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DYNAMIC ANALYSIS OF INELASTIC STRUCTURES WITH NONLINEAR VISCOUS DAMPERS

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ABSTRACT

A simple numerical algorithm for analyzing the dynamic response and energy transfer of inelastic structures with nonlinear fluid viscous dampers (FVDs) is proposed. This algorithm uses the force analogy method in determining the inelasticity of structures, while the nonlinear damping force is treated as an external applied force in the analysis. Energy response for the nonlinear FVD controlled structures is evaluated to demonstrate the simplicity and applicability of the proposed algorithm. The effectiveness of installing FVDs with different nonlinear power law coefficients and damping coefficients are also compared and studied.

Introduction

Earthquake is a type of natural disaster that can severely damage and destroy human lives, properties and belongings. Many effective passive, active, semi-active, and hybrid control methods (Symans and Constantinou 1999, Soong and Spencer 2002) have been proposed to reduce the damaging effects of earthquakes. The fluid viscous damper (FVD), as a type of passively controlled devices, has been studied in great detail and practically applied to reduce the damages caused by the earthquake ground motion (Constantinou and Symans 1993a & 1993b). Without any major structural modification, the FVD can be easily installed into structures and dissipate some portions of energy to reduce the energy dissipation demands in structural members.

Using a simple relationship between the damping force and relative velocity of linear FVDs (LFVDs), extensive numerical simulations, experiments, and theoretical analysis have been carried out (Reinhorn et al. 1995). At a later time, there are also some research works on the response of structures with non-linear FVDs (NLFVDs) installed. Terenzi et al. (1999) analyzed the effects of the NLFVD on the dynamic response of single degree of freedom (SDOF) systems based on the Newmark β numerical method. Martinez-Rodrigo and Romero (2003) used the Newmark β method to investigate the inelastic seismic response of multi-degree of freedom (MDOF) systems. Due to the complexity in modeling MDOF inelastic structures, the Newmark β method usually requires tremendous computational work that is very time consuming and costly simply because the nonlinear relationship between the damping force and relative velocity must be addressed implicitly.

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The objective of this study is to develop a simple explicit numerical method to predict the time history response of inelastic structures with NLFVD installed and define the transformation of various energy forms in the system. The force analogy method for inelastic structural analysis and energy balance equations are briefly introduced in this paper. The accuracy of the proposed method is then verified by using a SDOF structural model with LFVD. Finally, energy response of an inelastic multi-story frame is evaluated, and the contributions of FVDs with different power law coefficients and damping coefficients are analyzed and compared.

Inelastic Dynamic Response of Structure with NLFVD

Theoretical Background

A FVD generally attaches to one building floor at one end and connects to another floor at the other end. The difference in floor motions between the two ends of the FVD produces a damping force and a source of energy dissipation between the two floors. When the upper floor moves and has a relative velocity to the story below, the FVD damping force is produced and can be expressed in the form

$$f'(t) = c'\left\langle \dot{w}(t) \right\rangle^{\eta} \tag{1}$$

where $f'(t)$ is the manufactured damping force due to the FVD, c' is the manufactured damping coefficient of the FVD, $\dot{w}(t)$ is the relative velocity between two ends of the FVD, and η is a power law coefficient. In Eq. 1, the bracket $\left\langle \cdot \right\rangle^{\eta}$ term is defined as

$$\left\langle \dot{w}(t) \right\rangle^{\eta} = \begin{cases} \dot{w}(t)^{\eta} \\ -\left(-\dot{w}(t)\right)^{\eta} \end{cases} \tag{2}$$

The FVDs can be classified into the LFVDs, where the damping forces of which are linear functions of velocity, (i.e. $\eta = 1$), and the NLFVDs, where the damping forces exhibits a nonlinear relationship with velocity, (i.e. $\eta \neq 1$). For a LFVD, the manufactured damping force can be described as

$$f'(t) = c'\dot{w}(t) \tag{3}$$

For the more common case, $f'(t)$ is a nonlinear expression of velocity as given in Eq. 1, where η is generally less than 1.0. For simplicity, the definition using the bracket in Eq. 2 is here ignored, and Eq. 1 becomes:

$$f'(t) = c'\dot{w}(t)^{\eta} \tag{4}$$

Force Analogy Method

Changing the displacement, instead of changing the stiffness, to capture the yielding force is the basic concept of the force analogy method, which has been presented in detail by Wong and Yang (1999). This method is briefly described here.

Consider a moment-resisting frame with ‘n’ degrees of freedom (DOFs) and ‘m’ plastic hinge locations (PHLs). The total displacement $\mathbf{x}(t)$ of the structure at any time t can be described by the summation of elastic displacement $\mathbf{x}'(t)$, which can be recovered after unloading, and the inelastic displacement $\mathbf{x}''(t)$, which represents the permanent deformation once the structure exhibits inelastic behavior. This can be expressed as

$$\mathbf{x}(t) = \mathbf{x}'(t) + \mathbf{x}''(t) \tag{5}$$

Similar to the total displacement $\mathbf{x}(t)$ described in Eq. 5, the total moment $\mathbf{m}(t)$ at the PHLs of a typical moment-resisting frame can be separated into elastic and inelastic moments, i.e.,

$$\mathbf{m}(t) = \mathbf{m}'(t) + \mathbf{m}''(t) \tag{6}$$

where $\mathbf{m}'(t)$ is the elastic moment due to the elastic displacement, and $\mathbf{m}''(t)$ is the inelastic moment due to the inelastic displacement. The displacements in Eq. 5 and the moments in Eq. 6 are related by the following equations:

$$\mathbf{m}'(t) = \mathbf{K}'^{T}\mathbf{x}'(t) \tag{7}$$

$$\mathbf{m}''(t) = -(\mathbf{K}'' - \mathbf{K}'^{T}\mathbf{K}^{-1}\mathbf{K}')\mathbf{\Theta}''(t) \tag{8}$$

where $\mathbf{\Theta}''(t)$ is the plastic rotation at the PHLs, $\mathbf{K}$ is the $n \times n$ global stiffness matrix, $\mathbf{K}'$ is the $n \times m$ matrix describing the relationship between the plastic rotations at the PHLs and the forces at the DOFs, and $\mathbf{K}''$ is the $m \times m$ matrix that relates plastic rotations with the corresponding moments at the PHLs.

Substituting Eqs. 7 and 8 into Eq. 6 and rearranging the terms, the first governing equation of the force analogy method is obtained:

$$\mathbf{m}(t) + \mathbf{K}''\mathbf{\Theta}''(t) = \mathbf{K}'^{T}\mathbf{x}(t) \tag{9}$$

Based on the compatibility condition between plastic rotation $\mathbf{\Theta}''(t)$ and inelastic displacement $\mathbf{x}''(t)$, the second governing equation in the force analogy method is

$$\mathbf{x}''(t) = \mathbf{K}^{-1}\mathbf{K}'\mathbf{\Theta}''(t) \tag{10}$$

Dynamic Equilibrium Equation of Motion

When the NLFVD is installed in the structure, the dynamic equilibrium equation of motion becomes

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}'(t) + \mathbf{D}\mathbf{f}'(t) = -\mathbf{M}\ddot{\mathbf{g}}(t) \tag{11}$$

where $\mathbf{M}$ is the $n \times n$ mass matrix, $\mathbf{C}$ is the $n \times n$ damping matrix, $\dot{\mathbf{x}}(t)$ is the $n \times 1$ velocity vector, $\ddot{\mathbf{x}}(t)$ is the $n \times 1$ acceleration vector, $\ddot{\mathbf{g}}(t)$ is the $n \times 1$ ground acceleration vector, $\mathbf{D}$ is the $n \times p$ passive control force distribution matrix, $\mathbf{f}'(t)$ is the $p \times 1$ control force vector, and 'p' is the number of passive dampers installed in the structure. Replacing the elastic displacement $\mathbf{x}'(t)$ by the difference of total displacement $\mathbf{x}(t)$ and inelastic displacement $\mathbf{x}''(t)$ as given in Eq. 5, Eq. 11 becomes

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}\ddot{\mathbf{g}}(t) - \mathbf{D}\mathbf{f}'(t) + \mathbf{K}\mathbf{x}''(t) \tag{12}$$

Note that the control force $\mathbf{f}'(t)$ appears on the right side of Eq. 12, and it is therefore being considered and treated as an external applied force. Applying the state space method, Eq. 12 can be expressed as

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{H}\mathbf{a}(t) + \mathbf{G}\mathbf{f}'(t) + \mathbf{F}\mathbf{x}''(t) \tag{13}$$

where

$$\mathbf{z}(t)=\begin{Bmatrix}\mathbf{x}(t)\\ \dot{\mathbf{x}}(t)\end{Bmatrix}\ ,\ \mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}\ ,\ \mathbf{H}=\begin{bmatrix}\mathbf{0}\\ -\mathbf{1}\end{bmatrix}\ ,\ \mathbf{G}=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{D}\end{bmatrix}\ ,\ \mathbf{F}=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{K}\end{bmatrix} \tag{14}$$

and $\mathbf{0}$ is the zero matrix, $\mathbf{I}$ is the identity matrix, $-\mathbf{1}$ is the $n\times 6$ matrix that contains only 0's and –1's that relates the direction of ground acceleration to the direction of each DOF, and $\mathbf{a}(t)$ is the 6×1 earthquake ground acceleration vector representing the ground acceleration in all six directions, including both translations and rotations. If only translational ground acceleration in three directions is considered, $\mathbf{a}(t)$ becomes a 3×1 ground acceleration vector and $-\mathbf{1}$ becomes the $n\times 3$ matrix.

Solving for the first order linear differential equation in Eq. 13 by performing discretization, the discretized solution can be written as

$$\mathbf{z}_{k+1}=\mathbf{F}_s\mathbf{z}_k+\overline{\mathbf{H}}\mathbf{a}_k+\overline{\mathbf{G}}\mathbf{f}'_k+\overline{\mathbf{F}}\mathbf{x}''_k \tag{15}$$

where

$$\mathbf{F}_s=\mathbf{e}^{\mathbf{A}\Delta t}\ ,\ \overline{\mathbf{H}}=\mathbf{e}^{\mathbf{A}\Delta t}\mathbf{H}\Delta t\ ,\ \overline{\mathbf{F}}=\mathbf{e}^{\mathbf{A}\Delta t}\mathbf{F}\Delta t\ ,\ \overline{\mathbf{G}}=\mathbf{e}^{\mathbf{A}\Delta t}\mathbf{G}\Delta t \tag{16}$$

and 'k' is the discrete time step corresponding to time t. Equation 15 is a simple recursive equation, and the detailed recursive solution procedure has been discussed in Wong and Yang (1999).

Energy Balance

Based on the calculated response using the force analogy method, the energy transfers between various forms can then be calculated (Wong and Yang 2002). Define the absolute acceleration $\ddot{\mathbf{y}}(t)$ of the structure's degrees of freedom as

$$\ddot{\mathbf{y}}(t)=\ddot{\mathbf{x}}(t)+\ddot{\mathbf{g}}(t) \tag{17}$$

the dynamic equilibrium equation of motion as given in Eq. (11) can then be rewritten in the form:

$$\mathbf{M}\ddot{\mathbf{y}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}'(t)+\mathbf{D}\mathbf{f}'(t)=\mathbf{0} \tag{18}$$

Integrating both sides of Eq. 18 over the path of structural response from 0 to t_k gives

$$\int_{t=0}^{t=t_k}\ddot{\mathbf{y}}^T\mathbf{M}d\mathbf{x}+\int_{t=0}^{t=t_k}\dot{\mathbf{x}}^T\mathbf{C}d\mathbf{x}+\int_{t=0}^{t=t_k}\mathbf{x}'^T\mathbf{K}d\mathbf{x}+\int_{t=0}^{t=t_k}\mathbf{f}'^T\mathbf{D}^Td\mathbf{x}=0 \tag{19}$$

Since $d\mathbf{x}(t)=d\mathbf{y}(t)-d\mathbf{g}(t)$, where $\ddot{\mathbf{y}}(t)$ represents the absolute displacement of the structure and $\mathbf{g}(t)$ represents the ground displacement, Eq. 19 can be rewritten in the form:

$$\int_{t=0}^{t=t_k}\ddot{\mathbf{y}}^T\mathbf{M}d\mathbf{y}+\int_{t=0}^{t=t_k}\dot{\mathbf{x}}^T\mathbf{C}d\mathbf{x}+\int_{t=0}^{t=t_k}\mathbf{x}'^T\mathbf{K}d\mathbf{x}+\int_{t=0}^{t=t_k}\mathbf{f}'^T\mathbf{D}^Td\mathbf{x}=\int_{t=0}^{t=t_k}\ddot{\mathbf{y}}^T\mathbf{M}d\mathbf{g} \tag{20}$$

In addition, since $d\mathbf{x}(t)=d\mathbf{x}'(t)+d\mathbf{x}''(t)$ based on Eq. 5 rewritten in differential form, substituting this result into the third term of Eq. 20 gives

$$\int_{t=0}^{t=t_k}\ddot{\mathbf{y}}^T\mathbf{M}d\mathbf{y}+\int_{t=0}^{t=t_k}\dot{\mathbf{x}}^T\mathbf{C}d\mathbf{x}+\int_{t=0}^{t=t_k}\mathbf{x}'^T\mathbf{K}d\mathbf{x}'+\int_{t=0}^{t=t_k}\mathbf{x}'^T\mathbf{K}d\mathbf{x}''+\int_{t=0}^{t=t_k}\mathbf{f}'^T\mathbf{D}^Td\mathbf{x}=\int_{t=0}^{t=t_k}\ddot{\mathbf{y}}^T\mathbf{M}d\mathbf{g} \tag{21}$$

Note that the first three terms on the left side of Eq. 21 represent the kinetic energy (KE), natural damping energy (NDE) and strain energy (SE), respectively. According to the force analogy method, the fourth term on the left side of Eq. 21 can be rewritten in the form of plastic energy (PE) dissipation at individual plastic hinges based on Eqs. 7 and 10 as

$$PE = \int_{t=0}^{t=t_k} \mathbf{x}'^T \mathbf{K} d\mathbf{x}'' = \int_{t=0}^{t=t_k} \mathbf{x}'^T \mathbf{K}' d\mathbf{\Theta}'' = \int_{t=0}^{t=t_k} \mathbf{m}'^T d\mathbf{\Theta}'' = \sum_{i=1}^{m} \int_{t=0}^{t=t_k} m_i'(t) d\theta_i'' = \sum_{i=1}^{m} PE_i \tag{22}$$

where $m_i'(t)$ represents the elastic moment at the *i*th PHL, and $d\theta_i''(t)$ represents the incremental plastic rotation at the *i*th PHL.

The fifth term on the left side of Eq. 21 represents the manufactured damping energy (MDE). The term on the right side of Eq. 21 is the input energy (IE), which is due to the earthquake excitation. Finally, Eq. 21 can be written as

$$KE + NDE + SE + PE + MDE = IE \tag{23}$$

Verification of the Nonlinear Algorithm Using a SDOF System

To verify the accuracy of the proposed nonlinear algorithm for dynamic analysis of inelastic structures with NLFVDs, a comparison of response is performed using a SDOF elastic system with LFVD installed. The equation of the LFVD follows from Eq. 3 with $\dot{w}(t) = \dot{x}(t)$ that

$$f'(t) = c'\dot{x}(t) \tag{24}$$

Therefore, with a LFVD installed in a SDOF system, the analysis can be done using any classical numerical methods such as the Newmark β numerical method with inelastic capability. For this case, the SDOF dynamic equilibrium equation becomes:

$$m\ddot{x}(t) + [c + c']\dot{x}(t) + kx'(t) = -m\ddot{g}(t) \tag{25}$$

where the bolded notations are removed to denote scalar quantities. As a numerical verification of the nonlinear algorithm, consider a SDOF system with structural parameters shown in Fig. 1 with a yield moment of 1,690 kN-m at the PHL. Elastic-plastic behavior is used for the column's plastic hinge. The system is subjected to 1994 Northridge earthquake as shown in the first figure of Fig. 2. Based on this SDOF system with LFVD setup, Fig. 2 compares the structural response based on the "External Force" method described in Eq. 12 and the "Damping Force" method described in Eq. 25. It can be seen that the differences between the time history responses for the two methods are very small. Therefore, it is concluded that treating the force in FVD as an increase in damping coefficient (see Eq. 25) or a reduction in the external excitation force (see Eq. 12) is practically the same for evaluation of the time history response, and the proposed numerical algorithm gives satisfactory accuracy.

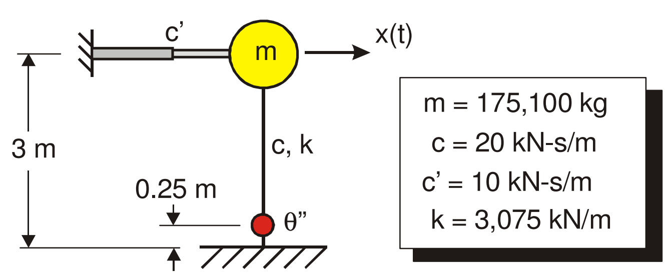

Figure 1. Single degree of freedom model.

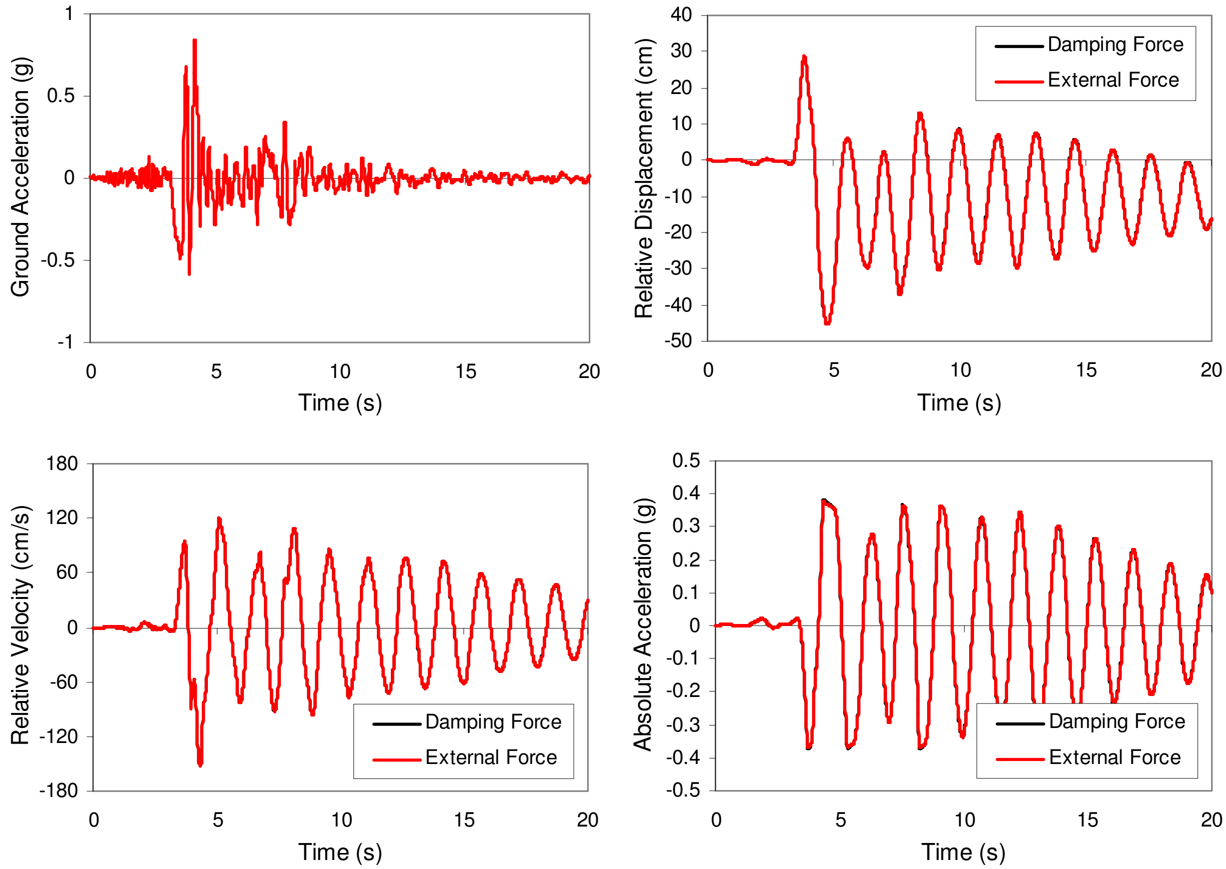

Figure 2. 1994 Northridge earthquake and comparison of responses between two methods.

Response and Energy Analysis of Multi-story Frame

For a multi-story passively controlled structural system, the FVDs are usually installed with one end of the FVD connects to the upper floor and another end of it attaches to the lower floor. In doing so, the damping forces of the FVD applied on the two floors will be of the same magnitudes but in opposite directions.

As a numerical example, consider a six-story passively controlled steel frame as shown in Fig. 3. Assume the mass of every floor is the same and equals to 630,400 kg, and the natural damping is a uniform diagonal matrix of the form $\mathbf{C} = c \times \mathbf{I}$, where $c = 210.1$ kN-s/m. The FVDs are installed between every floor as shown in Fig. 3, and they are labeled from C1 to C6. After performing static condensation to reduce the rotational degrees of freedom, the frame becomes a system with 6 DOFs and 40 PHLs as labeled in Fig. 3. Assume that plastic hinges exhibit elastic-plastic behavior. Yield moments are calculated by multiplying the plastic modulus of each section with the yield stress of steel. A uniform gravity load of 21.89 kN/m is assumed to act on the beams of every floor.

The horizontal component of the manufactured damping force along the direction of DOFs presented in Eq. 15 can be written as

$$\mathbf{f}'_k = \mathbf{l}'\mathbf{C}'\langle \dot{\mathbf{w}}_k \rangle^{\eta} \tag{26}$$

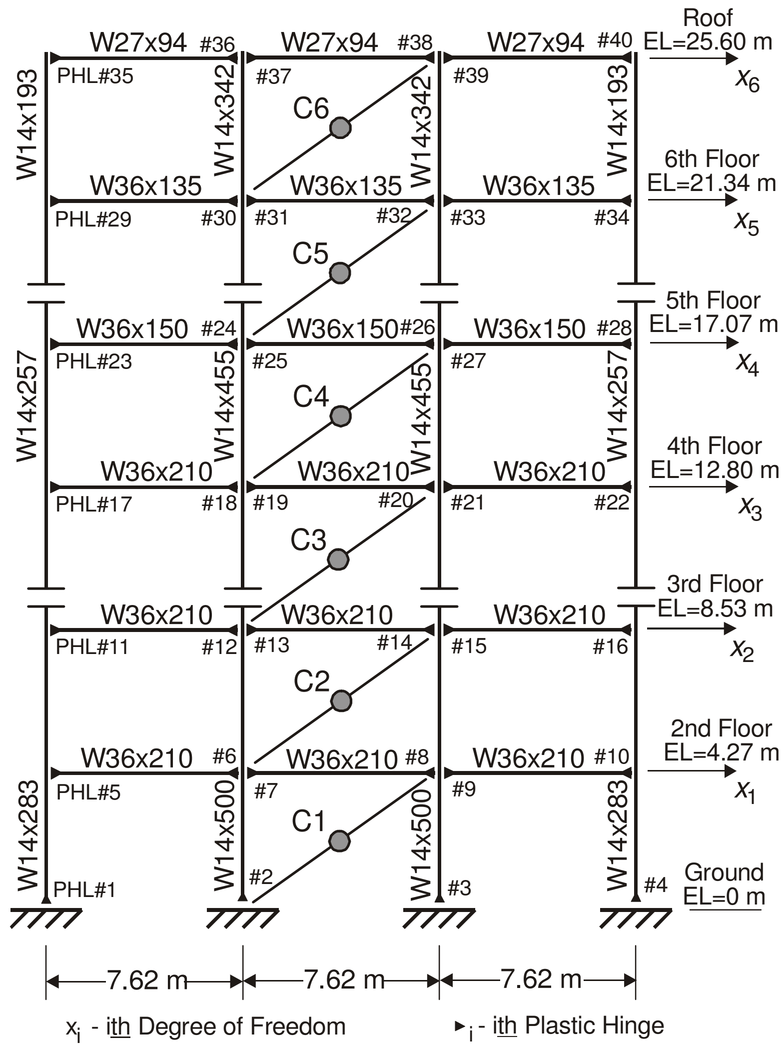

Figure 3. Six-story moment-resisting steel frame with NLFVD.

where $\mathbf{l}'$ is the $n \times p$ matrix that relates the effects of the FVDs on each DOF, and $\dot{\mathbf{w}}_k$ is the $p \times 1$ vector of relative velocities between the two ends of the dampers. Based on the setup shown in Fig. 3 where $p = 6$, the matrices can be written as:

$$\mathbf{l}' = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & -1 \\ 0 & \cdots & 0 & 1 \end{bmatrix}, \left\langle \dot{\mathbf{w}}_k \right\rangle^{\eta} = \begin{bmatrix} \left\langle x_{1,k} \right\rangle^{\eta} \\ \left\langle x_{2,k} - x_{1,k} \right\rangle^{\eta} \\ \vdots \\ \left\langle x_{6,k} - x_{5,k} \right\rangle^{\eta} \end{bmatrix} \tag{27}$$

Assume $\mathbf{C}' = c' \times \mathbf{I}$ is a 6×6 matrix, the energy time histories of the six-story passively controlled structure are now studied by subjecting it to the 1994 Northridge earthquake time history shown in Fig. 2 with different power law coefficients but with the same damping coefficient of $c' = 500$. The four cases for energy response comparisons include the six-story structure with no FVDs, linear FVDs (i.e., $\eta = 1$), low-nonlinear FVDs (i.e. $\eta = 0.7$), and high-nonlinear FVDs (i.e. $\eta = 0.35$). The comparisons of energy responses are plotted in Fig. 4.

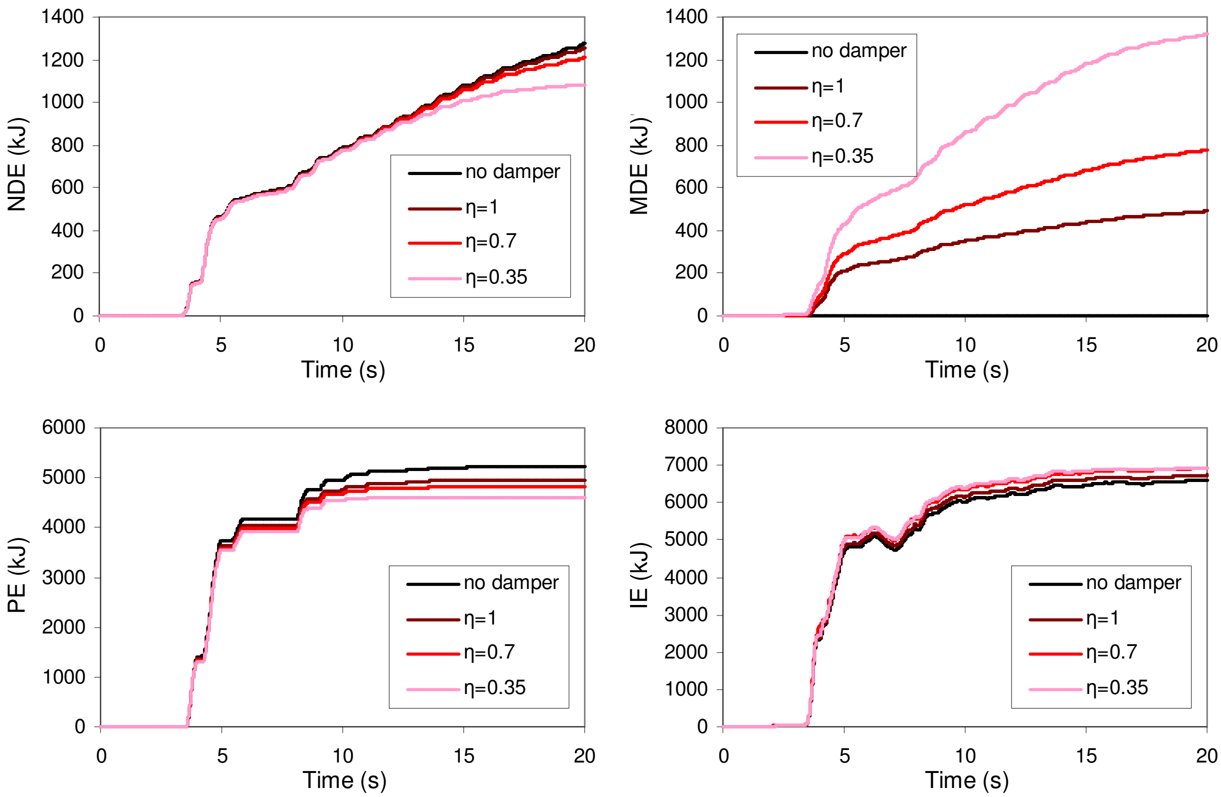

Figure 4. Comparisons of energy response of different power law coefficients.

As shown in Fig. 4, IE remains practically unchanged for the four cases while NDE and PE are reduced to different levels due to the presence of FVDs. The use of high-nonlinear FVDs (i.e. $\eta = 0.35$) gives the lowest structural energy response, while it also produces the largest FVD damping energy (MDE) response. The PE response shows little differences between the linear, low-nonlinear, and high-nonlinear FVDs cases, but it is reduced in all cases when compared to the case with no damper. This shows that it is always advantageous to install dampers for the reduction of plastic energy dissipation and thereby reducing the damage in the structure.

Based on these comparisons, it is concluded that the existence of the FVDs does not change the input energy, but it helps protect the structure by drawing some portions of the input energy and dissipating it in the form of manufactured damping energy (MDE), thereby reducing the plastic energy (PE) dissipation demands on the structure members. The properties of the FVDs (either linear, low-nonlinear, or high-nonlinear) have a small influence of the magnitude of the plastic energy in terms of percentage, while the difference is mainly dissipating more manufactured damping energy for high-nonlinear FVDs at the expense of reducing the dissipation of natural damping energy. Among the four cases, the high-NLFVDs dissipate the most energy.

Now the contributions of the FVDs with the same power law coefficient of $\eta = 0.35$ but with different FVD damping coefficients (i.e., $c' = 0$, $c' = 500$, and $c' = 1000$) are compared. Note that $c' = 0$ means no FVD is used. The results of energy response comparisons are presented in Fig. 5.

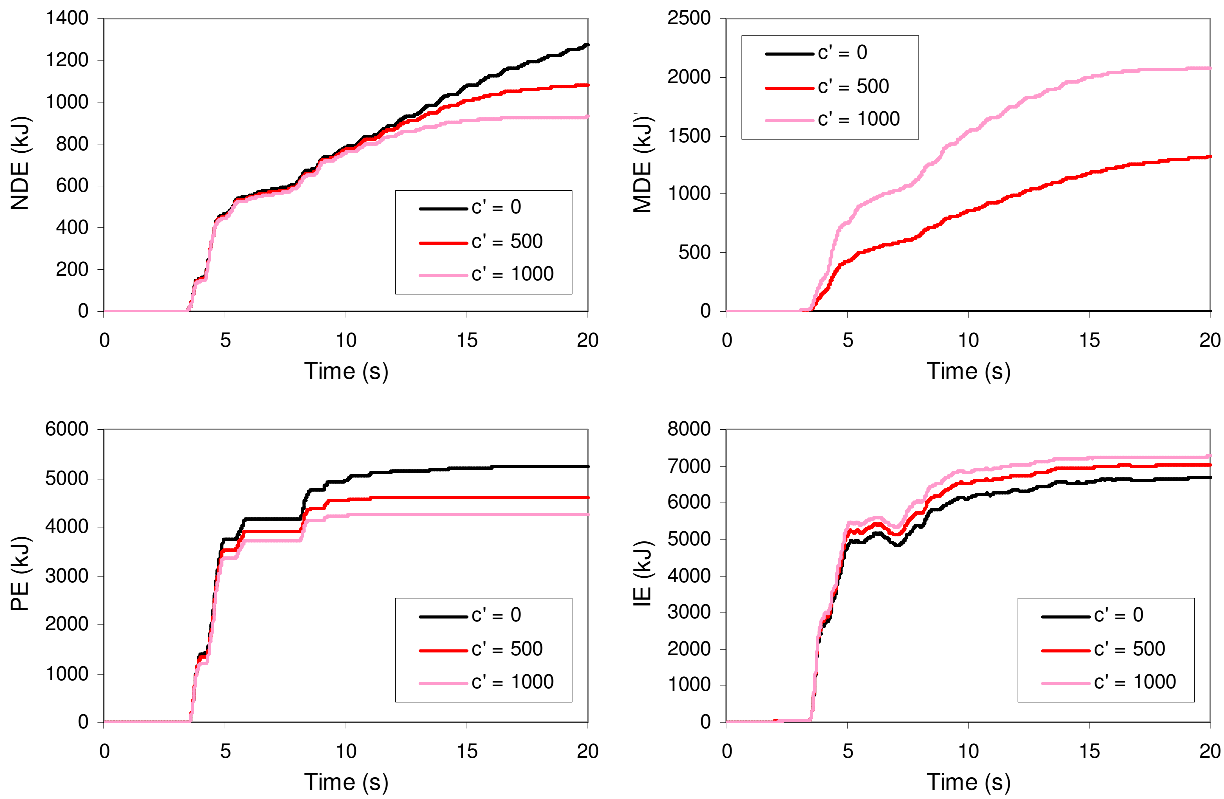

Figure 5. Comparisons of energy response of different manufactured damping coefficients.

As shown in Fig. 5, both NDE and PE are reduced due to the use of FVDs. Input energy does not change much for different FVD damping coefficients, which again proves that the input energy is not much affected by the presence of FVDs. With the increase of FVD damping coefficient, MDE increases while both PE and NDE decrease. This shows that the presence of FVDs extracts dissipative energy in the forms of PE and NDE and releases it in the form of MDE.

From the comparisons of different power law coefficients and FVD damping coefficients, it is observed that the control effect is more effective for high-nonlinear FVDs (i.e., small η) with high damping coefficient (i.e. large c').

Conclusions

In this research, a simple numerical algorithm based on the force analogy method is proposed to calculate the inelastic dynamic analysis of structures with nonlinear fluid viscous dampers. Study using a single degree of freedom system with a linear fluid viscous damper shows that the proposed algorithm is efficient and gives satisfactory accuracy. Numerical simulation is then performed on a multi-story passive controlled steel frame, and it is shown that the force analogy method is very suitable for the inelastic dynamic analysis of the passive controlled structures. Energy response of the passive controlled structures with different power law coefficients or FVD damping coefficients are also compared and some conclusions are drawn:

- The energy response of passively controlled structures can be effectively controlled with the installation of FVDs.

- Regardless of whether a structure has FVD installed or not, the input energy remains practically unchanged. The FVD is excellent in dissipating manufactured damping energy and releases the plastic energy demands of the structure members.

- The FVD, with lower power law coefficients (i.e., smaller η) and higher FVD damping coefficients (i.e., larger c') contributes more to the reduction of structure vibrations.

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