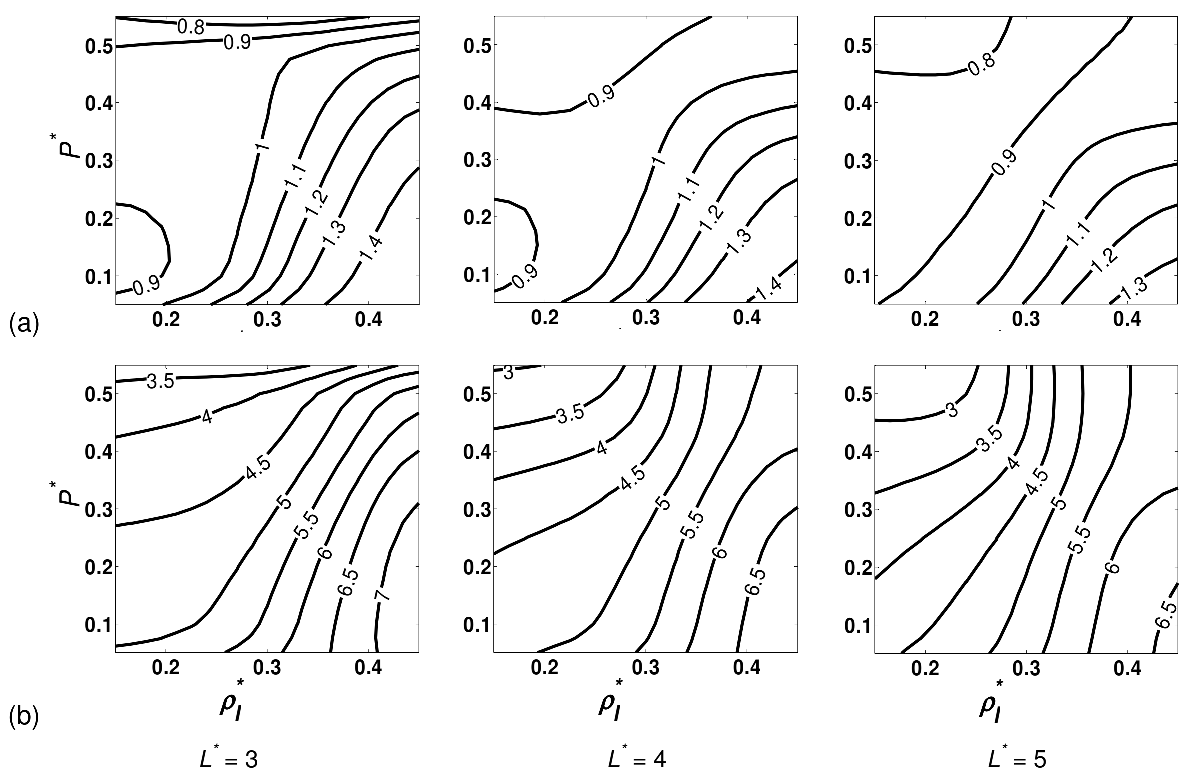


Figure 5. (a) Effective yield and (b) ultimate drifts as a function of five input parameters – variation of ρ_l^* and P^* (L^*=3, 4, 5, $\alpha\rho_s$*=0.07, f_c' = 30 MPa).

Figs. 5 and 6 show the effective yield and ultimate drifts estimated as a function of five input parameters. The results are presented in terms of isolines which are connecting the points of equal values of the estimated drift. The results in Fig. 5 show clear trends for both axial load and longitudinal reinforcement index and agree well with the results in Fig. 4. The same trend for axial load index can be seen also in Fig. 6. In the case of the index related to confinement, Fig. 6 suggests an increasing trend of the ultimate drift with increasing index and a decreasing trend for the effective yield drift. The later trend cannot be seen in Fig. 4, in which four out of five input parameters do not influence results.

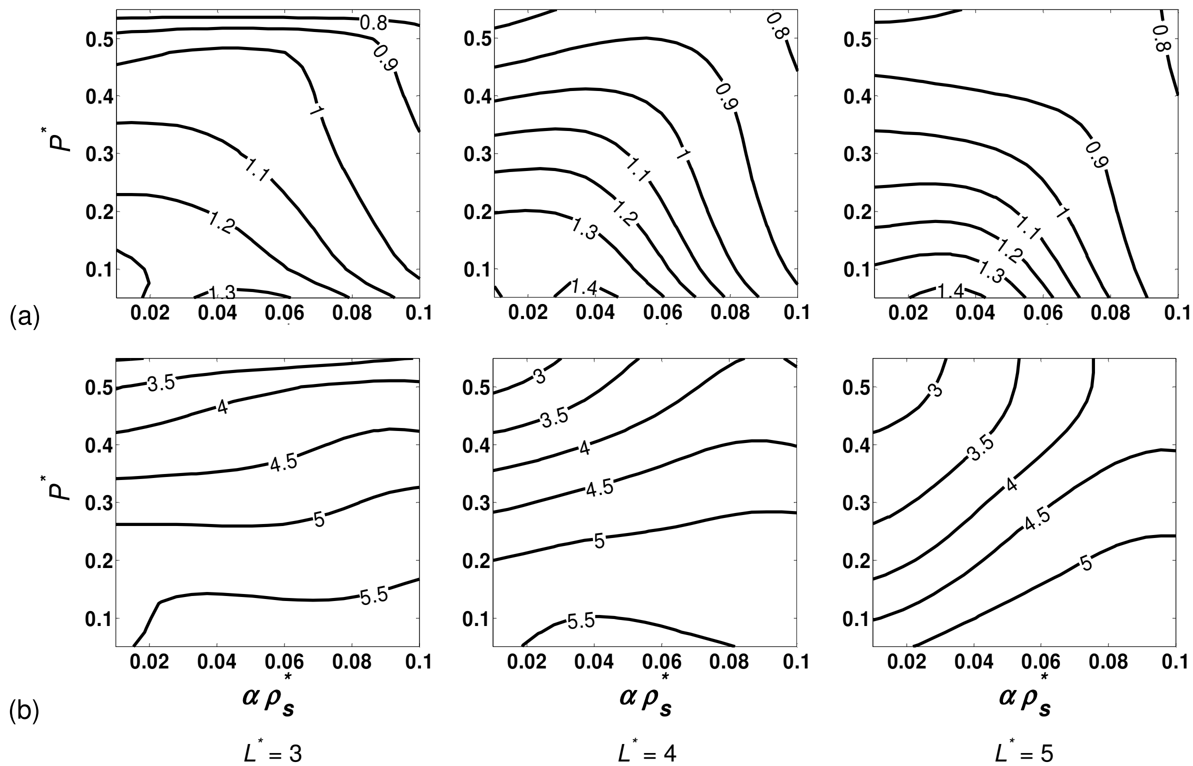


Figure 6. (a) Effective yield and (b) ultimate drifts as a function of five input parameters - variation of $\alpha\rho_s^*$ and P^* (L^*=3, 4, 5, ρ_l*=0.3, f_c' = 30 MPa).

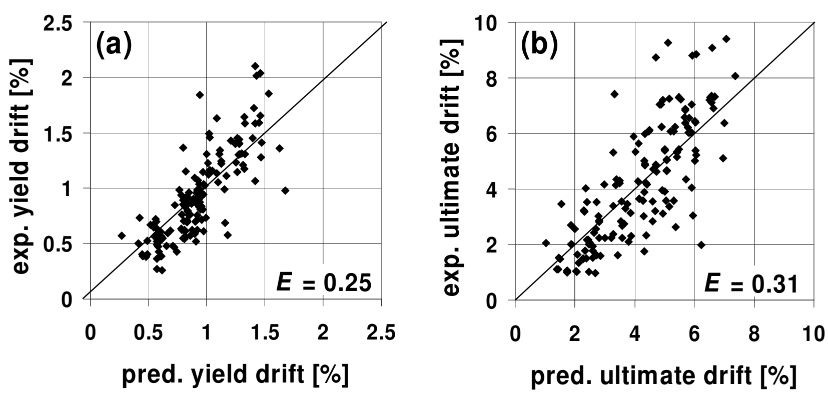


Figure 7. Comparison of predicted and experimental (a) effective yield drift and (b) ultimate drift.

Comparison of the predicted values obtained by the CAE approach with the values observed in tests is shown in Fig. 7. (»Test« results for effective yield drift apply to the bilinear idealization of the test force –

drift relation.) The values of the error measure E are also shown. The prediction error for the ultimate drift is larger than for the effective yield drift. On the other hand, it is smaller than in the case of the prediction with four input parameters, made in (Perus 2006, E = 0.35).

Hysteretic energy

In this section the influence of characteristics of columns on the energy dissipation capacity is shown and discussed. Normalized energy as a function of only one parameter is shown in Fig. 8 together with the estimation of local standard deviation.

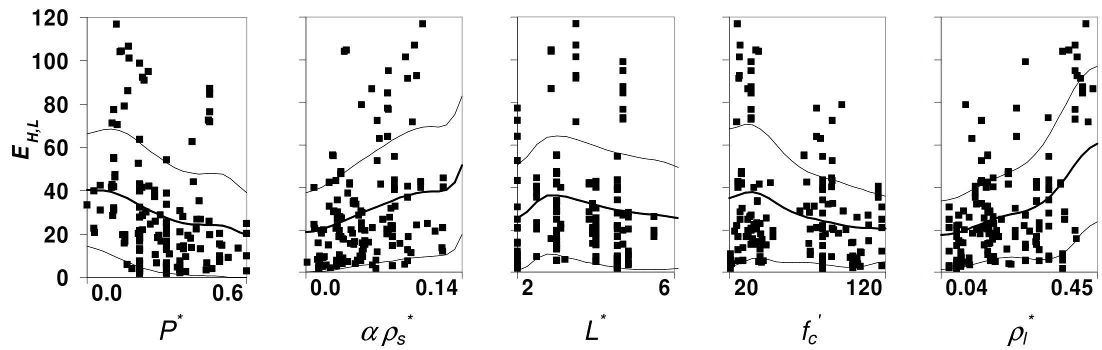


Figure 8. Normalized energy, determined by CAE, using only one input parameter. The local standard deviation is also shown.

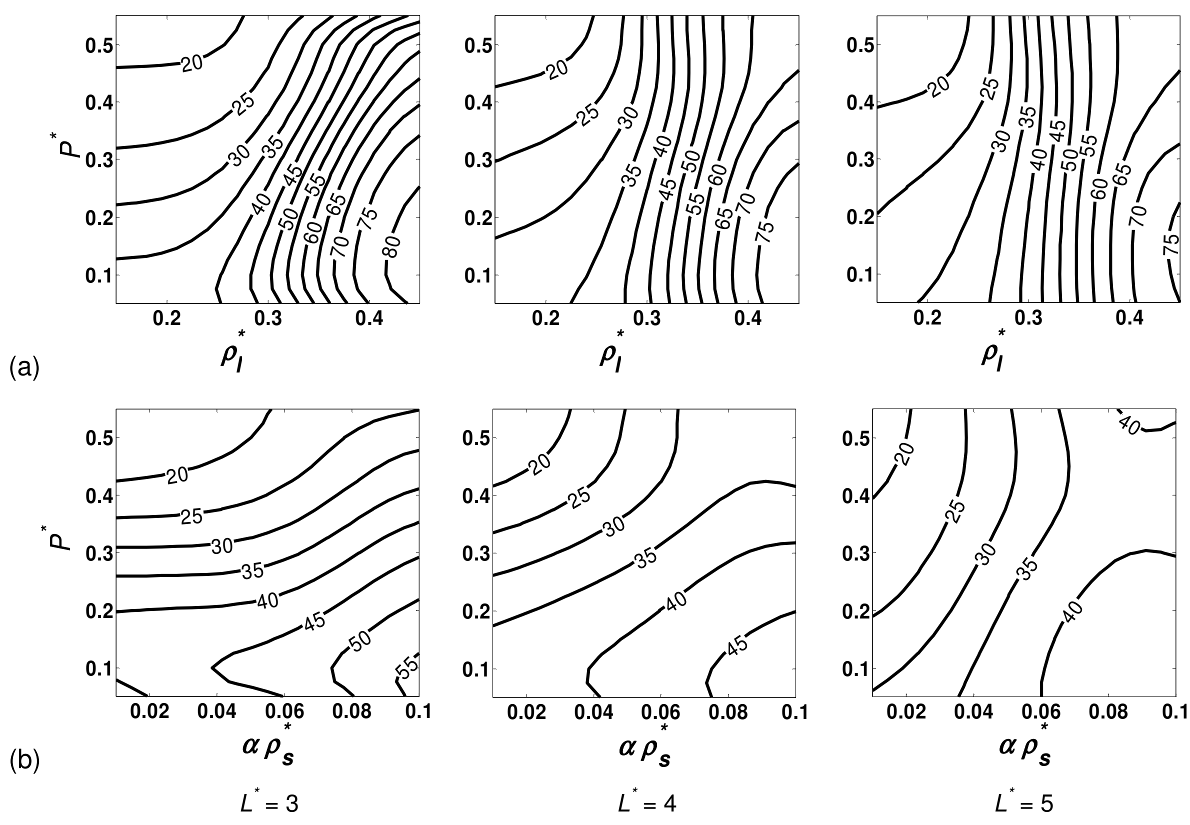


Figure 9. Normalized energy as a function of five input parameters: (a) Variation of ρ_l^* and P^* (L^*=3, 4, 5, $\alpha\rho_s$*=0.07, f_c' = 30 MPa), (b) Variation of $\alpha\rho_s^*$ and P^* (L^*=3, 4, 5, ρ_l*=0.3, f_c' = 30 MPa).

Energy capacity increases with increasing longitudinal reinforcement index and with increasing confinement, whereas it decreases with increasing axial load index and with increasing compressive strength above about 30 MPa. In the case of the shear span index trend is more complex. An increasing trend is observed with increasing shear span index only up to the value of about 3. For the values of shear span index above 3 a decreasing trend is observed. Observed trends are similar to trends for ultimate drift, with the exception of the influence of the concrete strength which is much more pronounced in the case of energy capacity than in the case of the drift capacity.

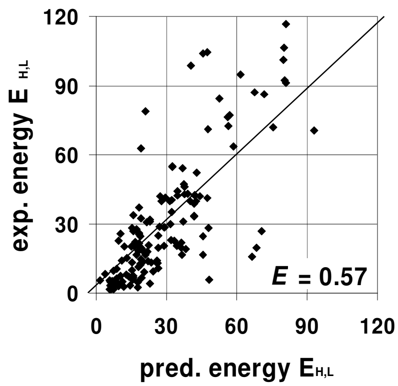


Figure 10. Comparison of predicted and experimental hysteretic energy.

Fig. 9 shows normalized energy determined as a function of five parameters. Two input parameters are presented as continuous values on the abscisa and ordinate of the two-dimensional graph like in Figs. 5 and 6. The results show similar trends as in Fig. 8. Moreover, the results for the normalized energy are qualitatively similar to the results for the ultimate drift (Figs. 5b in 6b). A comparison reveals a larger influence of confinement and longitudinal reinforcement in the case of normalized energy.

The error measure, shown in Fig. 10, is much larger than in the case of drifts.

Conclusions

A non-parametric empirical approach (the CAE method) was used for the estimation of the normalized force – drift envelopes and normalized energy dissipation capacity of RC columns as a function of their physical properties. The study demonstrated the ability of the CAE method to predict different structural characteristics together with measures of uncertainty from empirical data. Whereas some empirical and semi-empirical formulas exist for estimation of selected discrete quantities like ultimate drift, we have not yet seen in the literature predictions of complete normalized force – drift relations as a function of basic characteristics of columns.

Five input parameters were defined and used in calculations: axial load index, index related to confinement, shear span index, concrete compressive strength and longitudinal reinforcement index.

Based on the results of parametric study it can be concluded:

- The axial load index and the longitudinal reinforcement index exhibit substantial influence on yield drift, ultimate drift and hysteretic energy. All three quantities decrease with increasing axial load index and increase with increasing longitudinal reinforcement index.
- As expected, an increase of the index related to confinement increases the ultimate drift and the hysteretic energy. This index does not influence the yield drift.
- Yield drift, ultimate drift and hysteretic energy increase with increasing shear span index but only up to L^* = 3. With a further increase of this index a light decreasing trend can be observed for all three

quantities. Note that in the case of ultimate drifts and Fardis database (Panagiotakos 2001), the turning point was $L^* = 5$ (Perus 2006)

- The influence of the strength of concrete is substantial in the case of the hysteretic energy, which increases up to f_c'=40 Mp and then decreases with a further increase of f_c'. In the case of the yield and the ultimate drift a similar but very minor trend can be observed.
- Uncertainty in predictions for hysteretic energy is much larger than for drifts.
- Predictions may be less accurate especially in the case of input parameters with values near to the borders of the values of test specimens (i.e. for a high axial load index).

The quality of the prediction depends mainly on the quality of the database. By increasing the number of test specimens in the PEER database and/or by completing the Fardis database with the data on the hysteretic behaviour, more reliable estimates could be obtained. In principle, any number of input parameters (which are contained in the database) can be used in the CAE method. However, in practice the number of input parameters is limited by the size of the database. By increasing the size of the databases and by introducing new input parameters, new trends may be revealed, studied and incorporated in the mathematical models.

Acknowledgments

The results presented in this paper are based on work supported by the Slovenian Research Agency (project No. J2-6667-0792) and by the European Commission within the 6th Framework project LESSLOSS (No. 505448-GOCE). The original PEER database was downloaded from the University of Washington website.

References

CEN, March, 2005. Eurocode 8: Design of structures for earthquake resistance, Part 3: Assessment and retrofitting of buildings, EN 1998-3.

Fardis, M.N., and D. E. Biskins, 2003. Deformation capacity of RC members, as controlled by flexure or shear. *Performance-based engineering for earthquake resistant reinforced concrete structures*: A volume honoring Shunsuke Otani. Kabeyasawa T, Shiohara H, (eds.), University of Tokyo, 511-530.

Panagiotakos, T. B., and M. N. Fardis, 2001. Deformations of reinforced concrete members at yielding and ultimate, *ACI Structural Journal* 98 (2), 135-148.

Paulay T., and M. J. N. Priestley, 1992. *Seismic design of reinforced concrete and masonry buildings*, J.Wiley, New York.

PEER, University of Washington, *The UW-PEER Reinforced Concrete Column Test Database*, http://www.ce.washington.edu/~peera1/.

Perus, I., K. Poljansek, and P. Fajfar, 2006. Flexural deformation capacity of rectangular RC columns determined by the CAE method, *Earthquake Engineering and Structural Dynamics* 35, 1453-1470.