



## INTERRELATION OF AXIAL LOAD LEVEL AND CONFINING PRESSURE FOR MINIMUM DUCTILITY DESIGN OF CONCRETE COLUMNS

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### ABSTRACT

The current ductility design of reinforced concrete (RC) columns is based on some simplified deemed-to-satisfy rules that were derived some decades ago. These rules generally limit the minimum size and maximum spacing of the confinement that provide a minimum confining pressure to the RC columns. However, the rules are concrete strength independent, which have the drawback that the ductility level provided is lower for columns constructed of higher strength concrete or subjected to higher axial load level. To overcome the drawback and provide a consistent ductility level, an extensive parametric study based on nonlinear moment-curvature analysis that investigates the combined effects of concrete strength, axial load level and confining pressure on the ductility of RC columns is conducted in this paper. From the results, it is found that at a given concrete strength, the minimum ductility level actually depends on the axial load level at a fixed confining pressure, and vice versa. Hence, no fixed design value for axial load level and confining pressure can be proposed. Instead, an inequality and chart that ensures the provision of a consistent ductility level to RC columns are developed for practical design purpose.

### Introduction

The adoption of high-strength concrete (HSC) as construction materials for tall buildings is becoming more popular (Kwan 2000). From environmental point of view, the use of HSC in column construction of tall buildings would allow the member size and the material consumption to be significantly reduced. However, from structural safety point of view, HSC is more brittle than NSC, which could result in undesirable brittle failure. In a series of experimental tests, it has been proven (Li *et al.* 1991; Ho and Pam 2003) that the ductility of concrete columns decreases with the concrete strength. Therefore, in order to improve the ductility of HSC columns, more confinement should be provided to HSC columns than that provided to NSC columns previously. With better confinement, HSC columns would increase their chance of survival during earthquake attack.

In normal design of reinforced concrete (RC) members, engineers would design the members to have sufficient strength and stiffness to satisfy the ultimate and serviceability limit states respectively. However, only a certain level of ductility is provided by some empirical

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deemed-to-satisfy rules according to various concrete design codes (SA 2001; MC 2002; BD 2004; ECS 2004; ACI Committee 318 2008) limiting the minimum size and maximum spacing of confinement. Although these rules are satisfactory for NSC columns not subjected to earthquake loads, they may not provide adequate ductility to HSC columns and may trigger brittle failure under extreme events. This has already been verified by both theoretical (Li and Park 2004) and experimental (Li *et al.* 1991; Ho and Pam 2003) studies. Therefore, with a view to providing the same level of ductility in HSC columns, it is recommended that a more scientific approach of imposing a minimum ductility level should be adopted in the design of HSC columns.

In this paper, the results of an extensive parametric study is presented to report the combined effects of axial load, concrete strength, confining pressure and longitudinal steel on the flexural ductility of concrete columns. It will be shown in this study that in order to achieve the required minimum level of ductility, it is necessary to impose a limit on either the maximum axial load level or the minimum confining pressure in the column design. Lastly for practical design of HSC columns, an inequality and a chart have been produced.

### **Nonlinear Moment-curvature Analysis**

The stress-strain curves of unconfined and confined concrete developed by Attard and Setunge (1996) are adopted in the moment-curvature analysis. For steel reinforcement, a linearly elastic-perfectly plastic stress-strain curve is adopted. Since there could be strain reversal in the steel reinforcement at the post-peak stage despite monotonic increase of curvature (Pam *et al.* 2001), it is assumed that the unloading path is linear and has the same slope as the initial elastic portion of the stress-strain curve.

Three basic assumptions are made in the analysis: (1) Plane sections remain plane after bending; (2) The concrete has negligible tensile strength; (3) There is no bond slip between the concrete and steel; (4) The concrete core is confined while the concrete cover is unconfined. These assumptions are widely accepted in the literature (Park and Paulay 1975; Fafitis and Shah 1985; Pam *et al.* 2001). The moment-curvature behaviour of the column section is analysed by applying prescribed curvatures to the section incrementally from zero. At a prescribed curvature, the stresses developed in the concrete and steel are determined from the strain profile across the section depth and their respective stress-strain curves. Then the neutral axis depth and resisting moment are evaluated from the axial and moment equilibriums respectively. The above procedure is repeated until the resisting moment has increased to the peak and then decrease to lower than 50% of the peak moment.

### **Failure Modes and Flexural Ductility**

Three failure modes are observed. They are: (1) Tension failure – maximum tension steel strain is larger than its yield strain; (2) Compression failure – maximum tension steel strain is smaller than its yield strain; (3) Balanced failure – maximum tension steel strain is equal to its yield strain. Tension failure occurs in columns subjected to an axial load smaller than the balanced axial load level, while compression failure occurs when the subjected axial load is larger than the balanced axial load level. Balanced failure occurs in columns when it is subjected to the balanced axial load level denoted by  $(P/A_g f_{co})_b$ . It may be rigorously evaluated

using nonlinear moment-curvature analysis or by the following empirical equation:

$$\left(\frac{P}{A_g f_{co}}\right)_b = 3.1(f_{co})^{-0.5}(1 + 2f_r)^{0.3} \quad (1)$$

The flexural ductility of a column section may be expressed in terms of the curvature ductility factor  $\mu$  defined by Park and Paulay (1975):

$$\mu = \phi_u / \phi_y \quad (2)$$

where  $\phi_u$  and  $\phi_y$  are the ultimate and yield curvatures respectively.  $\phi_u$  is taken as the curvature when the resisting moment of the section has after reaching the peak moment  $M_p$  dropped to a value of  $0.8M_p$ . On the other hand,  $\phi_y$  is taken as the curvature at which the resisting moment would reach the peak moment  $M_p$  if the section has a constant stiffness equal to the secant stiffness at a resisting moment of  $0.75M_p$  (Watson and Park 1994).

### Factors Affecting Ductility of Columns

Based on the above definition, a parametric study on the effects of various factors on the ductility of concrete columns was conducted. The column sections analysed are shown in Figure 1. In the parametric study, the maximum uni-axial concrete strength  $f_{co}$  was varied from 40 to 100 MPa, the axial load level  $P/A_g f_{co}$  from 0.1 to 0.6, the longitudinal steel ratio  $\rho$  from 2 to 6% and the confining pressure  $f_r$  from 0 to 4 MPa. The steel yield strength  $f_y$  was fixed at 460 MPa.

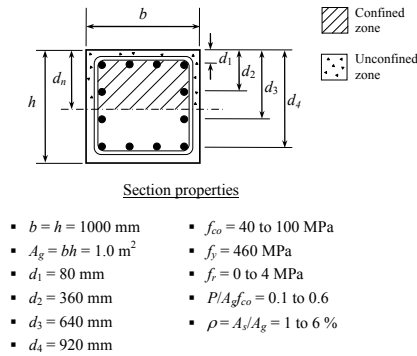


Figure 1. Column section analyzed

#### Axial Load Level

The ductility of column  $\mu$  is plotted against the axial load level  $P/A_g f_{co}$  for column sections having  $f_{co} = 40, 70$  or 100 MPa in Figure 2(a). It is seen that the ductility decreases as the axial load level increases. At a relatively low axial load level when tension failure occurs, the ductility drops rapidly with the axial load level. However, at a relatively high axial load level when compression failure occurs, the ductility drops slowly with the axial load level.

#### Concrete Strength

The ductility of column  $\mu$  is plotted against the concrete strength  $f_{co}$  at constant axial load

level of  $P/A_g f_{co} = 0.1, 0.3$  or  $0.6$  in Figure 2(b). It is observed that at all axial load levels, the ductility always decreases as the concrete strength increases. Hence, when HSC is used in place of NSC at the same axial load level, the flexural ductility could become a major concern.

### Longitudinal Steel Ratio

The ductility of column  $\mu$  is plotted against the longitudinal steel ratio  $\rho$  at constant  $P/A_g f_{co} = 0.1, 0.3$  or  $0.6$  in Figure 2(c). It can be seen from Figure 2(c) that at low axial load level ( $P/A_g f_{co} = 0.1$ ),  $\mu$  decreases with  $\rho$ . However, at high axial load level ( $P/A_g f_{co} = 0.6$ ),  $\mu$  remains fairly constant with  $\rho$  (Kwan *et al.* 2006). In general, it is evident that increasing longitudinal would not have significant beneficial effect on the ductility of columns.

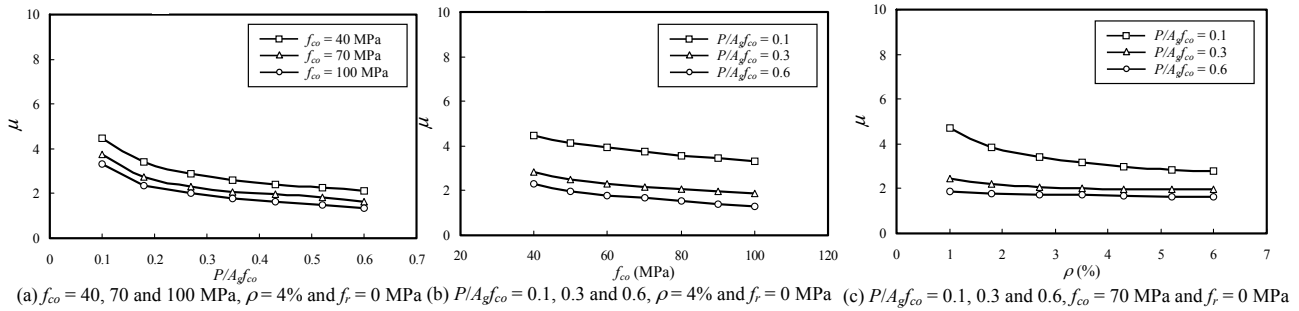


Figure 2. Effects of axial load level, concrete strength and longitudinal steel ratio on ductility

### Confining Pressure

The ductility of column  $\mu$  is plotted against the confining pressure  $f_r$  for  $f_{co} = 40, 70$  and  $100$  MPa in Figure 3(a). It is seen that an increase in  $f_r$  would always increase the column ductility. It is noted that at  $f_{co} = 40$  MPa, an increase in  $f_r$  from 0 to 1 MPa would increase the ductility from 3.0 to 8.3 (277%). At  $f_{co} = 70$  MPa, the same increase in  $f_r$  would increase the ductility from 2.0 to 4.0 (200%). At  $f_{co} = 100$  MPa, the same increase in  $f_r$  would increase the ductility from 1.6 to 2.9 (181%). Hence, the increase in flexural ductility with confining pressure diminishes as the concrete strength increases.

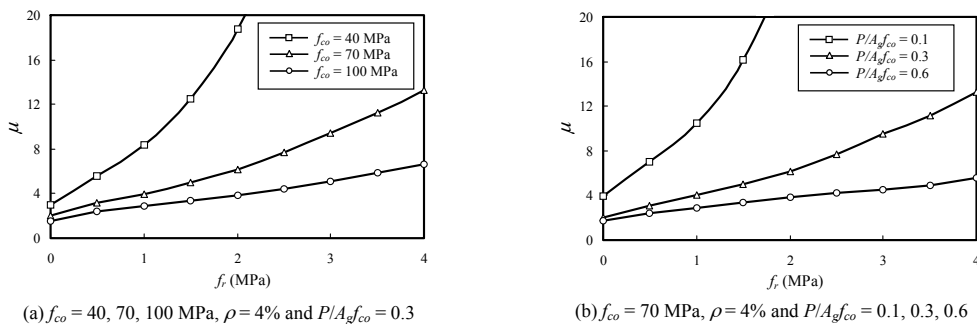


Figure 3. Effects of confining pressure on ductility

The value of  $\mu$  is plotted against  $f_r$  for  $P/A_g f_{co} = 0.1, 0.3$  and  $0.6$  in Figure 3(b). It is apparent that an increase in  $f_r$  would always increase the column ductility. It is noted that at  $P/A_g f_{co} = 0.1$ , an increase in  $f_r$  from 0 to 1 MPa would increase the ductility from 3.9 to 10.5

(269%). At  $P/A_g f_{co} = 0.3$ , the same increase in  $f_r$  would increase the ductility from 2.0 to 4.0 (200%). At  $P/A_g f_{co} = 0.6$ , the same increase in  $f_r$  would increase the ductility from 1.7 to 2.9 (171%). Hence, the increase in flexural ductility with confining pressure also diminishes as the axial load level increases.

### Direct Evaluation of Ductility of Concrete Columns

From the above discussion, it is evident that the effects of axial load level, concrete strength, longitudinal steel ratio and confining pressure on ductility of concrete columns are dependent on the failure mode. The failure mode of a given column may be determined by the axial load to balanced axial load ratio denoted by  $\gamma$ :

$$\gamma = \frac{(P/A_g f_{co})}{(P/A_g f_{co})_b} \quad (3)$$

The column would fail in tension if  $\gamma < 1$  and in compression when  $\gamma > 1$ . Having determined the failure mode, the flexural ductility may be evaluated directly using the formulas developed by regression analysis. The formulas for flexural ductility evaluation of concrete columns failing in tension and compression are given in Equations (4a) and (4b) respectively:

$$\mu = 10.7 (\lambda)^{F(f_r/f_{co})} (f_{co})^{-0.45} G(f_r/f_{co}) \text{ for } \gamma \leq 1.0 \quad (4a)$$

$$\mu = 14.0 (\gamma)^{-0.45} (f_{co})^{-0.45} H(f_r/f_{co}) \text{ for } \gamma > 1.0 \quad (4b)$$

$$\lambda = \frac{P + A_{st} f_y - A_{sc} f_y}{A_{sb} f_y} \quad (4c)$$

in which  $F(f_r/f_{co})$ ,  $G(f_r/f_{co})$  and  $H(f_r/f_{co})$  are functions of the confining pressure  $f_r$ :

$$F(f_r/f_{co}) = -1.25(1 + 5(f_r/f_{co})) \quad (5a)$$

$$G(f_r/f_{co}) = 1 + 2.5(f_r/\sqrt{f_{co}}) \quad (5b)$$

$$H(f_r/f_{co}) = 1 + 30(f_r/f_{co}) \quad (5c)$$

Eqs. (4) and (5) have been verified (Ho *et al.* 2009) using available column tests result obtained by other researchers (Sheikh and Yeh 1990; Sheikh and Khoury 1993; Sheikh *et al.* 1994).

### Improving Ductility of HSC Columns

As discussed before, the existing empirical rules stipulated in some design codes (SA 2001; MC, 2002; BD 2004; ECS, 2004; ACI Committee 318 2008), which are concrete strength and axial load independent, do not provide a consistent level of ductility to concrete columns. In some previous studies carried out by the authors (Ho *et al.* 2004; Au and Kwan 2006; Kwan *et al.* 2006), it has been found that for NSC beams, the minimum curvature ductility factor being provided as nominal ductility by these empirical rules is about 3.32. Since columns are also key

elements in building structures, it is proposed for consistency to adopt the same value of  $\mu_{\min} = 3.32$  for concrete columns not subjected to earthquake loads (Lam *et al.* 2009a, 2009b).

### Maximum Axial Load Level

From Figure 2(a), it is obvious that for any given set of concrete strength  $f_{co}$  and confining pressure  $f_r$ , there is a maximum axial load level  $(P/A_g f_{co})_{\max}$  for achieving a minimum curvature ductility factor of  $\mu_{\min} = 3.32$ . The values of  $(P/A_g f_{co})_{\max}$  for different combinations of concrete strength and confining pressure are evaluated and are summarized in Table 1. From the table, it is evident that  $(P/A_g f_{co})_{\max}$  decreases with  $f_{co}$  but increases with  $f_r$ . The values of  $(P/A_g f_{co})_{\max}$  tabulated in Table 1 are plotted against  $f_{co}$  for various  $f_r$  in Figure 4(a). It is noted that for unconfined columns ( $f_r = 0$  MPa), the maximum axial load levels are generally very low ( $\leq 0.26$  for  $f_{co} \geq 40$  MPa). With the provision  $f_r = 0.5$  MPa, the maximum axial load levels would increase dramatically by about 100%. Hence, at least some confinement should always be provided or otherwise the maximum axial load level would be too low to allow effective use of the strength potential of HSC.

Table 1. Maximum axial load levels

$f_{co}$ (MPa)	Maximum axial load level $(P/A_g f_{co})_{\max}$					
	$f_r = 0$	$f_r = 0.5$	$f_r = 1$	$f_r = 2$	$f_r = 3$	$f_r = 4$
40	0.26	0.56	0.75	0.97	> 1.0	> 1.0
50	0.20	0.35	0.62	0.82	0.97	> 1.0
60	0.16	0.32	0.53	0.71	0.86	0.94
70	0.12	0.27	0.39	0.63	0.76	0.85
80	0.10	0.27	0.32	0.57	0.68	0.77
90	0.09	0.23	0.29	0.50	0.61	0.70
100	0.08	0.22	0.26	0.42	0.56	0.63

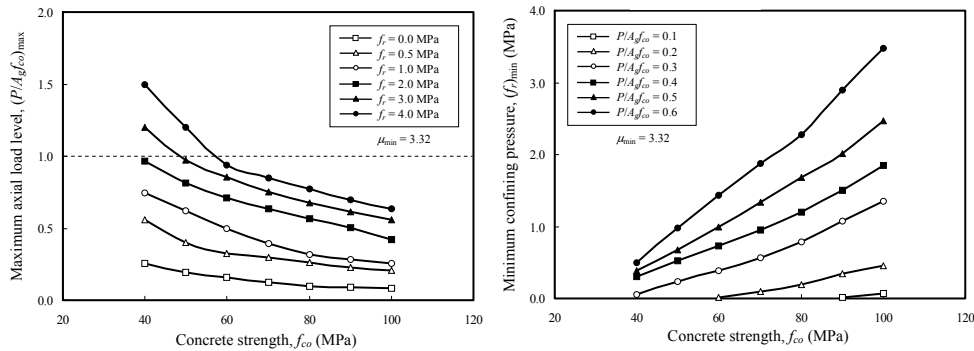


Figure 4. Variation of maximum axial load level and minimum confining pressure with concrete strength

### Minimum Confining Pressure

Likewise, for any given set of  $f_{co}$  and  $P/A_g f_{co}$ , there is a minimum confining pressure  $(f_r)_{\min}$  for achieving  $\mu_{\min} = 3.32$ . The value of  $(f_r)_{\min}$  so obtained for different  $f_{co}$  and  $P/A_g f_{co}$  are

summarized in Table 2. From the table, it is evident that  $(f_r)_{\min}$  increases with  $f_{co}$  or  $P/A_g f_{co}$ . The values of  $(f_r)_{\min}$  tabulated in Table 2 are plotted against  $f_{co}$  for various values of  $P/A_g f_{co}$  in Figure 4(b). The rate of increase of  $(f_r)_{\min}$  with respect to  $P/A_g f_{co}$  is generally higher at a higher concrete strength. This indicates the effectiveness of the confining pressure or reinforcement is lower at a higher concrete strength.

Table 2. Minimum confining pressure

$f_{co}$ (MPa)	Minimum confining pressures $(f_r)_{\min}$ (MPa)					
	$P/A_g f_{co} =$	0.2	0.3	0.4	0.5	0.6
40	0.00	0.00	0.06	0.30	0.39	0.50
50	0.00	0.00	0.23	0.52	0.68	0.98
60	0.00	0.02	0.39	0.73	1.00	1.43
70	0.00	0.09	0.57	0.95	1.34	1.88
80	0.00	0.20	0.78	1.20	1.68	2.27
90	0.02	0.34	1.07	1.51	2.02	2.89
100	0.07	0.46	1.35	1.85	2.47	3.47

### Design Formula and Chart

Design formula for direct evaluation of the maximum axial load level and minimum confining pressure for achieving  $\mu_{\min} = 3.32$  is developed in the following for practical design applications.

$$\frac{(P / A_g f_{co})}{(1 + 3.5 f_r)^{0.65}} \leq 24(f_{co})^{-1.2} \quad (6)$$

This inequality are best studied by plotting the corresponding values of axial load level and confining pressure in the form of contour lines for different concrete strengths as shown in Figure 5. It is noteworthy that the area underneath the contour line demarcates the scenario of  $\mu > 3.32$  and vice versa. This figure may be used as a design chart for determining the desirable combination of axial load level and confining pressure for achieving  $\mu_{\min} = 3.32$ .

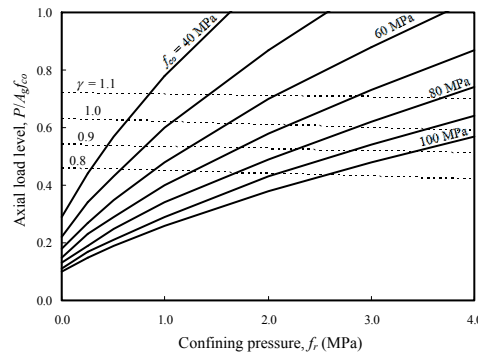


Figure 5. Design chart for axial load level and confining pressure

## Conclusions

The following conclusions are drawn from the study:

- Ductility of concrete columns decreases as axial load level increases.
- Increasing concrete strength will decrease the ductility of columns at constant axial load level.
- Increasing longitudinal steel content will not have significant beneficial effect on column ductility.
- Increasing confining pressure will always improve the ductility of concrete columns.
- Two equations were developed for rapid evaluation of ductility of concrete columns.
- A new method of designing a consistent minimum level of ductility  $\mu_{\min} = 3.32$  in all concrete columns was advocated. It limits the maximum allowable axial load level and minimum required confining pressure for column design.
- An inequality and chart were developed for designing concrete columns with minimum ductility.

## Acknowledgments

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Response-to-comments

Reviewer 1:

Comments:

This reviewer agrees with most of the reported conclusions. That said, one of the conclusions states "Increasing longitudinal steel content will not have significant beneficial effect on column ductility. In particular for columns fail in tension, the increase in longitudinal steel will decrease ductility." This reviewer fears that this conclusion can be interpreted incorrectly. The author should address this reviewer's concern by explaining the effect of the amount of longitudinal reinforcement on the yield displacement (or curvature) and ultimate/maximum displacement (or curvature.)

Response:

Thank you for the comment.

The conclusion is correct. For columns fail in tension, an increase in longitudinal steel content would increase the ratio of tension steel to balanced steel ratio and hence increase the degree of reinforcement  $\lambda$ . Therefore, the ductility of column section would be reduced as a result of increased  $\lambda$  (Lam *et al.* 2009a). However, for columns fail in compression, it can be easily seen from Figure 2(c) that the effects of increasing longitudinal steel content on ductility is quite insignificant.

However, the author agrees with the reviewer that it might create incorrect interpretation because the statement only applies to columns fail in tension. Therefore, the author has removed the second statement in the conclusions to avoid possible ambiguities in readers' understanding.