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INTRINSIC FREQUENCY AND ANALYSIS OF NON-STATIONARY SIGNALS

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ABSTRACT

This paper proposes a new concept of intrinsic frequency and a new analysis method for non-stationary signals based on it. Intrinsic frequency is an inherent characteristic of a signal. This paper gives the mathematic derivation of it, followed by two examples of continuous signals. The results are compared with theoretical frequencies, Fourier transform, instantaneous frequencies based on Hilbert transform, and instantaneous frequencies based on Hilbert Huang Transform. This new method provides new physical insight into the non-stationary, nonlinear phenomena than any other existing methods. The examples show that the intrinsic frequency has clear physical meanings and indicates the exact locations of the local maxima and local minima. It actually corresponds to a local fitting of the signal data into harmonic functions. The analysis method based on intrinsic frequency is thus very adaptive to the data, and provides local properties of the data. This method can be applied to a wide spectrum of challenging problems in earthquake engineering, including analysis of near-fault ground motion time histories, nonlinear structural responses, structural health monitoring, and post-disaster evaluation of existing structures by non-destructive methods.

Introduction

Signal processing is at the heart of many earthquake engineering problems. Analysis methods of stationary signals have been well established, especially those based on Fourier transforms (see Shinozuka 1991, Soong and Grigoriu 1992). But analysis of nonstationary signals has been a challenge. Both the magnitude and the frequency contents of nonstationary signals are varying with time. Fourier Transform, which is relied heavily on by traditional signal processing, decomposes the signal into many sinusoidal waves with fixed frequency and amplitude extending to infinity, thus can not disclose the time-varying nature of the signal and yields many spurious harmonics that do not have physical meaning.

In earthquake engineering, analysis methods for nonstationary signals are especially needed. First of all, the earthquake ground motions are nonstationary. In particular, the long duration acceleration pulses observed in many near-fault earthquake records have been shown to cause serious structural damage (Somerville, et al 1997). Analysis of such ground motions has to rely on methods of nonstationary signals so as to understand the effects of the long duration pulses. On the other hand, structural responses can also be nonstationary, either due to the nonstationary earthquake ground motions, or due to inelasticity or nonlinearity of the structure system itself. A

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good tool to analyze such nonstationary responses can help us better identify the system behavior, so as to perform structural health monitoring, damage evaluation, and structural control.

Contrary to the frequency concept in Fourier transform, frequency of a nonstationary signal changes instantly and locally. A full oscillation is no longer needed to define the frequency value. Frequency can change between oscillations (inter-wave oscillation); it can also change within an oscillation (so called intra-wave frequency modulation, a hallmark of nonlinearity, traditionally regarded as harmonic distortion). For analysis of nonstationary signals, the methods need to reveal the local feature of the signal, and be very adaptive to the data. A global representation of the data, such as the power spectral density generated by Fourier Transform, is not very useful.

If a signal can be expressed in the following form

$$x(t) = AM(t) * \cos(\int FM(t)\,dt + \alpha) \quad (1)$$

where α is a constant, then FM(t) is the theoretical frequency (modulation) function that describes the signal's frequency change with time , and AM(t) is the theoretical amplitude (modulation) function that describes the signal's amplitude change with time.

A rough way to estimate the FM(t) functions is by counting zero crossings, and AM(t) by spline fitting of the maxima or minima. This approach is only applicable to simple waveforms and yields only rough estimates (Yeh and Wen 1990).

Another method is by Hilbert transform. Taking Hilbert transform of $x(t)$

$$y(t) = \frac{1}{\pi} P \int \frac{x(\tau)}{t-\tau} d\tau \quad (2)$$

in which P indicates Cauchy principal value, one can form an analytical function

$$z(t) = x(t) + i\,y(t) \quad (3)$$

It can be further expressed in the form of

$$z(t) = a_{instantaneous}(t)\, e^{\gamma_{instantaneous}(t)} \quad (4)$$

Where $a_{instantaneous}(t) = \sqrt{x^2 + y^2}$ is usually called the instantaneous amplitude. $\gamma_{instantaneous}$ represents a phase angle, which is equal to $\mathrm{atan}\,(^{y}/_{x})$. Taking derivative of the phase angle, a frequency is calculated.

$$\omega_{instantaneous} = \frac{d\gamma_{instantaneous}(t)}{dt} \quad (5)$$

is usually called instantaneous frequency. For more details, please see Bendat and Piersol (2000).

However, most signals are so complicated that such a simple treatment yields nothing meaningful. Cohen(1995) limited the simple treatment to mono-component signals. For lack of a precise definition of the mono-component signal, “narrow-band” was adopted as a limitation on the data (Schwartz et al. 1966).

Huang, et al (1998) further developed a sifting procedure (i.e. empirical mode decomposition, EMD) to decompose complicated signals into simpler forms (i.e. intrinsic mode functions, IMF), then use the treatment to each of the IMFs, then form a Hilbert spectrum by plotting the instantaneous frequency and amplitude functions of all the IMFs together. The approach is called Hilbert Huang transform (HHT). HHT has been widely used in earthquake engineering since its introduction in 1998, from analysis of earthquake sources (Zhang et al 2003), to generation of three-directional near-fault uniform hazard ground motions (Gu and Wen 2007, Wen and Gu 2004), to structural system identification (Yang and Lei 2000), etc. In HHT, each IMF has to satisfy two conditions to ensure its simplicity; however, even so, the instantaneous frequency and amplitude are still not the theoretical ones, due to Bedrosan and Nuttul theorem, see Huang (2005).

In this paper, a new concept called "intrinsic frequency" is defined and a new method based on it is proposed for analysis of nonstationary signals. The intrinsic frequency is not necessarily equal to the theoretical frequency; however, it certainly is a useful and meaningful characteristic of the signal. As can be seen later in this paper, for continuous functions, for many cases, it has clear physical meanings and indicates the exact locations of the maxima and minima. For discrete functions, it is actually a local fitting of the data into harmonic functions. The method is thus very adaptive to the data, and provides local properties of the data.

Good results and new insights have been obtained by applying the new method to various data: from continuous wave functions, to the numerical results of the nonlinear equation systems, to discrete data from real world. Application to real data from a shake table test on a bridge pier showed clear indication of structural damage that corresponded well to observations. Since signal processing is at the heart of many engineering problems, this study most likely will turn out to be a great contribution to overcome many engineering challenges. However, exploration of the full physical interpretation of the "intrinsic frequency" for complicated data has just begun. There are still some difficulties and limitations of the method. Due to the length limit, only two examples of continuous signals are presented here; more examples and discussions will be presented in other papers.

In this paper, the intrinsic frequency is first defined; then, two examples of continuous signals are given. The intrinsic frequency is compared with Fourier transform, theoretical frequency, instantaneous frequency, and the results from HHT. The physical meaning and merits of the intrinsic frequency are clearly shown.

Mathematic Definition of Intrinsic Frequency

Given a time series $x(t)$, one first obtains its Hilbert Transform $y(t)$, then forms an analytical function as in Equation (3).

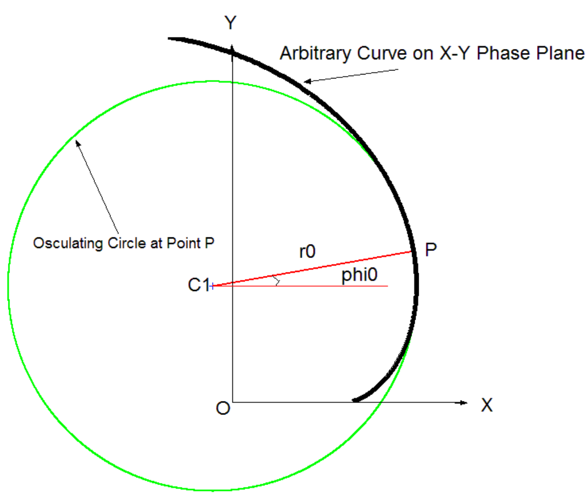

Figure 1 Illustration of Intrinsic Frequency Definition

The analytical function is a curve on phase plan X-Y. Figure 1 shows an arbitrary curve on phase plane. For each point on the curve, find its center of curvature $x_{c1}(t) + i\, y_{c1}(t)$, and consequently, the radius of curvature $r_0(t)$, and the angle $\varphi_0(t)$, which is the angle that the normal to the curve makes with the x-axis. Figure 1 also shows an arbitrary point P on the curve, the osculating circle at point P, the center of curvature C1 (x_{c1}, y_{c1}), the angle φ_0, and the radius of curvature r_0. From the figure, it is clear that Equation (3) can be rewritten as

$$z(t) = r_0(t)e^{i\,\varphi_0(t)} + x_{c1}(t) + i\,y_{c1}(t) \tag{6}$$

Then we continue doing this for the phase plane curve $x_{c1}(t) + i\,y_{c1}(t)$, and we can decompose the signal as

$$z(t) = r_0(t)e^{i\,\varphi_0(t)} + r_1(t)e^{i\,\varphi_1(t)} + x_{c2}(t) + i\,y_{c2}(t) \tag{7}$$

Continue doing this, we get

$$z(t) = r_0(t)e^{i\,\varphi_0(t)} + r_1(t)e^{i\,\varphi_1(t)} + +r_2(t)e^{i\,\varphi_2(t)} + x_{c3}(t) + i\,y_{c3}(t)$$

□

$$z(t) = r_0(t)e^{i\,\varphi_0(t)} + r_1(t)e^{i\,\varphi_1(t)} + \cdots + r_{n-1}(t)e^{i\,\varphi_{n-1}(t)} + x_{cn}(t) + i\,y_{cn}(t) \tag{8}$$

We define intrinsic frequency as

$$\omega_i = \frac{d\varphi_i}{dt} \quad \text{for i=0, 1 ,2,…n} \tag{9}$$

For any plane curve (x(t), y(t)), we have

$$x_c = x - \frac{\dot{y}\,(\dot{x}^2+\dot{y}^2)}{\dot{x}\,\ddot{y}-\ddot{x}\dot{y}} \tag{10}$$

$$y_c = y + \frac{\dot{x}\,(\dot{x}^2+\dot{y}^2)}{\dot{x}\,\ddot{y}-\ddot{x}\dot{y}} \tag{11}$$

$$r = \frac{(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}}}{\dot{x}\,\ddot{y}-\ddot{x}\dot{y}} \tag{12}$$

Substituting these equations into Equation (6), one can find that

$$\varphi_0(t) = \text{atan}\left[\frac{-\dot{x}(t)}{\dot{y}(t)}\right] \tag{13}$$

Similarly,

$$\varphi_1(t) = \text{atan}\left[\frac{-\dot{x}_{c1}(t)}{\dot{y}_{c1}(t)}\right] \tag{14}$$

Substituting Equations (10) and (11) into Equation (14), one can find that

$$\varphi_1(t) = \varphi_0(t) + \pi/2$$

By the same token, we have

$$\varphi_i(t) = \varphi_{i-1}(t) + \pi/2 \quad \text{for i=1, 2, …n.} \tag{15}$$

Thus we have

$$\omega_n(t) = \omega_{n-1}(t) = \cdots = \omega_0(t) \tag{16}$$

Since this is the unique function of the curve, we call it the intrinsic frequency function. From now on, we will denote it as $\omega_{intrinsic}$. It is an important and useful descriptor of the curve and thus the time series data (signal), as we will see in the following examples. And from now on, we will denote φ_0 as $\varphi_{intrinsic}$. And since

$$\dot{y}(t) = \frac{d[H(x)]}{dt} = H(\dot{x})$$

Equation (13) can also be written as

$$\varphi_0(t) = \text{atan}\left[\frac{-\dot{x}(t)}{H(\dot{x})}\right]$$

It is related to the angle used in the definition of instantaneous frequency, but on the signal $\dot{x}$, instead of x.

Let us find out its physical meaning. From Equations (9), (13), and (12), we can obtain that

$$\omega_{intrinsic}(t) = \frac{v_0(t)}{r_0(t)} \tag{17}$$

where

$$v_0(t) = \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2} \tag{18}$$

From Equation (17), we can see that $\omega_{intrinsic}$(t) is the angular velocity of the point rotating on the osculating circle around the instantaneous center of curvature. This is the physical meaning of the intrinsic frequency.

On the other hand, let us look at the instantaneous frequency defined in Equation (5). It can be proved that

$$\omega_{instantaneous} = \frac{x\dot{y} - y\dot{x}}{x^2 + y^2} \tag{19}$$

It does not have clear physical meaning; it is not even equal to $\frac{v_0(t)}{a_{instantaneous}}$, i.e., the angular velocity of the point rotating around the origin of the phase plane.

Now let us look at the result of the new analysis method, Equation (8). The real part of the equation becomes:

$$x(t) = r_0(t)\cos[\varphi_0(t)] - r_1(t)\sin[\varphi_0(t)] - r_2(t)\cos[\varphi_0(t)] + r_3(t)\sin[\varphi_0(t)] + r_4(t)\cos[\varphi_0(t)] + \cdots + x_{cn}(t) \tag{20}$$

In this expression, $r_0(t)\cos[\varphi_0(t)]$, $-r_1(t)\sin[\varphi_0(t)]$, $-r_2(t)\cos[\varphi_0(t)]$, … can be regarded as components of the signal, and $x_{cn}(t)$ can be regarded as the residue. The new signal processing method can be regarded as a new way to decompose the signal into a series of components. Each component has the same frequency function, alternating phase angle, but different amplitude function.

Equation (20) can also be rewritten as

$$x(t) = x_{cn}(t) + r_a(t)\cos[\varphi_{intrinsic}(t) + \alpha(t)] \tag{21}$$

Where

$$r_a = \sqrt{(r_0 - r_2 + r_4 - \cdots)^2 + (-r_1 + r_3 - r_5 + \cdots)^2} \tag{22}$$

$$\alpha = \text{atan}\left(-\frac{-r_1 + r_3 - r_5 + \cdots}{r_0 - r_2 + r_4 - \cdots}\right) \tag{23}$$

Expressed in the form of Equation (21), $x(t)$ can be regarded as a signal with a DC-term $x_{cn}(t)$ and a AC-term with amplitude $r_a(t)$ and frequency $\omega_{intrinsic}(t)$. Please note that in Equation (21), α is a function of time. $\omega_{intrinsic}(t)$ is the derivative of $\varphi_{intrinsic}(t)$ only, not the derivative of $[\varphi_{intrinsic}(t) + \alpha(t)]$, which is fundamentally different from the definition of the theoretical frequency. Equation(21) is the most general form of a sinusoidal wave that fits the data locally at time t, because all the four parameters that completely describe a sinusoidal wave: the DC term, the amplitude, the frequency, the initial phase angle, are all functions of t, i.e. are local at time t. Nothing is global, nothing is fixed beforehand, thus this expression is very adaptive to the data, and reveals the true local properties of the data.

Examples of Continuous Signals

Example 1: Superimposition of Two Sinusoidal Waves

Let us look at an example first: $x(t) = \cos(4\pi t) + \cos(5\pi t)$

A segment of the curve is shown in Figure 2. $x(t)$ can also be written as

$x(t) = 2\cos(0.5\pi t)\cos(4.5\pi t)$

Its Hilbert transform is $y(t) = 2\cos(0.5\pi t)\sin(4.5\pi t)$

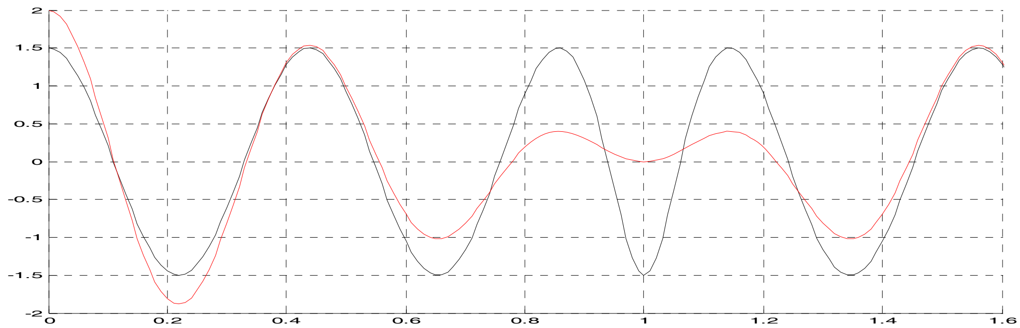

Figure 2 Beat Phenomenon (red line is the time series data, black line is cos(□) function)

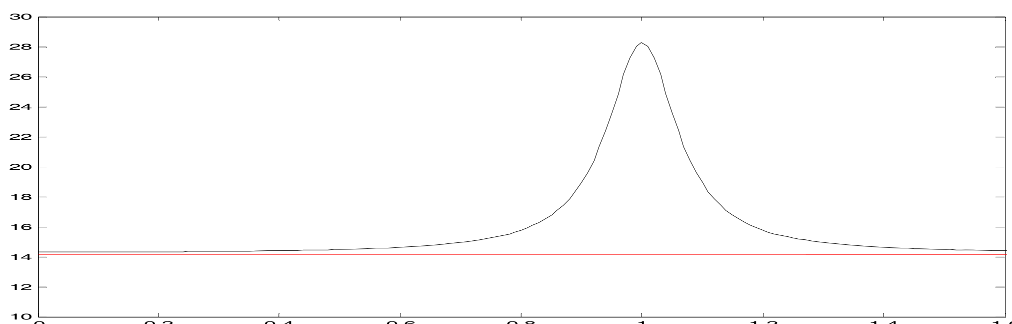

Figure 3 Intrinsic Frequency (black line) (red line is at 4.5π)

Since the frequencies of the two sinusoidal waves are close to each other, a phenomenon called Beat Phenomenon will occur, i.e. the amplitude of the signal will vary periodically. From the theoretical point of view, $2\cos(0.5\pi t)$ is the amplitude modulation function, and 4.5π is the theoretical frequency. The instantaneous frequency based on Hilbert Transform will yield the same result to us. Since the data is already an IMF for HHT, the HHT result is also the same. The new method in this paper offers an entirely different, yet very interesting and useful view of the data. The intrinsic frequency function is shown in Figure 3. It shows a peak around t=1, instead of a flat straight line at 4.5π. Looking at the data $x(t)$ in Figure 1, it is clear that visually, the frequency near t=1 is much higher than other places. Actually, the distances between the local maxima are [0.4380 0.4190 0.2860 0.4190], and the distances between the local minima are:[0.4350 0.3460 0.3460]. Obviously, the frequency is not a constant, not as the theoretical or instantaneous frequency suggested. Figure 2 also shows the function form of $1.5\cos(\varphi_{intrinsic})$. We can see that the local extrema of this curve are at the exact locations of the extrema of the original curve $x(t)$, showing that the intrinsic frequency function correctly describes the uneven spacing between local maxima/minima.

Although the traditional point of view is a powerful tool for understanding beat phenomenon, it completely fails to describe the local variation of the spacing between peaks/valleys. As we discussed before, the local features are of great importance to the analysis of non-stationary, nonlinear data. If we do Fourier transform to this data, the result will disclose the two frequency components, $\cos(4\pi t)$, and $\cos(5\pi t)$. Although this information is very useful, it does not give any information of the local variation, which is important for our purpose.

Now let us look at the general case $x(t) = cos(\omega_a t) + cos(\omega_b t)$. Since $cos(\omega_a t) + cos(\omega_b t) = 2\cos\left(\frac{\omega_a-\omega_b}{2}t\right)\cos\left(\frac{\omega_a+\omega_b}{2}t\right)$,. the theoretical frequency and instantaneous frequency of these signals are both $\frac{\omega_a+\omega_b}{2}$. Their intrinsic frequency function can be derived as $\omega_{intrinsic}(t) = \frac{\omega_a+\omega_b}{2}\left[1 + \frac{(\omega_a-\omega_b)^2}{2\omega_a\omega_b\{\cos[(\omega_a-\omega_b)t]+1\} + (\omega_a-\omega_b)^2}\right]$. The maximum value of the intrinsic frequency is $(\omega_a + \omega_b)$, and the minimum is $(\omega_a^2 + \omega_b^2)/(\omega_a + \omega_b)$.

Thus the intrinsic frequency is always greater than the theoretical frequency or the instantaneous frequency of $\frac{\omega_a+\omega_b}{2}$.

When ω_a and ω_b are close to each other, beat phenomena will occur, such as the previous example $x(t) = \cos(4\pi t) + \cos(5\pi t)$. On the other hand, when the two frequencies are far from each other, i.e., when $\omega_a \gg \omega_b$, the intrinsic frequency is approximately ω_a, whereas the theoretical frequency and instantaneous frequency are both approximately $\frac{\omega_a}{2}$. This is a big difference. Figure 4 shows a segment of $x(t) = \cos(5\pi t) + \cos(0.5\pi t)$ along with its two component sinusoidal waves $\cos(5\pi t)$ and $\cos(0.5\pi t)$. We can see that the waveform is basically $\cos(5\pi t)$, drifted up and down according to $\cos(0.5\pi t)$.

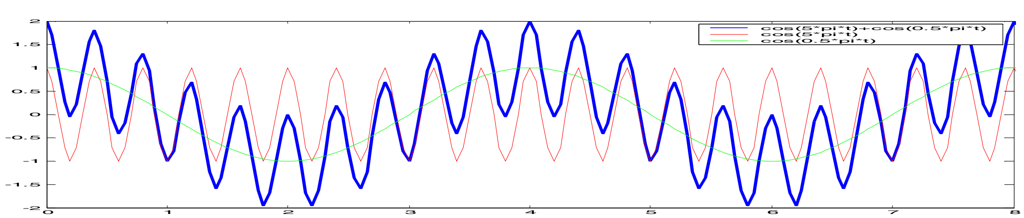

Figure 4 Signal of $x(t) = \cos(5\pi t) + \cos(0.5\pi t)$ (in blue) and its Two Components (in red and green, respectively)

The theoretical frequency and instantaneous frequency of 0.55*5π do not have any clear physical meaning. On the other hand, the intrinsic frequency is about 5π (varying between

0.92*5π and 1.1*5π), conforming with the waveform. Again, the intrinsic frequency yields the exact locations of the local extrema, and the best local fitting of the data into sinusoidal waveform.

Example 2: Involute of a Circle

Let us look at function: $x(t) = \cos(t) + t\sin(t)$. A segment of the function is shown in Figure 5. Its Hilbert transform is $y(t) = \sin(t) - t\cos(t)$.

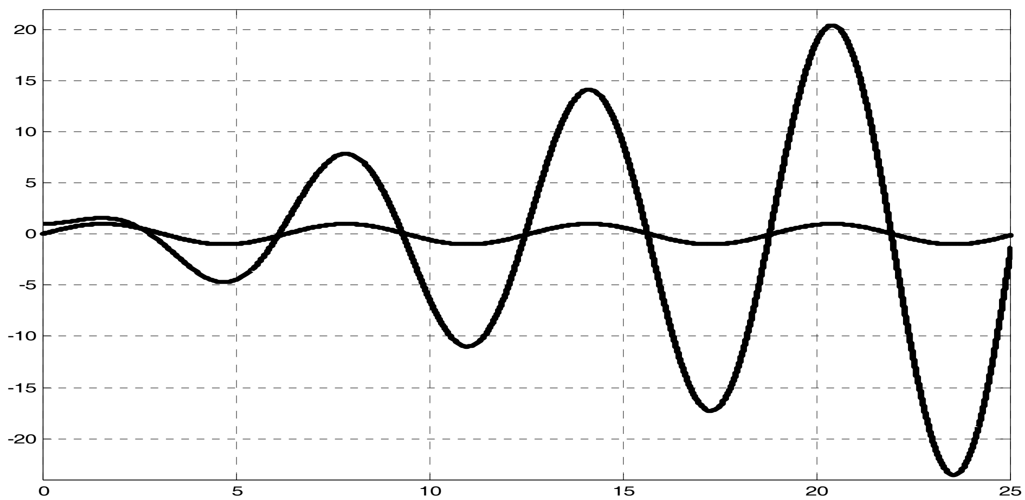

Figure 5 Involute of a Circle Along with the Curve sin(t)

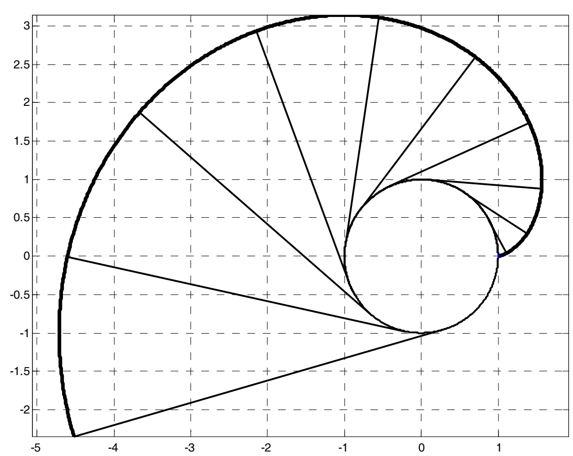

Figure 6 Phase Plane Curve of Involute of Circle

On phase plane, it is an involute of a unit circle, see Figure 6. Image there is a string wound on the circle and a pen is attached at the open end of the string. Now pull the pen to unwind the string from the circle, the trace of the pen point is the curve that we are studying.

The intrinsic frequency turned out to be $\omega_{intrinsic} = 1$. It is the frequency of the line rotating around the instantaneous rotation center, i.e. the tangent point of the line to the circle at the time instant. Thus it has clear physical meaning.

Further, we have $r_0 = t, r_1 = 1, r_i = 0$ for i=2, 3,…, and $r_a = \sqrt{1+t^2}$. So the new data analysis method yields two components, the first component, $r_0(t)\cos[\varphi_0(t)]$ is actually the term $t\sin(t)$, whereas the second component, $r_1(t)\sin[\varphi_0(t)]$, is actually the term $\cos(t)$. Here $\varphi_0 = \omega_{intrinsic}t - \pi/2$. The first component describes the movement of the rotating line as if it were rotating around the origin; whereas the second component is the circle. The two components are perpendicular with each other (90 degree out of phase), the vector sum is r_a. Both the two components rotate at the same angular speed, which is $\omega_{intrinsic} = 1$. The decomposition is physically meaningful. No other method can disclose so much information of the physical process or yield the constant angular speed.

For example, the Fourier transform cannot be applied to this signal, as it is clearly not stationary, and the Fourier transform will yield many meaningless superficial harmonic waves.

The instantaneous frequency defined by Hilbert transform can be derived as $\omega_{instantaneous} = 1 - \frac{1}{1+t^2}$. It begins from 0 at t=0 and approaches 1 as t approaches to infinity. It does not have obvious physical meaning. The instantaneous amplitude function is $a_{instantaneous} = \sqrt{1+t^2}$, which is equal to r_a.

Since $x(t)$ is already an IMF, the Hilbert Huang Transform is reduced to Hilbert transform and yields the same result as above.

Since $x(t)$ can also be written as $x(t) = \sqrt{1+t^2}\cos[t + \text{atan}(-t)]$, we have $\omega_{theoretical} = 1 - \frac{1}{1+t^2}$ and $a_{theoretical} = \sqrt{1+t^2}$. Again, the theoretical frequency does not have clear physical meaning, and the theoretical amplitude is the same as r_a.

Figure 5 also shows the curve $\sin(t)$, i.e. a constant-amplitude harmonic wave whose frequency function is the intrinsic frequency. We can see (and prove mathematically) that the extrema of the curve occur at the exactly same time instants as those of $x(t)$. If we construct a similar constant-amplitude harmonic wave, but use the instantaneous frequency or the theoretical frequency as the frequency function, the extrema of the curve will not occur at the same time instants as the extrema of $x(t)$. This further proves the correctness and usefulness of the intrinsic frequency.

Conclusions

This paper proposes a new concept of intrinsic frequency and a new analysis method of nonstationary signals based on it. As is demonstrated by the examples in the paper, intrinsic frequency is a useful and meaningful inherent characteristic of the signals. It has clear physical meanings and indicates the exact locations of the local maxima and minima. Compared with theoretical frequencies, Fourier transform, instantaneous frequencies based on Hilbert transform, and instantaneous frequencies based on Hilbert Huang Transform, intrinsic frequency provides us with new physical insight into the non-stationary, nonlinear phenomena that no other existing methods can provide. The new analysis method for nonstationary signals is very adaptive to the data, and provides local properties of the data. This method has also been applied to numerical results of nonlinear equation systems and discrete data from real world, and good results have been obtained. Wide applications of this method can be found in earthquake engineering, including analysis of near-fault ground motion time histories, analysis of nonlinear structural responses, structural health monitoring, active structural control, and post-earthquake evaluation of existing structures by non-destructive methods.

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