Proceedings of the 9th U.S. National and 10th Canadian Conference on Earthquake Engineering
Compte Rendu de la 9ième Conférence Nationale Américaine et
10ième Conférence Canadienne de Génie Parasismique
July 25-29, 2010, Toronto, Ontario, Canada • Paper No 1660

EVALUATION OF LIQUEFACTION POTENTIAL BY SIMPLIFIED METHOD WITH PROBABILISTIC GROUND ACCELERATIONS

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ABSTRACT

In the Seed-Idriss simplified method for evaluating liquefaction potential, the intensity and duration of seismic demand is specified by a deterministic peak ground acceleration that is associated directly with a particular design earthquake magnitude. The peak ground acceleration specified for a given location in the most recent National Building Code for Canada, NBCC 2005, is probabilistic, with a probability of exceedance of 2% in 50 years. A wide range of magnitudes is associated with all the relevant seismic sources that contribute to the probabilistic acceleration at a given location. The crucial question for implementation of the simplified method is what magnitude should be associated with the specified probabilistic acceleration. Two methods are presented for resolving the magnitude dilemma in a logical way: a weighted magnitude probabilistic analysis and a direct application of deaggregegated weighted magnitudes.

Evaluation of liquefaction potential

The generally accepted procedure in Canada for evaluating the potential for triggering liquefaction is the updated Seed-Idriss (1971) procedure described by Youd et al. (2001). Whether liquefaction occurs or not depends on the balance between the resistance to liquefaction of the soil and the seismic demand on the site represented by the intensity and duration of shaking. The intensity of shaking is defined by the peak ground acceleration and the duration is represented by earthquake magnitude. Adopting the notation recommended by Youd et al. (2001), the seismic intensity at a site is termed CSR, the cyclic stress ratio, and is defined by

$$CSR = \tau_{av}/ \sigma'_{vo} = 0.65 \, (a_{max}/g) \, (\sigma_{vo}/\sigma'_{vo}) \, (r_d) \qquad (1)$$

where a_{max} = peak horizontal ground acceleration at the ground surface; g = the acceleration due to gravity; σ_{vo}, σ'_{vo} = total and effective vertical overburden stresses respectively, r_d = stress reduction coefficient, and τ_{av} = average cyclic shear stress. The inherent resistance to liquefaction is represented in the Seed-Idriss method by either penetration resistance or shear wave velocity. Liquefaction potential may be determined from a liquefaction assessment chart such as that shown in Figure 1. Here the seismic demand is represented by the cyclic stress ratio, CSR, and the resistance by the normalized Standard Penetration Resistance, $(N_1)_{60}$. The curves shown in Figure 1 separate liquefiable from non-liquefiable sites for a given percentage of fines in the sand for a duration corresponding to M=7.5 Stress ratios on these lines are called cyclic resistance ratios, CRR. The factor of safety against liquefaction is given by CRR/CSR.

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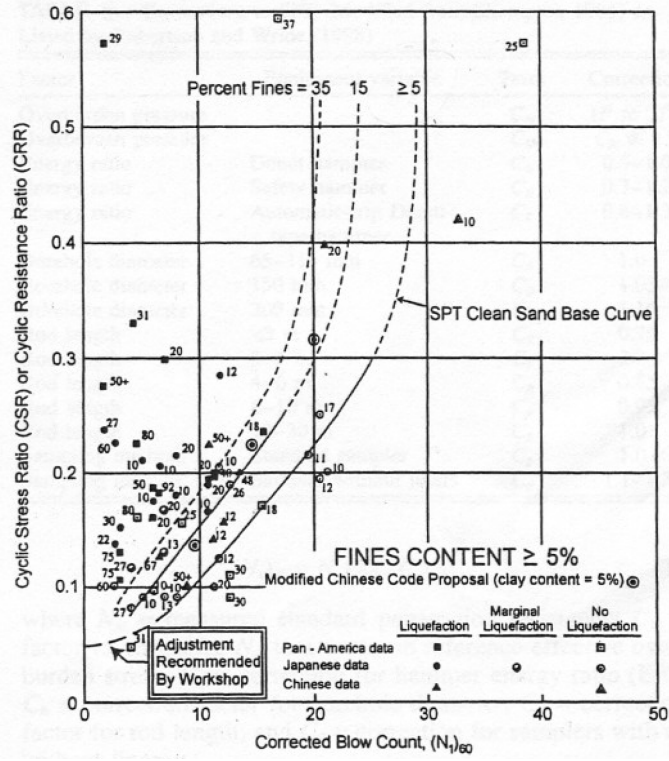[2]BGC Engineering Inc. Vancouver, BC, Canada

Figure 1. Liquefaction chart (from Youd et al, 2001).

The simplified method was originally used with scenario earthquakes in California. The design earthquake was usually located on a fault and the outcrop acceleration at the site to be used for site response analysis was determined by an attenuation relationship. There was a direct link between the design earthquake magnitude and the outcrop acceleration at the site. With the advent of probabilistic ground motion parameters, the direct link between site acceleration and design earthquake magnitude was lost because the probabilistic site acceleration is composed of the contributions of many different earthquakes. For liquefaction assessment in Canada, the site acceleration was assigned to one, somewhat arbitrarily selected, single earthquake magnitude without any assessment of how well the acceleration–magnitude pair simulated the combined effects of all the earthquakes affecting the site. As will be shown later, this procedure results often in the probability of triggering liquefaction being lower than the probability of the structural design motions being exceeded and therefore there may be an unintentional conservatism in evaluating the potential for triggering liquefaction. The degree of conservatism depends on the seismic environment.

The duration of shaking depends on the magnitude of the earthquake. This dependence was recognized by Seed and Idriss (1982) when they introduced Magnitude Scaling Factors, MSF, to relate the contributions of different magnitudes in generating liquefaction relative to the base magnitude, $M = 7.5$, which anchors the widely used liquefaction assessment chart shown in Figure 1. These scaling factors can be applied in two different ways; either to the liquefaction resistance or the seismic demand, when assessing the potential for triggering liquefaction. Youd et al. (2001) described a range of magnitude scaling factors that geotechnical engineers may adopt for use in practice. In this paper the factors recommended by Idriss as reported in Youd et al. (2001) are used. They are a lower bound to all the factors recommended by Youd et al.

(2001) and therefore their use is more conservative. These factors for magnitudes M are given in terms of the base magnitude M = 7.5 in Eq. 2.

$$MSF_M = 10^{2.24}/ M^{2.56} \tag{2}$$

They are also shown in Table 1.

Table 1. Idriss magnitude scaling factors (Youd, 2001).

Mag.	5.5	6.0	6.5	7.0	7.5	8.0
MSF	2.2	1.76	1.44	1.19	1.0	0.84

In this paper the seismic demand is scaled using the magnitude weighting factor, MWF, where MWF is the inverse of the scaling factor.

The effect of the magnitude weighting factor on CSR for a given magnitude is given by Eq. 3.

$$CSR = 0.65\ (a_{max}/g)\ (\sigma_{vo}/\sigma'_{vo})\ (r_d)\ (MWF) \tag{3}$$

When dealing with a scenario earthquake of magnitude M which has a direct link to the PGA at the site, the MWF for M can be applied directly in Eq. 3 without any ambiguity. However, if a probabilistic PGA is used, which is the result of the contributions of many magnitudes, what magnitude and hence what MWF should be used? In current practice a single magnitude is often selected which may be the maximum experienced earthquake or tends towards the maximum magnitude expected in the governing seismic source zone and its weighting factor is used with the NBCC 2005 PGA. Does this single magnitude represent adequately the collective effects of the many different magnitudes contributing to the probabilistic PGA? The answer to this question is sought using two methods that logically include the effects of weighting on the contributions of all magnitudes to the probabilistic PGA. These methods are; (1) a probabilistic seismic hazard analysis using weighted magnitudes and (2) a weighted magnitude procedure based on a magnitude deaggregation for the hazard level in NBCC 2005. The weighted magnitude probabilistic analyses were conducted using the computer program EZ-FRISK 4.3 (Risk Engineering, 1997). This paper is an update of a previous report (Finn and Wightman, 2006) and incorporates updated deaggregation data for Vancouver and Toronto supplied by Halchuk and Adams (2006) of the Geological Survey of Canada.

Weighted magnitude probabilistic analysis

The weighted magnitude probabilistic analysis approach was first proposed by Idriss (1985). He demonstrated the need for weighting the magnitudes and showed how for the same acceleration level the return period for the weighted response could be much longer depending on the seismic environment. As noted above, the weighting factors, MWF, used in the present study are the inverse of the MSF proposed by Youd (2001) and listed in Table 2.

The weighted magnitude probabilistic analysis is accepted in California as a procedure for implementing the requirements of the Division of Mines and Geology guidelines in DMG SP

117 and the Seismic Mapping Act for projects requiring review under the Seismic Mapping Act of California. DMG SP 117 states "The alternative approach calculating "magnitude-weighted accelerations" is considerably easier and it provides a unique magnitude to be used with the probabilistically derived accelerations" (SCEC 1999).

The weighted magnitude probabilistic analyses reported in this paper were conducted to obtain the magnitude–acceleration pair for evaluating liquefaction potential. In this context, the weighted hazard curves are called liquefaction hazard curves. The seismic hazard curve for Vancouver and the corresponding liquefaction hazard curve weighted for magnitude M = 7.5 are shown in Figure 2.

The acceleration for assessing liquefaction potential for an exceedance rate of 2% in 50 years is 0.32g for M=7.5 and the site factor C=1.0. For other values of C, the compatible acceleration is 0.32Cg. The liquefaction hazard acceleration should be used directly with the liquefaction resistance curve for magnitude M=7.5 without further scaling. As pointed out by Idriss (1985) the weighted probabilistic analysis can be done for any normalizing earthquake magnitude other than M=7.5 but the appropriate magnitude weighting factor for the chosen normalizing magnitude must be applied again, when calculating liquefaction resistance using Figure 1. Therefore, when evaluating liquefaction triggering only, the magnitude-acceleration pair to be used is the normalizing magnitude and the associated weighted acceleration.
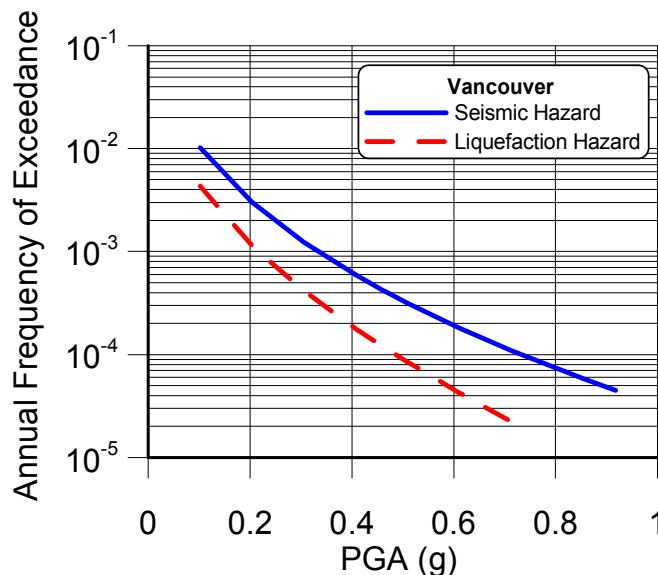

Figure 2. Hazard curves for Vancouver.

The unweighted and weighted PGA are for firm ground and, depending on the intensity of shaking, will be amplified or de-amplified at the surface by a site factor C on propagating through the softer soils often associated with liquefaction. The site factor C is usually determined by an appropriate site response analysis. Other options that are used are generalized amplification data such as provided in Idriss (1990), or the amplification factors in NBCC 2005. The factors of safety against liquefaction presented in the following were calculated by the simplified method for a range in $(N_1)_{60}$ values using the magnitude-acceleration pair from the weighted magnitude probabilistic analysis. Generic site conditions were assumed, consisting of

sand, with unit weight 20 kN/m^3, a water table at 2 m, and a range of $(N_1)_{60}$ values at 6 m depth. For these analyses the site factor was assumed to be C=1. The factors of safety are shown in Table 2.

Current practice in Vancouver for evaluating liquefaction potential is to use the NBCC 2005 accelerations with a magnitude M = 7.3. The factors of safety from this approach are also given in Table 2.

Table 2. Factors of safety against liquefaction for Vancouver.

SPT Blowcount, $(N_1)_{60}$	Liquefaction Triggering Safety Factors for Vancouver	
	Current Practice	Weighted Magnitude Analysis
	M7.3:0.48g	M7.5:0.30g
10	0.27	0.40
13	0.33	0.49
15	0.38	0.57
18	0.45	0.67
20	0.51	0.76
25	0.69	1.02
30	1.11	1.64

Over the range $10 \leq (N_1)_{60} \leq 30$, the factors of safety from the weighted magnitude probabilistic analysis are about 50% greater than the factors given by current practice in Vancouver.

If the magnitudes are weighted relative to M = 7.3, the recommended magnitude for Vancouver, the weighted magnitude probabilistic analysis gives a liquefaction acceleration of 0.35g. When M =7.3 (with MSF = 1.07) and a_{max} =0.35g are used in the simplified liquefaction assessment procedure, the factors of safety are similar to those shown for M = 7.5 and a_{max} =0.30g in Table 2.

Magnitude deaggregation method

The magnitude deaggregation method will be explained with reference to the magnitude-deaggregation for Vancouver shown in Figure 3 (Halchuk and Adams, 2006). In this case the magnitudes are collected in bins 0.25M wide and the central magnitude value is assigned to the bin. For example the bin labeled M = 5.125 contains all earthquakes in the range $5.0 \leq M < 5.25$. The contributions of the bin magnitude are sampled at various distances from the site. These contributions are shown by the row numbers in the magnitude contribution matrix in Figure 4.

The contributions are given per mil (1000) for convenience. If these numbers are divided by 10, the per cent contributions to the site acceleration at the various magnitude–distance combinations are obtained. The total contributions per magnitude bin are obtained by summing the distance contributions as suggested by Adams (2004), and Kramer and Mayfield (2005). The total bin contributions to the NBCC 2005 peak acceleration are given by the row numbers outside the matrix boundary. These contributions per magnitude bin are shown in the 2-D plot in Figure 5.

The sum of the bin contributions is 100%. The factor of safety against liquefaction at a site, taking into account the magnitude weighting factors is calculated as follows. The factor of safety of the site at the code acceleration level is computed for each binned magnitude and then multiplied by the contribution of the magnitude. The sum of all the contributions to the factor of safety gives the global factor of safety for the site. The calculation process for Vancouver is shown by the example in Table 3.
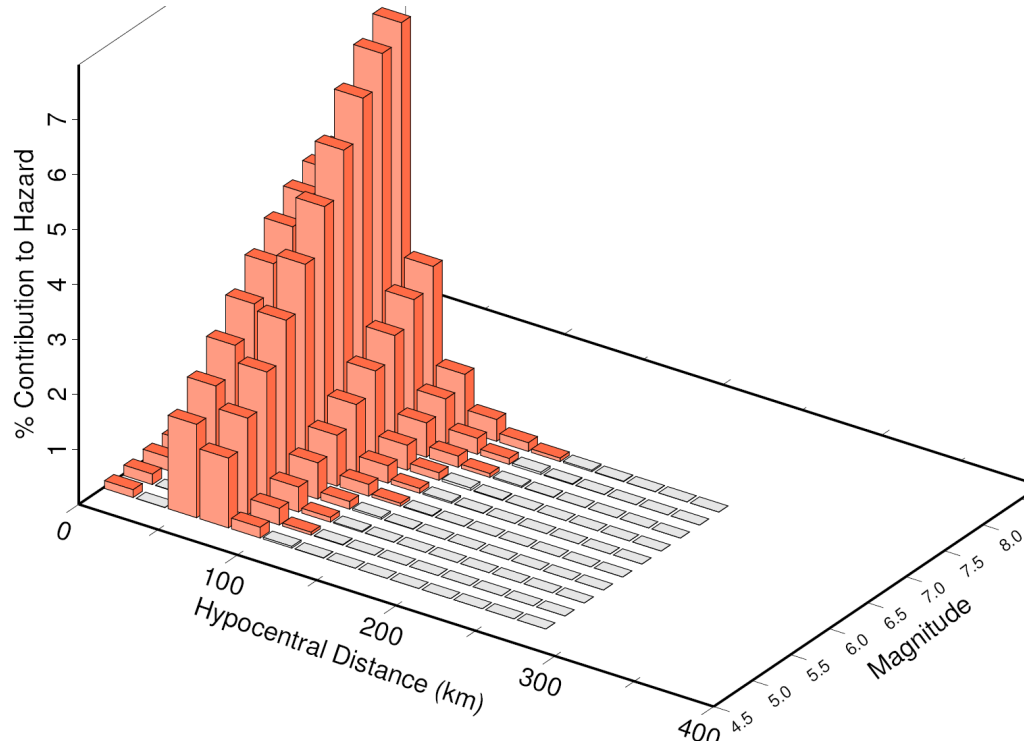

Figure 3. Magnitude-distance deaggregation for NBCC 2005 PGA in Vancouver.
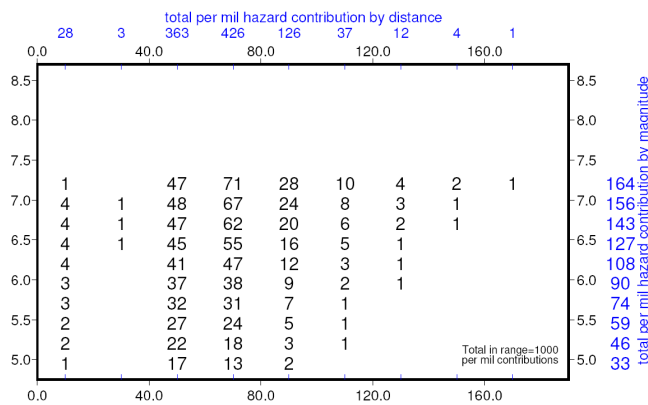

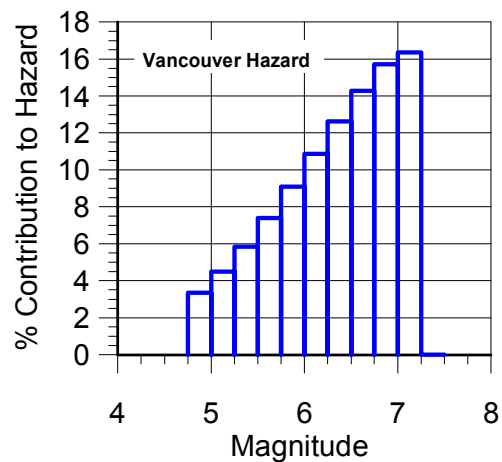Figure 4. Deaggregation matrix for NBCC2005 PGA in Vancouver.

Figure 5. Magnitude contributions to hazard in Vancouver.

Table 3. Sample calculation for factor of safety against liquefaction for Vancouver site with $(N_1)_{60} = 18$ at 6m depth.

Magnitude Bins	Central Magnitude	Contribution Factor	Liquefaction S.F.	S.F. Contribution
4.75 – 5.0	4.875	0.033	1.28	0.043
5.0 – 5.25	5.125	0.045	1.12	0.050
5.25 – 5.5	5.375	0.058	0.99	0.058
5.5 – 5.75	5.625	0.074	0.88	0.065
5.75 – 6.0	5.875	0.091	0.79	0.072
6.0 – 6.25	6.125	0.109	0.71	0.077
6.25 – 6.5	6.375	0.126	0.64	0.081
6.5 – 6.75	6.625	0.143	0.58	0.083
6.75 – 7.0	6.875	0.157	0.53	0.083
7.0 – 7.25	7.125	0.163	0.48	0.079
		Sum 1.000		Total Factor of Safety = 0.69

The factors of safety from the deaggregation method are compared in Table 4 with the factors obtained using the magnitude-acceleration pair from the magnitude weighted probabilistic analysis. The factors given by current practice in Vancouver and those arising from using mean and modal magnitudes with the code acceleration are also shown. The weighted magnitude probabilistic method and the deaggregation method give factors of safety within an average of 2% of each other. Note that the mean magnitude combined with the NBCC 2005 peak ground accelerations gives results very similar to the weighted magnitude probabilistic analysis in this seismic environment.

The deaggregation gives additional information on the statistics of the seismic environment. Of particular interest are the mean and modal magnitudes. For Vancouver these are M = 6.32 and M = 7.125 respectively. The mean magnitude in conjunction with the NBCC 2005 accelerations gives the same factors of safety as the two methods described above for Vancouver. The modal magnitude is the most likely event even though it usually contributes less than 20% of the hazard. For Vancouver, for example, it contributes about 16%. The modal magnitude is close to the M = 7.3 used in Vancouver practice and it also underestimates the factors of safety.

Table 4. Factors of safety against liquefaction in Vancouver for triggering options.

SPT Blow-Count $(N_1)_{60}$	Liquefaction Triggering Safety Factors for Vancouver				
	Current Practice	Modal Magnitude	Mean Magnitude	Deaggregation method	Weighted Magnitude Analysis
	M7.3:0.48g	M7.1: 0.48g	M6.32:0.48g	M7.25-4.75:0.48g	M7.5:0.30g
10	0.27	0.28	0.39	0.41	0.40
13	0.33	0.35	0.48	0.51	0.49
15	0.38	0.40	0.55	0.58	0.57
18	0.45	0.48	0.66	0.69	0.67
20	0.51	0.54	0.74	0.78	0.76
25	0.69	0.73	1.00	1.05	1.02
30	1.11	1.18	1.60	1.69	1.64

A deaggregation study was also conducted for Toronto. The GSC magnitude deaggregation for Toronto is shown in Figure 6 and the associated deaggregation matrix is shown in Figure 7 (Halchuk and Adams, 2006). The equivalent 2-D plot is shown as Figure 8.
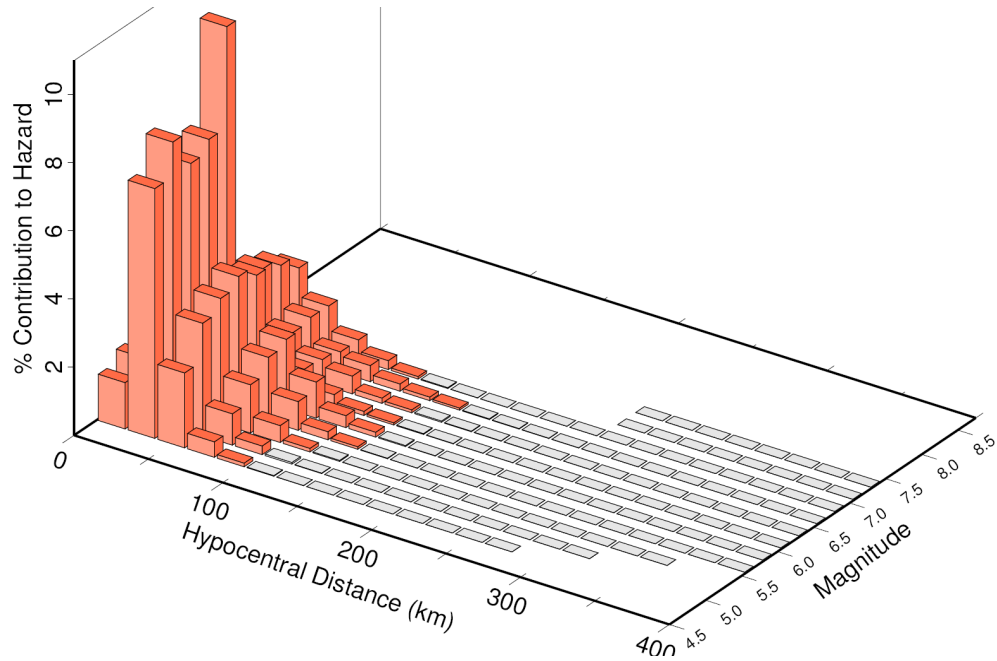

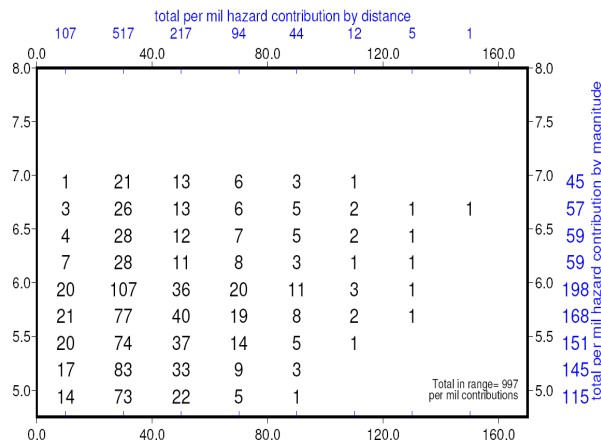Figure 6. Magnitude-distance deaggregation for NBCC 2005 PGA in Toronto.

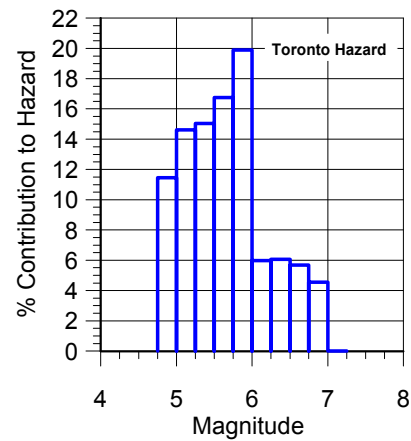Figure 7. Deaggregation matrix for NBCC2005 PGA in Toronto.

Figure 8. Magnitude contributions to hazard in Toronto.

The factor of safety for each binned magnitude was calculated for the previously prescribed range in $(N_1)_{60}$ values using the Seed-Idriss simplified method. The contribution of each magnitude bin to the total factor of safety was calculated using the data in the deaggregation matrix. The resulting factors of safety are given in Table 5. The magnitude deaggregation

method gives factors of safety an average of 7% greater than the factors given by the weighted magnitude probabilistic analysis. These differences are due to differences in the details of seismic hazard analysis between the GSC method on which the deaggregation approach is based and the weighted magnitude method..

Table 5. Factors of safety against liquefaction in Toronto for various triggering options.

SPT Blow-Count $(N_1)_{60}$	Liquefaction Triggering Safety Factors for Toronto			
	Modal Magnitude	Mean Magnitude	Deaggregation Method	Weighted Magnitude Analysis
	M5.875:0.2g	M5.67:0.2g	M7.0-4.75:0.2g	M7.5:0.10g
10	1.11	1.22	1.29	1.17
13	1.39	1.52	1.59	1.47
15	1.58	1.73	1.81	1.71
18	1.89	2.07	2.17	2.01
20	2.12	2.33	2.44	2.28
25	2.88	3.15	3.30	3.06
30	4.61	5.05	5.29	4.92

The magnitude used in practice, M = 6.0, gives results similar to the deaggregation or the weighted magnitude analyses. These analyses were conducted with an amplification/deamplification factor C=1.0 as in the case of Vancouver.

Conclusions

The weighted magnitude probabilistic analysis gives a unique magnitude-acceleration pair for any normalizing magnitude to use with the Seed-Idriss simplified method. In this study the normalizing magnitude was taken to be M = 7.5. Any other normalizing magnitude can be selected and a compatible magnitude-acceleration pair will be determined. All compatible magnitude-acceleration pairs determined by the weighted probabilistic analysis will yield the same factor of safety against liquefaction. The probabilistic acceleration from the weighted magnitude analysis must be multiplied by the site amplification/deamplification factor, C, to give the magnitude-acceleration pair to be used in evaluating liquefaction potential.

The approach based on a magnitude-distance deaggregation of the seismic hazard at a site uses the NBCC 2005 PGA. Here a 2-D magnitude deaggregation is developed which gives the contribution of each magnitude to the probability of exceeding the NBCC 2005 PGA. The code PGA is first multiplied by the amplification/deamplification factor C. Then the factor of safety against liquefaction for each magnitude bin is calculated for the modified acceleration and scaled by the contribution of that magnitude to the hazard. The scaled contributions to the factor of safety are summed to give the total factor of safety against liquefaction.

 The factors of safety given by weighted magnitude analysis are an average of 2% less than those given by deaggregation analysis for Vancouver sites and 7% less for Toronto sites. These differences result primarily from the different approaches to seismic parameter estimation. The weighted magnitude analysis does not account for epistemic uncertainty directly because it is not

included in EZ_FRISK analyses. Therefore "best estimate" seismic parameters given by Halchuk and Adams (2006) are used. The attenuation relationships are based on mean accelerations with no truncation of dispersions from the mean. The deaggregation method is based on site deaggregations given by the Geological Survey of Canada (Halchuk and Adams, 2006). The analyses leading to these deaggregations include the effects of epistemic uncertainty through the use of three sets of seismic parameters, the best estimates and upper and lower bounds on these estimates. The results from using these three sets are weighted and summed to give the code values for PGA and the associated deaggregations. The deaggregated procedure is recommended, if GSC deaggregations are available for the site. Otherwise the weighted magnitude approach is used. The differences between the methods as assessed for a number of sites to which both methods are applicable are of the order cited earlier.

Acknowledgements

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