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ANALYTICAL SOLUTIONS OF TSUNAMI WAVES INDUCED BY SEA FLOOR COLLAPSES

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ABSTRACT

This paper considers tsunami generation by sea floor collapses. Sea floor collapse is considered by both moving sea bottom or by rapid water discharge into a sea floor collapsing hole. The mathematical formation is done through the use of linear potential theory for fluid. The problem is specified in terms of velocity potential with appropriate boundary conditions imposed on water surface as well as at the sea bottom. Time dependence of impulsive movements of sea bottom is modeled as Dirac delta function. Green;s function method is used to provide a framework for analyzing arbitrary sea floor movement in a circular domain. Exact solutions were derived for some special cases of vertical sea bottom movements. In general, the solution is expressed in terms of infinite integral, but for the case of far field and long time solution, the asymptotic form of the integral can be evaluated by using Kelvin's stationary phase method. This corresponds to the leading waves of a tsunami generated by sea floor collapse. The solutions of the present study can provide the initial phase of sea surface conditions and can be used as input for tsunami simulations using TSUNAMI-N2, COST and COMCOT.

Introduction

The economic and infrastructural development in China in recent years has progressed rapidly, and most costal areas have been developed into urban areas with dense population. These recent developments make tsunami a real threat to the modern development. A repeat of 2004 Indian Ocean Tsunami along the coastline of China would be much more devastating than any historical tsunami occurred in China. As reported by Chau (2008), according to the historical records compiled by Lu (1984) there are over 200 tsunami of unknown causes occurred along China's coastline since 48 BC. At least six of these tsunami events killed more than 10,000 people (in 1045, 1329, 1458, 1536, 1776 and 1782). However, the number of historical tsunami events announced by State Oceanic Administration of China (Zhou and Adams 1988) are overly underestimated and therefore, current recognized tsunami hazard level in China is highly underrated (Chau et al. 2006).

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Although the exact formation mechanism of these tsunami of unknown origin is difficult to be identified without the tide gauge record, we postulate that some of the historical tsunami events in China are caused by submarine landslides. Submarine landslides have been identified as one of the sources for several destructive tsunamis, such as the devastating Papua New Guinea tsunami in 1998, which killed over 2,000 people (Lynett et al. 2003). There are five sea floor debris fans found at the continental self in the South China Sea (Liu 1992), ranging from 38-98 km in length and 11-39 km in width. In the Pearl River delta areas, there are also four debris fans ranging 54-66km in length and 14-50 km in width (Liu 1992). Based on 3.5 Hz-echograms conducted by the Lamont-Doherty Geological Survey ships from 1965-1980 in South China Sea, the largest single slump/debris flow complex near the edge of the continental self is up to 4,500 km^2 (Damuth 1980). These are potential evidences of submarine landslides and their occurrences might cause some historical tsunami of unknown sources (Chau 2009a, 2009b).

Another possibility of tsunami formation may be caused by sea floor collapse along the coastline. One may ask whether sea floor collapse can actually happen and cause tsunami. One vivid example of sea floor collapse is the Great Blue Hole (see Fig. 1). It is a large underwater sinkhole off the coast of Belize of Mexico, and lies near the center of Lighthouse Reef, a small atoll 100 km from the mainland and Belize City. The hole is a perfect circle with 305m in diameter and 123 m deep (see Fig .2). It was formed as a limestone cave during the last glacial period. It is believed that as the ocean began to rise again, the caves flooded, and the roof collapsed.

In South China, the Panyu 30-1 gas field, which is within 100 km of Hong Kong, was developed by the China National Offshore Oil Corporation (CNOOC). Gas extraction from sea bottom may also lead to sea floor collapse or movement, which in turn may lead to local tsunami events. As remarked by Guo (1991), submarine cave-in is not uncommon at where marine deposits are found in abundant. Bursting of submarine shallow-seated gas can also led to catastrophe. Therefore, we cannot rule out the possibility of sea floor collapse. Although tsunami has been linked to earthquakes, submarine landslides, volcanic eruption, or meteor impact (e.g. Bryant 2001), there is no theoretical formulation linking tsunami formation to sea floor collapse.

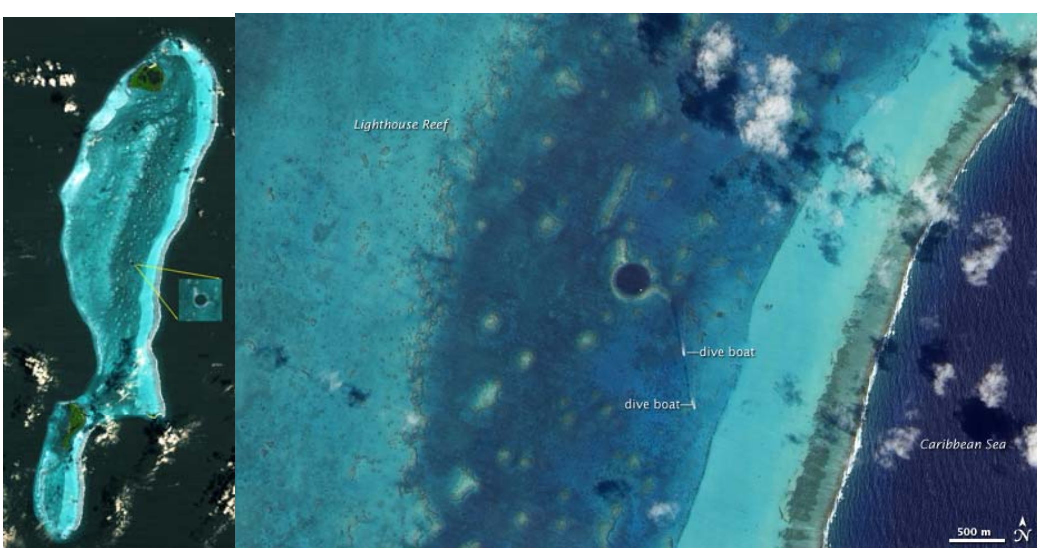

Figure 1. The Great Blue Hole in the Carribean

Figure 2. A close-up view of the Great Blue Hole near Belize (305m in diameter)

In this paper, we will formulate the problem using Green's function approach summarized by Mei (1989). In particular, the fundamental solution for an impulsive point source will be use to formulate the free surface velocity in terms of the sea floor sources in the form of vertical sea floor movements. Both the cases of axisymmetric and anti-symmetric source zones will be formulated by using the additional theorem of the Bessel functions. The axisymmetric solution corresponds to the case of symmetric sea floor collapse while the anti-symmetric solution corresponds to the case submarine landslides or anti-symmetric sea floor collapse. The time dependence is solved exactly using Laplace transform. A framework for solving arbitrary imposed bottom velocity field in a circular region is also given in terms of series of Bessel functions, sine and cosine. The leading wave characteristics were examined using Kelvin's stationary phase method.

Tsunami due to Sea Floor Collapse

Governing Equations

For inviscid irrotational flow, the velocity u and water pressure can be expressed as the gradient of a scalar potential Φ (Lamb 1975, Milne-Thomson 1996, Batchelor 2000, DeDontney and Rice 2008):

$$u = \nabla \Phi \tag{1}$$

$$p = -\rho g z - \rho \frac{\partial \Phi}{\partial t} - \frac{\rho}{2} \left| \nabla \Phi \right|^2 \tag{2}$$

where p is the pressure, $\vec{u}$ is the velocity field, ρ is the water density, g is the gravitational constant, and z is the vertical coordinate measuring downward from free surface as shown in Fig. 3. For sea floor collapse problems, the boundary condition imposed on $z = -h$ can be expressed as

$$\frac{\partial \Phi}{\partial z} = W(r,\theta,t) \tag{3}$$

which is normally nonzero within a finite domain on the sea bottom.

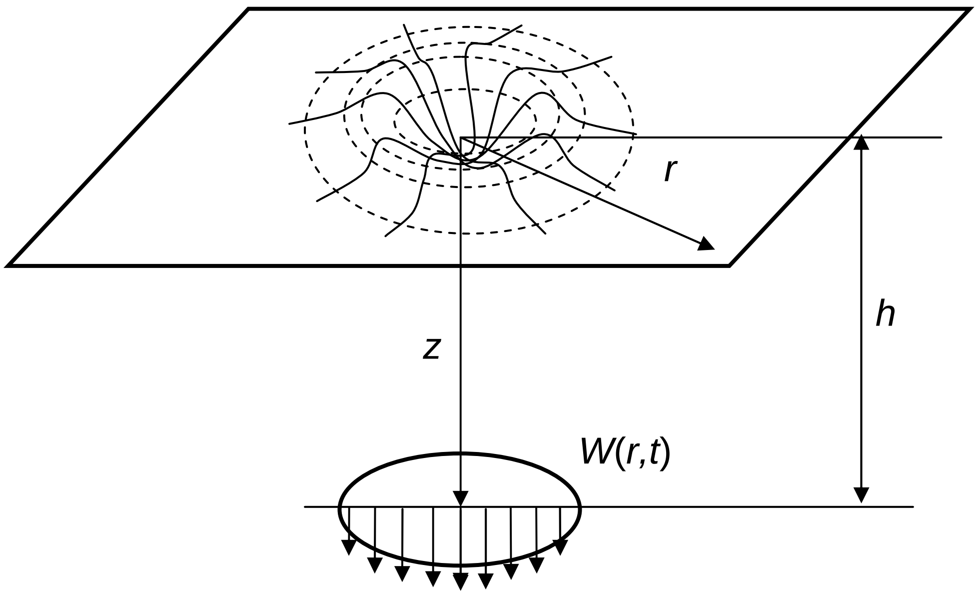

Figure 3. Water surface undulation due to sea bottom movement.

Using the principle of superposition, the Laplace transform of the velocity potential for the water of thickness h with unit sea bottom impulsive source can be written as by virtue of convolution theorem as (Mei 1989):

$$\overline{\Phi}(r,\theta,z,s) = \int_0^\infty r'dr' \int_0^{2\pi} d\theta \overline{W}(r',\theta,s) \int_0^\infty kJ_0(k|\vec{r}-\vec{r}'|)\hat{\overline{G}}(k,z,s)dk \tag{4}$$

where J_0 is the Bessel function of the first kind and zero order (Abramowitz and Stegun 1965), and G is the fundamental solution or Green's function of the water layer of thickness h subject to an impulsive source at sea bottom. The solution of G in the Hankel-Laplace transform space is given as (Mei 1989)

$$\hat{\overline{G}} = \frac{1}{2\pi}\frac{1}{s^2+\omega^2}\frac{s^2 \sinh kz - gk\cosh kz}{k\cosh kh} \tag{5}$$

Equation (4) can be specified on the free surface as

$$\bar{\zeta} = -\frac{s}{y}\bar{\Phi}\Big|_{z=0} = \frac{1}{2\pi}\int_0^{\infty} r'\int_0^{2\pi} \bar{W}(r',\theta',s)\int \; kJ_0(k|\vec{r}-\vec{r}'|)\frac{1}{\cosh kh}\left(\frac{s}{s^2+\omega^2}\right)dkd\theta' dr' \qquad (6)$$

$$|\mathbf{r}-\mathbf{r}'| \equiv \left[(x-x')^2+(y-y')^2\right]^{1/2} = \left[r^2+r'^2-2rr'\cos(\theta-\theta')\right]^{1/2} \qquad (7)$$

$$\omega^2 = gk\tanh kh \qquad (8)$$

where $\boldsymbol{r}'$ (r',θ, z) is the position of the source points where sea bottom velocity is imposed.

Impulsive Bottom Velocity

Consider the case of impulsive velocity being imposed on sea floor

$$W(r,\theta,t) = \Omega(r,\theta)\delta(t-0^+) \qquad (9)$$

For impulsive source, the Laplace transform of (9) is simply $\bar{W} = \Omega(r,\theta)$. By using the following formula of inverse Laplace transform

$$L^{-1}\{\frac{s}{s^2+\omega^2}\} = \cos(\omega t) \qquad (10)$$

Eq. (6) can be readily obtained as

$$\zeta = \frac{1}{2\pi}\int_0^{\infty} r'\int_0^{2\pi}\Omega(r',\theta')\int_0^{\infty} kJ_0(k|\vec{r}-\vec{r}'|)\frac{\cos(\omega t)}{\cosh kh}dkd\theta' dr' \qquad (11)$$

This equation can further be simplified by using the following Addition Theorem for Bessel function (Watson 1944):

$$J_0(k\left[r^2+r'^2-2rr'\cos(\theta-\theta')\right]^{1/2}) = \sum_{n=0}^{\infty}\varepsilon_n J_n(kr)J_n(kr')\cos n(\theta-\theta') \qquad (12)$$

where ε_n is the Jacobi symbol (ε_0 =1, ε_n =2, n=1,2,3,…). Substitution of (12) into (11) gives

$$\zeta(r,\theta,t) = \sum_{n=0}^{\infty}\varepsilon_n\int_0^{\infty} kJ_n(kr)\frac{\cos(\omega t)}{\cosh kh}[W_n^c(k)\cos n\theta + W_n^s(k)\sin n\theta]dk \qquad (13)$$

where

$$\begin{pmatrix} W_n^c(k) \\ W_n^s(k) \end{pmatrix} = \frac{1}{2\pi}\int_0^{\infty}\int_0^{2\pi} r'\Omega(r',\theta')J_n(kr')\begin{pmatrix}\cos n\theta' \\ \sin n\theta'\end{pmatrix}d\theta' dr' \qquad (14)$$

Once Ω(r,θ) in (9) is given, Eq. (14) can be used to find $W_n^c(k)$ and $W_n^s(k)$, and subsequently they can be substituted into (13) to give the free surface solution.

Axisymmetric Velocity

Consider the case that Ω(r,θ) is independent of θ, we have

$$\Omega(r,\theta) = W_0(r) \tag{15}$$

By applying the orthogonality of $\cos n\theta$ and $\sin n\theta$, (14) gives

$$W_0^c(k) = \int_0^\infty r' W_0(r') J_0(kr') dr' = \hat{W}_0(k) \tag{16}$$

with all other terms being zero. Therefore, (13) becomes

$$\zeta(r,\theta,t) = \int_0^\infty k J_0(kr) \frac{\cos(\omega t)}{\cosh kh} \hat{W}_0(k) dk \tag{17}$$

This axisymmetric solution should closely resemble the case of sea floor collapse.

Non-axisymmetric Velocity

For more general non-axisymmetric case, we can assume the following θ dependent sea bottom velocity

$$\Omega(r,\theta) = W_n(r)\cos n\theta \tag{18}$$

$$W_n^c(k) = \int_0^\infty r' W_n(r') J_n(kr') dr' = \hat{W}_n(k) \tag{19}$$

with all other terms being zero by orthogonality of sine and cosine functions. Therefore, (13) becomes

$$\zeta(r,\theta,t) = \cos n\theta \int_0^\infty k J_n(kr) \frac{\cos(\omega t)}{\cosh kh} \hat{W}_n(k) dk \tag{20}$$

When n = 1, Eq. (18) closely resemble the case of tsunami induced by submarine landslide (e.g. Ward 2001, Okal 2003, and Okal and Synolakis 2003). Physically, n = 1 predicts that the upper half of the submarine landslide is modeled by vertical downward movement whereas the lower half of it is modeled by vertical upward movement.

Specific Radial Distribution

To consider the solution, we have to specify the radial distribution of sea bottom velocity. In this paper, the following form is considered:

$$W_n(r) = \frac{A}{a}(a^2 - r^2)^{1/2} \qquad r < a$$
$$= 0 \qquad r > a \tag{21}$$

This can be substituted into both (15) and (18). When n = 0, we have the axisymmetric case and n = 1 for anti-symmetric case. To evaluate (16) and (19), we can use equations 1 and 9 of Section 6.567 of Gradshteyn and Ryzhik (1980) to yield the following formulas:

$$\int_0^\infty rJ_0(kr)(a^2 - r^2)^{1/2}dr = \frac{a^3}{ka}(\frac{\pi}{2})^{1/2}\frac{1}{(ka)^{1/2}}J_{3/2}(ka) \tag{22}$$

$$\int_0^\infty rJ_1(kr)(a^2 - r^2)^{1/2}dr = (\frac{\pi}{2})\frac{a^3}{ka}J_1^2(\frac{ka}{2}) \tag{23}$$

For the axisymmetric case, substitution of (22) into (16-17) yields

$$\zeta(r,t) = Aa(\frac{\pi}{2})^{1/2}\int_0^\infty \frac{\cos(\omega t)}{\cosh kh}\frac{1}{(ka)^{1/2}}J_0(kr)J_{3/2}(ka)dk \tag{24}$$

For the anti-symmetric case, substitution of (23) into (19-20) yields

$$\zeta(r,\theta,t) = Aa(\frac{\pi}{2})\cos\theta\int_0^\infty \frac{\cos(\omega t)}{\cosh kh}J_1(kr)J_1^2(\frac{ka}{2})dk \tag{25}$$

Equations (24-25) provide a close-from solution for evaluating the free surface displacement for an impulsive sea bottom velocity given by (9) and (15) for the axisymmetric case and (9) and (18) with n =1 for the anti-symmetric case.

Leading Waves of Tsunami of Different Sources

If a tide gauge station is located at a position that does not affect by bay seiches, the tsunami recorded at such a station can be used to study the source mechanism of a tsunami. Kajiura (1963) examined the leading wave of an earthquake-induced tsunami when it is at a far distance from the source. Other similar studies include that by Ben-Menahem and Rosenman (1972). For long times or large distance from the source area, we can set $r < t$ as $r \to \infty$. By recalling (8) that $\omega = (gk \tanh kh)^{1/2}$, the asymptotic forms of the integrals in (24-25) can be obtained by Kelvin's stationary phase method given as (Mei 1989, Erdelyi 1956):

$$f(x) = \int_{\alpha}^{\beta} F(k) \exp\{ixh(k)\}dk \tag{26}$$

$$f(x) \approx \left[\frac{2\pi}{xh''(k_0)}\right]^{1/2} F(k_0) \exp\{ixh(k_0) + \frac{i\pi}{4}\} \tag{27}$$

Note that Kelvin's stationary phase method states that the main contribution of the integration comes from the vicinity of points where the function *h(k)* is stationary (i.e. $h'(k_0) = 0$), and the contributions from the end points α and β are negligible. By applying Eq. (26-27), we can obtain the following asymptotic forms:

$$\zeta(r,t) \approx -Aa^2 (\frac{\pi}{2a})^{1/2} (\frac{J_{3/2}(k_0 a)}{k_0 \cosh k_0 h}) \left[\frac{1}{rt|\omega''(k_0)|}\right]^{1/2} \cos[k_0 r - \omega(k_0)t] \tag{28}$$

$$\zeta(r,\theta,t) = \cos\theta (\frac{\pi}{2})^{1/2} \frac{Aa}{\cosh k_0 h} J_1^2 (\frac{k_0 a}{2}) \left[\frac{1}{k_0 rt|\omega''(k_0)|}\right]^{1/2} \sin[k_0 r - \omega(k_0)t] \tag{29}$$

where $\omega'(k_0) = 0$. Comparison of Eq. (28) and (29) shows that the asymptotic form of *r* and *t* of sea floor collapse and submarine landslides (or anti-symmetric sea floor collapse) are the same. But there is no directivity effect from the axisymmetric sea floor collapse whereas there is a strong directivity effect from submarine landslide in Eq. (29). In addition, the leading wave from submarine landslides can either be positive or negative, depending on the relative orientation of the tide gauge station comparing to the slide direction whereas sea floor collapse always accompanies with receding water level before the arrival of the first surge. This information will be useful for studying the source mechanism of tsunami.

For the case of small source zone *a* comparing to *h*, we have $a/h \to 0$. For example, for sea floor collapse within a circular region of radius 20m in a sea of depth of 500m, $a/h \approx 0.04$. When *a* is small, Eq. (28) and (29) can further be simplified by expanding the Bessel functions in series of $k_0 a$.

Conclusions

In this paper, we have formulated a framework to consider sea floor collapse in the form of circular region. Velocity potential was used to formulate the initial boundary problem of tsunami induced by sea floor movements. Dirac delta function of time was used to model the impulsive sea floor velocity induced by sea floor collapse. By using Green's function approach summarized by Mei (1989), exact analytical solutions were obtained for the surface undulation induced by sea floor collapse for both axisymmetric and anti-symmetric cases. The axisymmetric solution corresponds to the case of symmetric sea floor collapse while the anti-symmetric solution corresponds to the case submarine landslides or anti-symmetric sea floor collapse. The leading wave characteristics were examined using Kelvin's stationary phase method.

In addition, computer simulation programs such as TSUNAMI-N2, COST and COMCOT use Okada's (1985) solution for sea bottom movement induced by faulting to directly estimate the sea surface undulation (i.e. assuming the same initial sea bottom and sea surface movements). The solution of the present problems can thus provide the initial phase of sea surface condition that can be used as input for these tsunami simulations.

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