9th US National and 10th Canadian Conference on Earthquake Engineering: *Reaching Beyond Borders*

toronto july 25-29, 2010

9ième Conférence Nationale Américaine et 10ième Conférence Canadienne de Génie Parasismique: *Au delà des Frontières*

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VULNERABILITY OF TANKS UNDER SEISMIC ACTIONS

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ABSTRACT

A procedure to examine the vulnerability of tanks under seismic load is presented. Seismic intensity, ground dynamic conditions, soil-structure interaction effects and structural dynamic behavior are considered. Tanks are characterized by their aspect ratio and capacity. The product of this work is a simplified procedure to predict the accelerations in which some failure modes are presented and to build vulnerability curves in terms of the probability that the identified failure modes would be presented.

Introduction

A procedure to determine vulnerability curves of seismic designed tanks under seismic load is presented. Seismic hazard conditions, ground dynamic conditions, soil-structure interaction effects and structural dynamic behavior are taken into account. Vulnerability is described as a probability of failure function, whose main parameter is the acceleration that leads to the failure mode of interest. A part of this study is focused on the description of the main failure modes, such as circumferential collapse, elastic failure and elastoplastic failure. Other part is related with the determination of the corresponding accelerations and the calculation of the vulnerability curves.

Failure probability analysis

The structural reliability is the probability that the structure does not reach a limit state (e.g. failure state) during a given period. One advantage of measuring the structural safety through this reliability is that it may be a representation of the overall level of safety of complex structural systems through a single number. Another advantage is that the uncertainties inherent in the design process are taken into account explicitly, objectively, and systematically. Such uncertainties are related mainly to the randomness of the intensities of loads which, in the case of our study, are referenced to earthquake accelerations and to a lesser extent to the variability in resistance of the material elements of the structure.

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Since 1969, the uses of reliability concepts to determine structural safety (Esteva, 1969 and Cornell, 1969), which are now implicit in the structural design specifications in Mexico, have been proposed for the first time. Recently, for the petroleum industry in Mexico (IMP and PEMEX, 2000), design specifications and assessment of offshore platforms and oil pipelines with reliability bases are being proposed. To handle the reliability of structures in a systematic way (Ang and Tang, 1984) and its application in major infrastructure works (Ang and De León, 2005, Der Kiureghian, 2007 and Ellingwood, 2007) proposals have also been presented for use in design standards.

In this work a proposed methodology is presented to define the failure probability as well as the vulnerability functions for tanks. The failure probability of a structural system is defined as the probability of occurrence of a failure event when the load C exceeds the resistance R; taking C and R as random variables, the failure probability is expressed as:

$$P_f = P(C > R) \tag{1}$$

When a structural system consists of only one member, or when their safety is governed by a single dominant failure mode, and if the load and resistance are considered as independent variables, the evaluation of this probability is simplified. In this way, the failure probability might be obtained through a safety margin, denoted as M, which also is practically considered as a random variable and having a normal distribution; this safety margin is evaluated by:

$$M = R - C \tag{2}$$

An event called failure occurs when R < C and thus the failure probability is related to the safety margin for M < 0. Thus,

$$P_f = F_M(0) \tag{3}$$

In this case $F_M(0)$ is the cumulative distribution of the safety margin M, and the probability of failure is represented in the following form:

$$P_f = \Phi[-\mu_M/\sigma_M] \tag{4}$$

where Φ is the normal cumulative standardized distribution and:

$$\mu_M = \mu_C - \mu_R \tag{5}$$

$$\sigma_M = (\sigma_C{}^2 + \sigma_R{}^2)^{1/2} \tag{6}$$

μ_C and μ_R being the mean values of load and resistance variables; and σ_C and σ_R their standard deviations respectively. However, if the variables C and R are considered lognormal (which occurs frequently because the lognormal variable is only defined for positive values and many quantities of engineering as well), the failure probability is defined by considering a safety factor, θ , which is usually expressed as:

$$\theta = R/C \tag{7}$$

while the probability of failure is defined as:

$$P_f = F_\theta(1) \tag{8}$$

The function $F_\theta(1)$ is the cumulative distribution of the safety factor θ . Then the failure probability gets the following form:

$$P_f = \Phi[-\lambda_\theta/\zeta_\theta] \tag{9}$$

where λ and ζ are the are the parameters of the lognormal distribution, and

$$\lambda_\theta = E(\ln\theta) \tag{10}$$

and

$$\zeta_\theta = \sqrt{\mathrm{Var}(\theta)} \tag{11}$$

$E(\bullet)$ being the expected value and $\mathrm{Var}(\bullet)$ the variance. Therefore, Eq. 9 gets the form:

$$P_f = \Phi(-\lambda_\theta/\zeta_\theta) \tag{12}$$

As a first approximation, it will be supposed that the failure event in tanks depends on the failure of only one structural member, and only one mode of failure will be studied at a time. If it is assumed that the probability if failure is computed as

$$P_f = 1 - \Phi(\beta) \tag{13}$$

then, if the reliability index $\beta = 0$, it is given that $\Phi(\beta)=1/2$, and therefore $P_f=1/2$. A complete, explicit form of the reliability index is

$$\beta = ln\,R/C + ln\sqrt{(1+Cv_C^2)/(1+Cv_R^2)}\Big/\sqrt{Cv_R^2+Cv_C^2} \tag{14}$$

Regards the working stress ratio as the inverse of the safety factor, defined as

$$R_T = C/R \tag{15}$$

the reliability index can be expressed as

$$\beta = \left[ln\sqrt{(1+Cv_C^2)/(1+Cv_R^2)} - ln\,R_T\right]\Big/\sqrt{Cv_R^2+Cv_C^2} \tag{16}$$

If the medians of the load and strength are equals, and the coefficients of variations of the load and strength are equals, then the vulnerability curve, or probability of failure, acquires the value 1/2. However, under independent uncertainties of the load and strength, just it can be ensured that the probability of failure reach a value close to 1/2 when load equals strength. Note that load and strength are equal when the intensity is equal to the design acceleration, that is

$$a_D = a\big|_{C=R} \tag{17}$$

Also, it can be shown that, if the coefficients of variation Cv_C and Cv_R are small, the vulnerability curve, or probability of failure, grows rapidly. Contrarily, if they are big, the vulnerability curve grows slowly. Without uncertainties (deterministic case), the vulnerability curve is a Heviside function, defined as

$$P_f = \begin{cases} 0 & \text{si } a < a_D \\ 1 & \text{si } a \geq a_D \end{cases} \tag{18}$$

A typical form of the working stress ratio is a linear relation with the intensity, given in terms of the Peak Ground Acceleration (PGA). It means that the load acting on the element has a variation that is directly proportional to the seismic intensity. As in the vulnerability curve, there is a point of major importance in the working stress ratio. It is such that its value is 1, and it is presented when $a = a_D = a\big|_{C=R}$. There is another point of interest, and it is presented when the acceleration is null. For this condition it follows that the working stress ratio is $R_T = R_{T_0}$. It corresponds to the permanent loads, which act without seismic acceleration, as the self weight. In tanks, could be considered also the effects of hydrostatic pressure.

Collapse condition for seismic designed tanks

It is common to use the infinite length pipe theory to determine the thickness of the plates of a tank. The expression used to relate the internal pressure with the circumferential stress σ_x in a pipe is

$$\sigma_x = pr/t \tag{19}$$

where p is the internal pressure, r the radius, t the thickness of the plate. The pressure p is composed by

$$p = p_H + \sqrt{p_i^2 + p_c^2} \tag{20}$$

where $p_H = \gamma H$ is the hydrostatic pressure (γ and H are the volumetric weight and the height of the fluid, respectively), p_i is the impulsive pressure and p_c is the convective pressure. Then, for the design it will have to equal the maximum stress of the plate and the developed stress due to the action of the internal pressure. Note that the circumferential stress σ_x is a tensile stress. According to Eq. 19 the thickness is given by

$$t = pr/\sigma_x \tag{21}$$

For A36 steel, the permissible stress S_D is 160 MPa, the yielding stress F_y is 250 MPa and the ultimate stress F_U is 410 MPa. The first one is the permissible stress, the second one is the limit of elastic behavior, and the third one is related with the material ductility and the rupture. The design of tanks is done with permissible stresses, but they are reviewed with yielding stresses.

Overstrength Factor

An interesting result is related with a tank designed for the permissible stress when, in fact, its safety is controlled by the actual yielding stress. When the acceleration is zero, the only circumferential stress generated in the plate is due to hydrostatic pressure, which has the form

$$\sigma_H = p_H r / t \tag{22}$$

Working stress ratio for the permissible and yielding stresses is, respectively

$$R_{T,S_D} = \sigma_x / S_D \tag{23}$$

$$R_{T,F_y} = \sigma_x / F_y \tag{24}$$

Circumferential stresses have this linear dependence with the acceleration, $\sigma_x = \sigma_H + \alpha a$. If $R_T = 1$, one can write

$$\left(\sigma_H + \alpha a_D\right) / S_D = 1 \tag{25}$$

$$\left(\sigma_H + \alpha \bar{a}_D\right) / F_y = 1 \tag{26}$$

where a_D and $\bar{a}_D$ are the design acceleration and the design acceleration with overstrength. The ratio between these accelerations provides the theoretical collapse overstrength factor.

$$FSR_T^c = \bar{a}_D / a_D = \left(F_y - \sigma_H\right) / \left(S_D - \sigma_H\right) \tag{27}$$

The theoretical collapse acceleration regarding the overstrength factor, is

$$\bar{a}_T^c = \bar{a}_D = a_T^c \times FSR_T^c \tag{28}$$

where $a_T^c = a_D$ is the theoretical collapse acceleration without overstrength, that is, the design acceleration.

Site and soil-structure interaction effects in the vulnerability of tanks

It is widely recognized that the ground and foundation characteristics can have very important effects in the structural seismic response. These are commonly known as site and soil-structure interaction effects. Soil conditions amplify the motion in frequencies that are controlled by the dynamic properties of the materials of the ground. These frequencies can coincide with the frequencies of vibration of the structure, and lead it to the resonance condition. Besides, the dynamic interaction condition between the foundation and the ground modify the dynamic properties of the structure, particularly the fundamental period of vibration and the material damping. It is usual to take into account these aspects in the seismic structural design.

Internal pressure variation with the peak ground acceleration (PGA)

The objective in the seismic design of tanks is to determine the thickness of the steel plates using the Eq. 19, 20 and 21. Note that all the spectral information, that is, the site seismic information relevant to design is accounted for in Eq. 20. This equation is an approximation to determine the hydrostatic and hydrodynamic actions on the plates of the tank. The former term is related with the hydrostatic pressure, as well as the later is a quadratic combination of the impulsive (associated to the adhered mass to the plate) and convective (due to the fundamental mode of vibration of the fluid) pressures. To increment the internal pressure by seismic load, the PGA is incremented. By simplicity, the evolution of the vulnerability and working stress ratio curves has been referred to this parameter of intensity: the PGA. For these reason, it is convenient to have a consistent form to build design spectra. This would allow preserving the site effects due to the soil conditions during the spectral scaling. The total pressure in the tank can be written in the following form:

$$p_{tot} = p_H + a_o\sqrt{B_i^2 F_i^2 + B_c^2 F_c^2} \tag{29}$$

where F_i and F_c are form factors of the impulsive and convective modes and depend on the geometry of the tank. The factors B_i and B_c are the spectral ordinates for the impulsive (T_i) and convective (T_c) periods, normalized with the PGA. They have the form $B_i = a(T_i)/a_o$ and $B_c = a(T_c)/a_0$. In Eq. 29 can be noted that the internal pressure grows linearly with the PGA. It is a straight line with origin ordinate given by the hydrostatic pressure p_H and slope given by the term $\sqrt{B_i^2 F_i^2 + B_c^2 F_c^2}$, that is constant for a given tank and a given site (design spectrum). As in the pressure, the stress follows a straight line with origin ordinate given by the hydrostatic stress $p_H r/t$ and slope given by the term $\sqrt{B_i^2 F_i^2 + B_c^2 F_c^2}\ r/t$.

Relationship between the collapse accelerations of a tank in soil and rock

Suppose that a tank is designed for a soft soil site. The design acceleration in the soil is known (a_D^{soil}) and correspond to the PGA of the site design spectrum. What is the collapse acceleration? The answer can be described in terms of the behavior of tanks in rocky ground and in that knowledge of the seismic response in soft soil given by the site design spectrum. Consider the linear relationship between maximum pressure in the tank and the PGA shown in fig 1 (left).

Note that the pressure p_{max} could correspond to the design pressure (p_D) or to the collapse pressure (p_c). These pressures are independent of the site, just depend on the tank. For each site, the maximum pressure p_{max} is related to the PGA, that correspond, for each case, to the design (a_D) or collapse (a_c) acceleration. Linear equations for total internal pressures, in terms of the PGA for each site, are respectively

$$p^{soil} = p_H + a_o^{soil} \sqrt{B_i^{soil\,2} F_i^2 + B_c^{soil\,2} F_c^2} \tag{30}$$

and

$$p^{rock} = p_H + a_o^{rock} \sqrt{B_i^{rock\,2} F_i^2 + B_c^{rock\,2} F_c^2} \tag{31}$$

PGA for each site is obtained by equaling both expressions to the maximum pressure

$$a_{max}^{soil} = (p_{max} - p_H) \Big/ \sqrt{B_i^{soil\,2} F_i^2 + B_c^{soil\,2} F_c^2} \tag{32}$$

$$a_{max}^{rock} = (p_{max} - p_H) \Big/ \sqrt{B_i^{rock\,2} F_i^2 + B_c^{rock\,2} F_c^2} \tag{33}$$

and

$$a_{max}^{soil} \Big/ a_{max}^{rock} = \sqrt{B_i^{rock\,2} F_i^2 + B_c^{rock\,2} F_c^2} \Big/ \sqrt{B_i^{soil\,2} F_i^2 + B_c^{soil\,2} F_c^2} \tag{34}$$

is the ratio of both expressions. This relation shows that the ratio is independent of the maximum pressure. For that, this ratio can be particularized to see the relationship between design and collapse accelerations.

$$a_C^{soil} \Big/ a_C^{rock} = a_D^{soil} \Big/ a_D^{rock} = \sqrt{B_i^{rock\,2} F_i^2 + B_c^{rock\,2} F_c^2} \Big/ \sqrt{B_i^{soil\,2} F_i^2 + B_c^{soil\,2} F_c^2} \tag{35}$$

Finally, a relation that allows knowing the collapse acceleration in soft soil site starting from the collapse acceleration in rock is obtained, and it is

$$a_c^{soil} = a_c^{rock}\, a_D^{soil} \Big/ a_D^{rock} \tag{36}$$

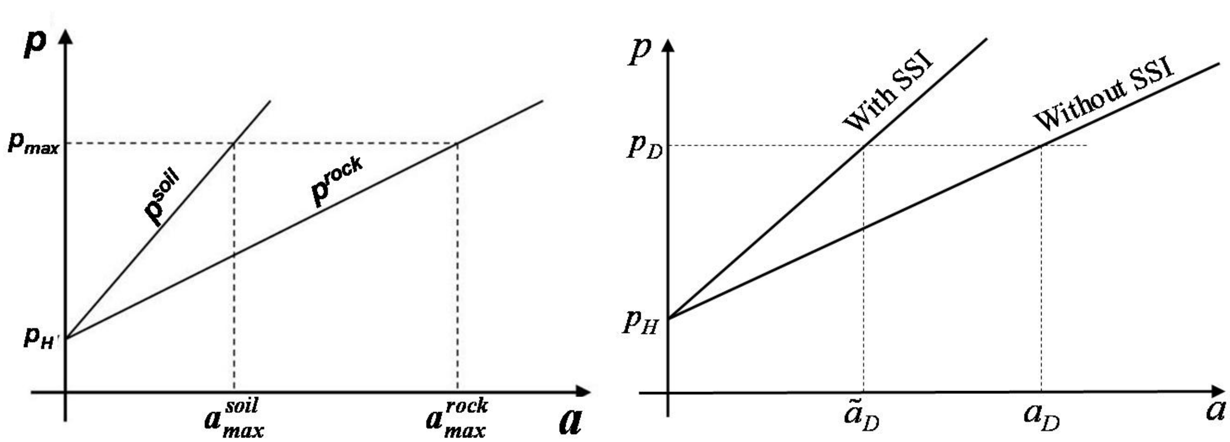

Fig 1. Linear relation between the internal pressures in the tank and the PGA for (Left) two arbitrary sites, one on rock and the other on soil, and (Right) regarding that the base is rigid (without soil-structure interaction)and flexible (with soil-structure interaction).

Soil-structure interaction effects in the collapse acceleration of a tank

It is well known that the soil-structure interaction (SSI) has effects only in the impulsive short period mode, and that these effects on the convective long period mode can be neglected. Avilés and Pérez-Rocha (2003) have shown that the actual structure of flexible base with natural period T_e and damping ratio ζ_e can be replaced adequately by an equivalent oscillator of rigid base defined by the period and damping of the coupled system, $\tilde{T}_e$ and $\tilde{\zeta}_e$. These parameters are given by (Avilés and Pérez-Rocha, 1996)

$$\tilde{T}_e = (T_e^2 + T_h^2 + T_r^2)^{1/2} \tag{37}$$

$$\tilde{\zeta}_e = \zeta_e \frac{T_e^3}{\tilde{T}_e^3} + \frac{\zeta_h}{1+2\zeta_h^2}\frac{T_h^2}{\tilde{T}_e^2} + \frac{\zeta_r}{1+2\zeta_r^2}\frac{T_r^2}{\tilde{T}_e^2} \tag{38}$$

where $T_h = 2\pi(M_e/K_h)^{1/2}$ and $T_r = 2\pi(M_e(H_e+D)^2/K_r)^{1/2}$ are the natural period associated with the translation and rocking of the structure, as well as $\zeta_h = \pi C_h/\tilde{T}_e K_h$ and $\zeta_r = \pi C_r/\tilde{T}_e K_r$ are the corresponding damping factors of the soil. After computing the dynamic effective parameters of the impulsive component, $\tilde{T}_i$ and $\tilde{\zeta}_i$, the spectral accelerations are determined for these new effective parameters in the original design spectrum, using an appropriated scaling such that the total pressure would be the same that the one corresponding to the rigid base condition, that is

$$p_{tot} = p_H + \tilde{a}_o\sqrt{\tilde{B}_i^2 F_i^2 + B_c^2 F_c^2} \tag{39}$$

here $\tilde{B}_i = \tilde{a}(\tilde{T}_i)/a_o$, where $\tilde{T}_i$ is the impulsive period modified by SSI and $\tilde{a}(\tilde{T}_i)$ is the spectral ordinate to that period. The relationship between the internal pressures in a tank, with and without SSI effects, and the design accelerations (PGA) are shown in the right part of figure 1. It is of interest to know what is the collapse acceleration with SSI, $\tilde{a}_C$. For that reason, an appropriately scaled spectrum such that for the effective parameters provide the same total pressure that the one without SSI. The PGA of that scaled design spectrum is, precisely, the design acceleration whit SSI, $\tilde{a}_D$. Following the same way that in the previous case, one can write the next relationship

$$\tilde{a}_C/a_C = \tilde{a}_D/a_D = \sqrt{B_i^2 F_i^2 + B_c^2 F_c^2} \Big/ \sqrt{\tilde{B}_i^2 F_i^2 + B_c^2 F_c^2} \tag{40}$$

The relation that allows knowing the collapse acceleration with SSI starting from the collapse acceleration without SSI, for a given site (design spectrum) is

$$\tilde{a}_C = a_C\,\tilde{a}_D/a_D \tag{41}$$

Example

Consider a tank with V=40000 m^3 and aspect ratio H/D=0.2. Suppose that the collapse acceleration for three design accelerations on rock (a_D=0.1, 0.3, and 0.5) are known, as shown in table I. Now consider a tank with same volume and aspect ratio, designed for a soft soil with the next design spectrum for 5% of critical damping: $a(T_e) = a_o + (c - a_o) T_e / T_a$ if $T_e \leq T_a$, $a(T_e) = c$ if $T_a < T_e \leq T_b$, and $a(T_e) = c(T_e / T_b)^r$ if $T_e \geq T_b$. Regard that a_o=0.25, c=0.75, T_a=0.35, T_b=1.2, and r=1. What is the collapse acceleration? what is the collapse acceleration with SSI, if $\tilde{T}_i / T_i = 1.41$ and $\tilde{\zeta} = 0.05$?

Table I. Design and collapse accelerations for a tank on rock with V=40000 m^3 and H/D=0.2 (accelerations in parts of the acceleration of the gravity)

Design acceleration a_D	0.1	0.3	0.5
Collapse acceleration a_C	0.384	0.694	1.006

First, one has to determine the periods and spectral accelerations for the impulsive and the convective modes. These periods are $T_i = 1.41$ s and $T_c = 10.46$ s, and the corresponding spectral accelerations are $a_i = 0.65$ s and $a_c = 0.129$ s. The hydrostatic pressure is $p_H = 0.104$ MPa, the hydrodynamic pressure is $p_E = 0.072$ MPa, and the total internal pressure is $p_T = p_H + p_E = 0.176$ MPa. From eq 33, it is had that

$$a_D^{rock} = a_o^{rock} = (p_T - p_H) \Big/ \sqrt{B^{rock}{}_i^2 F_i^2 + B^{rock}{}_c^2 F_c^2}\text{, and therefore } a_D^{rock}\Big|_{p_T = 0.176\text{ MPa}} = 0.233$$

By interpolation in table I, it is had

Design acceleration a_D	0.1	**0.233**	0.3
Collapse acceleration a_C	0.384	**0.591**	0.694

Eq 36 yields $a_c^{soil} = a_c^{rock}\, a_D^{soil} \big/ a_D^{rock} = (0.591)\,0.250/0.233 = 0.632$, meanwhile the rigorous solution (using finite element method) is $a_c^{soil} = 0.666$.

Eq 40 allows getting that $\tilde{a}_D = a_D \sqrt{B_i^2 F_i^2 + B_c^2 F_c^2} \Big/ \sqrt{\tilde{B}_i^2 F_i^2 + B_c^2 F_c^2} = 0.22$ and eq 41 yields that $\tilde{a}_C = a_C \tilde{a}_D / a_D = (0.632)0.22/0.25 = 0.557$. The rigorous solution (using finite element method) is $\tilde{a}_C = 0.560$.

Conclusions

A simplified model of vulnerability of tanks under seismic action has been described. It has been pointed out that the working stress ratio, defined as the ratio between the load and the strength, has a linear variation with the input acceleration, and this one is the peak ground acceleration (PGA). As well, a consistent scaling law for design spectra has been used to examine the variations of the load with the seismic intensity.

The most important point of the working stress ratio is that one where its value is 1, and occurs when the input acceleration is equal to the collapse acceleration.

A theoretical overstrength factor has been obtained, and it relates the design and collapse acceleration. As well, a relation between the collapse acceleration of two tanks, one in rock and the other in soil, has been obtained. Following the same kind of relations, an equation to get the collapse acceleration regarding soil-structure interaction was obtained in terms of the collapse acceleration regarding rigid base.

An example that shows the good accuracy of the useful relations was illustrated. In this example, the collapse acceleration suit corresponding to rock condition was used to get the collapse acceleration of a tank in soft soil without and with soil-structure interaction. Errors were less than 5%.

Acknowledgments

Author appreciates a lot all supports provided by the Comisión Federal de Electricidad, especially to Enrique Mena Sandoval.

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