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EXAMPLE COLLAPSE PERFORMANCE EVALUATION OF STEEL CONCENTRICALLY BRACED SYSTEMS

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ABSTRACT

Conventional special concentrically braced frames (SCBF) have been widely used due to their efficiency in resisting lateral forces. Under severe lateral loading, braces will buckle, laterally and locally, leading to a deterioration of the strength and stiffness of the SCBF and under repeated excursions of cyclic inelastic deformation to fracture of braces. On the other hand, buckling restrained braced frames (BRBF) provide more stable hysteretic behavior. The collapse resistance of SCBF and BRBF systems is examined. A series of 2, 3, 6, 12 and 16 story tall, double story X braced frame archetypes designed using the provisions of ASCE-7/05 for seismic design category D_{min} and D_{max} are analyzed using the ATC-63 methodology. The paper examines modeling of steel braced frames, including brace buckling and rupture due to low cycle fatigue, as well as the application of the ATC 63 methodology. Results of representative static pushover and dynamic analyses are presented. For the assumptions in the ATC 63 methodology and the current ability to model braced frame behavior, it was found that, except for lowrise SCBF structures, a high confidence of achieving the collapse prevention limit state was provided. Reasons for the behavior predicted are presented, along with recommendations for improved design and evaluation methods.

Introduction

The system design requirements of ASCE/SEI 7-05 are used as the basis of the design of concentrically braced steel frame archetypes. The relevant seismic performance factors (SPFs) include the R factor, C_d factor, and Ω_0 factor. In current study, C_d is taken as being equal to R. A set of archetypes is developed based on the design requirements of ASCE/SEI 7-05. Nonlinear computer models of the resulting designs are developed using OpenSees to simulate structural responses to different ground motions. Each of the archetypes was subjected to the far-field ground motion set as described in FEMA P695 (ATC-63). The ground motions were applied with successively increasing intensity according to the FEMA P695 (ATC-63) methodology until collapse was observed. Results are analyzed to predict the collapse capacity of each archetype for each ground motion, and collapse margin ratios (*CMR*) are computed and

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compared to acceptance criteria using the FEMA P695 (ATC-63) methodology.

The evaluations do not constitute a complete study of all possible concentric braced frame behavior, as only a small subset of structural configurations, proportions and detailing options is considered herein. The seismic response of representative SCBFs and BRBFs is examined.

Recent research and design practice favors the use of double-story X-braced frame configurations. Double-story X-braced frames help avoid the large unbalanced transverse beam loads that can occur in chevron or inverted chevron braced frames when braces buckle. While the hysteretic and failure modes of braces dominate the behavior of concentrically braced frame systems, previous analytical studies have shown that the moment frame behavior of the beams and columns in a braced frame structure can play an important role in determining the ultimate behavior of the structure after the braces buckle or fail. Thus, in the analyses, global buckling of braces and possible rupture of braces, beams and columns are explicitly considered in the computational models. Other critical non-simulated failure modes are indirectly accounted for in beams, columns and other components based on available test data. Details of the modeling assumptions are provided later.

Structural System Information

Design Requirements

The archetype design carefully follows ASCE/SEI 7-05 design requirements. The beamcolumn connections and brace-to-framing connections of the archetypes are not designed in detail. It is assumed that these connections have adequate stiffness and strength, and are detailed so they will not fail before the braces rupture. Since the ASCE/SEI 7-05 requirements mainly focuses on braces, beams and columns, many assumptions are needed to design the beamcolumn connections, gusset-to-framing connections and brace-to-gusset connections -- these assumptions have an effect on the overall behavior of the system. As a consequence, it is believed that a FEMA P695 (ATC-63) rating of "B --Good" reflects the degree of confidence in the design requirements for both the SCBF and BRBF archetypes.

Test Data

There have been many studies on the test of braces, braced frame components, and braced frames (Black et al. 1980, 2004; Zayas et al. 1980; Tremblay 2002, 2008; Powell et al. 2008; Yoo et al. 2008; Yang and Mahin 2005; Uriz 2005). Many of these tests were conducted decades ago. The material properties may vary depending on the date and location of the tests. To calibrate the analytical models used herein, data from recent tests on braces and braced frames (Yang and Mahin 2005; Uriz 2005) at Pacific Earthquake Engineering Research Center are adopted. The test data include the cyclic responses of tube braces, pipe braces, buckling restrained braces, conventional buckling chevron braced frames, and chevron BRB frames. Considerable test data are available, but there are important limitations that should be considered such as: 1) the variations in member sizes, 2) the variations in loading condition, 3) the variations in slab details and 4) the variations in drift range.

In summary, the available test data does provide sufficient data to have high confidence in the model parameters and failure criteria. However, due to limitations and variations of experimental procedures and conditions, for the purpose of assessing the total uncertainty, the test data is given at this time a FEMA P695 (ATC-63) rating of "B--Good".

Identification of Archetype Configurations

Table 1 summarizes the properties of the SCBF and BRBF archetype designs needed to evaluate SDC D, including code-calculated structural period, fundamental period of analytical model, and the design base shear. Ten archetypes represent the combination of two seismic demand intensities and five building heights, namely 2, 3, 6, 12 and 16 stories. Fig. 1 illustrates the typical layout of the archetypes. The archetypes intend to cover braced frames in the short and long period range, and are evaluated for high- and low-seismic demands, which are represented by the minimum and maximum demands imposed for Seismic Design Category (SDC) D. The archetypes are designed considering a soil site (Site Class D) condition and the resulting design lateral loads are based on $S_s = 1.5g$ and $S_1 = 0.6g$ for SDC D_{max} , and $S_s = 0.55g$ and $S_1 = 0.13g$ for SDC D_{min} .



Figure 1. Typical layout of SCBF and BRBF archetypes.

All of the braces in both the SCBF and BRBF structures are assumed to have pin connections to the framing. This is for the sake of simplicity for these analyses. Rigid in plane offsets are assumed at the beam-column connections and brace-to-framing connections. The effective length of the braces corresponds to 70% of the work-point-to-work-point length. The effective stiffness of the BRBs are modified to 1.4 times the stiffness computed using only the steel core to account for the stiffness contribution from tapered and connection areas typically encountered with BRBs.

The gravity load only framing system is simplified as being a leaning column in the design. The P-delta effects are considered by applying gravity load on the leaning columns. The gravity columns are assumed to be axially rigid, but to have no lateral resisting capacity.

The braced frames are idealized as 2-D frames, and are designed assuming that all structural members are adequately braced laterally to avoid adverse torsional behavior. Any rotation of the floor diaphragms about a vertical axis is also ignored. As such, torsional effects due to mass and stiffness eccentricities, or premature deterioration of bracing on one side of the building, are not accounted for in the design or response analyses.

Archetyne	No. of Stories	Key Archetype Design Parameters									
Design ID Number		Analysis Procedure	Seism								
			SDC	R	T [sec]	T_1 [sec]	<i>V/W</i> [g]	$S_{MT}(I)$ [g]			
Performance Group No. PG-1SCB											
2SCBFDmax	2	ELF	D _{max}	6	0.26	0.40	0.167	1.50			
3SCBFDmax	3	ELF	D _{max}	6	0.49	0.58	0.167	1.50			
Performance (Group No.	PG-2SCB									
6SCBFDmax	6	ELF	D _{max}	6	0.82	1.02	0.122	1.10			
12SCBFDmax	12	ELF	D _{max}	6	1.38	1.91	0.073	0.65			
16SCBFDmax	16	RSA	D _{max}	6	1.71	3.16	0.059	0.53			
Performance C	Group No.	PG-3SCB									
2SCBFDmin	2	ELF	D _{min}	6	0.28	0.55	0.083	0.75			
3SCBFDmin	3	ELF	D _{min}	6	0.52	0.80	0.064	0.58			
Performance (Group No.	PG-4SCB									
6SCBFDmin	6	ELF	D _{min}	6	0.88	1.51	0.038	0.34			
12SCBFDmin	12	ELF	D _{min}	6	1.47	2.64	0.023	0.20			
16SCBFDmin	16	RSA	\mathbf{D}_{\min}	6	1.83	4.67	0.022	0.16			
Performance (Group No.	PG-1BRB					_	_			
2BRBFDmax	2	ELF	D _{max}	8	0.40	0.50	0.125	1.50			
3BRBFDmax	3	ELF	D _{max}	8	0.73	0.80	0.103	1.23			
Performance (Group No.	PG-2BRB						_			
6BRBFDmax	6	ELF	D _{max}	8	1.23	1.35	0.061	0.73			
12BRBFDmax	12	RSA	D _{max}	8	2.06	2.82	0.044	0.44			
16BRBFDmax	16	RSA	D _{max}	8	2.56	3.73	0.044	0.35			
Performance (Group No.	PG-3BRB						_			
2BRBFDmin	2	ELF	D _{min}	8	0.43	0.68	0.059	0.70			
3BRBFDmin	3	ELF	D _{min}	8	0.78	1.25	0.032	0.38			
Performance C	Froup No.	PG-4BRB	i	,	1						
6BRBFDmin	6	ELF	D _{min}	8	1.31	2.34	0.022	0.23			
12BRBFDmin	12	RSA	D _{min}	8	2.21	3.49	0.022	0.14			
16BRBFDmin	16	RSA	D _{min}	8	2.74	4.83	0.022	0.11			

 Table 1.
 Special steel concentrically braced frame archetype design properties.

Nonlinear Model Development

The archetype structures are simulated using two-dimensional plane frame models with a leaning column, as shown in Fig. 2. The analytical models of the archetypes are implemented in OpenSees (McKenna 1997). The columns are assumed continuous and are fixed to the base for all the nonlinear models. The beams are rigidly connected to the columns. At connections with gusset plates, the behavior is very nearly fixed, even if such connections are not detailed as being fully restrained. The braces including the gusset plates in the ends were modeled with force-based nonlinear beam-column element. Fiber sections were used for the critical sections where

yielding might occur. The beam and columns were modeled similarly to capture inelastic behavior. A corotational formulation was used to model member buckling while local buckling was not explicitly modeled. An empirical cycle counting method was used to simulate rupture due to low-cycle fatigue (Uriz 2005). The vertical floor mass tributary to the braces intersecting a beam or column was included in the models. Earlier studies (Khatib et. al. 1988) showed that this vertical mass has a significant effect on dynamic response during brace buckling. P- Δ effects were represented using a leaning column. The leaning column was constrained to have the same lateral displacement as the most adjacent column at a level in the braced bay. The axial and flexural stiffness of the columns are assumed to be large, but a pin was introduced at the bottom of the column in each story.



Figure 2. Two-dimensional plane frame model with a leaning column.

The analytical models of BRBF archetypes are essentially the same as those of SCBF archetypes, except for the modeling of the braces. Adopting the fatigue model, the BRBs are modeled to conform to the code requirement on the ductility capacity for testing of braces. That is, the BRBs are assumed to achieve a cumulative inelastic axial deformation of 200 times the yield deformation before fracture. As noted previously, rigid end zones are used to increase the stiffness computed using only the steel core to get more accurate stiffness of BRBs accounting for the stiffness contribution form tapered and connection areas.

Non-Simulated Failure Modes

In the analytical models of archetypes, the failure modes due to the damage of the braces are calibrated with test data and accurately simulated by the model. Although the brace failure is one of the most critical response parameters related to potential archetypes collapse, there are other non-simulated failure modes that are not included in the analysis models. These nonsimulated modes are incorporated in this through post-processing or prevented through design details. These failure modes include:

- Beam-column connections failure
- Net reduced section failure of SCBF braces at gusset plates
- Fracture of gusset plate to framing
- Column local fracture (at edges of gusset plate, shear tabs or at splices)
- Column global buckling
- Local buckling of beams, columns and braces (mimicked imperfectly by low cycle fatigue model)
- Lateral torsional buckling related failures

- 3D response of system (such as accidental torsion)
- Panel zone deformation

Many of the non-simulated failure modes can be avoided by detailed design and quality control during construction.

Earlier tests (Newell and Uang 2006) on wide flange columns under cyclic interactions of axial and lateral loading demonstrate that the columns begin to loss their capacity after 7% to 9% story drift ratio. The member sizes of the column test specimens are quite similar to those of the archetypes. The critical story drift capacity of columns is modified to be incorporated in component limit state checks for collapse modes. Because the boundary conditions of the test specimens are more constrained than those of the archetypes, the story drift capacity for the non-simulated failure of columns is modified to 10% radians and is used in the evaluation as the collapse criteria. If the archetype does not show collapsing response at large ground motion intensity, the criteria of 10% drift will be used to check if the archetype should be regarded as having collapsed.

Uncertainty due to Quality of the Computational Model

For the purpose of assessing uncertainty, the model used herein is given a FEMA P695 (ATC-63) rating of "B-Good." The brace behavior is believed to control the failure modes of the SCBFs and BRBFs. The brace models are calibrated with test data and capture the failure responses satisfactorily. Moreover, the braced frame models incorporated with the brace models also coincide with the test data of two-story braced frames. Although there are non-simulated failure modes that are not explicitly included in the analysis models, they are taken into consideration based on the test data. However, the fiber-based element has limited capacity to simulate the local buckling behavior. The capabilities for modeling of capturing the local buckling behavior can be significantly improved. Also, many local and global 3-D effects are not explicitly modeled. Recognizing that the modeling approach used is able to directly simulate structural response up to collapse, and is well calibrated to large amounts of data, but there are still some need for improvement in the simulated and non-simulated failure modes, this model is rated as "B-Good."

Nonlinear Structural Analyses

To compute the system overstrength factor (Ω_0) and to help verify the structural model, monotonic static pushover analysis is used. This pushover is based on the lateral load pattern prescribed in ASCE/SEI 7-05. Fig. 3 shows examples of the pushover curve for the three story archetypes.

For Archetype 3SCBFDmax, brace buckling occurs at about 0.002 roof drift ratio, which is also the drift ratio at the maximum strength. The strength then drops quickly because the braces rapidly lose their compression capacity with increasing lateral displacement. The P-delta effect adds to the negative tangent stiffness of the archetype until complete collapse occurs. The overstrength factor is computed as $\Omega = 730k/519k = 1.41$, and the building ductility capacity can be computed as $\mu_c = 0.012/0.002 = 6.01$ using the ATC-63 methodology. For Archetype

3BRBFDmin, the braced frame yields at about 0.003 roof drift ratio. The maximum strength of 210k occurs at around 0.01 roof drift ratio. The negative tangent lateral stiffness comes mainly from the P-delta effect.

To compute the collapse capacity for each archetype design, the incremental dynamic analysis (IDA) approach is used with the far-field ground motion set and ground motion scaling method. Fig. 4 illustrates how the IDA method is used to compute the collapse capacity of Archetype 3SCBFDmax. The spectral acceleration at collapse (S_{CT}) is computed for each of the 44 ground motions of the Far-Field Set, and then the median collapse level (\hat{S}_{CT}) is computed, which is 2.4 g for Archetype 3SCBFDmax. The collapse margin ratio (*CMR*), defined as the ratio of \hat{S}_{CT} to the Maximum Considered Earthquake ground motion demand (S_{MT}), is 1.60 for Archetype 3SCBFDmax.





Figure 3. Pushover curve for three story archetypes. Figure 4. IDA of archetype 3SCBFDmax.

Performance Evaluation

Table 2 shows the final results and acceptance criteria for each of SCBFs and BRBFs. The table include the collapse margin ratio (*CMR*) computed from IDA, the spectral shape factor (*SSF*), and the final adjusted collapse margin ratio (*ACMR*). The acceptable margins are determined from the quality ratings of design requirements, test data and modeling. Each archetype is shown to either pass or fail the acceptance criteria by comparing the *ACMR* and acceptable *ACMR* values for particular performance groups.

Generally, the SCBF archetypes with short periods have lower *ACMR* values and may thus be the most susceptible to collapse. The sixteen-story archetypes are designed by RSA procedure and have lower *ACMR* values than would be expected from the trend of values for the shorter structures designed according to the ELF method. The two-story SCBF designed for high seismic (SDC D_{max}) does not pass the FEMA P695 (ATC-63) test. The results also show that the archetypes designed for high seismic hazards tend to have lower *ACMR* values than those designed for low seismic hazards.

Arch. Design	Design Configuration		Compu Collap	uted Ov se Marg	Acceptance Check					
ID No.	No. of Stories	SDC	Static Ω	CMR	μ_T	SSF	ACMR	Accept. ACMR	Pass/ Fail	
Performance Group No. PG-1SCB										
2SCBFDmax	2	D _{max}	1.44	1.00	4.3	1.22	1.22	1.56	Fail	
3SCBFDmax	3	D _{max}	1.41	1.60	6.1	1.28	2.05	1.56	Pass	
Mean of Perfor	mance Gr	oup:	1.42	1.30	5.2	1.25	1.63	1.9	Fail	
Performance G										
6SCBFDmax	6	D _{max}	1.34	1.64	6.6	1.36	2.23	1.56	Pass	
12SCBFDmax	12	D _{max}	1.60	3.23	3.2	1.32	4.26	1.56	Pass	
16SCBFDmax	16	D _{max}	2.11	2.64	18	1.21	3.20	1.46	Pass	
Mean of Perfor	mance Gr	oup:	1.69	2.50	3.8	1.30	3.23	1.96	Pass	
Performance G	roup No.	PG-3SC	B							
2SCBFDmin	2	D _{min}	1.38	1.73	5.8	1.12	1.94	1.56	Pass	
3SCBFDmin	3	D _{min}	2.41	3.62	3.0	1.08	3.91	1.56	Pass	
Mean of Perfor	mance Gr	oup:	1.90	2.68	4.4	1.10	2.93	1.96	Pass	
Performance G	roup No.	PG-4SC	B							
6SCBFDmin	6	D _{min}	1.86	3.53	3.9	1.15	4.06	1.56	Pass	
12SCBFDmin	12	D _{min}	2.20	6.00	3.4	1.23	7.38	1.56	Pass	
16SCBFDmin	16	D _{min}	1.56	3.53	1.2	1.06	4.64	1.40	Pass	
Mean of Performance Group:			1.87	4.63	2.8	1.15	5.36	1.96	Pass	
Performance G	roup No.	PG-1BR	В							
2BRBFDmax	2	D _{max}	1.31	1.73	11.9	1.33	2.31	1.56	Pass	
3BRBFDmax	3	D _{max}	1.48	4.63	22.7	1.39	6.44	1.56	Pass	
Mean of Performance Group:			1.40	3.18	17.3	1.36	4.37	1.96	Pass	
Performance G	roup No.	PG-2BR	В							
6BRBFDmax	6	D _{max}	1.47	3.29	15.5	1.53	5.03	1.56	Pass	
12BRBFDmax	12	D _{max}	1.17	2.27	4.0	1.4	3.18	1.56	Pass	
16BRBFDmax	16	D _{max}	1.00	3.14	3.1	1.32	4.15	1.56	Pass	
Mean of Performance Group:			1.21	2.90	7.5	1.42	4.12	1.96	Pass	
Performance G	roup No.	PG-3BR	В							
2BRBFDmin	2	D _{min}	1.44	1.71	6.6	1.13	1.94	1.56	Pass	
3BRBFDmin	3	D _{min}	2.11	5.53	10.5	1.2	6.63	1.56	Pass	
Mean of Performance Group:			1.77	3.62	8.5	1.17	4.28	1.96	Pass	
Performance G	roup No.	PG-4BR	B							
6BRBFDmin	6	D _{min}	1.28	3.04	6.4	1.28	3.90	1.56	Pass	
12BRBFDmin	12	D _{min}	1.44	2.86	3.1	1.21	3.46	1.56	Pass	
16BRBFDmin	16	D _{min}	1.15	4.55	2.0	1.15	5.23	1.46	Pass	
Mean of Perfor	1.29	3.48	3.8	1.21	4.19	1.96	Pass			

 Table 2.
 Summary of collapse performance evaluations of special steel concentrically braced frame archetypes.

The results for BRBF in Table 2 show more variation, but the BRBF archetypes with short periods generally have lower *ACMR* values and may also be more susceptible to collapse. The two-story BRBFs have especially low *ACMR* values compared to the other BRBF archetypes. For the long period group, the twelve-story archetypes have the lowest *ACMR* values. The results of two-story and six-story BRBF archetypes show that the archetypes designed for high seismic demand (SDC D_{max}) do not necessarily have lower *ACMR* values.

Comparing the results for SCBF and BRBF archetypes, the SCBF have smaller *ACMR* values than do the BRBFs. The two-story SCBF designed for SDC D_{max} cannot even pass the test. Adjustment of design requirements may be warranted for short period SCBF (or BRBF) systems.

Conclusions

The archetypes of BRBF pass the FEMA P695 (ATC-63) evaluation protocol. With the exception of the two-story SCBF designed for SDC D_{max} , all of the SCBF archetypes pass as well. As such, the seismic performance factors (SPFs) of ASCE/SEI 7-05 seem appropriate for BRBF based on the methodology and the archetypes considered herein, but may need adjustment for two-story (low rise) SCBF structures.

The Equivalent Lateral Force Analysis (ELF) and Model Response Spectrum Analysis (RSA) methods result in significantly different member sizes for high- and mid-rise braced frames. This difference is not so pronounced for low-rise braced frames. For example, for Archetype 6SCBFDmax the difference in the member forces determined by ELF and RSA is within 10%. For the taller Archetype 16SCBFDmax, the difference in member forces goes up to 50%. This seems counter to the requirement that minimum design forces using the RSA method should not be less than 80% of the values from the ELF method. However, when the structural design becomes controlled by drift, member sizes are controlled by the member stiffnesses needed to satisfy code drift requirements. Less restrictive provisions are applied when using the RSA method to compute drift, and this results in far smaller member sizes.

The SCBF archetypes tend to use brace sizes, which are larger than commonly considered in previous test programs. The local and global behavior, especially low cycle fatigue characteristics, of large braces may differ from those observed for smaller braces. Ultimate behavior is also likely controlled by factors such as fracture of beams and columns near gusset plates, and lateral torsional response of members. Additional test data is becoming available that can be used to improve the confidence in the quality of test data and analytical models.

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