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METHOD OF LINES SIMULATIONS OF TSUNAMI AND UNDULAR BORE PROPAGATION IN RIVERS

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ABSTRACT

The paper presents results of an analytical and numerical study of the tsunami propagation upstream in river channels. This phenomenon was observed in 1983 and more recently in 2003 when tsunami waves ascended several rivers in Japan in the form of a hydraulic bore. A mathematical model for tsunami propagation in rivers as both undular-type and non undular-type bores is proposed. The model equation is based on the regularized long wave equation that includes an additional Burgers term to account for the effects of the dissipation of energy caused by internal viscosity and bottom friction. Exact solutions in the form of a kink-profile for the evolution equation using the hyperbolic tangent method are derived analytically. A numerical solution procedure based on the method of lines is also presented. The exact solution is used to specify initial data for the incident tsunami waves in the numerical model. Several errors are monitored in order to assess the conserved properties and the numerical scheme. Numerical simulations of tsunamis ascending in rivers as an undular bore are also presented.

Introduction

Tsunami coastal effects include penetration of the tsunami waves into rivers and estuaries. Tsunami waves travel through these coastal waters until their energy is totally dissipated. Tsuji et al. (1991) mentioned that during the 1983 Japan Sea Tsunami, the waves travelled into several river systems. The tsunami wave advance through the rivers occurred in two different forms: either as a strong bore or in the form of an undular bore.

Following Tsuji et al. (1991), these two different forms are schematically illustrated in

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Fig. 1. The authors mentioned that a strong bore appears as a step with a flat water surface behind it, whereas an undular bore consists of an initial wave, followed by a train of smaller waves.

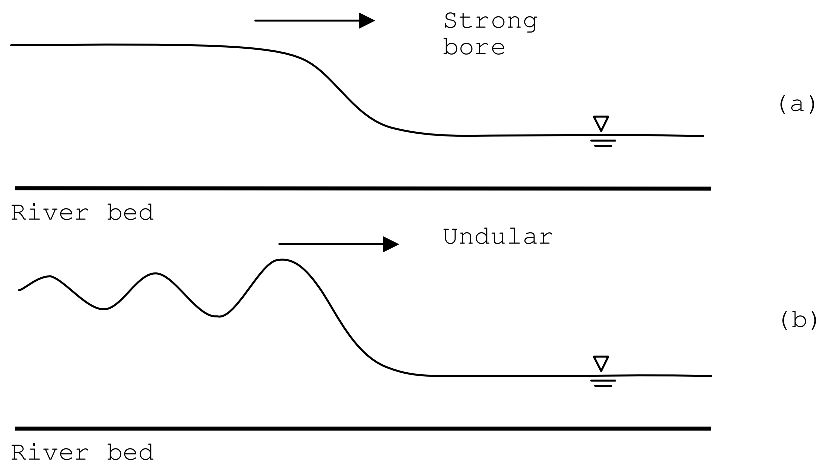

Figure 1. Schematic of a strong bore (a) and of an undular bore (b) (Tsuji et al., 1991).

In the present study, only the undular bore structure is considered. Two different methods used to obtain the solution are presented: an analytical one and a numerical solution of the equal width (EW) wave equation, making use of the method of lines (MOL) with an adaptive grid.

Hydrodynamics of Bores

According to Johnson (1970, 1972), the surface profile of a bore in a viscid fluid is given by the following equation:

$$\eta_t + \eta\eta_x + \beta\eta_{xxx} = p\eta_{xx} \tag{1}$$

where η is the height of the water surface, t is time, x is the direction of propagation, p represents internal or bottom friction and β is a dispersion parameter. In the ideal case of zero viscosity, the asymptotic solution of Eq. (1) is a solitary or a cnoidal wave.

Pelinovskii (1982) used a general form of the KdV-Burgers equation and showed that the following equation can be used:

$$\eta_t + c_0\left(1 + {}^{3\eta}\!/_{2D}\right)\eta_x + c_0 D/6\eta_{xxx} = -F(\eta) \tag{2}$$

Here,

$$c_0 = \sqrt{gD} \tag{3}$$

where D is the water depth and $F(\eta)$ is a friction function.

Chester (1966) showed that the velocity profile follows a parabolic distribution in a Poiseulle flow for a viscid fluid. The water surface can be approximated by a hyperbolic tangent equation in both the up and downstream directions. Under certain conditions, the bore assumes the form of an Airy function. For some recent work on modelling of tsunami undular bores, see Yaacob *et al.* (2008).

Analytical Solution

Hamdi et al. (2005) developed analytical solutions that can be used as input for numerical models for application to the propagation of tsunami waves in river systems in the form of undular bores. They proposed a new model equation for simulating both undular-type and non undular-type bores. The model equation is based on the regularized long wave equation (RLW) that includes an additional Burgers term to account for the effects of the dissipation of energy caused by internal viscosity and bottom friction. Exact solutions in the form of a kink-profile for the evolution equation using hyperbolic tangent method are derived analytically. A numerical solution procedure based on the method of lines is also presented. The exact solution is used to specify initial data for the incident tsunami waves in the numerical model. Several error measures are monitored in order to assess the conserved properties and the numerical scheme. Numerical simulations of tsunamis ascending in rivers as an undular bore are presented.

The Korteweg-de Vries (KdV) equation, $u_t + uu_x + u_{xxx} = 0$, is a well-known nonlinear partial differential equation (PDE) originally formulated to model unidirectional propagation of shallow water gravity waves in one dimension; it describes the long time evolution of weakly nonlinear dispersive waves of small but finite amplitude.

Because of its role as a model equation in describing a variety of physical systems, and because of its interesting mathematical properties, the KdV equation has been widely investigated in recent decades. As early as the 1960s, it was discovered that the KdV equation forms a completely integrable Hamiltonian system and admits an infinite number of conservation laws and invariants of motion (see Miura *et al.*, 1968). An important property of the completely integrable system is the exact interactions of its solitary wave solutions which retain their original shapes and speeds after collision and exhibit only a small overall phase shift. These special solitary waves are named solitons and their clean interactions are called elastic interactions.

More recently, similar models to the KdV equation have been proposed. Benjamin *et al.* (1972) advocated that the PDE, $u_t + u_x + uu_x - \mu u_{xxt} = 0$, modeled the same physical phenomena equally well as the KdV equation, given the same assumptions and approximations that were originally used by Kortweg and de Vries (1895). This PDE is now often called the *regularized* long wave equation, although it is also known as the Benjamin, Bona and Mahoney (BBM) equation. The dispersive term u_{xxt} confers more practical mathematical properties to the RLW equation and therefore makes it a preferable model to the KdV equation. The RLW equation also has explicit second-power hyperbolic secant (sech^2) solitary wave solutions, but it is a non-integrable system since small dispersive effects can be observed when its solitary wave solutions collide.

Hamdi *et al.* (2004b) presented exact solitary wave solutions for general types of the RLW equation with nonlinear terms of any order *p*, $u_t + au^p u_x - \mu u_{xxt} = 0$. The solutions were obtained by integrating a first order nonlinear ordinary differential equation (ODE) with symbolic computation using the mathematical software Maple. In their study, they first showed how this approach could also be applied to derive the exact solutions for the classical generalized Korteweg–de Vries (gKdV) equation, which includes a nonlinear term of any order *p* and a cubic dispersion term, that is,

$$u_t + au^p u_x + \mu u_{xxx} = 0 \tag{4}$$

By coupling this gKdV equation with the quintic regularized long wave (qRLW) equation, which includes fifth order dispersion, a new evolution equation called the gKdV-qRLW equation is obtained; this equation can model the effects of a high order singular perturbation (in the limit $\varepsilon \to 0$) to the gKdV equation,

$$u_t + au^p u_x + \mu u_{xxx} + \varepsilon u_{txxxx} = 0 \tag{5}$$

Exact solitary wave solutions for this new model equation are derived in Hamdi *et al*. (2005). These analytical and explicit solutions are obtained for any p, μ and ε. The approach presented in Hamdi et al. (2005) is general and can also be applied to find exact solitary wave solutions for similar nonlinear wave equations such as KdV-like and Boussinesq-like equations (see Hamdi *et al.* 2004a, 2004b). The exact solitary wave solutions can be used to specify initial data for the incident waves in the numerical model and for the verification of the associated computed solutions (Hamdi *et al.* 2001). Analytical expressions for three conservation laws and for three invariants of motion for solitary wave solutions of this new equation are also derived Hamdi *et al*. (2005). The invariants of motion can be used as verification tools to investigate the conservation properties and performance of numerical schemes for the approximate solution of this new class of PDEs. More details on the derivation of the exact solutions can be found in Hamdi *et al.* (2005).

Numerical Model

Hamdi *et al.* (2001) developed numerical solutions of the EW wave equation using an adaptive method of lines (MOL). As part of the input to the numerical model, the analytical solutions discussed in the previous section were used. They then applied this numerical solution to the formation of an undular bore. This approach is quite relevant for propagation of tsunami waves in rivers.

In this section, the numerical solution is briefly addressed. The EW wave equation is a model PDE for the simulation of one-dimensional wave propagation in media with non-linear wave steepening and dispersion processes.

Hamdi *et al.* (2001) solved the EW wave equation by using an advanced numerical MOL with an adaptive grid, whose node movement is based on an equidistribution principle. These authors presented several numerical solutions to illustrate important features of the propagation of a solitary wave, the inelastic interaction between two solitary waves and the development of an undular bore. The KdV equation shown below,

$$u_t + uu_x + u_{xxx} = 0 \tag{6}$$

is the first classical nonlinear PDE that has been successful for the description of wave propagation with nonlinear wave steepening and dispersion effects. On the other hand, Benjamin *et al.* (1972) suggested that the following equation,

$$u_t + u_x + uu_x - \mu u_{xxx} = 0 \qquad (7)$$

can reproduce the same physical processes, just as well as the KdV equation, provided the same assumptions and approximations as in the KdV equation are made. Eq. 7 is now generally referred to as the RLW equation.

Whereas the KdV equation can be solved by analytical methods for some particular problems, and also in general by the inverse scattering transform (IST) technique, and also by spectral methods (SMS), the RLW and EW equations cannot be solved by the IST. However, the RLW, EW wave and KdV equations can be solved through numerical methods, such as the MOL. The EW wave equation can be written as:

$$u_t + uu_x + \mu u_{xxt} = 0 \qquad (8)$$

The main numerical difficulty in solving this equation is due to the dispersive term, u_{xxt}. This term couples the space and time derivatives. Hamdi *et al.* (2001) solved Eq. 8 by using an advanced MOL with adaptive gridding. The details of this technique and the numerical solutions can be found in Hamdi *et al.* (2001).

Formation of an Undular Bore

Following Hamdi *et al.* (2001), the formation of an undular bore is now briefly described. A long wave, with a gradually and monotonically sloped front, can propagate in deep water without significant change in shape, when the nonlinear effects of steepening are balanced by dissipation and dispersion. However, as a long wave travels into shallow water, the smoothly varying front can steepen further, and this type of a steepening wave is called a bore. Ocean tides can produce large bores (e.g., with amplitudes of 3m) that propagate upstream in river channels and attract bore watchers. When the surface elevation of the water behind a long bore is less than 0.28 times the water depth in front, the steepening front of a bore that is initially smoothly varying will develop surface ripples that grow into a train of large oscillations or undulations, and this type of wave is called an undular bore.

The new equation given by Eq. 8 is solved with $\mu = 1/6$ by the MOL described in the previous section. Numerical results from the MOL solution of the EW wave equation with $\mu = 1/6$ are given first in the form of a time-distance diagram in Fig. 2 for the reduced spatial interval [-20,46], which is the main part of the entire computational domain [-20,55]. The front of the initially smooth bore begins to steepen as it propagates to the right, and this front eventually breaks into an ever-increasing number of undulations, one after the other, forming what is called an undular bore. This bore continues to advance in space and time as a train of oscillatory waves. A close observation of the formation of each undulation will reveal that its peak amplitude increased from an initial value of 0.10 for the initial bore to a somewhat larger amplitude at larger distances and times. This train of undulatory waves carries the mass, momentum, and energy of the initial bore forward in space and time.

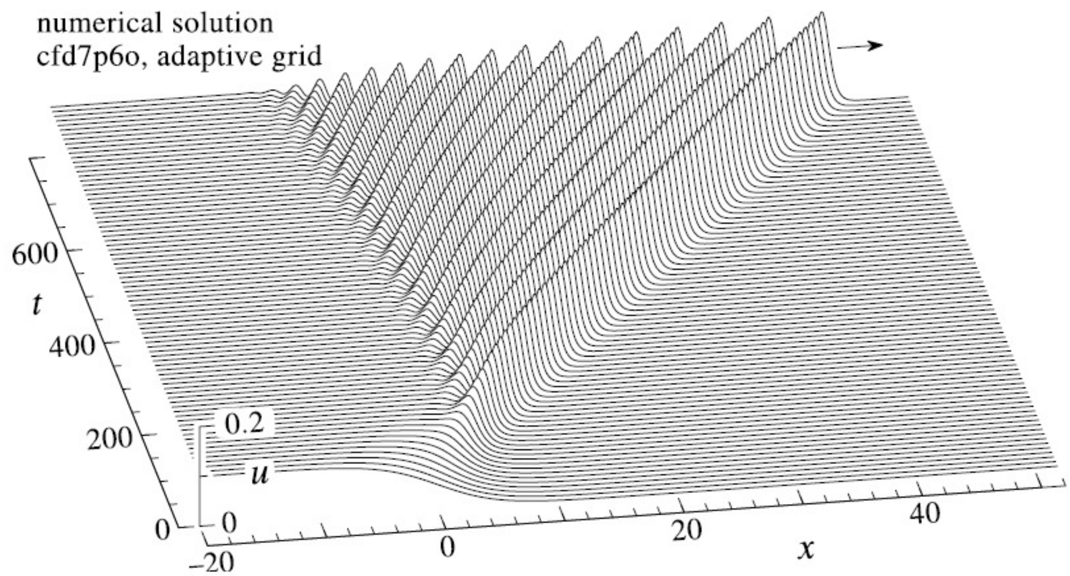

Figure 2. Formation of an undular bore.

A clearer view of the cross-section of the undular bore is presented in Fig. 3, where three spatial distributions at the times t = 0s, 300s, and 600s are depicted.

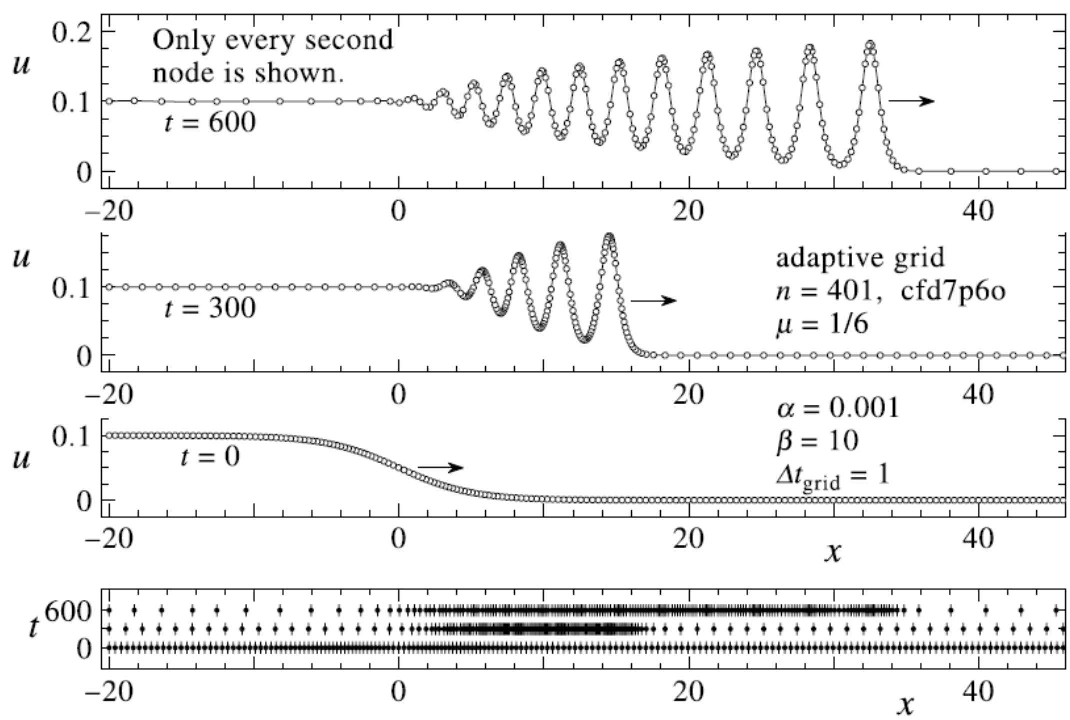

Figure 3. Cross-sections through an undular bore.

An adaptive grid with a large number of nodes is required to obtain an accurate solution at later times because of the increasing number of undulations. For the computational time interval of [0,800], the use of 401 grid nodes is more than sufficient, as can be seen by the node distribution in the undulations at the later time of t = 600s. The grid nodes are well clustered in regions of large gradients and curvatures. Note that only every second node is shown for clarity. The invariants of motion were calculated during the numerical computations to help verify if the solution is computed accurately. The exact values were reproduced and remained constant to four significant digits. This provides more assurance that the solution is computed accurately.

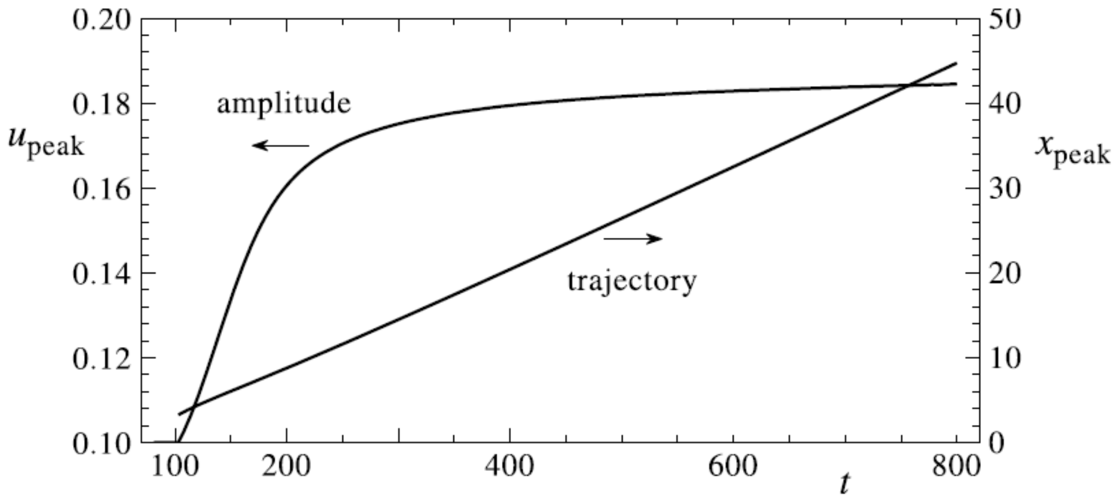

Figure 4. Amplitude growth and trajectory of the leading wave of an undular bore.

The increase in peak amplitude with time of the leading wave of the undular bore and the corresponding slightly concave trajectory are both shown in Fig. 4. The peak amplitude of this leading wave can be clearly seen to increase rapidly at first from the initial value of 0.10 of the original bore, and then it rises more slowly to what appears as an asymptotic limit that is at least 0.182. The trajectory of the peak amplitude of the leading wave accelerates, more quickly at smaller times and then asymptotically to a final speed. The slope of the trajectory at later times, which is the asymptotic speed of the leading wave, is about 0.061, which is about one-third of the peak amplitude.

Summary and Conclusions

An advanced numerical method of lines for solving the equal width equation on uniform and adaptive grids was used to illustrate the formation of an undular bore. Observations show that tsunami waves can propagate upstream in coastal estuarine and river systems in the form of either a regular bore or an undular bore. The numerical simulation tools, that the authors developed based on the method of lines, can be used to perform a rapid forecast of tsunami propagation in rivers and prediction of water levels and amplification effects of bores. These tools will provide a better planning of river or channel walls of both sides of a stream to prevent water overflow into residential areas along the river banks. The model presented in this study can be used by river engineers to produce tsunami inundation maps and evacuation plans of riverside areas.

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