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SCENARIO SPECTRA FOR DESIGN GROUND MOTIONS AND RISK CALCULATION

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ABSTRACT

A common method for developing design spectra based on the probabilistic approach is uniform hazard spectra (UHS). This paper reviews the basics of UHS and conditional mean spectra and presents a new approach for developing a suite of realistic scenario earthquake spectra that account for the correlations and variability of the peaks and troughs in the spectra at different spectral periods. These scenario spectra can be assigned rates that will allow reconstituting the hazard curves at different periods. The conditional mean spectra and the conditional spectra provide methods for breaking the UHS down into realistic earthquakes. The choice of using a UHS rather than multiple spectra for the different scenarios should be the decision of the engineering analyst, not the hazard analyst. If it is worth the additional analysis costs to avoid exciting a broad period range in a single evaluation, then the engineer should request multiple scenario spectra from the hazard analyst. For risk calculation for structures that have a response that depends on widely separated spectral frequencies, then the conditional spectral approach can be used.

Introduction

A common method for developing design spectra based on the probabilistic approach is uniform hazard spectra. A uniform hazard spectrum (UHS) is developed by first computing the hazard at a suite of spectral periods using response spectral attenuation relations. That is, the hazard is computed independently for each spectral period. For a selected return period, the ground motion for each spectral period is measured from the hazard curves. These ground motions are then plotted at their respective spectral periods to form the uniform hazard spectrum. This process is shown graphically in Figure 1.

The term "uniform hazard spectrum" is used because there is an equal probability of exceeding the ground motion at any period. Since the hazard is computed independently for each spectral period, in general, a uniform hazard spectrum does not represent the spectrum of any single

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earthquake. It is common to find that the high frequency (f>5 Hz) ground motions are controlled by nearby moderate magnitude earthquakes, whereas, the long period (T>1 sec) ground motions are controlled by distant large magnitude earthquakes.

The "mixing" of earthquakes in the UHS is often cited as a disadvantage of PSHA. There is nothing in the PSHA method that requires using a UHS. Based on the disaggregation, multiple spectra (for each important source) can be developed. The reason for using a UHS rather than using multiple spectra for the individual scenarios is to reduce the number of engineering analyses required. A deterministic analysis has the same issue. If one deterministic scenario leads to the largest spectral values for long spectral periods and a different deterministic scenario leads to the largest spectral values for short spectral periods, it is common practice to develop a single design spectrum that envelopes the two deterministic spectra. In this case, the design spectrum also does not represent a single earthquake.

The choice of using a UHS rather than multiple spectra for the different scenarios is the decision of the engineering analyst, not the hazard analyst. If it is worth the additional analysis costs to avoid exciting a broad period range in a single evaluation, then the engineer should request multiple scenario spectra from the hazard analyst.

In this paper, we will review the basics of uniform hazard spectra and conditional mean spectra and present a new approach for developing a suite of realistic scenario earthquake spectra that account for the correlations and variability of the peaks and troughs in the spectra at different spectral periods. These scenario spectra can be assigned rates that will allow reconstituting the hazard curves at different periods.
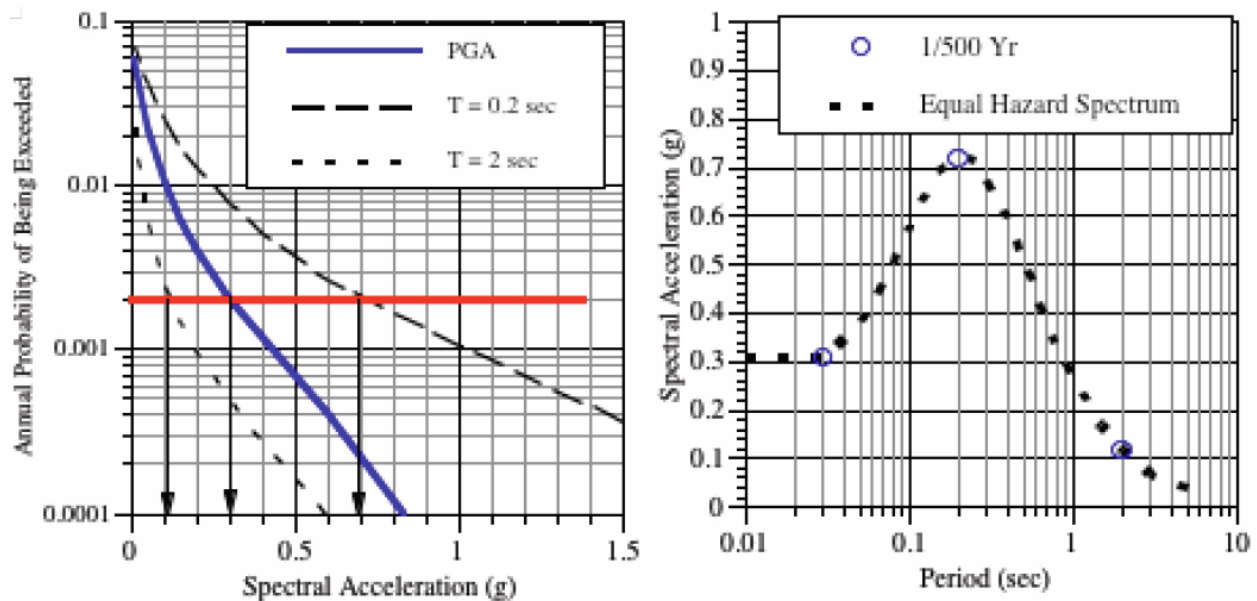

Figure 1. Procedure for developing a uniform hazard spectrum with a return period of 500 years

Uniform Hazard Spectra and Deaggregation of the Hazard

The hazard curve gives the combined effect of all magnitudes and distances on the probability of exceeding a given ground motion level. To provide insight into what events control the hazard at a given ground motion level and spectral period, the hazard curve is broken down into its contributions from different earthquake scenarios. This process is called disaggregation (Bazzurro and Cornell 1999) whereby the fractional contribution of different scenario groups to the total hazard is computed. The most common form of deaggregation is two-dimensional in magnitude and distance bins. Formally, it is the conditional probability of the ground motion that exceeds a target value being generated by an earthquake in the specific magnitude and distance ranges. For a specified return period and spectral period, the deaggregation in magnitude and distance allows the dominant earthquake scenario (magnitude and distance) to be identified. The controlling earthquake scenario will be different for different return period and spectral periods.

An example of a disaggregation of the T=0.2 and T=1.5 sec spectral accelerations for a site in southern Illinois is shown in Figure 2. At the short spectral periods, the ground motion is controlled by local earthquakes and at the long spectral periods the ground motion is controlled by large magnitude earthquakes on the New Madrid zone. The UHS for a return period of 2500 years, represents an envelope of the ground motion from these two different earthquakes. Therefore, using the UHS leads to an unrealistic spectrum that cannot occur in a single earthquake. As an alternative, multiple spectra for different controlling scenarios can be developed and used.
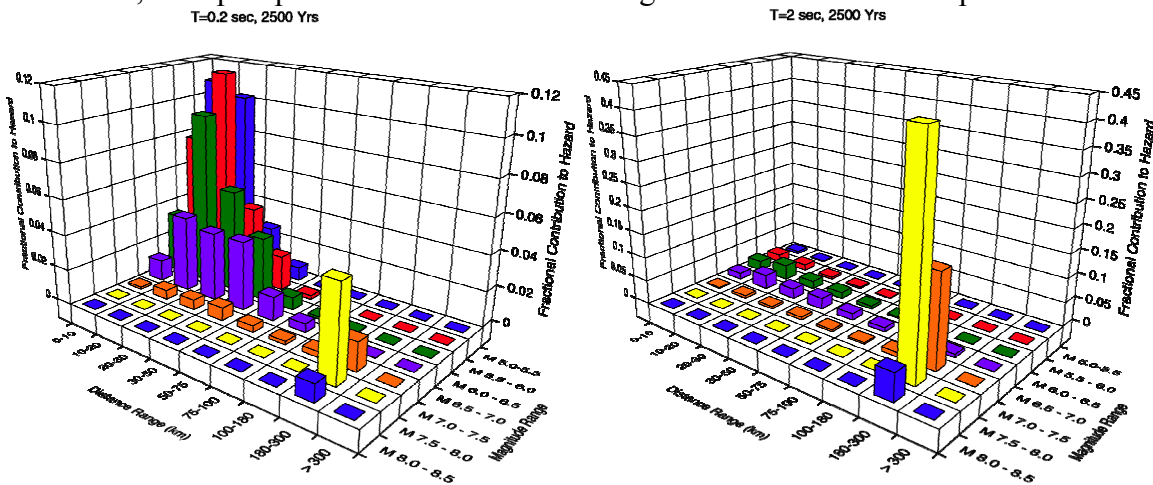

Figure 2. Example disaggregation of the hazard for short period (T=0.2 sec) and long period (T=1.5 sec) spectral acceleration for a site in southen Illinios.

Conditional Mean Spectra

As an alternative to using UHS, Baker and Cornell (2006) developed the Conditional Mean Spectra (CMS). The term "conditional" refers to the spectrum being anchored at a selected spectral acceleration at a reference period of interest (such as the fundamental period of the structure). The term "mean" refers to the mean value of the number of standard deviations (epsilon) of the ground motion model at different periods given epsilon at the reference period, described later. The CMS represents an average spectrum that would occur if the selected spectral acceleration at the reference spectra period occurs.

To construct a CMS, the first step involves disaggregating the hazard at a reference spectral period, T_0, and a specified return period, RP. The hazard leads to a spectral acceleration Sa(T_0,RP). The disaggreation will determine the magnitude and distance of the controlling earthquake scenario. Using the parameters of the controlling earthquake scenario, ground motion attenuation relationships are used to determine the median and standard deviation of the ground motion at reference period T_0 and at all other spectral periods T. The number of standard deviations, $\varepsilon(T_0,RP)$, that the UHS at spectral period T_0 is above the corresponding median spectral acceleration for the controlling earthquake scenario is related to the median and standard deviation of the ground motion for the controlling source and the spectral acceleration from the UHS:

$$SA(T_0,RP) = \hat{S}a(M,R,T_0,RP)\exp\big(\varepsilon(T_0,RP)\sigma(M,T_0)\big) \tag{1}$$

where $\hat{S}a(M,R,T_0,RP)$ is the median ground motion, M and R are the magnitude and distance from of the controlling source determined from the disaggregation, and σ is the standard deviation of the ground motion at period T_0. The mean value of epsilon at other spectral periods given the epsilon at T_0 is then found taking into account the correlation of the variability of the ground motion at different spectral period:

$$\bar{\varepsilon}(T,T_0) = \varepsilon(T_0)\rho(T,T_0) \tag{2}$$

where ρ is the correlation coefficient of the residuals from ground motion models. The CMS is given by

$$CMS(T,T_0,RP) = \hat{S}a(M,R,T,RP)\exp\big(\bar{\varepsilon}(T,T_0)\sigma(M,T)\big) \tag{3}$$

An as example for the southern Illinois site (disaggregation shown in Figure 2), the CMS computed at T=0.2 sec, T=1.5 sec, and T=3.0 sec are compared to the 2500 year return period UHS in Figure 3. This figure demonstrates how the UHS is an envelope of many different earthquakes and will not occur in a single earthquake.
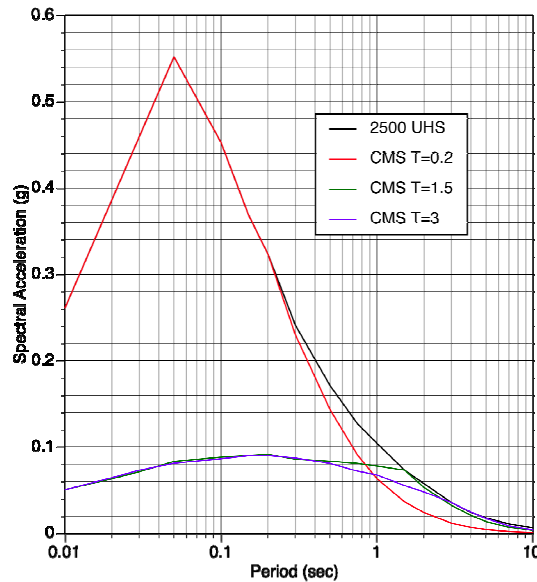

Figure 3. Example comparing the CMS at different spectral periods with the UHS for a return period of 2500 years for a rock site in southen Illinios.

An issue with the application of the CMS has been that it is tied to a single reference spectral period, T_0. In the work by Baker and Cornell (2006), the fundamental period of the structure has been used for the reference period, but what is to be done for structures that do not have a well defined fundamental period? The concept of the CMS is not restricted to developing a single spectrum based on the fundamental period of the structure. We can break the UHS down into a suite of CMS at different T_0 values that, when taken together, reproduce the UHS.. The problem is that you need the CMS at many different T_0 values if the full UHS is to be covered. For example, the three CMS in Figure 3 do not meet the UHS at periods between 0.3 and 1.5 seconds. To keep the CMS limited to a reasonable number of cases, the CMS can be broadened into scenario spectra, such that the envelope of the CMS is equal to the UHS.

The choice of using a UHS rather than multiple spectra for the different scenarios is the decision of the engineering analyst, not the hazard analyst. If it is worth the additional analysis costs to avoid exciting a broad period range in a single evaluation, then the engineer should request multiple scenario spectra from the hazard analyst.

In practice, the hazard analyst often only provides the UHS to the engineer in the hazard report. A hazard report should also include a comparison of the UHS with the spectra from the individual representative events indentified by the disaggregation. This gives the engineer the information needed to make a decision whether to evaluate multiple scenarios one at a time or to envelope the spectra from the multiple scenarios to reduce the number of analyses required.

Conditional Spectra

The use of CMS as a target design spectrum has the advantages of being representative of realistic earthquake scenarios, but the CMS has two shortcomings: (1) the CMS represents mean values and does not capture the peak to trough variability and (2) a suite of CMS cannot be developed that reproduce the hazard curve (Abrahamson and Yunatci, 2010). For risk analyses, we need to provide suites of ground motions with rates of occurrence that are both realistic and reproduce the hazard. To address this need, we propose the development of conditional spectra (CS). These are similar to the CMS, but the use of the mean epsilon has been replaced with multiple realizations of the epsilons. The procedure for developing CS is given below.

The standard deviation of the peaks and troughs variability around a specific CMS can be computed from the correlation of the epsilon values:

$$\sigma_{PT}(T,T_0,M) = \sigma(M,T)\sqrt{1-\rho^2(T,T_0)} \tag{4}$$

An example of the peak to trough range around a CMS is shown in Figure 4. This quantifies the amount of peak to trough variability that should be included in the CS. At the reference period, T_0, there is no peak to trough variability because the spectrum is conditioned on this spectral value. For spectral periods close to T_0, there is only a small peak to trough variability because the epsilon

values are strongly correlated for nearby spectral periods. That is, there is typical width to the peak and troughs. The conditional spectral can be developed by generating suites of correlated epsilon values that are consistent with the peak to trough range shown in Figure 4. The procedure for developing the correlated epsilons is given in the next section.
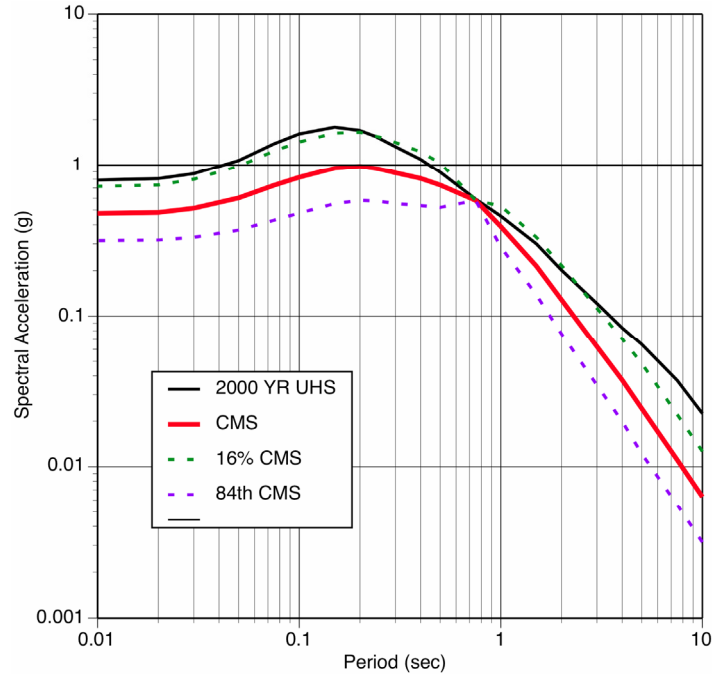

Figure 3. Example of the range of peak to trough variability about the CMS

Once the CS are developed, the second issue for risk analyses is the estimating of the rate of each CS. The rate of occurrences of the CS are derived from the hazard curve. First, for the reference period T_0, the rate of occurrence of $Sa(T_0)$ between z_1 and z_2 can be compute from the hazard:

$$Rate(z_1 < Sa(T_0) < z_2) = Haz(T_0, z_1) - Haz(T_0, z_2) \qquad (5)$$

Next, each realization of the epsilons is equally likely, so the rate for each realization of the CS is just the rate given in eq. (5) divided by the number of realizations of epsilons. If enough realization are included, then using the CS and their rates will reproduce the hazard curves at all spectral periods.

Generation of Correlated Epsilon Values

Spectral accelerations of earthquake ground motions are usually obtained from empirical ground motion models at different spectral periods. The residuals of the ground motion prediction models are correlated across spectral periods. The correlation of the residuals is needed to develop conditional mean spectra and to perform vector hazard calculations. Correlation coefficients can be

obtained from Abrahamson and Silva (2008) or Baker and Cornell (2006). Table 1 shows the Abrahamson and Silva (2008) unsmoothed correlation coefficients for the normalized intra-event residuals. Using these correlation coefficients for the development of the conditional mean spectra gives the mean epsilon values. Figure 5 shows an example of the relationship between the epsilon values at two periods (0.1 and 0.2 seconds). The black line in Figure 4 represents the average correlation between the epsilon values; however, a large scatter is observed around the average. This scatter needs to be sampled for the development of scenario spectra.

The adopted procedure for the generation of different realizations of correlated epsilon values is based on the principles of generating spatially correlated random fields described in Walling (2009). Here we replace space and use spectral period. The goal is to generate different sets of correlated epsilon values that can reproduce an experimental semi-variogram estimated using a set of empirical data.

Variogram Analysis

In spatial statistics, a variogram is a function describing the degree of spatial dependence or continuity of a spatial random field or stochastic process. It can be defined as:

$$2\gamma(\Delta h) = E\left[\left(Z(h + \Delta h) - z(h)\right)^2\right] \tag{6}$$

where $\gamma(\Delta h)$ is the semi-variogram, $z(h)$ is the value of the random variable at location h and Δh is the location separation or lag. A variogram analysis consists of developing an experimental variogram from a set of empirical data and then fitting a variogram model to the data. For ground motion residuals, we find that a spherical model is applicable. The functional form of the spherical model is given as:

$$\gamma(\Delta h) = \begin{cases} c\left[1.5\dfrac{\Delta h}{a} - 0.5\left(\dfrac{\Delta h}{a}\right)^3\right] & \text{for } \Delta h \leq a \\ c & \text{otherwise} \end{cases} \tag{7}$$

where the sill, c, is the value at which the semi-variogram levels off and the range, a, refers to the lag value at which the semi-variogram reaches the sill value.

Applying the variogram analysis to epsilons, the random field to be modeled is epsilon and the spatial lag, Δh, is replaced by the difference of the natural logarithm of spectral periods, $ln(T_1/T_2)$. Using the unsmoothed correlation coefficients, ρ, of the intra-event residuals given in Table 1 between periods of 0.1 and 5 seconds, the experimental semi-variogram was calculated over all pairs of observations with different lag periods as:

$$\gamma\left[\ln\left(\frac{T_1}{T_2}\right)\right] = 1 - \rho\left[\ln\left(\frac{T_1}{T_2}\right)\right] \tag{8}$$

The empirical data were then fitted with the spherical semi-variogram model as shown in Figure

6 and the coefficients of the model were determined. Since epsilon values have a mean of zero and variance of 1, the spherical model was constrained to level off at a value of 1.

Table 1. Unsmoothed correlation coefficients for intra-event residuals (Source: Abrahamson and Silva, 2008)

	PGA	T=0.02	T=0.03	T=0.04	T=0.05	T=0.075	T=0.10	T=0.15	T=0.20	T=0.25	T=0.30
PGA	1.000	0.999	0.992	0.982	0.965	0.921	0.898	0.888	0.885	0.879	0.864
T=0.02	0.999	1.000	0.995	0.986	0.970	0.926	0.902	0.889	0.882	0.874	0.858
T=0.03	0.992	0.995	1.000	0.993	0.980	0.936	0.909	0.886	0.872	0.857	0.837
T=0.04	0.982	0.986	0.993	1.000	0.990	0.949	0.917	0.885	0.863	0.840	0.815
T=0.05	0.965	0.970	0.980	0.990	1.000	0.961	0.927	0.883	0.849	0.818	0.788
T=0.075	0.921	0.926	0.936	0.949	0.961	1.000	0.962	0.893	0.837	0.785	0.740
T=0.10	0.898	0.902	0.909	0.917	0.927	0.962	1.000	0.918	0.850	0.789	0.733
T=0.15	0.888	0.889	0.886	0.885	0.883	0.893	0.918	1.000	0.913	0.843	0.780
T=0.20	0.885	0.882	0.872	0.863	0.849	0.837	0.850	0.913	1.000	0.917	0.848
T=0.25	0.879	0.874	0.857	0.840	0.818	0.785	0.789	0.843	0.917	1.000	0.925
T=0.30	0.864	0.858	0.837	0.815	0.788	0.740	0.733	0.780	0.848	0.925	1.000
T=0.40	0.826	0.817	0.792	0.766	0.734	0.668	0.649	0.688	0.749	0.820	0.885
T=0.50	0.785	0.775	0.748	0.718	0.682	0.609	0.582	0.614	0.672	0.744	0.803
T=0.75	0.679	0.669	0.642	0.611	0.573	0.498	0.467	0.479	0.529	0.598	0.651
T=1.0	0.591	0.581	0.554	0.524	0.488	0.416	0.383	0.392	0.427	0.491	0.545
T=1.5	0.501	0.492	0.467	0.439	0.405	0.344	0.307	0.317	0.353	0.409	0.456
T=2.0	0.404	0.396	0.376	0.353	0.325	0.278	0.246	0.250	0.276	0.319	0.355
T=3.0	0.328	0.320	0.301	0.277	0.255	0.217	0.198	0.202	0.223	0.264	0.289
T=4.0	0.276	0.269	0.251	0.228	0.211	0.183	0.170	0.181	0.194	0.229	0.245
T=5.0	0.204	0.197	0.181	0.164	0.149	0.119	0.113	0.124	0.135	0.152	0.171
T=7.5	0.179	0.174	0.164	0.153	0.142	0.117	0.107	0.098	0.103	0.105	0.121
T=10	0.220	0.219	0.211	0.203	0.187	0.158	0.146	0.146	0.123	0.139	0.164

|	T=0.40	T=0.50	T=0.75	T=1.00	T=1.50	T=2.00	T=3.00	T=4.00	T=5.00	T=7.50	T=10
PGA	0.826	0.785	0.679	0.591	0.501	0.404	0.328	0.276	0.204	0.179	0.220
T=0.02	0.817	0.775	0.669	0.581	0.492	0.396	0.320	0.269	0.197	0.174	0.219
T=0.03	0.792	0.748	0.642	0.554	0.467	0.376	0.301	0.251	0.181	0.164	0.211
T=0.04	0.766	0.718	0.611	0.524	0.439	0.353	0.277	0.228	0.164	0.153	0.203
T=0.05	0.734	0.682	0.573	0.488	0.405	0.325	0.255	0.211	0.149	0.142	0.187
T=0.075	0.668	0.609	0.498	0.416	0.344	0.278	0.217	0.183	0.119	0.117	0.158
T=0.10	0.649	0.582	0.467	0.383	0.307	0.246	0.198	0.170	0.113	0.107	0.146
T=0.15	0.688	0.614	0.479	0.392	0.317	0.250	0.202	0.181	0.124	0.098	0.146
T=0.20	0.749	0.672	0.529	0.427	0.353	0.276	0.223	0.194	0.135	0.103	0.123
T=0.25	0.820	0.744	0.598	0.491	0.409	0.319	0.264	0.229	0.152	0.105	0.139
T=0.30	0.885	0.803	0.651	0.545	0.456	0.355	0.289	0.245	0.171	0.121	0.164
T=0.40	1.000	0.904	0.743	0.629	0.538	0.420	0.335	0.266	0.205	0.172	0.199
T=0.50	0.904	1.000	0.838	0.722	0.619	0.501	0.409	0.332	0.248	0.207	0.220
T=0.75	0.743	0.838	1.000	0.878	0.753	0.634	0.494	0.393	0.291	0.229	0.259
T=1.0	0.629	0.722	0.878	1.000	0.853	0.727	0.581	0.471	0.359	0.287	0.313
T=1.5	0.538	0.619	0.753	0.853	1.000	0.875	0.707	0.601	0.497	0.415	0.450
T=2.0	0.420	0.501	0.634	0.727	0.875	1.000	0.824	0.714	0.638	0.551	0.562
T=3.0	0.335	0.409	0.494	0.581	0.707	0.824	1.000	0.875	0.779	0.658	0.634
T=4.0	0.266	0.332	0.393	0.471	0.601	0.714	0.875	1.000	0.905	0.788	0.720
T=5.0	0.205	0.248	0.291	0.359	0.497	0.638	0.779	0.905	1.000	0.871	0.805
T=7.5	0.172	0.207	0.229	0.287	0.415	0.551	0.658	0.788	0.871	1.000	0.921
T=10	0.199	0.220	0.259	0.313	0.450	0.562	0.634	0.720	0.805	0.921	1.000
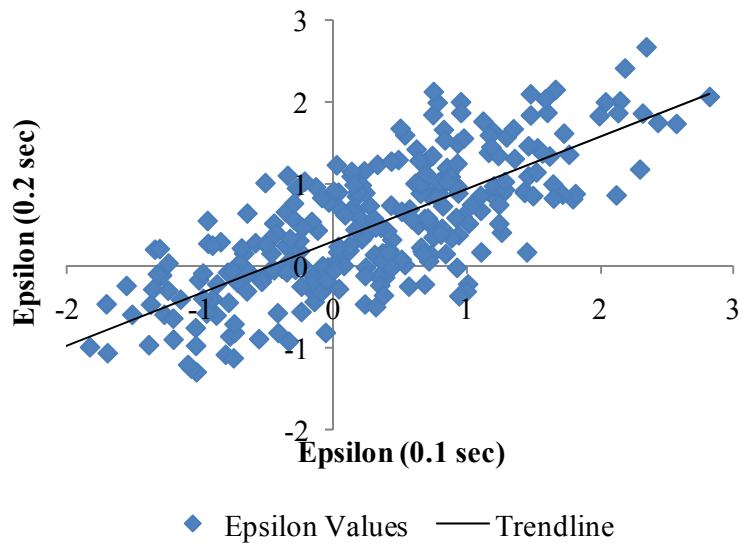
Figure 5. Correlation of epsilon values at periods of 0.1 and 0.2 seconds
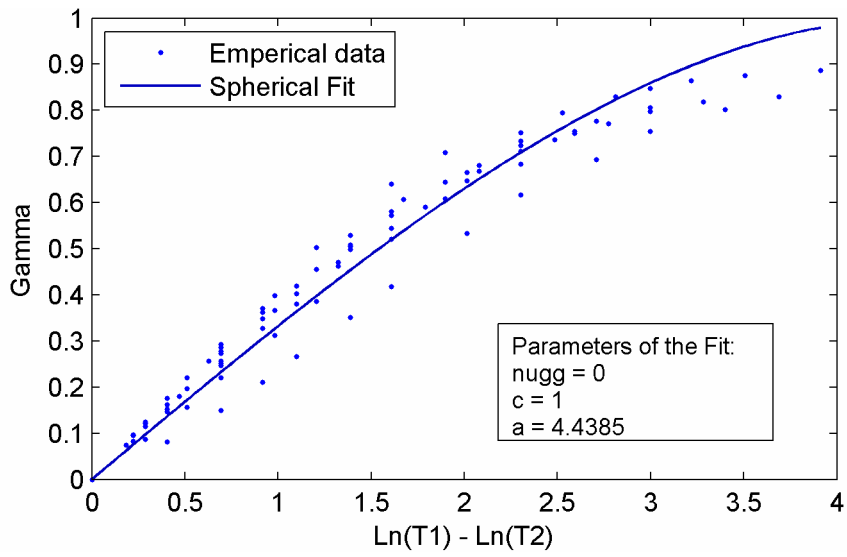

Figure 6. Semi-variogram model for periods of 0.1 to 5 seconds

The procedure for generating one suite of the correlated epsilon is as follows:
1.   Compute the covariance function given by 1-γ.
2.   Extend the covariance function to negative lags to define an even function
3.   Compute the Fourier transform of the extended covariance function
4.   Sample random phase angles
5.   Compute the inverse Fourier transform

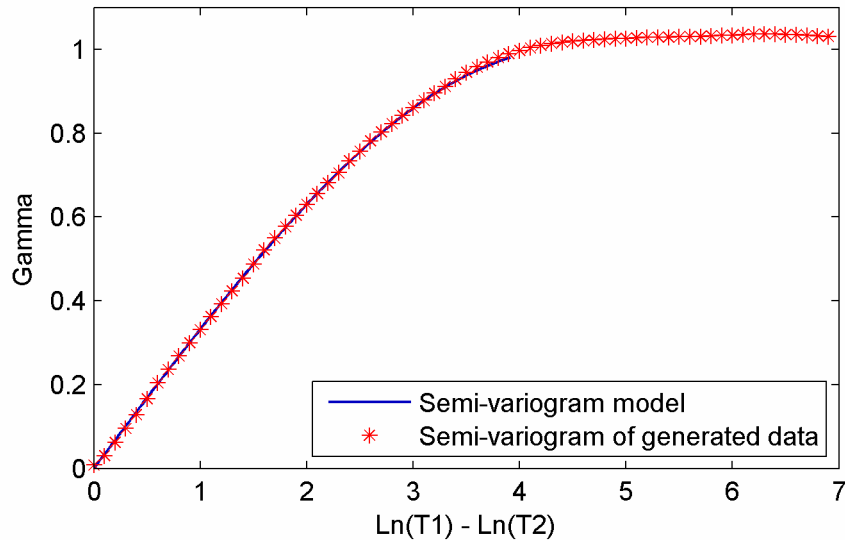As a test of the method, the semivariogram of the generated epsilons can be compared to the model (Figure 7).

Figure 7. Comparison of the semi-variogram model and the semi-variogram of the generated epsilon values

Conditional Spectra Example

As an example, CS are developed for a structure with a fundamental period of 0.5 seconds for a site in the San Francisco Bay area. Using the procedure described above, suites of CS can be developed. It s easy to generate a large number of realizations, but there are limits to the number of scenarios that the engineering analysis can consider. In this example, we limited the number of realizations of epsilons to 10 for each return period. Figure 8 shows the 10 realizations for a single return period. The CS have the same value at the reference period of 0.75 seconds, but they have peaks and troughs in the spectra. Here, the sampling with period has only 20 spectral periods from 0.1 to 5 seconds, limiting the resolution of the peaks and troughs.

As a check, the hazard from the suite of CS can be computed and compared to the original hazard curves (Figure 9). The hazard for the reference period is reproduced closely as this is used to set the rates (eq. 5). At T=0.2 seconds, the hazard computed from the small set of CS is similar to the hazard from the PHSA for return periods from 200 to 5000 years. The underestimation of the

hazard at low return periods results from not considering more small return periods in defining the CS. If a larger number of realizations is used, for example 30 per return period, then the agreement between the hazard from the CS and the PSHA is much better, but that leads to a larger number of cases than most projects will be willing to consider. There is a trade-off between accuracy of the hazard at all spectral periods and the number of realizations that can be considered in the structural evaluations.

This method can also be used to select time histories for dynamic analyses of structures. If the scaling approach is used, then the CS can be used to define the range of the peak to trough variability after scaling to the spectrum at T_0. If spectral matching is used, then the CMS can be used as a target with weak matching criteria to preserve the peak to trough variability given by the CS.
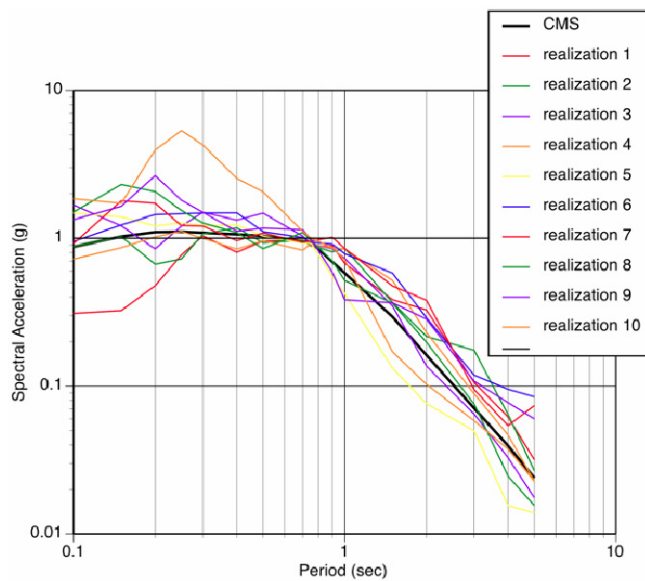

Figure 8. Example of 10 conditional spectra for a ground motion level of 0.93g at a spectral period of 0.75 seconds showing peak to trough variability.
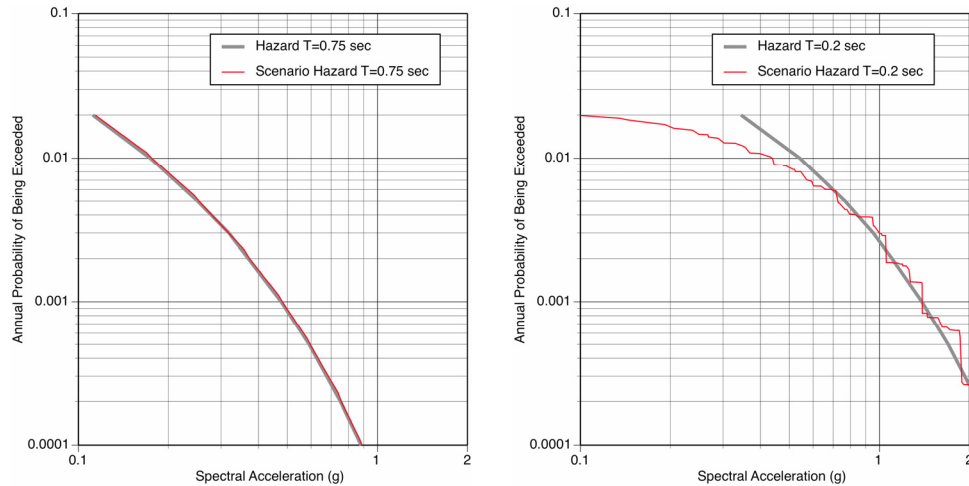
Figure 9. Comparison of the hazard fro the PHSA and the hazard from the CS using 10 realizations for each ground motion level. The left plot is for the reference period T_0=0.75 sec and the right hand plot is for a period of 0.2 sec.

Conclusions

The conditional mean spectra and the conditional spectra provide methods for breaking the UHS down into realistic earthquakes. The choice of using a UHS rather than multiple spectra for the different scenarios should be the decision of the engineering analyst, not the hazard analyst. If it is worth the additional analysis costs to avoid exciting a broad period range in a single evaluation, then the engineer should request multiple scenario spectra from the hazard analyst. For risk calculation for structures that have a response that depends on widely separated spectral frequencies, then the conditional spectral approach can be used.

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