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A STUDY ON THE DEPENDENCY OF SEISMIC INPUT ENERGY ON THE CHARACTERISTICS OF STRUCTURAL HYSTERETIC BEHAVIOR BY USING AN EXPLICIT HYSTERETIC MATHEMATICAL MODEL

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ABSTRACT

The variation of input energy with characteristics of various structural systems, particularly in hysteretic states, has not been studied to such extent that creates enough confidence for proposing energy-based design criteria. In this paper, first a mathematical model for expressing the hysteretic behavior of structures is introduced, which has a simple mathematical form, and uses only three parameters, including the initial stiffness, the ultimate strength, and a parameter which controls the rate of change of curvature of the hysteretic curves. These parameters are all based on the real physical characteristics of structures. Then, to find out how the seismic input energy depends on the hysteretic characteristics of structures, a computer program has been developed in MATHEMATICA environment based on the proposed mathematical hysteretic model, and several time history analyses have been performed by using a variety of accelerograms. Regarding the three main parameters of the hysteretic model, three sets of analyses have performed in each of which just one parameter of the hysteretic model has been considered as variable to find out its effect independently. It should be noted that although the mathematical hysteretic model is simple, using it in a time history analysis program needs special attention to some criteria which should be met to keep the realistic behavior of the model. Numerical results show that in some cases the amount of input energy varies remarkably with the characteristics of the system, particularly the parameter which control the rate of change of stiffness. On this basis it can be claimed that this parameter can be used as a controlling tool for limiting the amount of earthquake input energy, and accordingly the level of overall damage to the structure.

Introduction

Earthquake input energy has been discussed by researchers since mid 70s (Kato and Akiyama 1975), and several efforts have been made to use this concept in the seismic design of

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building systems since early 90s (Surahman and Merati 1992), and have been continued during recent years (Kinugasa, and Nomura 1996; Chai and Fajfar 2000; Jiang and Zhu 2006). As one of the first works in this regard, Kato and Akiyama (1975) have studied the energy input and damage in structures subjected to severe earthquakes. Mentioning that Housner's assumption that the energy input contributing to structural damage can be expressed as half the product of the mass of the structure and the velocity response spectrum, they have expressed that structural damage corresponds to the energy absorption due to plastic deformation, and the energy input causing damage may correspond to the sum of the energy absorption due to plastic deformation and the elastic vibrational energy. They have evaluated each component in the above law from some numerical analyses of inelastic vibrational systems, and have found that Housner's assumption is basically valid, however, they have not given any suggestion for the use of energy in seismic design.

Surahman and Merati (1992) have discussed the input energy based seismic design code for shear buildings up to four stories high, subjected to different earthquake loadings [4]. They have employed a Newmark linear acceleration direct integration method for computation of deformations, forces, and energy in the elasto-plastic ranges, and have concluded that as a basis for a seismic design code, the energy approach is relatively more consistent than the base shear approach.

Kinugasa and Nomura (1996) have studied on the development of seismic design based on energy concept. As the first part of their study they have worked on checking the performance of earthquake-proofing by considering energy input velocity, and have proposed a performance check method of earthquake-proofing based on the concept of energy balance between input energy and absorbed energy. The feature of their study is to be able to consider the rapidity of energy input by introducing the idea of "Energy Input Velocity". In that study, the destructive power of an earthquake is expressed by the quantity of the input energy and its input velocity. They have expressed seismic capacity of a building by the quantity of energy that can be absorbed within the continuation time of an earthquake on condition that the deformation is limited to the design maximum deformation. They have calculated 'the amount of that energy' by considering the balance between energy input velocity of the earthquake and energy absorption velocity of the building. They have suggested the comparison of 'the amount of that energy' with the quantity of input energy caused by ground motion as a criterion for judgment on the seismic safety.

Chai and Fajfar (Oct. 2000) have proposed a procedure for estimating input energy spectra for seismic design. Mentioning that the damage potential of an earthquake ground motion is evaluated in terms of the total power of the acceleration of the ground motion, and by assuming an appropriate spectral shape for the input energy spectrum, and using the well-known Parseval theorem for evaluating the total power of a random signal, they have determined the peak amplification factor for the equivalent input energy velocity spectrum. They have shown that the peak amplification factor for the input energy spectrum depends on the peak ground acceleration to peak ground velocity ratio and duration of the strong motion phase of the ground motion. Values for the equivalent input energy velocity amplification factor vary from about 2 to 10 for most of the recorded ground motions used in that study. They have claimed that a considerable scatter of data is observed, however, the peak amplification factor predicted by the Fourier amplitude spectrum of the ground acceleration provides a fairly good estimate of the mean value of the peak input energy compared to that determined from inelastic dynamic time history analyses, particularly for systems with high damping and low lateral strength. They have expressed that the peak amplification factor derived in their study provides a more consistent approach for estimation of seismic demand when compared to an earlier empirical expression used for the formulation of duration-dependent inelastic seismic design spectra, even though only a slight difference in the required lateral strength results from the use of the new formula.

Jiang and Zhu (Sept.-Oct. 2006) have presented energy input design spectra for near-fault regions and application in energy-based seismic design. Mentioning that the reliable definition of input energy spectra is an essential foundation for energy-based seismic design and evaluation method, and considering the influence of soil type and fault distance, they selected a world-wide ensemble of 224 records within 15km of fault projective distance as a data base, and have derived the energy spectra for seismic design with the shape and amplitude adjustment according to different seismicity groupings. They have compared their proposed energy input design spectra (EIDS) with that from the Japanese Building Code (1985 and 2001) and the actual energy demand of earthquakes that occurred near faults. They have claimed that the proposed spectra can meet the practical earthquake energy demand, and have advised a procedure for energy-based seismic evaluation and design and have tried to confirm it by 3 RC bridge piers.

Recently Hosseini and his colleagues (2009) have discussed a new insight to the 'input energy' concept and its usage for more reliable seismic design of buildings. In that study the input energy, calculated as the work done by the shear force at the building foundation during an earthquake, is divided into two parts, a positive part and a negative part. The positive part can be considered as the energy which is transferred from ground to the building, while the negative part can be considered as the energy which is returned back from the building to the ground. Obviously, if the positive part of the work or input energy can be reduced by giving some specific structural and dynamical features to the building system, it will be helpful for the building safety, and this would possibly lead to the optimum design of buildings subjected to earthquakes. They have analyzed some sample buildings with various distributions of parameters along their heights and have calculated the values of "received energy" and "returned energy" for them by using some accelerograms with various frequency content from low to high. In each case just one parameter of the building characteristic has been considered variable along the building height, and others have been considered as constant, to find out the effect of each characteristic independently.

It is seen that in spite of several studies on earthquake input energy, up to now no straightforward method has been introduced for using this concept in seismic design, and none of the existing codes have such approach. It is believed that the reason behind this fact is that the variation of input energy with characteristics of various structural systems in both linear and nonlinear states has not been studied to such extent that there is enough confidence for proposing energy-based design criteria. Therefore, still more investigation in this regard is necessary. On the other hand, several mathematical models, both implicit and explicit, have been proposed for expressing the hysteretic behavior of structural systems in both deterministic and probabilistic approaches. In some of these models the back bone or virgin curve of the hysteretic behavior is stated by a single mathematical relationship, but in some others several functions are used. The number of parameters used in these relationships varies from just two to more than ten. Clearly, as the number of model parameters increases the compatibility of the model with real cases increase as well, however, at the same time the determination of model parameters becomes more and more complicated. Some of these models are not based on the real physical characteristics of structure, and are just mathematical functions, and therefore, in some cases they lead to unrealistic behaviors.

In this paper at first a mathematical model for expressing the hysteretic behavior of structures is introduced, which has a simple mathematical form, and uses only three parameters, including the initial stiffness, the ultimate strength, and a parameter which controls the rate of change of curvature of the hysteretic curves. These parameters are all based on the real physical characteristics of structures. Then, to find out how the seismic input energy depends on the hysteretic characteristics of structures, a computer program has been developed in MATHEMATICA environment based on the proposed mathematical hysteretic model, and several time history analyses have been performed by using a variety of accelerograms covering a wide range of PGA, frequency content and duration. Regarding the three main parameters of the hysteretic model, three sets of analyses have performed in each of which just one parameter of the hysteretic model has been considered variable to find out the effect of each characteristic independently.

The Explicit Hysteretic Mathematical Model

Hosseini and Ghafory-Ashtinay (1991) introduced an explicit hysteretic behavioral model for structural system which has just three main parameters, including the initial stiffness coefficient, k, the ultimate generalized resistance or strength, r_u, from which the generalized yielding displacement, x_y, is defined as $x_y= r_u/k$, and the parameter p, which specifies how the generalized resistance of the system in case of the virgin curve, $r_v(x)$, or return curves, $r_i(x)$, as a function of generalized displacement of the system, x, approaches the ultimate strength value, r_u. The backbone or virgin curve of the model, and its return curves are given respectively by:

$$r_v(x) = r_u \times sign(x)\{\exp[-A/(|x|/x_y+B)^P] - C\}/(1-C) \quad (1)$$

$$r_i(x) = r_{r,i} + 2r_u \times sign(x - x_{r,i}) \times \{\exp[-A/(|x - x_{r,i}|/2/x_y + B)^P] - C\}/(1-C) \quad (2)$$

where:

$$A = B^P(1+1/P) \qquad B = (P+1)/(1/C-1) \qquad C = 1/\exp(1+1/P) \quad (3)$$

In Eq. (2) $r_{r,i}$ and $x_{r,i}$ are respectively the generalized resistance and the generalized displacement values at the i[th] return point. Sample curves of the proposed nonlinear model and its hysteretic loops are shown Fig. 1.

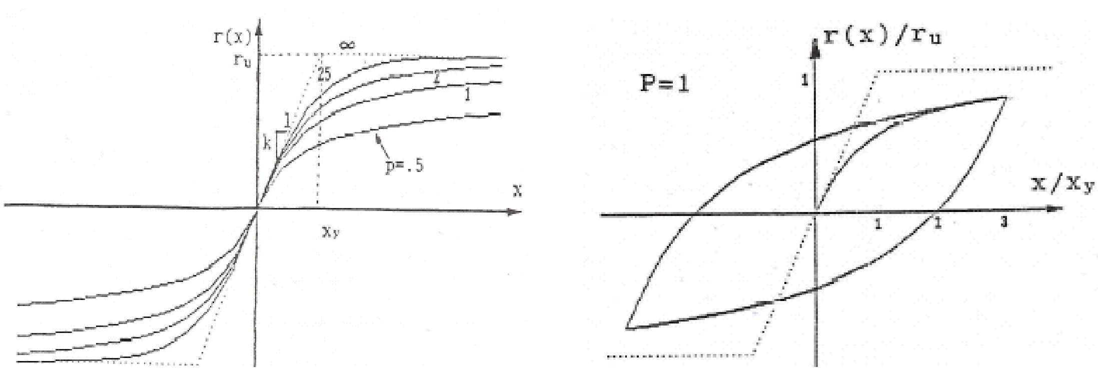

Figure 1. The virgin curves and a closed loop of the proposed hysteretic model

It should be noted that although the hysteretic model is expressed by a relatively simple and explicit mathematical function, using it in a time history analysis program needs special attention to some criteria which should be met to keep the realistic behavior of the structural models. These criteria are related to aiming pints of the new return curves, and also shifting from one curve to another when one curve passes over one previous return point, as shown in Fig.2.

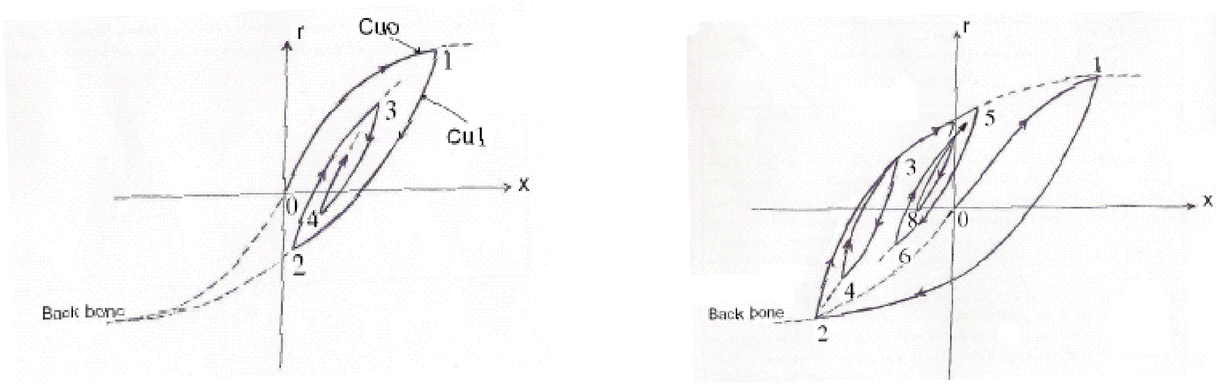

Figure 2. The conditions of aiming of the new curves after each return or “first criterion” (left), and passing over the previous return points or “second criterion” (right)

In Fig. 2 Cu0 and Cu1 show the virgin curve and the first return curve respectively, and the numbers refer to return points. The abovementioned criteria should be considered in the computer program with some proper IF statements. Authors developed a program in MATHEMATICA (Version 5.1) environment for seismic response analysis of SDOF system by the proposed model. The flowchart of the program is shown in Fig. 3.

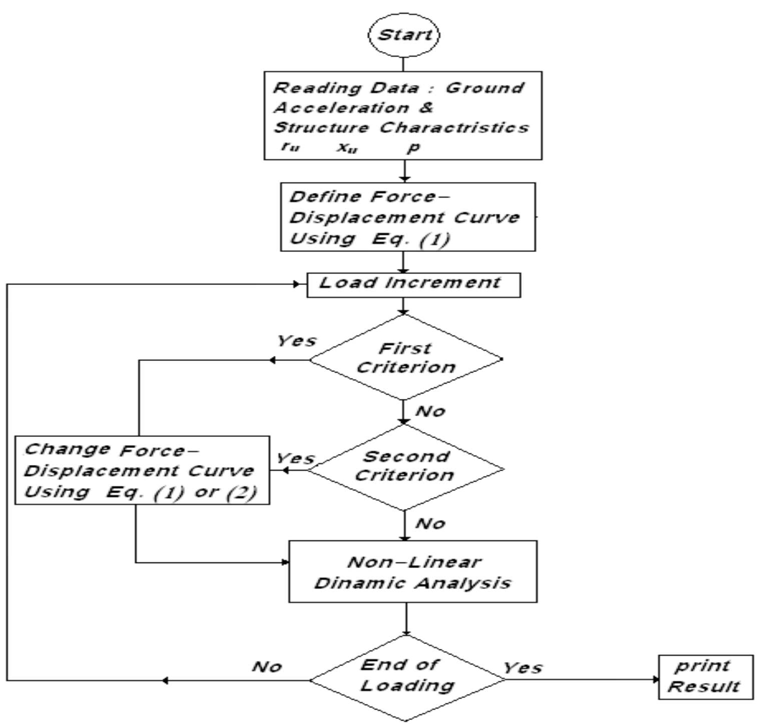

Figure 3. The flowchart of the developed program for seismic response analysis of hysteretic SDOF systems by the proposed mathematical model

A sample of the hysteretic response curves, and the related energy time history, obtained by the developed program are shown in Fig. 4.

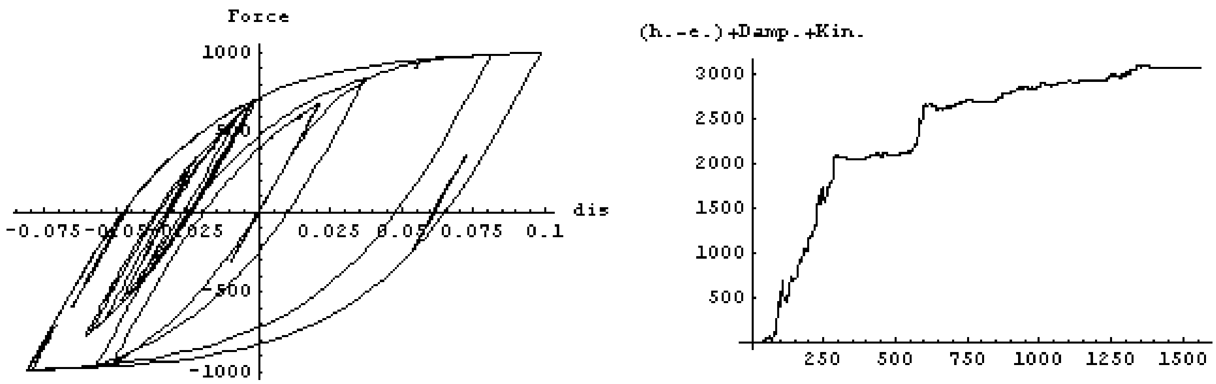

Figure 4. A sample of hysteretic response by using the proposed explicit model (left) and the energy time history obtained by it (right)

To verify the developed computer program, the Wen nonlinear hysteretic model, employed in SAP version 14 was used, in which the force displacement relationship is:

$$F = \text{ratio}\ \mathrm{k}\, x + (1 - \text{ratio})\ \text{yield}\ z \tag{4}$$

where 'ratio' is ratio of post-yielding stiffness on the initial stiffness, k, and 'yield' is the yielding force, and z is the hysteretic variable ($|z| \leq 1$) given by the following equation:

$$\dot{z} = \frac{\mathrm{k}}{\text{yield}} \begin{cases} \dot{x}\,(1 - |z|^{\text{exp}}) & \text{if } \dot{x}\, z > 0 \\ \dot{x} & \text{otherwise} \end{cases} \tag{5}$$

The initial value of z is zero, and after yielding happens its value equals to one. In Eq. (5) as the value of 'exp' gets larger the behavior of the model gets closer to the bilinear behavior. To make the hysteretic loop of the proposed model similar to that of Wen model, the value of r_u in the proposed model is set equal to the value of r in Wen model, the value of 'ratio' is set equal to zero, and if the value of p in the proposed model is set to 5 and the value of 'exp' in Wen model is set to 1, the two model will result in very similar curves. By using these values and assuming k=300N/m, m=100kg, c=10N.s/m, and using two accelerograms shown in Fig. 5 the response of the two models have been calculated and compared in Figs. 6 and 7.

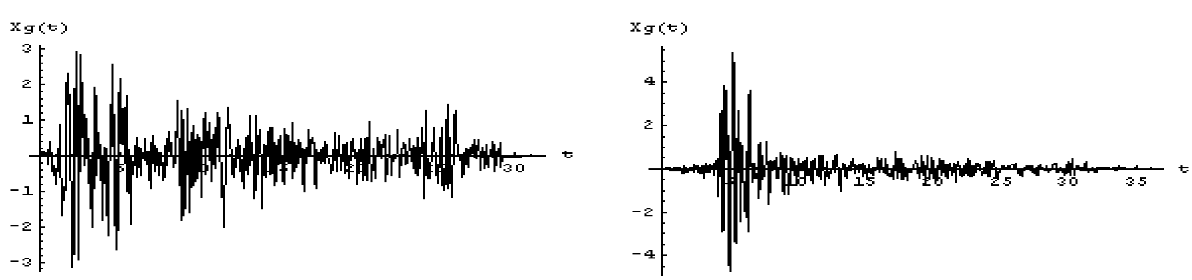

Figure 5. Accelerograms used for comparing the proposed model and Wen model, one with long strong motion duration (left) and one with short duration strong motion (right)

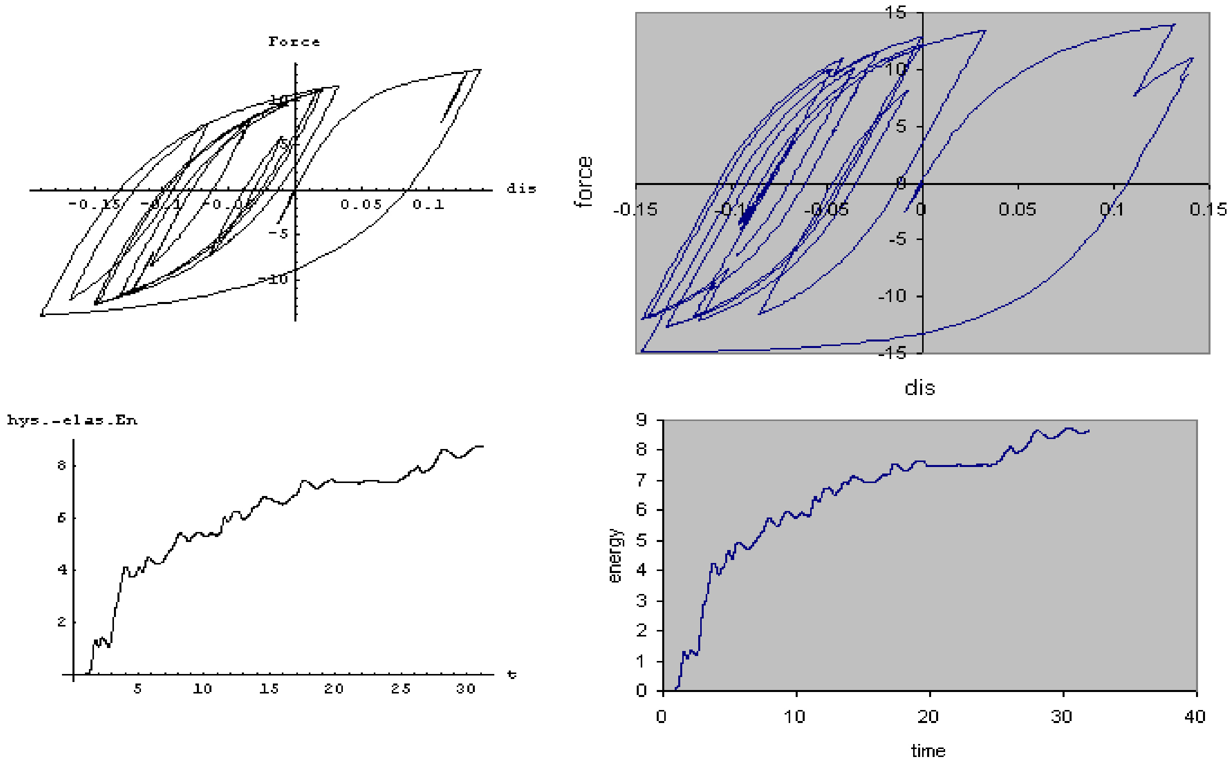

Figure 6. Comparison of hysteretic responses and energy time histories of the suggested model (left) and Wen model (right) for earthquake with long strong motion duration

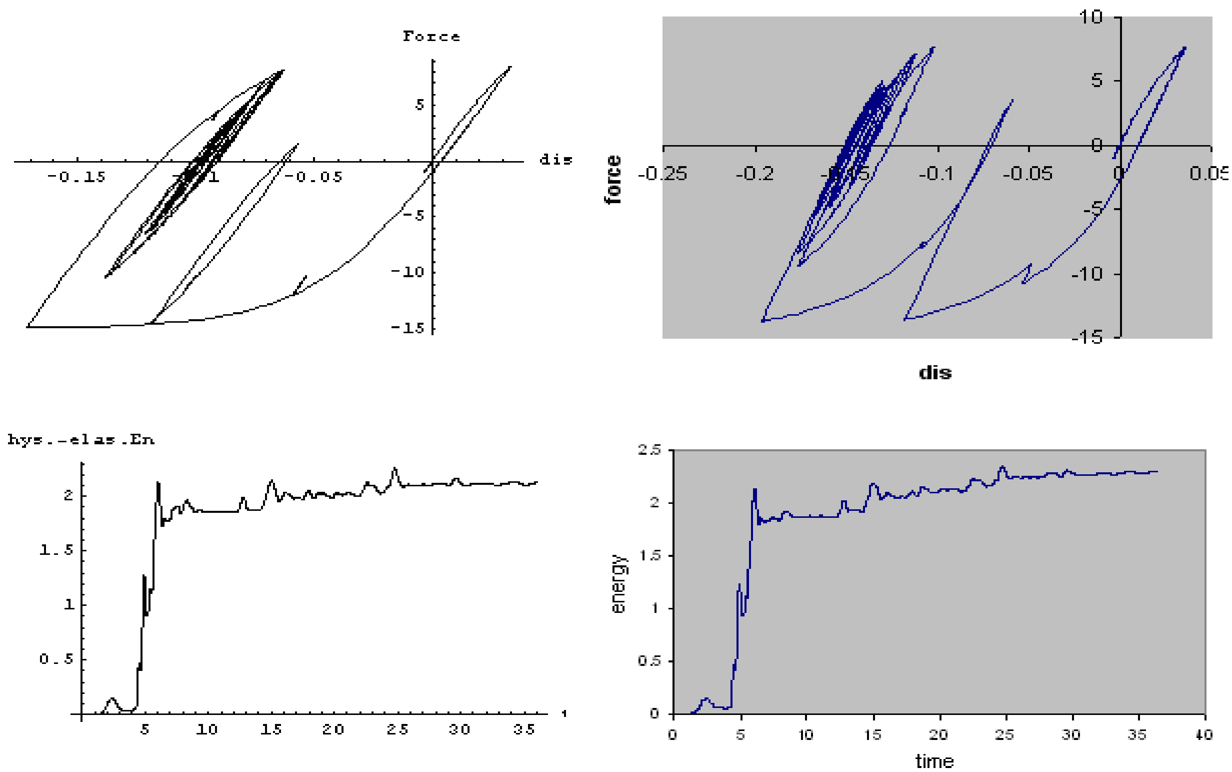

Figure 7. Comparison of hysteretic responses and energy time histories of the suggested model (left) and Wen model (right) for earthquake with long strong motion duration

It is seen in Figs. 6 and 7 that although the hysteretic responses obtained by using the suggested explicit model and Wen model, are not exactly the same (which is basically due to the non-physical behavior of Wen model, while the proposed explicit model is based on two basic physical concepts, namely the initial stiffness and the ultimate strength of the structural system), the energy time histories are very similar. This shows that the amount of absolute input energy and its variations are not so much sensitive to the slight changes in the response time history.

Numerical Results with Regard to Energy Time History

To find out how the system characteristics, including stiffness, strength, damping, and the yielding trend affect the amount of maximum hysteretic energy and its variation, several time history analyses were performed by using a variety of accelerograms covering a wide range of PGA, frequency content and duration. Regarding the three main parameters of the hysteretic model, three sets of analyses have been performed in each of which just one parameter of the hysteretic model has been considered variable to find out the effect of each characteristic independently. Fig. 8 shows variations of the maximum hysteretic energy with respect to the yielding displacement, ultimate strength, and model parameter for El Centro earthquake, as a sample.

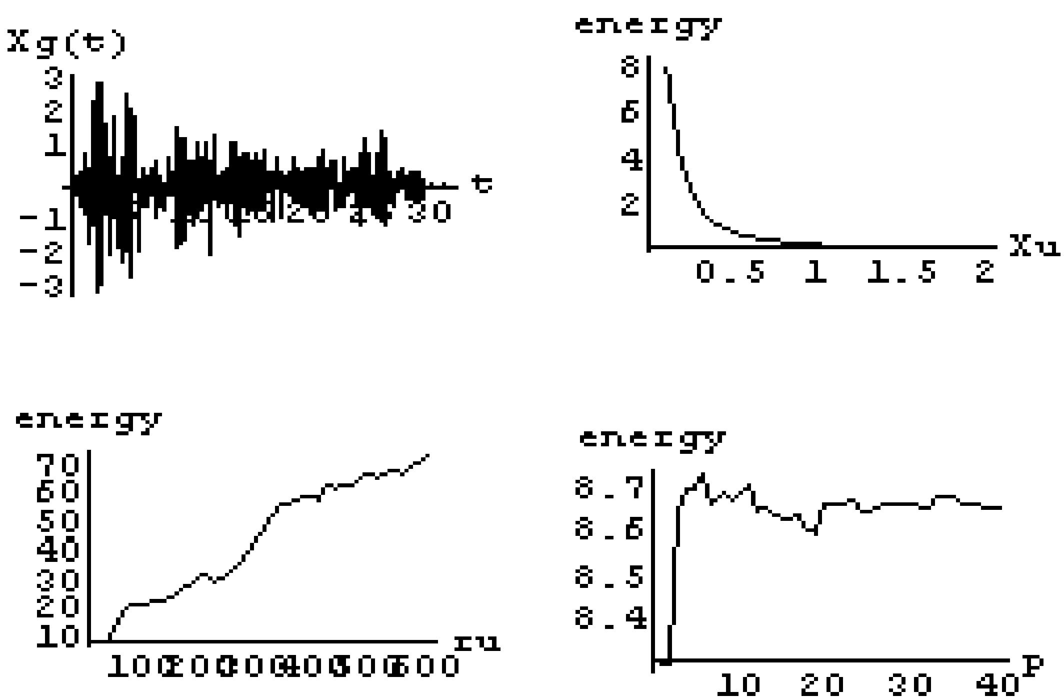

Figure 8. Variation of the maximum hysteretic energy versus the system characteristics for El Centro earthquake

It is seen in Fig. 8 that the amount of maximum hysteretic energy decreases as the yielding displacement of the system increases, but increases with increase in the ultimate strength of the system and also increase in the model parameter. However, it can be observed that for model parameter values larger than a specific value the maximum hysteretic energy remains almost constant. This is because as the model parameter p increases the system's behavior approaches more and more the elastic perfectly plastic behavior.

Fig. 9 shows the smoothed curves based on the average values of the results obtained from all employed accelerograms.

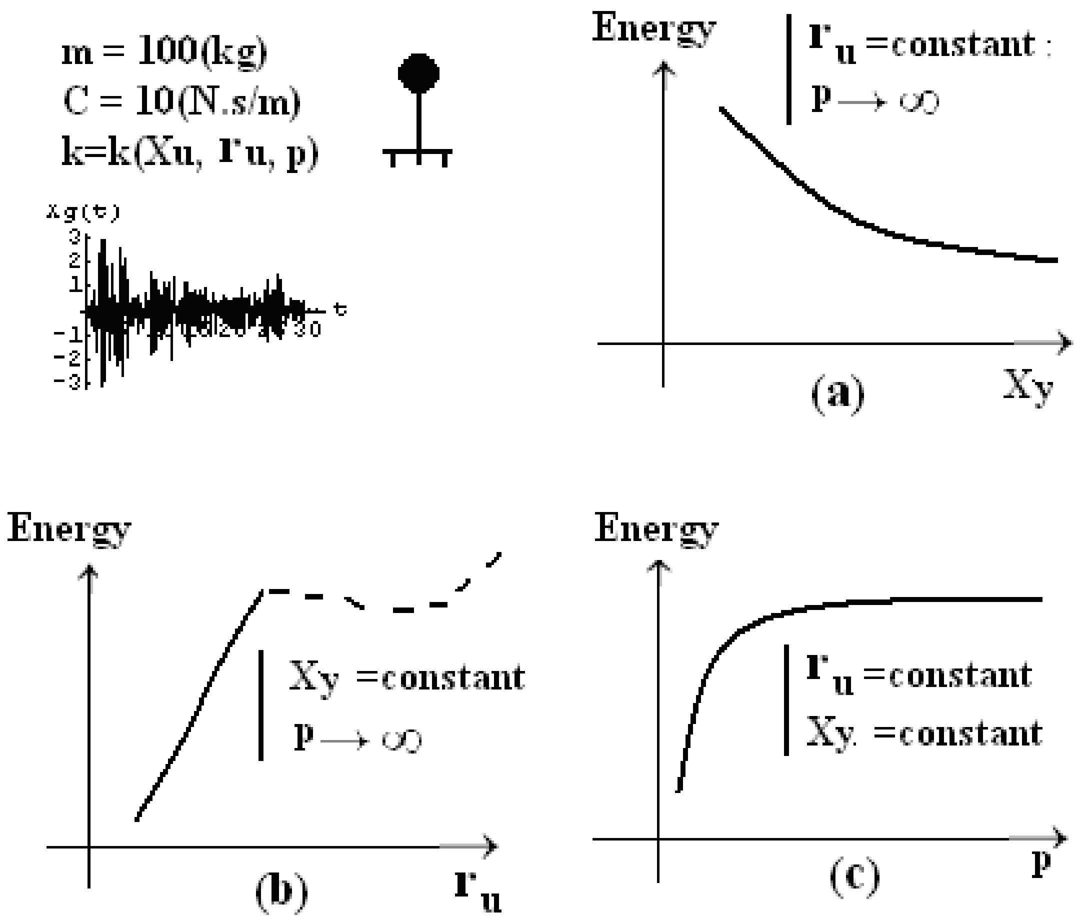

Figure 9. Average curves showing the variation of the maximum hysteretic energy versus the system characteristics, obtained by using several accelerograms

It is seen in Fig. 9 that the maximum hysteretic energy decreases with increase in the yielding displacement of the system, and increases with increase in the yielding or ultimate resistance of the system and also increase in the model parameter. With regard to the ultimate resistance the variation of hysteretic energy is linear first, and for resistance values more than a specific level the variation of hysteretic energy does not show any specific trend. The reason behind this fact is that with low ultimate strength value all system goes to plastic range, while with high ultimate strength values, depending on the earthquake intensity, in some cases the system experiences plastic deformation, while in some other cases it remains elastic.

Conclusions

Based on the numerical results obtained by several accelerograms, having a wide rage of PGA, duration of strong motion, and frequency content it can be concluded that:

- Given the values of ultimate strength and model parameter, the maximum hysteretic energy decreases with increase in the yielding displacement of the system.
- Given the values of yielding displacement and model parameter, the maximum hysteretic energy increases with increase in the ultimate strength to some specific value, and above than that value the variation of energy does now show any specific trends.
- Given the values of yielding displacement and ultimate strength of the system, the maximum hysteretic energy increases with increase in the model parameter. The increase is very fast for low values of the parameter, but for parameter values of more than a specific value the maximum hysteretic energy remains almost constant. On this basis it can be claimed that this parameter can be used as a controlling tool for limiting the amount of earthquake input energy, and accordingly the level of overall damage to the structure.

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