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A geometric model for propagation of tsunami waves

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ABSTRACT

To model the spread of a tsunami wave, its perimeter can be regarded as a continuous curve, or more simply as a discrete number of points, which approximate the curve. Each point on the perimeter can be regarded as point source which expands as small ellipse. The parameters of each ellipse depend on the energy focusing effect, travel path of the waves, coastal configuration, offshore bathymetry and the time step, only to name a few. According to Huygen's wavelet principle, the envelope of these ellipses describes the new perimeter. With discontinuous wave front the perimeter may become complex, developing concavities surrounding area of low rate of spread. An algorithm which defines this envelope has been developed.

Key words: Huygen's wavelet, tsunami propagation, Simulation

Introduction

The enormous destruction of tsunamis to mankind highlights the need for real-time simulation systems accurately predicting their spread. These will be used in the computer-based decision support system that incorporates real-time assimilation of the phenomenon, prompt and expedious collection of data from various sources. By utilizing different spread algorithms prompt warnings may be issued for a program of preparedness. This will be used as a real time technical aid and training tool for controlling disasters.

The aim is to provide a graphical representation of the development of tsunami waves that spread under spatially variable topographic, slope and meteorological conditions. The development of a suitable planning and allocation of resources is a critical phase in mitigating tsunamis. This can be enhanced by reliable simulations of the propagation of a reported tsunami, spreading under actual or forecast conditions. This emphasizes the necessity to find out the perimeter expansion of a tsunami wave under various conditions.Constantin et al**[1]** discussed the range of validity of nonlinear dispersive integral equation for the modeling of propagation of tsunami waves. By considering a three layer system, the generalized governing equations for multilayered long wave system was developed by Imteaz et al**[2].**Recently, Marchuk **[4]** investigated numerically the tsunami wave behavior above the ocean bottom ridges using finite difference method. Prasad Kumar et al **[5]** used the Huygen's method for computing the tsunami travel times based on isochrones table.Lehfeldt et al**[3]** studied the propagation of a tsunami wave in the north sea by performing mumerical simulations and found out the most affected areas in the north sea and the German bight..

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Basic Concept

A simple geometric model, based on Huygen's principle, incorporating elliptical spread at each point on the wave front, is proposed .We assume that each point on the wave front at time t expands as a small ellipse. The new wave front at time t +Δt is defined as the outer envelope formed by small ellipses (Figure1).

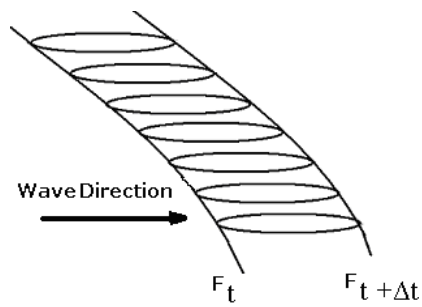

Figure1: Huygen's principle

The propagation ellipse at each point can be expressed in form $\frac{(x-a)^2}{a^2}+\frac{y^2}{b^2}=1$

where the forward, flank and back rates of spread are defined as $\frac{(a+c)^2}{\Delta t}, \frac{b^2}{\Delta t}$ and $\frac{(a-c)^2}{\Delta t}$ respectively. If we constrain c as per equation $a^2=b^2+c^2$, the focus of the propagation ellipse and the origin of the coordinate system coincide with the point on the old perimeter. In this form, the wave direction is aligned with the x-axis. The ellipse parameters for each point on a wave front can be estimated from the forward rate of spread and other parameters.

There are two critical assumptions implicit in this application of Huygen's principle. Firstly, it is assumed that each point propagates independently of its neighbors as a small ellipse with the ellipse parameters only dependent on how the energy is focused, the travel path of the waves, the coastal configuration and the offshore topography. Secondly, it is assumed that the spatial variables which effect rate of spread are constant beneath the whole ellipse for the period Δt. Hence, errors will be minimized as $\Delta t \rightarrow 0$ and as number of points tend to ∞.

The purpose of the algorithm described in this paper is to define the envelope in a reliable and speedy manner. The algorithm is presented as follows.

Algorithm description

The concept of perimeter expansion algorithm is based on the method of determining points on the new perimeter. The wave front (perimeter) is maintained as a set of points $\mathbf{W_t}$,

in clockwise order. The algorithm first accesses the database for information at each point $\mathbf{P_{t,i}}$ in the parent perimeter $\mathbf{W_t}$. Parameters such as energy, travel path, coastal configuration are retrieved for the selected rate of spread at each old perimeter point we define the ellipse parameters a,b,c using the estimated forward rate of spread and the length to breadth ratio.

A point or points on the propagating ellipse are then selected for inclusion in the new perimeter $\mathbf{W_{t+\Delta t}}$ .At each old Pont, we define the deflection, D as line change of direction of the old perimeter such that a point in a concave part of the perimeter has a negative deflection. We define four curvature categories accordingly to the deflection, D, at the point on the old perimeter.

Concave when $D \leq \frac{-\pi}{4}$

Low curvature when $\frac{-\pi}{4} < D < \frac{\pi}{4}$

Moderately convex when $\frac{\pi}{4} \leq D \leq \frac{\pi}{2}$

Sharply convex when $D > \frac{\pi}{2}$

If a point on the old perimeter is on the section of perimeter of low curvature we select only one point on the propagating ellipse to contribute then we identify three points on the perimeter and in the case of moderately convex points and concave points we identify two points on the perimeter of the propagating ellipse for inclusion. To identify these points we use some additional ellipse geometry.

The locus of the ellipse with respect to the old perimeter point, can be represented in the parametric form

$$X = b \sin S \sin\theta + C\cos\theta - a\cos S \cos\theta$$
$$Y = b \sin S \cos\theta - C \sin\theta - a\cos S \sin\theta \qquad \textbf{...(1)}$$

where θ is the angle of the wave direction and the origin coincides with the old perimeter point. The gradient at any point on an ellipse can be expressed in terms of the parametric variable, S.

$$Gradient = \frac{dy}{dx} = \frac{dy}{dS}\frac{dS}{dx}$$

Thus, for some gradient G, the required value of the parameter S is given by

$$S = \tan^{-1}\left[\frac{b(G\sin\theta - \cos\theta)}{a(G\cos\theta - \sin\theta)}\right]$$

These are two solutions for S, where

$$0 \le S \le 2\pi$$

So, given a gradient, the coordinates of two tangent points can be determined using (1).The two tangent points $\mathbf{p}_1$ **and** $\mathbf{p}_2$ are on the opposite sides of the ellipse. Here we are only interested in the tangent toward the outside of the old perimeter.

Using vector algebra, for a given vector **u**, we can identify two points $\mathbf{p}_1$ and $\mathbf{p}_2$ on an ellipse such that the gradient of the ellipse at those two points are equal to the gradient of the vector. We define the outward normal **n**, by rotating the vector **u** by $\frac{\pi}{2}$ radians in an anticlockwise direction (Figure 2)

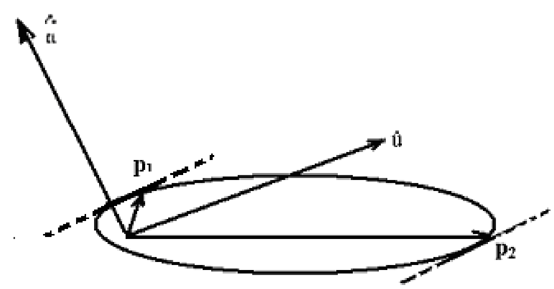

Figure 2: Selection of the outside tangent point

The scalar product of the vector joining the focus and the point $\mathbf{p}_1$ with the outward normal is greater than 0,so $\mathbf{p}_1$ is the required tangent point.So,given a vector, or its gradient, we can identify which of the tangent points on the propagation ellipse is toward the outside of the old perimeter.

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Detection and Deletion of Overlaps

If a section of perimeter is retarded by an area of low spread, the advancing perimeters either side of that area can eventually overlap (Figure 3)

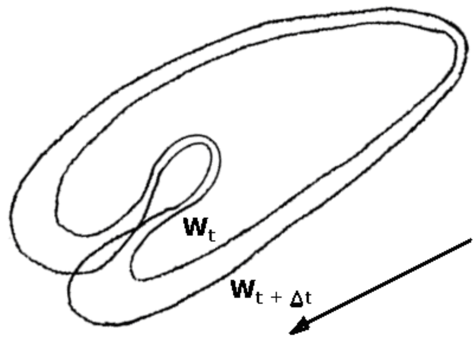

Figure 3: Formation of internal loops

Our interest is in the external perimeter of the waves, rather than the internal untraversed area which may or may not, eventually traversed. The easy approach would be to check every line for an intersection with every other line, a laborious but not impossible task for the computer. Instead, we divide the perimeter into segments.

Segments are defined by a division of the wave front into horizontal zones(Figure 4).A segment is defined as all the points in a length of perimeter as it crosses a zone. The first and last point in a segment actually resides in the adjacent zones.

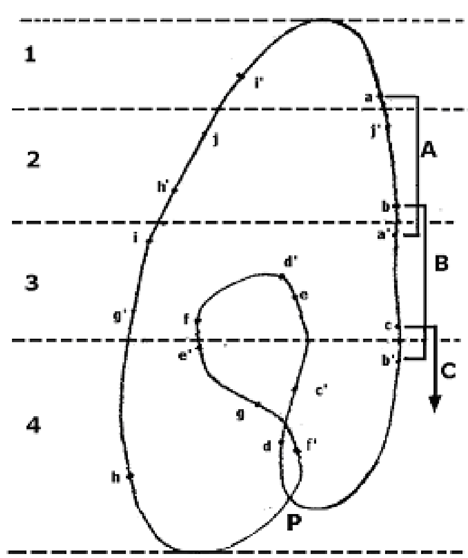

Figure 4: Overlap detection and removal

Each segment is compared against each other segment, except for itself and adjacent segments which by definition can provide no intersections. In the figure the perimeter is divided into four zones and has ten segments. Segment A includes all points from point a to a'.Similarly, B includes all points from b to b' etc, Although segments B and D,B and E, and B and H share the same zone numbers, their maximum and minimum x values do not overlap, hence no line intersections can exist. Segments C and G share the same zone numbers and their maximum and minimum x values do overlap so we check for intersections between the line segments. The intersection point P is identified and the internal loops clipped out of the perimeter. An overlap which occurs completely within one zone may persist until it grows large enough to spread across more than one zone.

Merging of perimeters

A related task in the system with multiple wavefronts is the merging of two different perimeters. If more than one wave front is being propagated then a rectangle aligned with the coordinate system which just spans the perimeter is formed. If rectangle associated with separate perimeters overlap then we can define overlap rectangle (Figure 5).

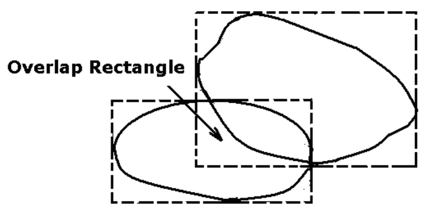

Figure 5: Identification of overlap rectangle

A line of intersection check is performed to identify the intersection points. If any are found the two perimeters are merged by clipping out those points between the intersection points.

Validity of the model

We can validate our model by considering the simulation of tsunami waves under different cases. When the initiating wave has a small irregular perimeter, the algorithm produces a large elliptical wave front under uniform energy conditions as is commonly observed (Figure 6).

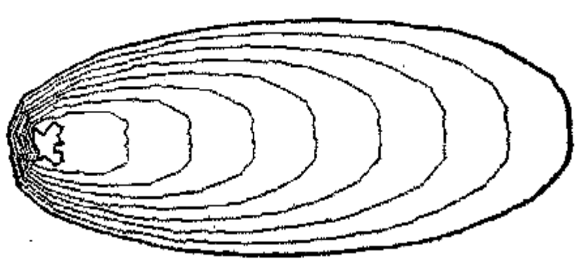

Figure 6: Under uniform energy, slope conditions, an initial perimeter begins to approximate an ellipse after a number of time steps.

The spread of the algorithm is satisfactory when predicting wave spread which is occurring on a scale of seconds under homogeneous and non-homogeneous oceans (Figure 7).The results obtained by our model are in

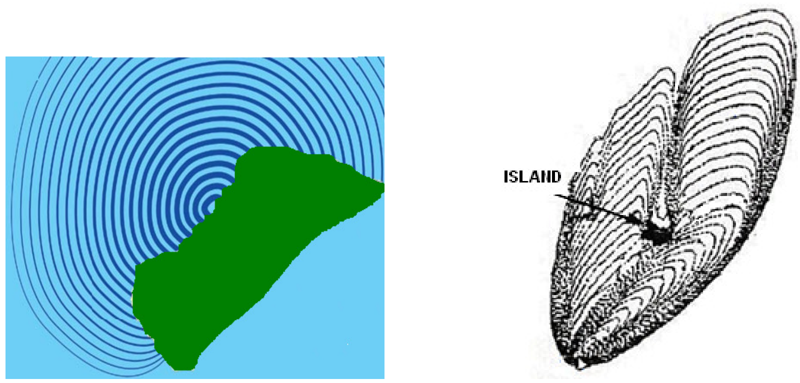

Figure 7: The performance of the algorithm under different cases.

commensuration with the tsunami data available in literature(for example [**6**]) of Solomon island tsunami of 2007 and others.

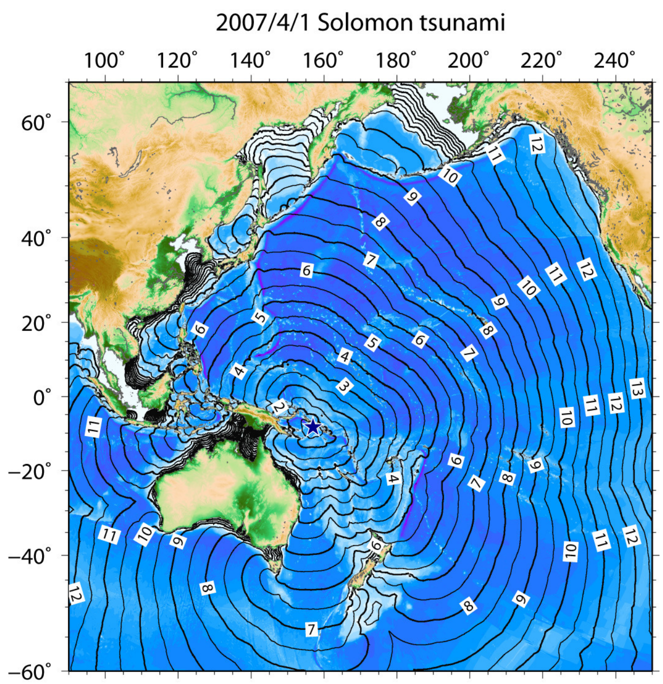

Figure 8: Solomon island tsunami on 2007/4/1 **[6]**

With a simple geographical database consisting of one large polygon of ocean with high tsunami danger and uniform energy type, a single point explosion developed into an elliptical perimeter. When the density of points on a part of the perimeter becomes sparse we can add intermediate points, thus increasing the number of points in defining the perimeter.

Limitations

In addition to the inaccuracy related to the models used to generate the propagating ellipses, the application of Huygen's principle to the propagation of wave fronts rests on two assumptions. Firstly, that points propagate independently of their neighbors and that rate of spread varies linearly between adjacent points, and secondly that the spatial variables effecting rate of spread are constant for the period Δt.

The performance of this algorithm in defining the enveloping curve is limited by the assumption made regarding the location and number of points on each propagating ellipse that are included in the new wave front. The ideal enveloping curve would consist entirely of the common tangents between neighboring propagating ellipses connected with arcs from the ellipses. We use the gradient of the old perimeter. That gradient is only equal to that of the common tangent between the ellipses if the ellipses are identical. Hence the tangent points we derive only approximate the true tangent points. In some cases we may have to use iteration method to determine the tangent points. The magnitude of the errors associated with the enveloping curve approximation are trivial that are inherent in the practical application of Huygen's principle to the propagation of a wave front.

Conclusion

An algorithm is described based on Huygen's frontal propagation in a straightforward manner. Points on the perimeter of propagating ellipses are selected to form a new perimeter which approximates the ideal enveloping curve. Each newly defined perimeter is corrected to remove rotations which are generated at concave points .Further, the overlapping sections of the perimeter are identified efficiently and any internal loops are clipped out of the perimeter leaving only the outer perimeter which is of principal interest. The algorithm is fast enough to be useful in real time simulation of the spread of tsunami waves.

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