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# A GENERAL FORCE-BASED HYBRID SIMULATION FORMULATION

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## ABSTRACT

This paper proposes a general formulation for force-based Hybrid simulation, which is a structural seismic response simulation method combining the use of numerical simulation of computational substructure and physical testing of experimental substructure. By introducing a set of splitting coefficient matrices to the general equation of motion of the experimental substructural model, a group of force-based hybrid simulation methods can be formulated, including real time pseudo-dynamic substructure testing, effective force substructure testing and shake table substructure testing. This paper first reviews the recent development in hybrid simulation methods; especially those displacement-based ones and introduces the force-based hybrid simulation concept. Then the general formulation is presented with a detailed discussion of the splitting coefficient matrices. Hardware components necessary to implement the general formulation and the correspondingly developed simulation controller are integrated into a general test platform. A small-scale pilot setup was used in the verification tests. Test results which validated the concept of the proposed general formulation and the feasibility of the corresponding testing platform are discussed at last.

## Introduction

Laboratory seismic testing of civil structural components and systems includes Quasi-static testing (QST), Pseudo-dynamic testing (PSD), Shake table testing (STT), Effective force testing (EFT) and the newly developed Real time dynamic hybrid testing (RTDHT). RTDHT shown in Fig. 1 combines the use of shake tables, actuators, and computational engines for the seismic response simulation of structures. The structure to be simulated is divided into a *experimental substructure* and one or more *computational substructures*. The interface forces between the substructures are imposed to the experimental substructure by actuators while the shake table introduces the earthquake ground motions, or motions of other computational substructures. Meantime, the displacement and velocity responses of the experimental substructure are fed back to the computational engine to determine the dynamic responses of the computational substructures and the interface loading conditions to be applied as well. A

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controller platform was developed by Reinhorn et al (2005) to implement the RTDHT, consisting of multiple PCs to perform the functions as illustrated in Fig.1.

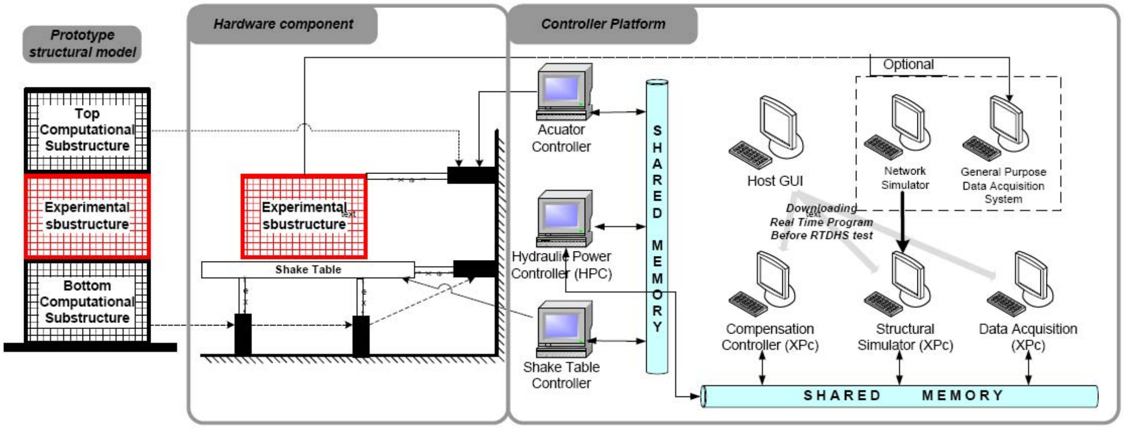

Figure 1. Real time dynamic hybrid testing (RTDHT)

Table 1. Summary of laboratory seismic testing methods

| | | *Fast/ Real time* | *Substructure* | ***Dynamic effect*** | *Loading device* | |
|---|---|---|---|---|---|---|
| | | | | | Act. | Shake Table |
| **QST** | | No | Yes | Actuator imposes predefined displacement or force quasi-statically, *inertial effect negligible*. | Displ. / force | N/A |
| **PSD** | ***Slow*** | No | Yes | Dynamic responses and interaction with the computation substructure is *numerically simulated* from the equation of motion and applied by actuator in displacement. | Displ. | N/A |
| | ***Fast*** | Yes | Yes | | | |
| **STT** | | Yes | No | *Realistic dynamic effect* achieved in the structural assembly. | N/A | Accel. |
| **EFT** | | Yes | Yes | Effective force directly applies to the *lumped* mass of the structural model. | Force | N/A |
| **RTDHT** | | Yes | Yes | *Realistic inertial force* achieved in the experimental substructure by shake table imposed motion and interface forces between substructures applied by actuator. | Force | Accel. |

Table 1 summarizes the speed of applying load, compatibility with substructure techniques, achieved dynamic effects and loading devices used in various laboratory seismic testing methods. QST does not generate any dynamic effect in the specimen while EFT is limited to test structural model with lumped mass. STT is not suitable for testing of large scale model due the capacity limitation of shake table. Although both fast PSD and RTDHT experiments employ substructures in physical testing with online computations to simulate the global system response. The latter technique produces inertial effects naturally in the physical system while the former one simulates computationally such effects. Therefore RTDHS allows a researcher to focus on specific problems represented in the substructure under the most realistic conditions using emerging computational power in tandem with state-of-the-art control systems. Moreover, such procedures and configurations may extend significantly the testing capabilities with the proposed general formulation in that various seismic testing methods in Table 1 can be conducted by the developed RTDHT system without individual numerical algorithm and control system modifications.

## General Formulation of Hybrid Simulation

A derivation for substructure formulation in Real time dynamic hybrid testing (RTDHT) can be obtained by partitioning the equation of motion describing the global structural model, the equation of motion for the experimental substructure then becomes (Shao, 2006):

$$\mathbf{M}_e\ddot{\mathbf{x}}_e + \mathbf{C}_e\dot{\mathbf{x}}_e + \mathbf{f}_e\left(\mathbf{x}_e, \dot{\mathbf{x}}_e\right) = -\mathbf{M}_e\mathbf{R}\ddot{\mathbf{u}}_g + \mathbf{T}_e \tag{1}$$

where $\mathbf{M}_e$, $\mathbf{C}_e$ are the mass and damping matrices, $\mathbf{f}_e(\mathbf{x}_e, \dot{\mathbf{x}}_e)$ represents the inelastic response of the experimental substructure. $\ddot{\mathbf{x}}_e$ , $\dot{\mathbf{x}}_e$ and $\mathbf{x}_e$ are the vectors of experimental substructure's acceleration, velocity and displacement/rotation associated with each degree-of-freedom (DOF) relative to the ground reference frame. Terms on the left side of Eq. 1 are the idealized model of the experimental substructure that will actually be physically replicated from the prototype structure during a hybrid testing. Therefore there is no need to develop an accurate numerical model of the experimental substructure from the RTDHT point of view. The terms on the right of Eq. 1 represent the dynamic input to the physical specimen, consisting of the ground acceleration excitation $\ddot{\mathbf{u}}_g$ and the interface force vector $\mathbf{T}_\mathrm{e}$ due to the substructures' interaction. $\mathbf{R}$ is the ground motion scale and direction vector. When RTDHT was first proposed, the ground acceleration excitation was designated to be applied by the shake table and the interface forces applied by the force-controlled dynamic actuators attached at the interface DOFs. Alternative loading configurations were identified with the progress of RTDHT development. These alternatives, representing various hybrid simulation methods, can be expressed by the general formulation that will bring about different loading cases performed by the shake tables and the actuators while ideally resulting in the same experimental substructure's response as formulated in Eq. 1. The general formulation is shown in Eq. 2 as,

$$\begin{aligned}\mathbf{M}^p{}_e\ddot{\mathbf{x}}_e + \mathbf{C}^p{}_e\dot{\mathbf{x}}_e + \mathbf{f}_e{}^p\left(\mathbf{x}_e, \dot{\mathbf{x}}_e\right) = -\mathbf{M}^p{}_e\mathbf{R} \quad & \underbrace{\left(\mathbf{E} - \boldsymbol{\alpha}_l\right)\ddot{\mathbf{u}}_g}_{\text{Shake table applied acceleration}} \\ + \underbrace{\underbrace{\mathbf{T}_e}_{\substack{\text{Force input due}\\ \text{to interaction}}} - \underbrace{\boldsymbol{\alpha}_m\mathbf{M}_e\left(\mathbf{R}_e\ddot{\mathbf{u}}_g + \ddot{\mathbf{x}}_e\right)}_{\substack{\text{Inertial forces of virtual}\\ \text{mass of the specimen}}} - \underbrace{\boldsymbol{\alpha}_c\mathbf{C}_e\dot{\mathbf{x}}_e}_{\substack{\text{Damping force of virtual}\\ \text{damping of the specimen}}} - \underbrace{\boldsymbol{\alpha}_f\mathbf{f}_e\left(\mathbf{x}_e, \dot{\mathbf{x}}_e\right)}_{\substack{\text{Restoring force of virtual}\\ \text{stiffness of the specimen}}} - \underbrace{\mathbf{M}^p{}_e\mathbf{R}\boldsymbol{\alpha}_l\ddot{\mathbf{u}}_g}_{\substack{\text{Effective force of partial}\\ \text{dynamic excitation}}}}_{\text{Actuator applied forces}} & \end{aligned} \tag{2}$$

in which $\boldsymbol{\alpha}_m$, $\boldsymbol{\alpha}_c$ , $\boldsymbol{\alpha}_f$ and $\boldsymbol{\alpha}_l$ are the *mass, damping, restoring force and dynamic load splitting coefficient matrices*. By setting different values of these matrices, a variety of loading cases can be formulated including those listed in Table 1. In the following sections, each of these splitting coefficient matrices will be discussed.

## Mass Splitting Coefficient Matrix $\boldsymbol{\alpha}_m$

In a conventional dynamic testing such as EFT, STT, full mass $\mathbf{M}_e$ is usually required in the physical specimen. The inertia forces are therefore developed naturally during the testing. However, for structural models that are large with respect to the loading devices, such masses may be difficult to be built or safely supported by the testing rig. To overcome the limitation in specimen's weight, a portion of the mass can then be modeled numerically in a computer to reduce the size of the physical mass being fabricated, installed and tested (see also Kausel, 1998 and Chen et al. 2006). The mass that is modeled analytically is defined as *virtual mass*. A mass splitting coefficient matrix $\boldsymbol{\alpha}_\mathrm{m}$ is defined herein as a diagonal matrix representing the ratio of the virtual mass ($\mathbf{M}_e^v$) to the total mass of the experimental substructure ($\mathbf{M}_e$) as required in dynamic simulation.

$$\boldsymbol{\alpha}_m = \mathbf{M}^v{}_e \cdot \mathbf{M}^{-1}{}_e = \left(\mathbf{M}_e - \mathbf{M}^p{}_e\right) \cdot \mathbf{M}^{-1}{}_e \tag{3}$$

The physical mass matrix $\mathbf{M}_e^p$ is then expressed as $\mathbf{M}^p{}_e = \left(\mathbf{E} - \boldsymbol{\alpha}_m\right)\mathbf{M}_e$, in which $\mathbf{E}$ is a diagonal identity matrix. With only partial mass presented in the physical specimen, inertia effects related to the virtual mass has to be included as additional input that can be formulated as $\mathbf{T}_e^v = -\boldsymbol{\alpha}_m \mathbf{M}_e \left(\mathbf{R}_e \ddot{\mathbf{u}}_g + \ddot{\mathbf{x}}_e\right)$. This inertial force is then added to the right side of Eq. 2 that must be applied at each DOF of the reduced mass specimen. Note that the force vector contains either all, or a portion of the inertia forces, depending of the magnitude of $\boldsymbol{\alpha}_m$ and different test methods are formulated:

1) $\boldsymbol{\alpha}_\mathrm{m} = \mathbf{E}$ represents the case of a massless specimen; all the inertia force is numerically simulated in the computer and applied to the physical substructure as an external force, known as a force-based PSD hybrid simulation conducted in real time speed;

2) $\boldsymbol{\alpha}_\mathrm{m} = \mathbf{0}$ defines that full mass is included in the experimental substructure without virtual mass in the numerical model. This test condition, with full physical mass, is defined as Dynamic hybrid simulation where the inertia effects within the experimental substructure are developed physically (or "naturally") during the testing;

3) $\mathbf{0} < \boldsymbol{\alpha}_\mathrm{m} < \mathbf{E}$ (not all the diagonal entries in $\boldsymbol{\alpha}_m$ equal to zero or unity), the required mass is divided between the physical mass in the specimen and the virtual mass simulated in the computer. This is defined as Quasi-dynamic hybrid simulation, a hybrid testing method combining the Dynamic and the PSD tests. Part of the inertia effects are simulated numerically while the remaining developed naturally. This method allows application of part of the inertial forces required by dynamic simulation to the physical substructure when the loading devices have limited capacities.

### Damping and Restoring Force Splitting Coefficient Matrices ($\boldsymbol{\alpha}_c$ and $\boldsymbol{\alpha}_f$)

Similar to the mass splitting coefficient matrix dividing the required mass to virtual and physical mass, the damping and restoring force splitting coefficient matrices $\boldsymbol{\alpha}_c$ and $\boldsymbol{\alpha}_f$ split the required damping and restoring force components into virtual and physical counterparts respectively. They are defined the same way as $\boldsymbol{\alpha}_m$, the ratio of the virtual part to the total dynamic simulation required quantities (i.e. $\mathbf{C}_e, \mathbf{f}_e\left(\mathbf{x}_e, \dot{\mathbf{x}}_e\right)$),

$$\boldsymbol{\alpha}_l = \mathbf{C}^v{}_e \cdot \mathbf{C}^{-1}{}_e \tag{4}$$

$$\boldsymbol{\alpha}_f = \mathbf{f}^v{}_e \cdot \mathbf{f}^{-1}{}_e \tag{5}$$

With only physical damping and restoring force component exist in the specimen, damping force and restoring forces related to the virtual counterparts are numerically simulated and applied by the actuators, as the terms of $-\boldsymbol{\alpha}_c \mathbf{C}_e \dot{\mathbf{x}}_e$ and $-\boldsymbol{\alpha}_f \mathbf{f}_e\left(\mathbf{x}_e, \dot{\mathbf{x}}_e\right)$ shown in Eq. 2. The introduction of these two splitting coefficient matrices will greatly enhance the flexibility in designing the experimental substructure and allow the researchers to focus the physical experiments on the complex components within the structural model. For instance, when the effect of a new damper system to the structural seismic response is the object of a hybrid simulation, researchers can build the specimen including the physical damper with the remaining damping effect related to structural system numerically simulated and applied by the actuators. Also if the dynamic behavior of an innovative brace system with complex restoring force pattern is being studied, hybrid simulation can be conducted on this brace system integrated with the proper mass and damping components and the remaining restoring force component such as columns in the same story can then be numerically simulated. In this case, the virtual restoring force components usually have a simple and predictable dynamic response and cab be modeled as a linear stiffness.

### Dynamic load splitting coefficient matrix ($\boldsymbol{\alpha}_l$)

During an RTDHT test the dynamic loading formulated by the right side of Eq. 1 is simultaneously applied by shake tables and actuators. The load sharing can be determined by the load splitting coefficient matrix $\boldsymbol{\alpha}_l$ in Eq. 2. The ground acceleration excitation is divided into two components, with one component assigned to the base excitation $\left(\mathbf{E} - \boldsymbol{\alpha}_l\right)\ddot{\mathbf{u}}_g$ (shake table) and the other to the actuators as effective force ($-\mathbf{M}^p{}_e \mathbf{R} \boldsymbol{\alpha}_l \ddot{\mathbf{u}}_g$). Several cases are notable:

1) $\boldsymbol{\alpha}_l = \mathbf{0}$, the shake table (or base) does not move and the entire dynamic loading is applied to the experimental substructure using the actuators attached to the specimen at each DOF. This is the Effective force hybrid simulation;

2) $\boldsymbol{\alpha}_l = \mathbf{E}$, the ground motion is applied at the base without contribution from effective forces. For substructure testing, the interface forces with the complementary

computational substructure are introduced by actuators at the appropriate interface DOFs shown by term $\mathbf{T}_e$ in Eq. 1. This is the conventional RTDHT;

3) $\mathbf{0} < \boldsymbol{\alpha}_l < \mathbf{E}$, the ground acceleration or its effects are applied in part by shake tables (or another form of base movement) and in part by actuators. Several strategies may be used for splitting the driving function between the shake table and the dynamic actuators as proposed by Kausel (1998). In fact, the characteristics of the splitting coefficients can be chosen to optimize the total power needed by the testing system or to achieve other mechanical advantages.

Therefore, Eq. 2 represents the general formulation defining the loading configurations applied to the experimental substructure during the force-based hybrid simulation. The three types of tests (pseudo-dynamic, dynamic and quasi-dynamic), and the associated load application splitting between the shake tables and the actuators, are identified and listed in Table 2, assuming the structural test model containing all required damping and restoring force components ($\boldsymbol{\alpha}_c = \mathbf{0}$ and $\boldsymbol{\alpha}_f = \mathbf{0}$). All seven cases shall produce the same responses of the global structural model including both computational substructures and experimental substructure when subjected to ideal real time loading conditions.

Table 2. Experimental substructure loading configurations in hybrid simulation

| **TEST MODEL** | | **TOTAL DYNAMIC LOAD** | | | |
|---|---|---|---|---|---|
| $\mathbf{M}^p{}_e\ddot{\mathbf{x}}_e + \mathbf{C}_e\dot{\mathbf{x}}_e + \mathbf{f}_e(\mathbf{x}_e, \dot{\mathbf{x}}_e)$ | | $-\mathbf{M}^p{}_e\mathbf{R}(\mathbf{E}-\boldsymbol{\alpha}_l)\ddot{\mathbf{u}}_g + \left(\mathbf{T}_e - \boldsymbol{\alpha}_m\mathbf{M}_e\left(\mathbf{R}\ddot{\mathbf{u}}_g + \ddot{\mathbf{x}}_e\right) - \mathbf{M}^p{}_e\mathbf{R}\boldsymbol{\alpha}_l\ddot{\mathbf{u}}_g\right)$ | | | |
| *Test type* | *Mass splitting* | *Load splitting* | *Table accel.* | *Actuators forces* | *Test methods* |
| **PSD** | $\boldsymbol{\alpha}_\mathrm{m} = \mathbf{E}$ | $\boldsymbol{\alpha}_l = \mathbf{E}$ | $\mathbf{0}$ | $\mathbf{T}_e - \mathbf{M}_e\left(\mathbf{R}\ddot{\mathbf{u}}_g + \ddot{\mathbf{x}}_e\right)$ | Force-based PSD hybrid simulation |
| **Dynamic** | $\boldsymbol{\alpha}_m = \mathbf{0}$ | $\boldsymbol{\alpha}_l = \mathbf{E}$ | $\mathbf{0}$ | $\mathbf{T}_e - \mathbf{M}_e\mathbf{R}\ddot{\mathbf{u}}_g$ | Effective force hybrid simulation |
| | | $\boldsymbol{\alpha}_l = \mathbf{0}$ | $\ddot{\mathbf{u}}_g$ | $\mathbf{T}_e$ | Conventional RTDHT |
| | | $\mathbf{0} < \boldsymbol{\alpha}_l < \mathbf{E}$ | $(\mathbf{E}-\boldsymbol{\alpha}_l)\ddot{\mathbf{u}}_g$ | $\mathbf{T}_e - \mathbf{M}^p{}_e\mathbf{R}\ddot{\mathbf{u}}_g$ | Advanced RTDHT |
| **Quasi-dynamic** | $\mathbf{0} < \boldsymbol{\alpha}_m < \mathbf{E}$ | $\boldsymbol{\alpha}_l = \mathbf{E}$ | $\mathbf{0}$ | $-\mathbf{M}^p{}_e\mathbf{R}\ddot{\mathbf{u}}_g + \mathbf{T}''_e$ | A hybrid method of PSD and Dynamic hybrid simulation. |
| | | $\boldsymbol{\alpha}_l = \mathbf{0}$ | $\ddot{\mathbf{u}}_g$ | $\mathbf{T}''_e$ | |
| | | $\mathbf{0} < \boldsymbol{\alpha}_l < \mathbf{E}$ | $(\mathbf{E}-\boldsymbol{\alpha}_l)\ddot{\mathbf{u}}_g$ | $\mathbf{T}''_e - \mathbf{M}^p{}_e\mathbf{R}\boldsymbol{\alpha}_l\ddot{\mathbf{u}}_g$ | |

## Test Platform for General Force-based Hybrid Simulation

The test platform developed to implement the general formulation is a force-based platform as illustrated in Fig. 2 (Shao, 2006). The platform uses multiple physical and computational systems including: (i) high-performance servo-hydraulic Structural and seismic testing controllers; (ii) Data acquisition and information streaming; (iii) Real time hybrid simulation controller that includes a computational model based Real time structure simulator to perform computational substructure numerical simulation/interface loading calculation; and a force-based Compensation controller. The Compensation controller has two functions. One is to determine the load command based on the general formulation Eq. 2 and the other is to conduct the necessary compensation of the hydraulic loading devices (i.e. time delay in responses). The intent of this test platform design was to integrate and coordinate various hardware components during a hybrid simulation and the modularized configuration makes it flexible for future development of individual components without modifying the platform architecture.

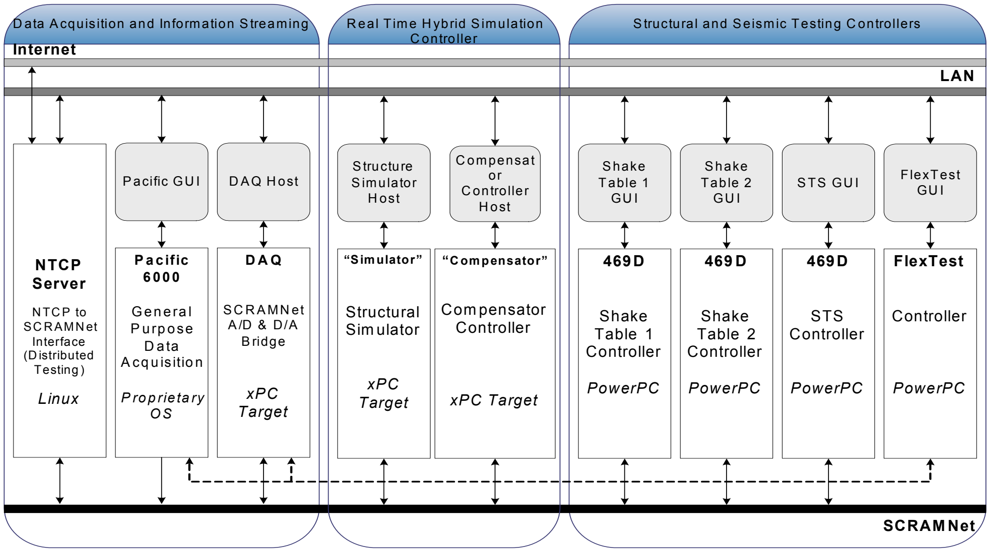

Figure 2. Hardware components of general force-based hybrid simulation test platform

The flowchart to implement the general formulation (Eq. 2) in the Compensation controller is shown in Fig. 3. The input to the controller must include the interface force $\mathbf{T}_e$ and the ground acceleration excitation $\ddot{\mathbf{u}}_g$. The responses of the test model are feedback to determine the corresponding force related to the virtual components of the experimental substructure while the splitting coefficient matrices are predefined and remain constant during the simulation. The outputs are the applied force and acceleration command that need to be executed by the dynamic actuators and shake tables respectively.

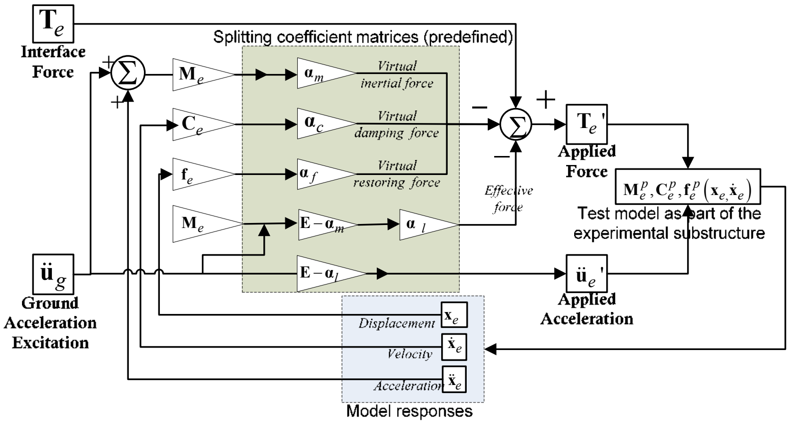

Figure 3. Flowchart to implement the general formulation of hybrid simulation

## Verification Test

The concept of the proposed general formulation for force-based hybrid simulation and the corresponding test platform was then experimentally verified using a small-scale pilot test setup as is shown in Fig. 4, including a SDOF frame structure, a force controlled actuator (Sivaselvan et al. 2008) and a unidirectional shake table. However the shake table used here is controlled in displacement instead of acceleration as assumed in the general formulation resulting in imperfect simulation responses. The full mass of the structure required for dynamic simulation is 79.1kg. By removing the lead bricks, a reduced mass specimen was obtained which was used for Quasi-dynamic hybrid simulation where the virtual mass is 77% of the full mass. The white noise acceleration time history excitation was created by a function generator, using a frequency range of 0.1~10Hz and unity amplitude.

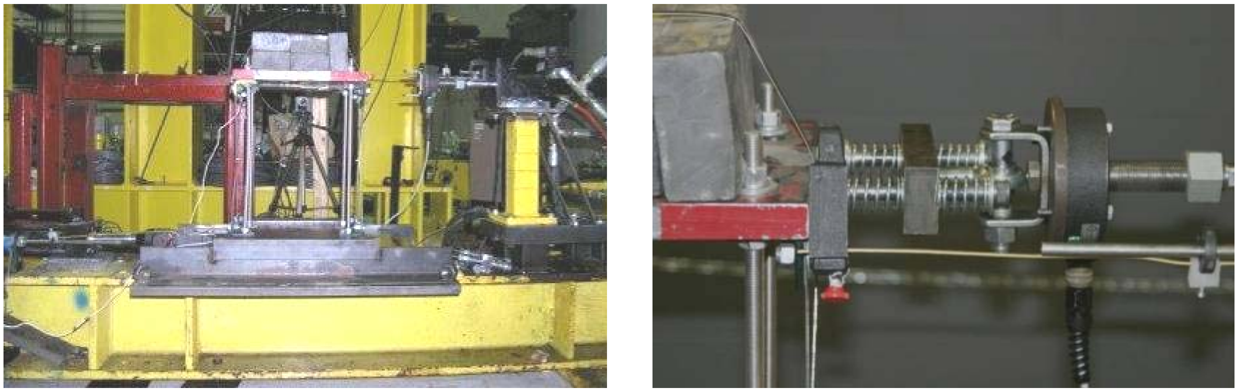

Figure 4. Hybrid simulation test setup

Table 3. Hybrid simulation loading cases

| TEST NAME | CASE | $\alpha_m$ | $\alpha_l$ | TB ACCEL. | ACT FORCE |
|---|---|---|---|---|---|
| *Shake table* | **1** | 0 | 1 | $\ddot{u}_g$ | 0 |
| *Dynamic* | **2** | 0 | 1 | $\ddot{u}_g$ | 0 |
|  | **3** |  | 0 | 0 | $-M\ddot{u}_g$ |
|  | **4** |  | 0.5 | $0.5\ddot{u}_g$ | $-0.5M\ddot{u}_g$ |
| *Quasi-dynamic* | **5** | 0.77 | 1 | $4.45\ddot{u}_g + 3.35\ddot{x}_e$ | 0 |
|  | **6** |  | 0 | 0 | $-M\left(\ddot{u}_g + 0.77\ddot{x}_e\right)$ |
|  | **7** |  | 0.5 | $2.17\ddot{u}_g + 1.67\ddot{x}_e$ | $-0.5M\left(\ddot{u}_g + 0.77\ddot{x}_e\right)$ |

Seven loading cases defined by the general formulation were tested assuming $\mathbf{T}_e = \mathbf{0}$ and $\boldsymbol{\alpha}_c = \boldsymbol{\alpha}_f = \mathbf{0}$, as listed in Table 3. The measured specimen's responses from different loading cases are presented in Fig. 5 compared with the numerical simulation result (the thinner line). The top left plot shows the structural displacement history obtained from the numerical simulation. The simulation response was computed using Matlab/Simulink$^{\text{TM}}$, which used a continuous transfer function representing the dynamic properties of the test model. The top right plot is the measured data from Case 2 during which the shake table is used while the dynamic actuator is not connected. The remaining three plots on the left are the response measured from tests with the full mass specimen representing a Dynamic hybrid simulation, while the three right plots show the response of the reduced mass specimen considered as Quasi-dynamic hybrid simulation. Except for the result obtained from the pure shake table test (due to the controller limitation of the shake table), all other measured displacement responses exhibit a good match to the simulated response, demonstrating that different loading cases were able to generate true dynamic responses in the specimen. Therefore the general formulation was experimentally verified to be correct and the corresponding test platform developed is feasible and effective. The actuator was capable of applying the desired force to the specimen using the developed force control strategy.

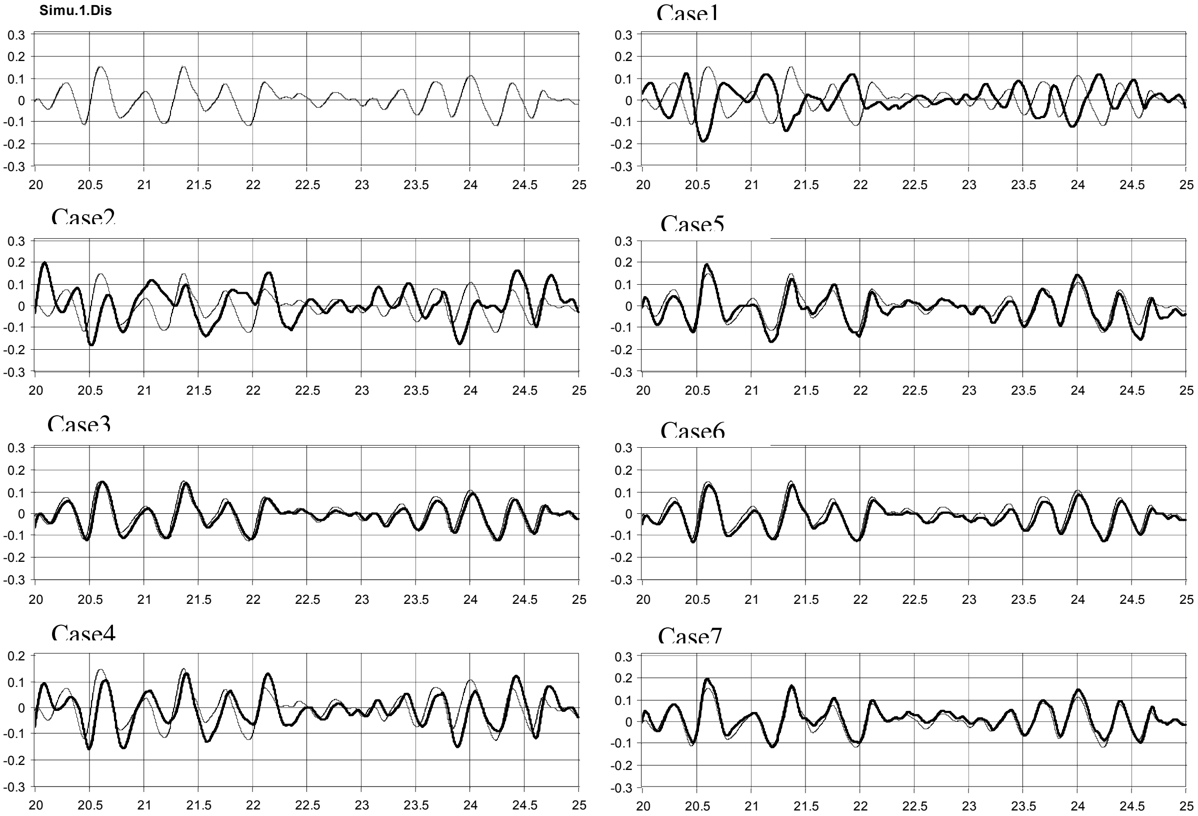

Figure 5. Measured structural displacement response compared to simulation results

However it is also observable that responses involving shake table input only approximately matches the simulation results. For example, when the reduced mass specimen was tested where more dynamic loading was imposed by the actuator than from the shake table (the inertia force related to the virtual mass was applied by the actuator), the displacement responses are more consistent compared to the ones measured from the tests using the full mass specimen. Moreover, the tests conducted using the actuator only show the best match to the simulation (the third row in Fig. 5), where in these cases, the results are nearly identical in the amplitude to the simulated response with a delay of approximately 13 milliseconds. This is equivalent to the time delay of the table actuator's response.

## Concluding remarks

A general formulation is proposed for force-based hybrid simulation, which is a seismic laboratory testing method combining the shake tables, dynamic actuators and numerical simulation in one test procedure. Using the splitting coefficient matrices in the equation of motion of the experimental substructure, the general formulation not only broadens the application range of Real time dynamic hybrid simulation (RTDHT) to include all the current modern seismic simulation methods, but also enhance the flexibility in physical specimen design to accommodate the limitations in loading equipment and test space. A corresponding test platform was developed to implement the general formulation. Both the concept of the general formulation and the test platform were verified experimentally by a simple one DOF specimen real time hybrid simulation.

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